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Exploring problem-solving through the intervention of technology and Realistic Mathematics Education in the Calculus content course

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Abstract

In Thai mathematics classrooms, there is a lack of attention to support students' mathematical problem-solving skills by working from real-world contexts that make sense to students. This study investigated how pre-service mathematics teachers' problem-solving can be explored in their content course, intervening with technology and Realistic Mathematics Education (RME) through the Mean Value Theorem (MVT) lesson. The study included nine pre-service mathematics teachers purposively selected from a public university in Thailand who attended a Calculus course. Data was collected from classroom artifacts, observation notes, and interviews. It was found in this study that the intervention of technology and RME in pre-service mathematics teachers' content courses has the potential to build pre-service mathematics teachers' problem-solving abilities. It was also discussed that the intervention could use RME to conceptualize mathematics theorem and cultivate Polya's problem-solving steps. The findings provide light on the efficacy of using technology and RME in enhancing problem-solving skills among other content courses. They could be used to inform the creation of mathematics curricula and instructional strategies in undergraduate content courses for mathematics education programs.

Keywords: Pre-service Teacher Education, Problem-solving, Realistic Mathematics Education, Technology

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Education in Thailand has evolved over time, resulting in what some refer to as "Thailand 4.0," an innovation- or technology-driven economy (Thailand 1.0 is about agriculture; Thailand 2.0 is about light industries; and Thailand 3.0 is about heavy industries). The Office of the Education Council announced a new revision of strands and learning standards for mathematics, sciences, and social studies, religion, and culture in Thailand in 2017 because of this movement over the best approach to learn mathematics, the sciences, and technology. For mathematics, this new set of strands and learning standards required students to acquire skills, such as being problem-solvers and communicating their mathematical work using such technology. With the release of this revised curriculum, the Bureau of Academic Affairs and Educational Standards published additional grade level indicators that Thai students were now required to demonstrate a comprehension of mathematical principles and justify their reasoning to solve complicated daily contexts more than originally expected (Bureau of Academic Affairs and Educational Standards, 2017).

As the world becomes more complex, it becomes increasingly necessary for students to have the ability to think critically and solve problems in a variety of contexts (Weintrop et al., 2016; Erdogan, 2019; Cheng & Li, 2020). This requires students to analyze data, recognize patterns, draw connections, and apply their knowledge to novel and challenging circumstances. However, traditional educational institutions may not always emphasize these competencies. To prepare students for the future, schools should allow them to engage in real-world problem-solving activities and build critical thinking skills through hands-on learning opportunities (Brundiers, Wiek, & Redman, 2010; Snyder, & Snyder, 2008; Baharin, Kamarudin, & Manaf, 2018). Additionally, schools must cultivate an environment that stimulates creativity, independent thought, and teamwork so that students are ready to tackle the complex difficulties they will confront in their personal and professional lives. Reformed pedagogy was proposed since it requires students to own their learning and build knowledge when interacting with a challenging setting (Chan Chun Ming, 2009; Pino-Fan et al., 2015). In recent challenges highlighted in mathematical problem solving, there was concern about students' readiness to solve problems in an increasingly complicated environment and the necessity for students to function in unfamiliar contexts to elicit critical problem-solving abilities (Tabach & Friedlander, 2013; Kaiser et al., 2017).

Realistic Mathematics Education (RME) is an instructional method that stresses the development of problem-solving skills in mathematics because it emphasizes relating mathematical concepts to real-world problems (Ardiyani, Gunarhadi, & Riyadi, 2018; Özkaya & Yetim, 2017), encouraging students to engage in hands-on problem-solving activities, and fostering critical thinking and deductive reasoning (Uyen et al., 2021; Ndiung, 2021; Sitorus, 2022; Hasbi, Lukito, & Sulaiman, 2019). In RME, students are provided with real-world problems and expected to solve them mathematically (Yilmaz, 2020; Van den Heuvel-Panhuizen & Drijvers, 2014). This strategy enables students to recognize mathematical topics' significance and practical applications, boosting their motivation and engagement with learning. In addition, by tackling issues from the real world, students can strengthen their problem-solving abilities and learn to apply mathematical principles in novel and complex settings (Mancosu, Jørgensen, & Pedersen, 2005; Ersoy, 2016; Chong, Shahrill, & Li, 2019). Additionally, RME emphasizes the significance of students building their understanding of mathematical subjects instead of simply memorizing formulas or methods (Scherer, 2020). This results in a deeper and more meaningful grasp of mathematics and the development of transferable critical thinking and deductive reasoning skills. The relationship between RME and mathematical problem-solving skills rests in the approach's emphasis on fostering problem-solving and critical thinking skills, which are crucial for success in both academic and practical settings. Thus, the RME approach to mathematical problem-solving prioritizes students' representational fluency through the universal application of mathematical concepts to real-world situations (Sembiring, Hadi, & Dolk, 2008).

In problem-solving, while finding a solution, students frequently link mathematical visualization with making images or diagrams as a starting point for solving difficulties. Moreover, visualizing plays a significantly broader role in problem-solving, including promoting the formation of ideas and facilitating the transmission of results and comprehension (Drijvers et al., 2010; Thomas et al., 2017). Technology-based visualization has a far broader function in problem-solving, including helping formulate ideas and transfer results and comprehension (Chen & Chang, 2021; Buentello-Montoya, Lomelí-Plascencia, & Medina-Herrera, 2021).

We cannot refute that the most essential aspect of a vital education is effective teaching, and teachers recognize the significance of schooling. To strengthen prospective teachers, colleges of education must encourage pre-service teachers to spend as much time as possible with the topic they will be teaching and to investigate how students form conceptions and misconceptions about this content (Burton, Daan, & Giesen, 2008)—additionally, changing from teacher-led learning to self-study learning using technology to allow students to acquire knowledge on their own, which one of the core ideas of RME, requires a well-defined technological plan by the teacher to support student learning. However, the efficiency of technology integration depends on the teaching and technological skills of the teacher. Therefore, teachers must be equipped with the skills necessary to implement this strategy effectively (Ertmer, 2005; Williams et al., 2023; Ifinedo, Rikala, & Hämäläinen, 2020; Wilson, Ritzhaupt, & Cheng, 2020).

Furthermore, as prospective mathematics teachers, pre-service teachers must also comprehend how theorems and proofs provide the foundation for the complicated processes that comprise these problem-solving computation strategies. According to previous studies, students often consider calculus as consisting primarily of procedures and a few quantitative computations independent of and unrelated to the axioms and theorems that underlie the results. Therefore, Calculus teaching must and should emphasize theorem to meet the aims of mathematical learning: the development of reasoning and communication skills, an appreciation of rigor, and the capacity to adopt problem-solving procedures (Ashraf, 2020; Gravemeijer & Doorman, 1999).

This study investigated how pre-service mathematics teachers' problem-solving can be explored in their content course, intervening with technology and Realistic Mathematics Education (RME). Specifically, this study explored how pre-service teachers built problem-solving skills while engaging in the Mean Value Theorem (MVT) lesson. The findings of this study could provide insight and impetus for further research into the relationship between the problem-solving of pre-service mathematics teachers and the intervention of technology and RME.

Realistic Mathematics Education (RME) Approach

Few educational solutions have been proffered for how students can consistently acquire mathematics. Freudenthal asserts that students acquire mathematics by dealing with real-world problems and reconfiguring their mathematical understanding with the support of teachers (Freudenthal, 1991, as cited in Tong et al., 2022). Freudenthal also regards mathematics as a human endeavor (Freudenthal, 1973), and mathematical practice is viewed as a human activity (Kusumaningsih et al., 2018; Laurens et al., 2017; Peni, 2019).

Realistic Mathematics Education (RME) is an approach to mathematics teaching that emphasizes the development of students' mathematical understanding and reasoning through problem-solving and mathematical modeling (Yuanita, Zulnaidi, & Zakaria, 2018; Tamur, Juandi, & Adem, 2020). This strategy is founded on a set of fundamental concepts and characteristics, including:

1. Context-based learning: RME emphasizes the significance of learning mathematics through problems and life situations that are meaningful and relevant to students. This assists students in comprehending the relevance of mathematics to real-world situations and enhances their comprehension of mathematical concepts (Van Den Heuvel-Panhuizen, 2005).
2. Active learning: Through hands-on activities, problem-solving, and exploration, students are actively engaged in the learning process in RME. This helps students gain a deeper comprehension of mathematical topics and procedures, as well as critical thinking and problem-solving abilities (Tinh et al., 2021; Chairil Hikayat, Hairun, & Suharna, 2020).
3. Development of mathematical reasoning: The RME curriculum heavily emphasizes the development of students' mathematical reasoning skills. This entails helping students develop the

capacity to interpret mathematical conditions, describe mathematical concepts in various forms, and apply mathematical reasoning to solve problems (Uyen et al., 2021; Ndiung, 2021).

4. Concentrate on processes and concepts: In RME, both procedures and concepts are deemed essential. Concepts are regarded as the basis for comprehending mathematical structures and relationships, whereas procedures are perceived as methods for addressing issues. Through hands-on exercises and problem-solving, both methods and concepts are developed (Sitorus, 2022).
5. Connections between mathematical concepts: RME highlights the relationships between various mathematical disciplines and the significance of viewing mathematics as a connected and integrated whole. This helps students gain a broader and more comprehensive understanding of mathematics as well as an appreciation for the interconnectedness of mathematical topics (Uyen et al., 2021; Hasbi, Lukito, & Sulaiman, 2019).
6. Emphasizing the constructive nature of mathematics: RME acknowledges that mathematics is a creative and constructive subject and that mathematical understanding grows through a process of conjecture, testing, and refining. This helps students grasp mathematical research's iterative and reflective character and improves their capacity to solve mathematical problems creatively and critically (Gravemeijer, 2020).

Since RME lays a heavy focus on problem-solving and encourages students to apply mathematical reasoning to find solutions, introducing pre-service teachers to this method will allow them to acquire problem-solving skills that they may use in their classrooms. Moreover, RME provides students with relevant and context-based alternatives to explore mathematical concepts and ideas. This helps pre-service teachers develop a deeper understanding of mathematical concepts and relationships, which they can subsequently utilize to teach their students more effectively (Tinh et al., 2021).

RME emphasizes the significance of teaching mathematics in context and teaches pre-service teachers how to apply mathematical concepts to real-world problems. This can help them create compelling and relevant lessons for their students. Furthermore, RME can recognize that students arrive to the classroom with different backgrounds, experiences, and ways of thinking about mathematics. Pre-service teachers will be more ready to meet the different needs of their students if they are exposed to a diversity of problem-solving strategies and approaches to thinking about mathematics (Sembiring, Hadi, & Dolk, 2008).

RME also understands that mathematical understanding evolves through a process of speculation, testing, and refining. Pre-service teachers will develop a growth mindset and an appreciation for the iterative and reflective character of mathematical enquiry if they are exposed to this method. Additionally, RME promotes students to think creatively and critically about mathematical problems and supports pre-service teachers in developing these abilities. This will benefit not only their future students, but also their own continued professional development as teachers (Tinh et al., 2021). Therefore, the use of RME in pre-service mathematics teacher education programs has the potential to generate well-prepared, self-assured, and successful mathematics teachers who can support the mathematical development of their students.

To support RME in practical level, mathematical modeling is a crucial component of RME because it allows students to use and reinforce their mathematical knowledge in a setting that is meaningful and relevant (Riyanto, & Putri, 2017). Students can see the connections between mathematics and the real world and build problem-solving and critical thinking abilities using mathematical models (Asempapa, 2015; Gainsburg, 2008). Considering RME is typically described in a natural environment, mathematical modeling has been acknowledged as a technique for elaborating on problem-solving strategies. For mathematical modeling, the terms "model for" and "model of" relate to two distinct forms of mathematical models (Gravemeijer,1999, 2007).

A "model for" is a mathematical representation used to predict real-world phenomena. This model type is used to simulate or predict the behavior of a system based on particular inputs and assumptions. For instance, a model for predicting the weather would consider parameters such as temperature, wind speed, and air pressure to predict future circumstances. On the other hand, a "model of" is a mathematical representation used to describe or explain an actual phenomenon. This model type is employed to aid comprehension of a system's fundamental relationships and patterns. To illustrate how a population varies over time, a model of population growth might consider elements such as the birth rate, mortality rate, and migration trends (Gravemeijer,1999, 2007).

However, the purpose of mathematical modeling in both instances is to describe real-world phenomena in a manner that is simple, abstract, and susceptible to mathematical analysis. Researchers can acquire insights into complicated systems and make more informed judgments based on the results of their simulations and analysis when they employ mathematical models.

In mathematical modeling, the goal is to formulate a mathematical representation of a real-world situation that accurately represents its pertinent characteristics (Riyanto et al., 2019). This requires the application of mathematical concepts and processes to abstract from the particulars of the real-world problem and simplify it into a mathematical form that can be analyzed and comprehended. Thus, mathematization, which entails the use of mathematical symbols, equations, and models to describe real-world events, connections, and occurrences, is fundamental to mathematical modeling. Moreover, mathematization utilizes mathematical concepts and processes to describe, analyze, and comprehend real-world phenomena (Stillman, 2011; Stillman et al., 2007).

The mathematization procedure requires numerous steps, including understanding the problem, formulating the problem mathematically, solving the mathematical problem, and interpreting the solution. The first stage is to comprehend the real-world situation and determine the essential parts of the issue that pertain to the modeling procedure. The subsequent step is to convert the real-world issue into a mathematical representation. This may require identifying pertinent variables, relationships, and restrictions, as well as employing mathematical symbols and equations to describe these components of the problem. After the real-world problem has been mathematically represented, the next stage is to solve it using mathematical concepts and processes. Finally, the solution to the mathematical problem must be understood in terms of the corresponding real-world issue. This requires converting the mathematical answer to a real-world form and assessing its applicability and precision regarding the original problem (Suh, Matson, & Seshaiyer, 2017).

Mathematization is an essential component of mathematical modeling because it allows mathematicians and other scientists to explain and comprehend issues and events in the actual world using mathematical concepts and techniques. Mathematical models can be used to create predictions, test hypotheses, and inform decision making in numerous domains, including science, engineering, economics, and medicine, through the process of mathematization (Stillman, 2011; Suh, Matson, & Seshaiyer, 2017).

Horizontal mathematization and vertical mathematization are two distinct methods of acquiring mathematical comprehension in mathematics education. Horizontal mathematization refers to the development of mathematical knowledge within a specific subject or topic. This entails expanding and refining students' mathematics knowledge and abilities within a certain topic (Menon, 2013). An example of horizontal mathematization in mathematics is the development of an understanding of fundamental arithmetic operations, such as addition, subtraction, multiplication, and division.

Vertical mathematization refers to the development of mathematical knowledge across multiple mathematical domains and disciplines. Transferring mathematical knowledge and skills from one mathematical domain to another (Menon, 2013). As an example of vertical mathematization in mathematics, consider the application of geometric notions and reasoning to solve algebraic problems (Surat, Sukendra, & Juwana, 2023).

In mathematics education, both horizontal and vertical mathematization are essential because they contribute to students' general mathematical development and their ability to apply mathematics effectively in the real world. Horizontal mathematization provides students with a foundation of mathematical knowledge and skills within a certain subject area, but vertical mathematization enables students to recognize the connections between diverse mathematical areas and apply their knowledge to new and challenging situations. Thus, the goal of mathematics education should be to promote both horizontal and vertical mathematization, so that students can build a comprehensive and adaptable grasp of mathematics and use it effectively in a variety of contexts.

METHODS

The qualitative approach was implemented in this study to explore how pre-service mathematics teachers' problem solving performed in the Calculus course. This methodology is the most applicable method to investigate problem solving abilities of pre-service mathematics teachers through the designed sessions as a common phenomenon.

The Participant

The participants were purposively selected from 63 pre-service mathematics teachers in a public university in Lampang, Thailand, who attended the Calculus course in the first semester of their first year at the time of the study. Before entering the program, all pre-service mathematics teachers took the entrance examination in digital technology skills. Additionally, they were given some problems to find solutions in classroom activities. As a result, there were 32 pre-service mathematics teachers who passed the required level in digital technology from the examination. According to the activities, 17 of them showed that they could implement normal processes but failed to tackle complex or non-routine situations.

Furthermore, from the survey distributed before starting this study, 11 pre-service mathematics teachers had yet to previously gain experience in RME activities and mathematical modeling from their classrooms. However, nine of them committed to attending this study as our subjects. Prior to this study, we obtained consent from the participants to participate in this study.

Procedure

In this study, one 4-hour session was conducted outside the classroom, and the mathematics activities differed from standard teaching practices in the school. The seven stages of the activity were presented so that participants could build the concept of MVT by solving the given real-world problem.

Adapting the process of mathematics modeling by Stillman (2011) involves all five features of RME with precisely delineated transitional stages between each characteristic. Stillman (2011) stated the process of mathematical modeling in Figure 1.

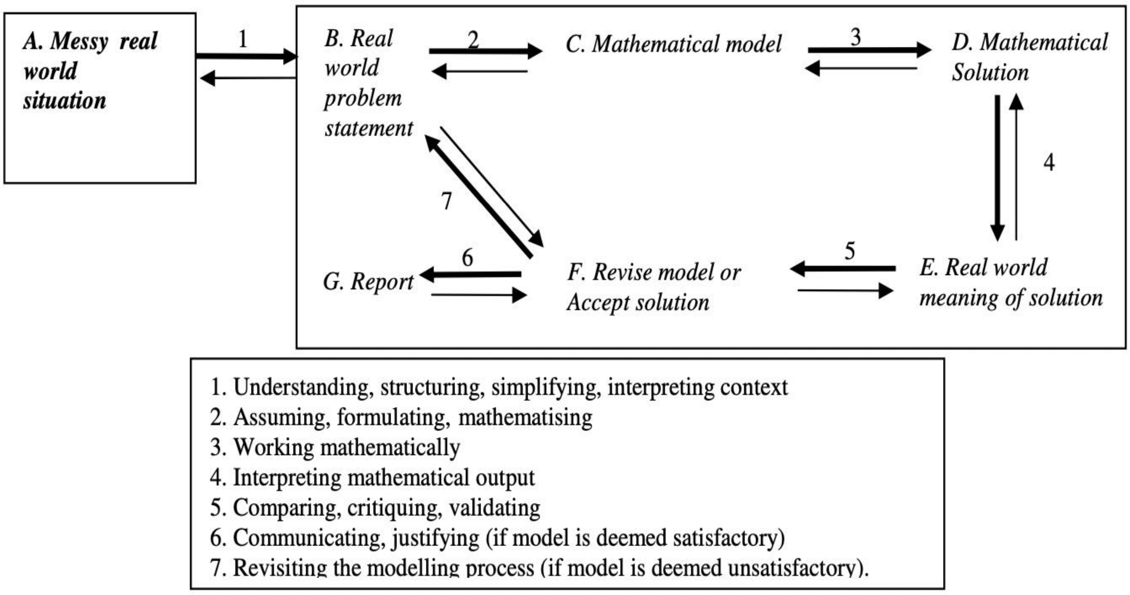

Figure 1. The process of mathematical modeling
(Stillman, 2011: retrieved from Geiger, Stillman, Brown, Galbriath, & Niss, 2017)

In Figure 1, the typical entries A through G indicate stages of the modeling process, with the bolder arrows denoting transitions amongst stages. The double-headed light arrows emphasize that the modeling is not a linear or unidirectional process. Additionally, it suggests the existence of metacognitive activity, which can look forwards and backward to stages in the process. Descriptors 1–7, in Figure 1, illustrate the cognitive activities modelers engage in transition from one stage to the following stage, and an overview of these cognitive activities is provided in the following summary.

Understanding, organizing, simplifying, and interpreting the context is required to formulate a mathematically viable problem from a real-world scenario (Durmus & Karakirik, 2006; Gravemeijer et al., 2017). This is accomplished by making assumptions and identifying features relevant to solving a problematic real-world situation (Seino et al., 2005; de Almeida, Sousa, & Tortola, 2021; Zbiek & Conner, 2006). Real-world situation refers to the real-world context of the modeling challenge frequently defined using common knowledge. The modelers must comprehend the task to create a mental representation of the scenario, i.e., how the persons think about the problem. They must then structure, simplify, and interpret the information from the task to create an actual model (Csaszar & Levinthal, 2016; Anhalt & Cortez, 2016; Şen Zeytun, Çetinkaya, & Erbaş, 2017). By mathematizing these requirements and developing a mathematical model, this realistic model has been moved from the 'real world' to the 'mathematics world' (Guler, 2018; Abassian et al., 2020; Boz-Yaman, 2022). Next, the modelers operate mathematically in the mathematics world to identify solutions in the form of mathematical results (Suh et al., 2021). Finally, the mathematical results are translated into real-world meaning by returning to the real world, and the actual results are confirmed (Boz-Yaman, 2022). If the validation reveals invalid results and additional essential components are missing, the model must undergo the modeling cycle again (Segaran & Eu, 2022; Sekulic et al., 2020).

The session was videotaped, and the contents of the videos were analyzed using by content analysis method to identify problem solving of participants' engagement with the activity. During the activity, data also included classroom artifacts, such as participants' works and interviews to elicit further information relating to their problem solving from the session to confirm or reject the prior collected data. Relevant interview sections were transcribed and coded. The code explicitly focused on participants' interaction with the mathematical modeling process, representing problem-solving abilities.

The Sequence of a Designed Activity

Mathematics consists of many different operations. The derivative is the most important used by mathematicians, scientists, and engineers. In calculus, The Mean Value Theorem (MVT) is the key to recovering functions from derivatives; its corollaries are the path that leads to integral calculus. There are many consequences in terms of other theories and applications, and the valuable content in calculus has been used to determine the solution to that everyday life problems. According to Stillman's process in Figure 1, the class activity was planned in each stage.

Stage I. Messy Real-World Problem

- Post the recent news from the Internet. For example, it was found that, in some cases, cars were driven over the speed limit of 60 km/h, which warns us by the limit speed sign.
- Discuss with the class how to control the car speed in this area to prevent accidence that causes serious injury to travelers.

Stage II. Real World

- Introduce the speed limit camera and let students discuss their accuracy. Two speed cameras on roads instantaneously measure car speed as they pass. The cameras photograph every car that passes the first set and decode its number plate. Then they look for the same car passing the second set a little while later. The time difference and work out the average speed. The drivers get a ticket in the mail if it exceeds the limit. Drivers routinely slow down for the instantaneous speed cameras and accelerate as soon as they pass them. Nevertheless, everyone knows how the average speed check cameras work, so everyone drives below the speed limit the whole time in the average speed check areas.
- Then, ask, "Should we decide you use a limit speed camera to prevent this incident on the highway in this area? How is this camera accurate?"

Stage III. Mathematical Model

- Suppose a car accelerates from B to G in Figure 2 in an hour. We must change this representation to a distance graph with respect to time. However, the focus of this activity is generating such knowledge, so, to reduce complication, we will set the coordination of axes as we want to figure out.

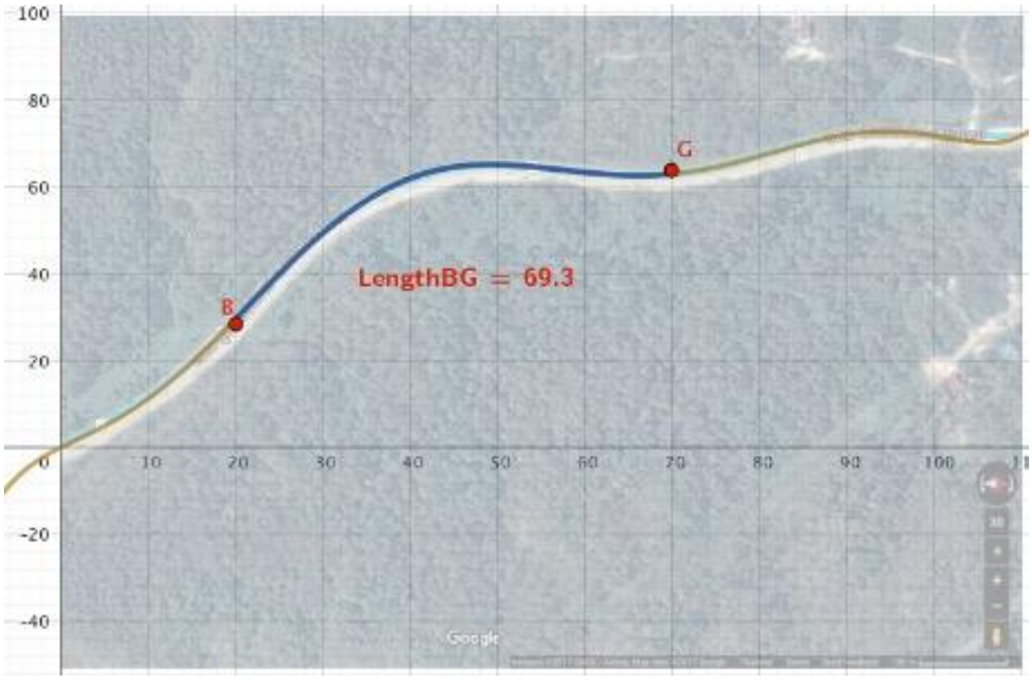

Figure 2. The picture of the road that car travel from B to G

- So, construct a curve as fit as possible to the road curve in the closed interval [0,2.4] of the x-axis as in Figure 3. Also, construct a curve as fit as possible to the road curve in the closed interval [0,2.4] of the x-axis as in Figure 4 by using the FitPoly command.

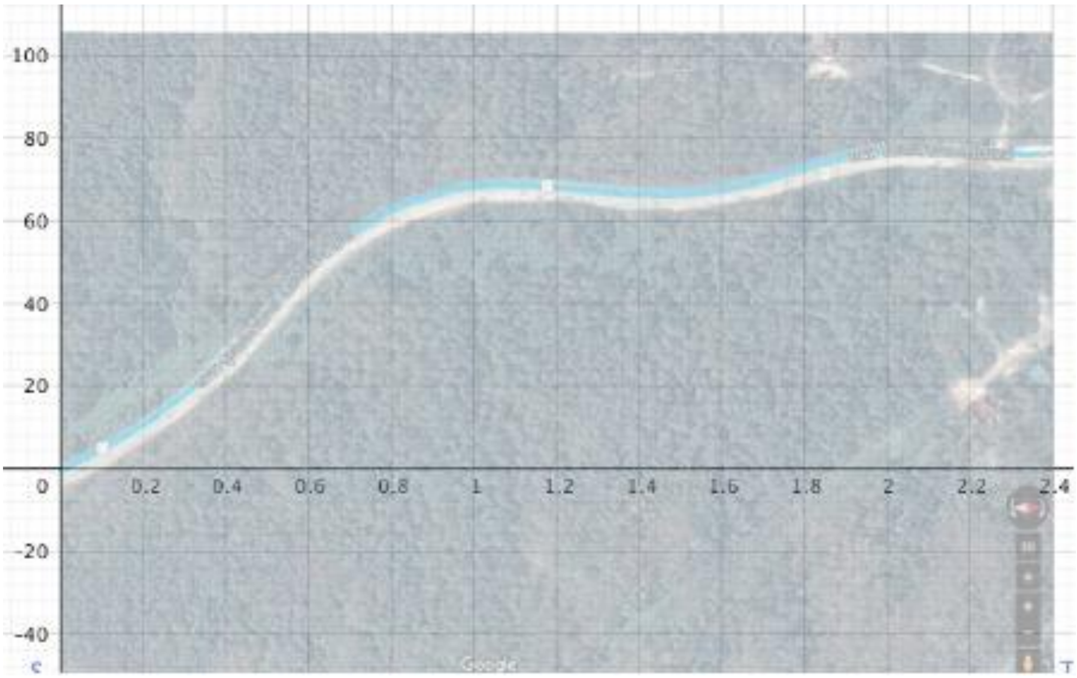

Figure 3. The adjusted coordination

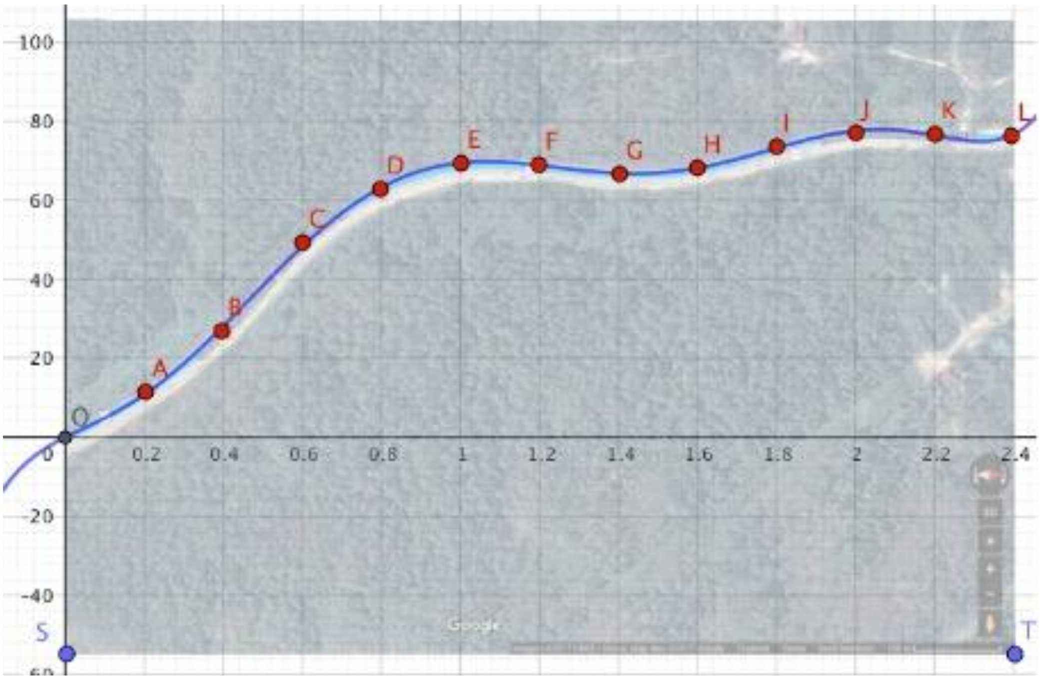

Figure 4. The constructed curve

- In this coordinate setting with hour as its unit, from M to N, this car took about 0.6 hours to reach the distance 69.51 – 28.18 = 41.33 km.

Using GeoGebra, we have the average change of the distance of this car (m) equal to 68.31 km/h, as shown in Figure 5.

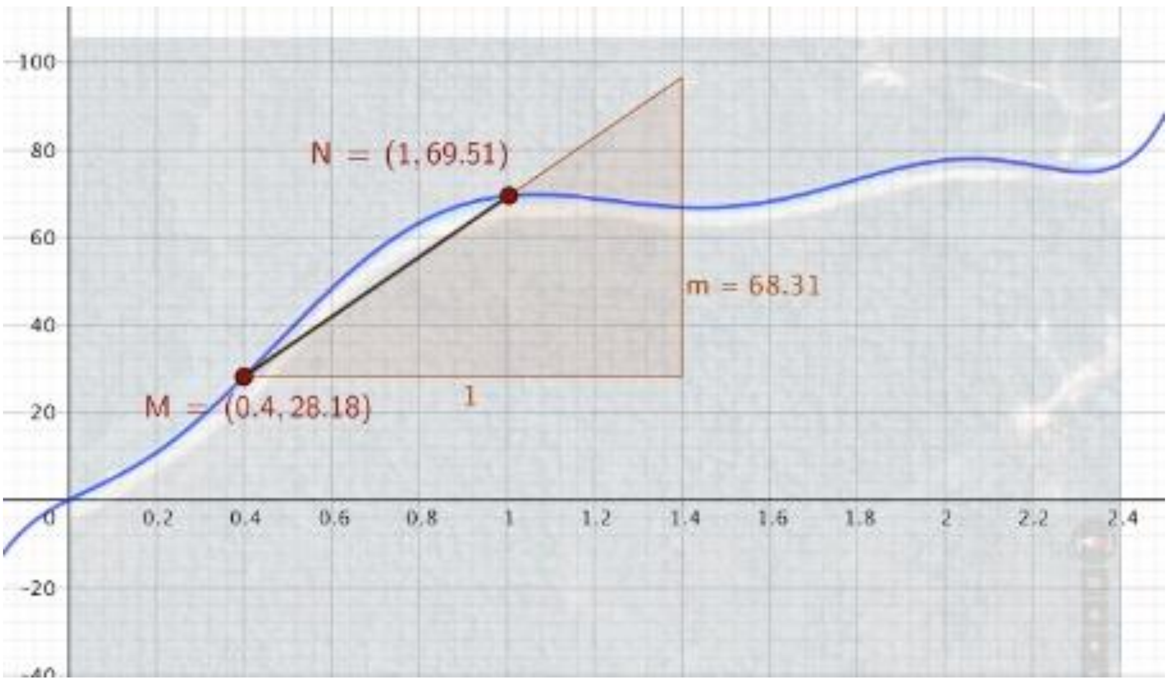

Figure 5. Example of finding an average speed by GeoGebra

Stage IV. Mathematical Solution

- Find the coordinate of the point P by GeoGebra. At P, the car's speed is equal to the average speed of more than 60 km/h, from M to N, which exceeds the limit speed in this area, as in Figure 6.

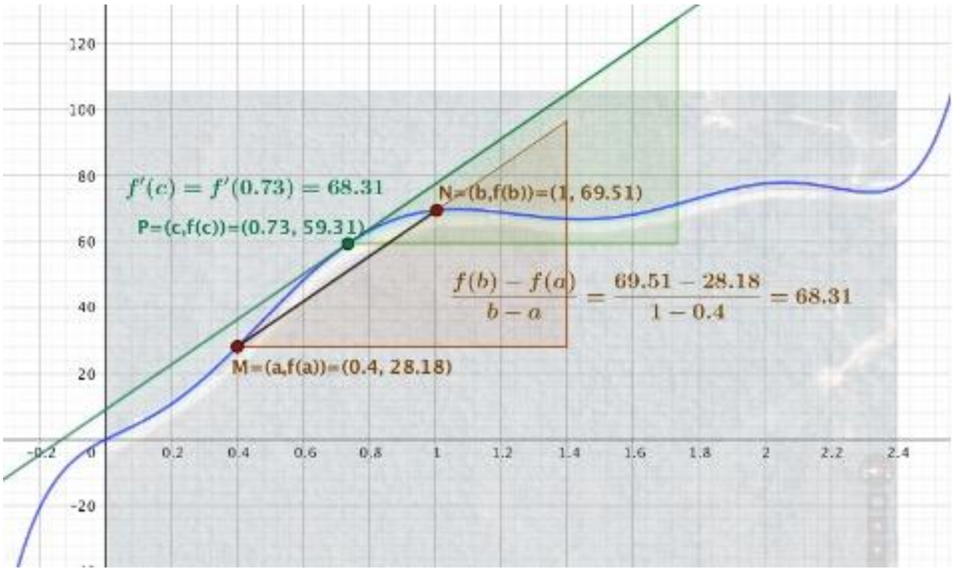

Figure 6. Example of finding the point with its instantaneous speed equal to the average speed

Stage V. Real-World Meaning

- At point P on f in Figure 6, the car has an instantaneous speed equal to 68.31 km/h, which is over the speed limit. Instead, if we recognize this graph as a function of distance (s) with respect to time (t), we have that, from M to N, its average speed is 68.31 km/h.

Stage VI. Revise Model or Accept Solution

- Choose another couple of points to reassure that the model applies to any point in the continuously differentiable curve as in Figure 7.

Stage VII. Report

- Answer the central question: During the drive, the car traveled 68.31 km/h, exceeding the speed limit. So, the car driver must pay for the police penalty ticket. And conclude that if y = f(x) is continuous at every point of the closed interval [a,b] and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

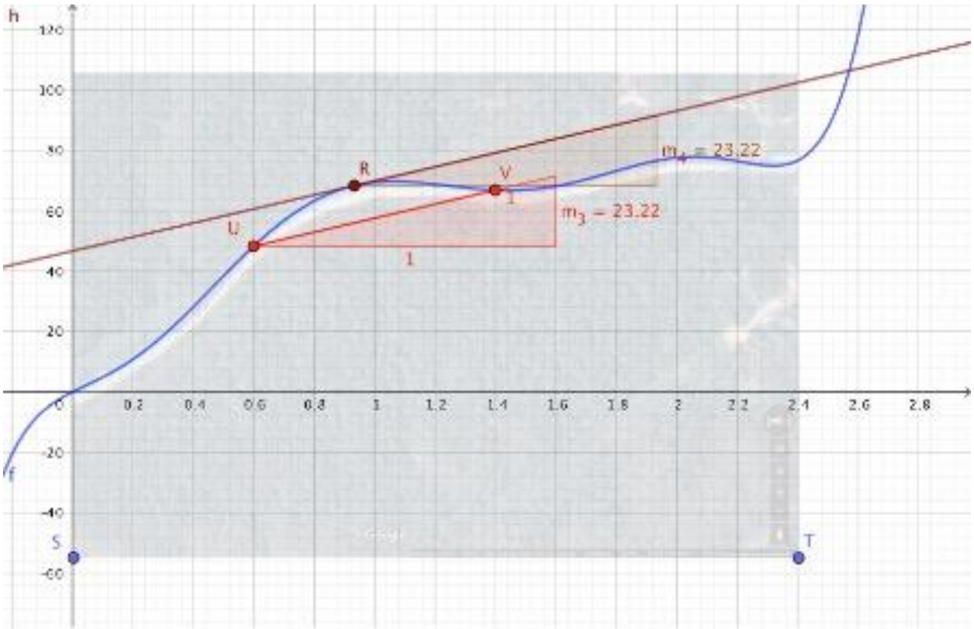

Figure 7. Apply the idea at another couple of points

RESULTS AND DISCUSSION

MVT Conceptualization by RME from the Activity

Showing stages of modeling illustrates the transition steps between stages according to Stillman's diagram. From the activity in class, the stages from I – VII were illustrated in nodes, and transition steps were illustrated by arrows in Figure 8, identifying primary stages in the modeling process and providing mental steps for a solution path. Pre-service mathematics teachers' learning experiences on the modeling process are presented. Each stage of activity is concluded in each node, and the kinds of cognitive transition steps from that activity example are shown in the descriptors Steps 1 – 7 as the following.

Step 1

Pre-service mathematics teachers understood and interpreted the context of the problem from the messy real-world situation. To intervene RME in this content, the messy real-world situations in which many car accidents occurred because of unawareness of over-limit drivers' speed were presented. Then, the pre-service mathematics teachers inspected that this speed might cause severe injury in the given situations. During the discussion, the following questions were considered to help support pre-service mathematics teachers' thinking processes in establishing the concept of MVT:

"If the polices recorded a car traveling between two cameras in such a period, the pictures and timestamps would be forwarded to decide whether to issue a penalty ticket."
"While it was possible to speed for short periods and keep the speed under the limit at the camera points, this system would check to speed to get somewhere faster."
"How it was ensured that people driving under the limit might not be subjected to erroneous tickets."

So, pre-service mathematics teachers' written and oral presentations of modeling tasks in this stage showed their ability to interpret the real-world situations for modeling tasks, point out considerable quantities involved in the speed of a driven car, and make realistic assumptions, such as continuously driving or the road condition, for the more amenable formulation of the mathematics model.

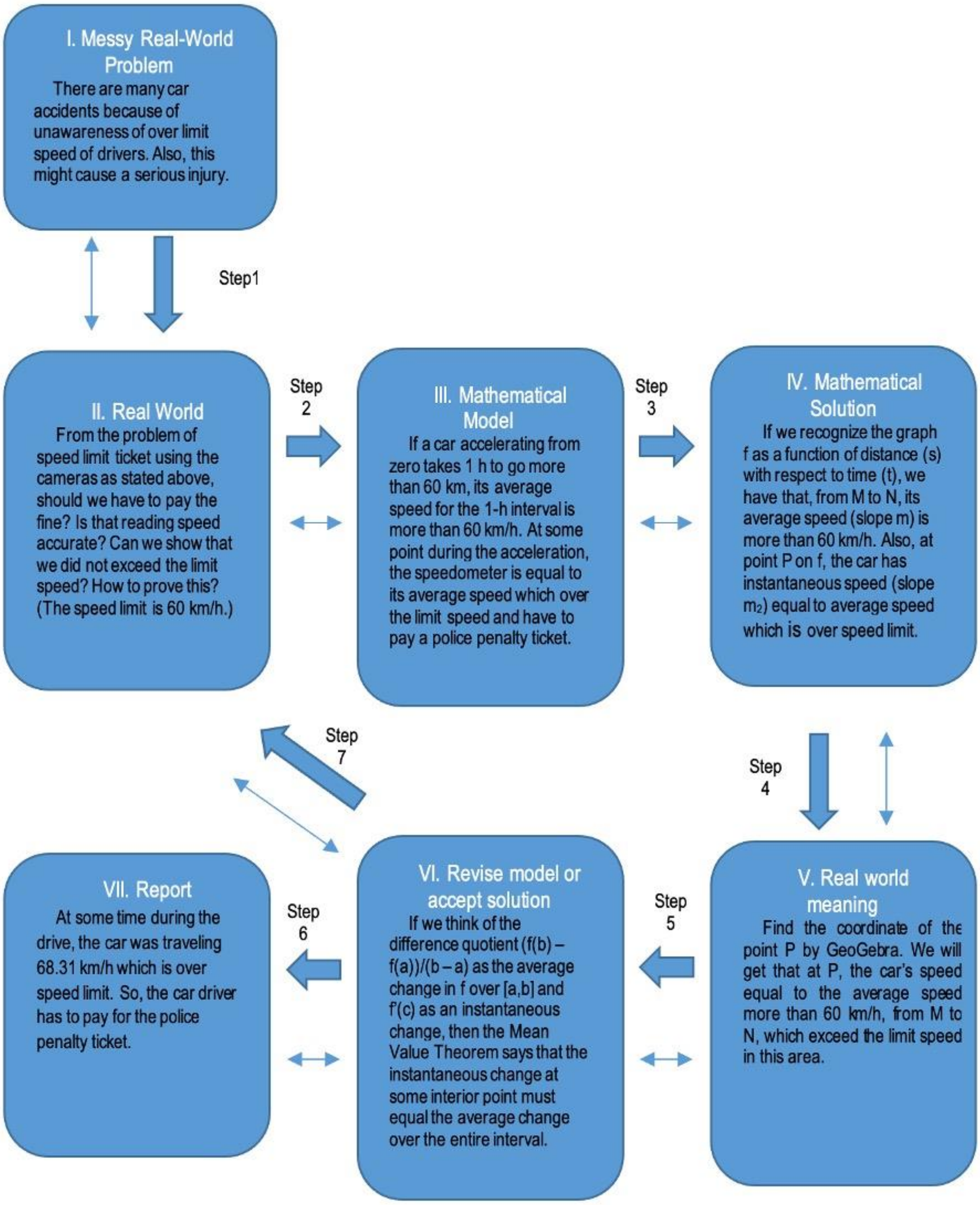

Figure 8. The process of mathematical modeling of the activity

Step 2

Pre-service mathematics teachers formulated and mathematized the real-world problem into a mathematics model. As shown in Figure 6, since the car's position was continuous, there were no discontinuous on the position graph; that is, the car passed through every point on the way from the start. Then, the conditions satisfied the theorem. The participants discussed the following.

"The car in this example took about 0.6 hours to reach the distance 69.51 – 28.18 = 41.33 km."
"Using GeoGebra, we have the average change of this car equal to 68.31 km/h."

The researcher further asked, "along the driving, did the driver of this car reach this speed at least once?", and let students discuss and present their idea. In this transition stage, the pre-service mathematics teachers made basic errors, such as a unified representation of speed in their formulation of the car distance. However, they were asked to deliberate the consequence of incorrect decisions using variables of interest value on the predictive prospected modeling function.

When they progressed to the limit speed ticket task, they noticed that the variables of interest were identified appropriately, and their relevant relationships were also formally expressed so that formulas could be applied. Thus, pre-service mathematics teachers realized that finding relationships between interested variables can assist in better understanding the real-world problem.

Step 3

Pre-service mathematics teachers worked mathematically with the intervention of technology by using GeoGebra to find the solution visually with validated mathematics. From Figure 8, the discussion was initiated as the following.

"From M to N, its average speed (slope m) is 68.31 km/h. Also, at point P, the car has an instantaneous speed (slope m2) equal to 68.31 km/h, which is over the speed limit."
"By calculating with GeoGebra, let M = (a,f(a)) and N = (b,f(b)). Then, define it as d. We will get the number of the value d= (f(b)-f(a))/(b-a) = 68.31."
"During the acceleration, the speed meter must read exactly 68.31 km/h. By GeoGebra, we can find a point P = (c, f(c)) which has the instantaneous car speed of 68.31 km/h"

The participants discussed and tested their intuition with the researcher, getting the instantaneous car speed as the following Figure 9.

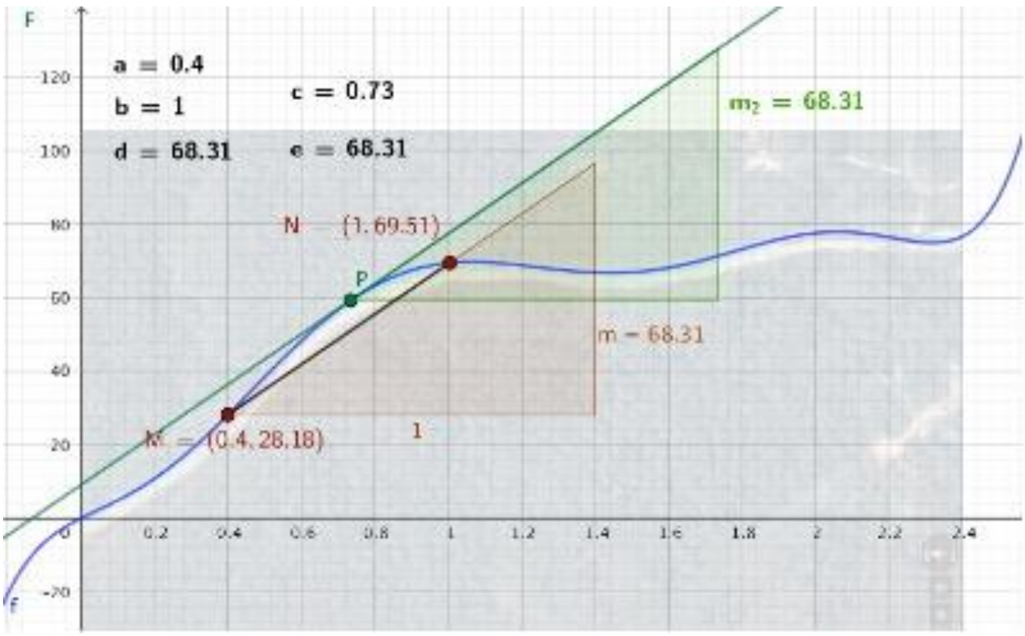

Figure 9. Example of testifying Mean Value Theorem intuition by GeoGebra

They continued that

"We got e = f'(c) which was also equal to 68.31, and c was in (0.4, 1). We could find that d = e. So, we have that there is c in (a, b) such that f'(c) equal to (f(b)-f(a))/(b-a)."

Then, they related all ideas to answer the main question "Does the driver of this car have to pay the bill for a fine over the speed limit of 60 km/h in this area?". The further discussion included the following.

"Conclude that if y = f(x) is continuous at every point of the closed interval [a,b] and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which f'(c)=(f(b)-f(a))/(b-a)."

To obtain the mathematical solution, finding the average and instantaneous speed of a car would be done and satisfied the given initial data condition of that car. Pre-service mathematics teachers were encouraged to explore how the genre in which a mathematical model was formulated might influence its interpretation and learning. Using GeoGebra for detailed solving, pre-service mathematics teachers were then engaged in studying the average and instantaneous speeds pattern before carefully presuming the general form of the MVT. So, pre-service teachers had to apply appropriate formulae to determine the functional relationship of f'(c) and (f(b)-f(a))/(b-a).

Step 4

Interpreting the situation in the context of the real-world situation was evident as follows.

"Traveling from M to N, if a car accelerating from f(a) = 28.18 km takes about b – a = 0.6 h to go f(b) = 69.51 km, calculating by GeoGebra, its average speed for the interval is 68.31 km/h."
"So, we can interpret the output that at least once a time, the car was going 68.31 km/h."

Although pre-service mathematics teachers could find the functional relationships between the interested variables representing the MVT, they did not be able to interpret this result of the mathematical model in the context of speed limit tickets concerning over speed limit driven cars. So, they integrated mathematical arguments to explain their more convincing explanation.

Step 5

Pre-service mathematics teachers compared with other results and critiqued the process with others by applying the MVT. In contrast to the MVT, pre-service mathematics teachers did not determine their model by comparing it with the computing data by hand. They found other sources of explanation, such as online sources, to be compared with the model data.

Step 6

In the following discussion, pre-service mathematics teachers communicated and justified the model and whether it satisfied the original expectation.

"At point P on f (in Figure 8), if we recognize this graph as a function of distance (s) with respect to time (t), we have that, from M to N, its average speed is 68.31 km/h."

"So, the car has an instantaneous speed equal to 68.31 km/h when it was driven 43.8 minutes from the origin point."
"During the drive, the car traveled 68.31 km/h, which is over the speed limit. So, the car driver has to pay for the police penalty ticket."

Step 7

Pre-service mathematics teachers revised the model process if it was unsatisfactory. They attempted to clarify the error difference between the value from the graph and the calculation by hand (it is about digit estimation). In such a learning mathematics modeling process, pre-service mathematics teachers checked their model in other data with the same setting condition, such as on other points on the curve. When they repeatedly validated the variables' relationships, they could intuitively understand and identify the theorem.

RME and Problem-Solving Steps from the Activity

After relating all ideas that pre-service mathematics teachers got to answer, the main question was, "Does the driver of this car has to pay the bill for a fine over the speed limit of 60 km/h in this case?" we allowed them to explain their supporting reasons. It was found that they performed Polya's step of defining the problem from the transition step of modeling (Polya, 1973; Noble, 1982; Son, Darhim, & Fatimah, 2020), which was understanding and interpreting context.

After that, they developed various solution schemes, the second step of Polya (Polya, 1973; Noble, 1982; Son, Darhim, & Fatimah, 2020), when they tried formulating and mathematizing the problem while in the transition step of modeling. It was evident that they connected a function's average rate of change over an interval with the instantaneous rate of change of the function at a point within the interval as in Figure 10.

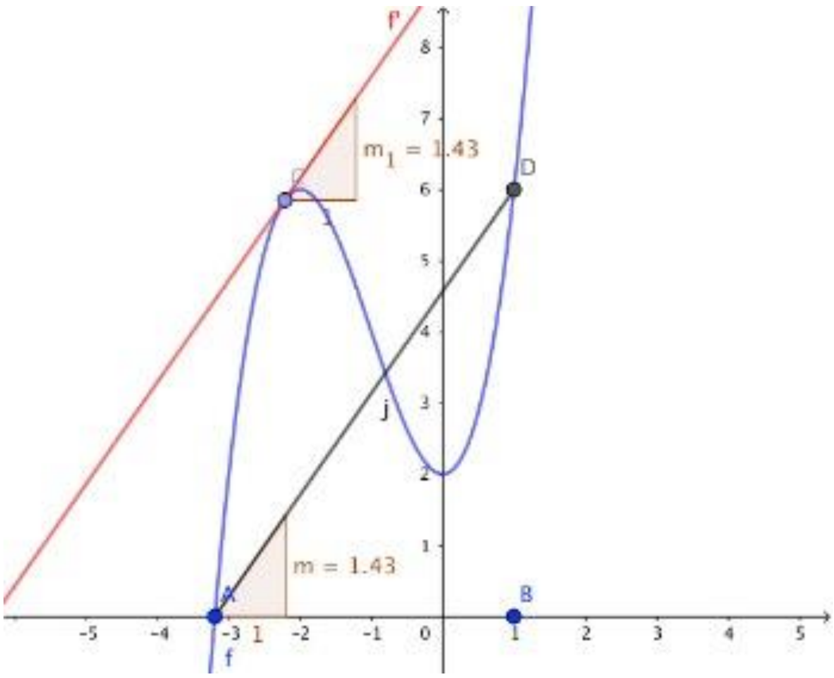

Figure 10. Example of one participant's formulation by connecting the problem to a mathematical concept

Then, they implemented the solution, Polya's third step (Polya, 1973; Noble, 1982; Son, Darhim, & Fatimah, 2020), when they were in modeling's transition step of mathematically work and interpreted mathematical output. Pre-service mathematics teachers reviewed the part of MVT and were very explicit about what the theorem says, describing it verbally and with mathematical symbols to answer the question. For example,

"At some point during the acceleration, the speed meter must read exactly 68.31 km/h which is over the speed limit of 60 km/h By GeoGebra, we can find a point P = (c,f(c)) which has the instantaneous car speed of 68.31 km/h"

Moreover, they specified that the car has an instantaneous speed of more than 60 km/h that should pay the penalty bill. Finally, they checked the result, Polya's fourth step (Polya, 1973; Noble, 1982; Son, Darhim, & Fatimah, 2020), when they were in modeling's transition step of validating the solution and communicating the result to the real-world solution. It was found that they tried to apply the MVT on other points of the given curve as in Figure 11.

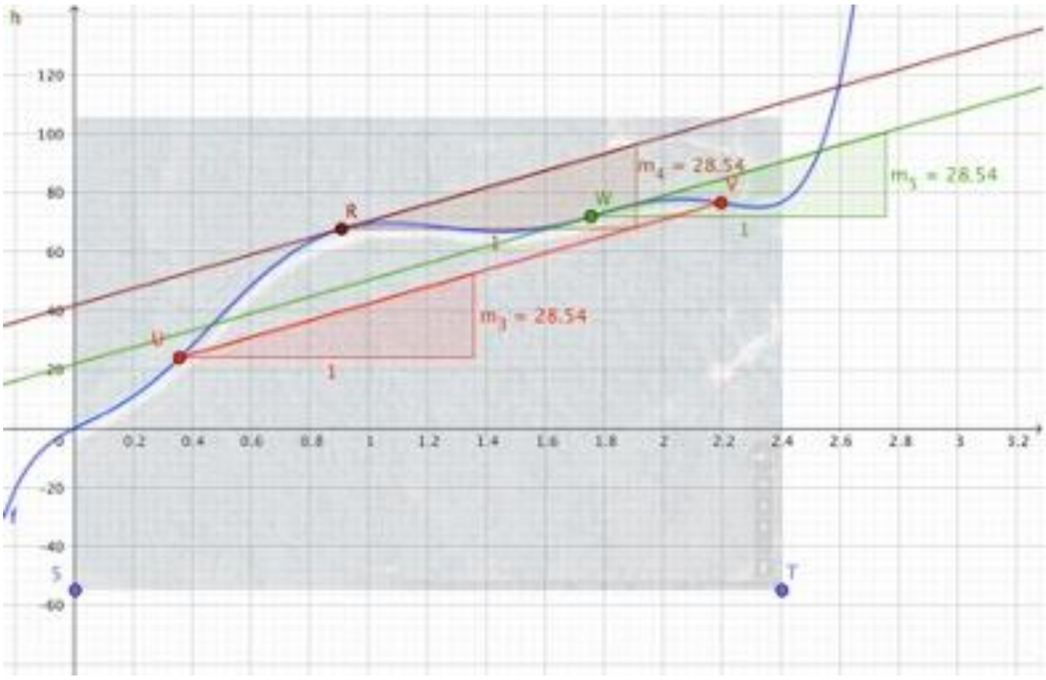

Figure 11. Example of one participant's validation of the solution

For instance, Figure 11 illustrates that the pre-service mathematics teacher correctly found the average rate of change by correctly determining the tangent line of f at any point in the expected interval. She also correctly determined the line parallel to the segment UV, which has a common point (intersect point) with the function curve f, and precisely described the slope of the tangent line. As a result, she correctly interpreted the meaning of the coordinate of R in this alternative problem situation.

The proposed design was implemented as shown in Figure 12. It shows the classroom activities in sessions, including individual problem solving and discussion among pre-service mathematics teachers.

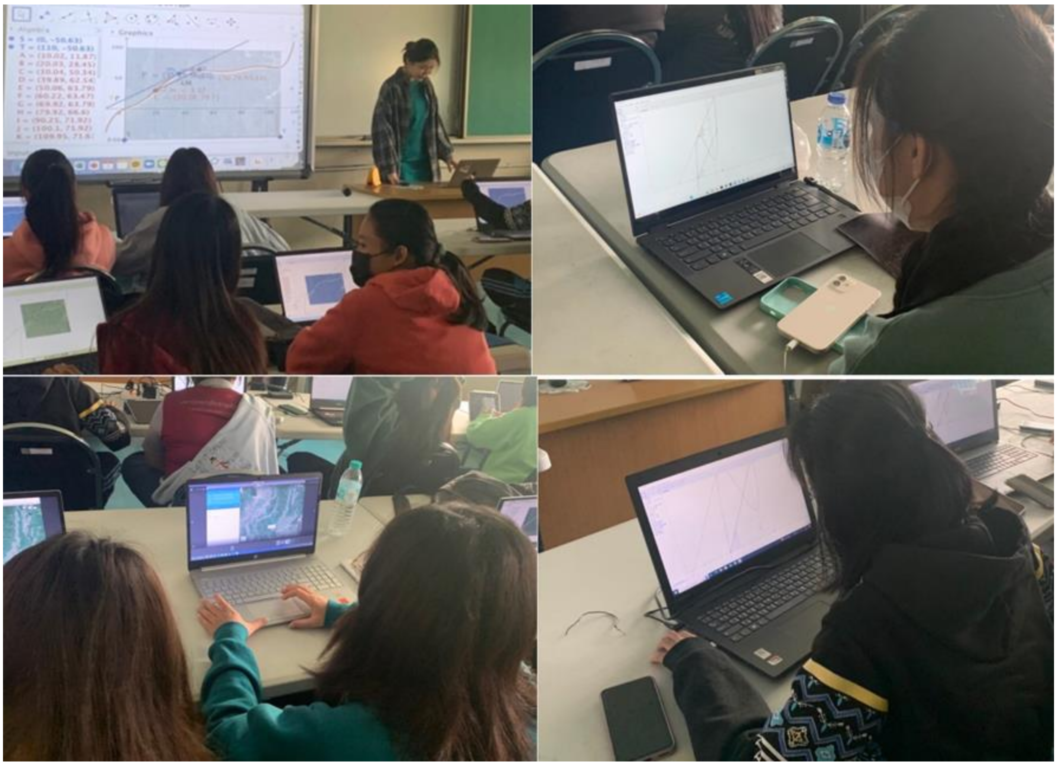

Figure 12. The classroom activities, individual problem solving and discussion

In sum, pre-service mathematics teachers developed the mathematical modeling process and used their efforts to tackle real-world problems during the process. In addition, the capacity to validate the model, critically analyze it, evaluate it and its results, and explain the model were included. The mathematics modeling process and transition between stages were very orderly with the concept of problem solving. The modeling process contained the four elements outlined by Polya (1973) as follows.

Table 1. Comparison of Polya's problem solving and modeling process

Polya's problem solving	Transition between stages of mathematics modeling
1) Define the problem	- Understanding, interpreting context
2) Developing various solution schemes	- Assuming, formulating, mathematizing
3) Implement the solution	- Work mathematically - Interpret the mathematical output
4) Check the result	- Comparing, critiquing, validating - Communicating, justifying/revising the model process

Table 1 shows the iterative nature at any step in the problem-solving process, and each step is amplified with the example in B from typical mathematical modeling transition steps. Steps in solving mathematical problems from Polya were involved as the following. Firstly, pre-service mathematics teachers had access to vast amounts of real-world data using technology, which facilitated the understanding to define the problem. They utilized technology to visualize and analyze data and uncover patterns and relationships that could inform their strategies for problem-solving during RME implementation. Secondly, the intervention of technology and RME enabled students to collaborate and communicate with others, which was valuable during the "Developing various solution schemes" phase of problem-solving. For instance, students can utilize technology to design and test mathematical models collaboratively and receive comments and support from their classmates and teachers. Furthermore, the intervention allowed pre-service mathematics teachers to experiment and visualize mathematical models, which was advantageous during the "Implement the solution" and "Check the result" phases of the problem-solving process. For instance, students can use technology to construct and evaluate virtual simulations and investigate the dynamic behavior of mathematical models.

CONCLUSION

The focus of this study was how pre-service mathematics teachers' problem-solving can be explored in their content course, intervening with technology and RME. The process of mathematical modeling as an RME approach was utilized in this study to conceptualize mathematics theorem, such as MVT, and cultivate problem-solving steps. According to the result, technology was utilized to produce virtual simulations and interactive models that assist pre-service mathematics teachers in comprehending complex mathematical ideas in RME. Moreover, the intervention of technology and RME benefit the development of problem-solving skills by providing pre-service mathematics teachers with an opportunity to experiment and test their mathematical comprehension. They can, for instance, use technology to construct and test mathematical models and examine real-world data and relationships in a safe, controlled environment.

Moreover, the intervention of technology and RME can improve students' mathematics comprehension and problem-solving abilities by providing them with creative tools and resources. The steps of Polya's problem-solving method are still applicable and beneficial in this environment, and the use of technology can enhance Polya's process by allowing students to access and analyze real-world data, collaborate, communicate, experiment, visualize, and receive feedback and assistance.

The finding of this study may contribute to the existing literature on the intervention of technology and RME in mathematics education, providing light on the efficacy of technology and RME in enhancing problem-solving skills among pre-service mathematics teachers. This study could assist teacher educators in discovering the most successful technology and RME strategies in other content courses to increase pre-service mathematics teachers' problem-solving skills, which could then be implemented in their classrooms.

For implication, the finding from this study could be used to inform the development of mathematics curricula and teaching approaches in schools and universities. Moreover, it could be used to advocate for the broader use of technology and RME in mathematics education, which could ultimately lead to improved problem-solving abilities and critical thinking skills in students.

In addition, it is impractical to cover all the mathematical concepts they may encounter in their future teaching using mathematics modeling. Then, we should develop independent, reflective pre-service mathematics teachers who can address new content and pedagogies presented in the activity with any mathematics content, such as Geometry or Graph theory, and technology they prefer and independently make sense of these new ideas.

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Declarations

Author Contribution	:	RS: Conceptualization, Writing - Original Draft, Editing and Visualization. PI: Review & Editing, and Validation. SS: Review & Editing, and Validation. NJ: Writing - Review & Editing, Formal analysis, Methodology, Validation, and Supervision. Y-YH: Methodology, and Supervision.
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