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Identification of characteristics of didactic and meta-didactic mathematical knowledge of novice and expert teachers when reflecting on class episodes

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Abstract

This article describes the didactic analysis carried out by an expert teacher and a novice teacher, based on the didactic suitability criteria (DSC) of the Onto Semiotic Approach (OSA) when analyzing episodes of a high school mathematics class in Costa Rica. This is a qualitative investigation with an interpretive hermeneutic approach. First, a questionnaire was applied to the two participants, to classify them as novice and expert, based on certain defined characteristics. Both teachers then analyzed three video segments using a guide for didactic analysis and, finally, an interview was conducted to elaborate on the reflections previously obtained. The data analysis focused on identification, classification, and comparison of elements in each of the video segments with reference to the OSA's DSC indicators. The results show that there are differences and similarities between the didactic analysis carried out by these teachers. It is concluded that the novice teacher provides a more descriptive analysis, in which errors are not specified, and focuses on the activities carried out by his students, while the expert teacher, who provides a thoughtful evaluative analysis of, identifying errors in the concepts and focusing his analysis on the teacher's actions and performance in the video segments.

Keywords: Class Episodes, Didactic Suitability Criteria, Expert Teacher, Novice Teacher, Reflection

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Several investigations are currently seeking to strengthen the training of mathematics teachers to improve their professional performance (Brown, 2022; Strutchens et al., 2017; Wittmann, 2021). Furthermore, greater preparation in teachers' didactic, pedagogical, and mathematical content is demanded by the social and school context. For adequate training, one of the necessary reference points is the link with the working environment, where the knowledge and experience of in-service teachers are vital. Taking advantage of this expertise is ideal in guiding the development of certain skills.

One way to investigate aspects related to teacher education is by learning more about the characteristics of teacher expertise in an effective mathematics class, how to recognize expert teachers, and how these expert teachers differ from others. As Li and Kaiser (2011) indicate, collecting and analyzing information about the characteristics of expert teachers allows us to understand what novice

teachers need to develop before they can become experts themselves. Rojas, Carrillo, and Flores (2012) also point out the importance of identifying the characteristics that make a teacher an expert, so that these characteristics can be eventually developed by other teachers.

Some authors have identified certain types of knowledge that an expert teacher must have, including knowledge of mathematical topics, knowledge of the characteristics of learning, knowledge of the teaching of mathematics (e.g., Berliner, 2001; Rojas, Carillo, & Flores, 2015; Wolff et al., 2015) and meta-didactic-mathematical knowledge (Breda, Pino-Fan, & Font, 2017). In general, experts have more specialized knowledge that allows them to make better decisions to facilitate student learning (Auerbach et al., 2018; Berliner, 1988; Wolff et al., 2015). It is in this way that this research can contribute to the knowledge about didactic analysis. carried out by an expert teacher and a novice secondary school teacher when analyzing episodes of a high school mathematics class in Costa Rica, based on the didactic suitability criteria (DSC) (Breda, Font, & Pino-Fan, 2018) of the Onto Semiotic Approach (OSA).

Expert and Novice Teachers

When studying phenomena related to expert and novice teaching staff, there is no definition of knowledge strictly oriented towards this subject, since it will necessarily depend on the implicit and explicit interests of different educational systems and their possibilities (Chi, 2011; Li & Kaiser, 2011; Russ, Sherin, & Sherin, 2011). However, certain differences between expert teachers and novice teachers can be clearly specified. For example, Berliner (1988) proposes the existence of five stages that categorize teacher development according to professional performance; they occur gradually, showing that teachers who are considered to have expertise have acquired it in stages since they began their professional careers. In the first stage are teachers-in-training and beginning teachers. In this stage they learn the tasks to be performed and a series of context-free rules to guide classroom behavior (Berliner, 1988). Secondly, the novice and advanced novice lack some responsibility for their actions, they try to follow the established rules, without considering their decisions, accepting responsibility for their actions occurs when the teacher makes personal decisions in the classroom. and so on, the model describes the teacher's condition. Berliner's model describes this process in teaching and makes it possible to specify the different characteristics, skills, and knowledge that characterize teachers in each stage (Schempp et al., 1998).

The main characteristics of expertise as outlined in the international literature are presented below. It has been decided to use the classification of content knowledge and pedagogical knowledge to organize it. It is important to note that some types of knowledge from different models such as TPACK's TK are included in one of the two groups. Since, Table 1 allows us to summaries this knowledge, it is not possible to adapt it to a specific model.

Table 1. Characteristics of expert and novice teachers

Types of knowledge	Novice	Expert
Content Knowledge	Limited knowledge of content.	Knowledge of the content.
	Fragmented and incomplete knowledge, unable to make connections between different elements and concepts of the subject being taught.	Structured and easily accessible knowledge, which allows recognition, interpretation, and interrelation of information.
	Due to lack of experience and limited knowledge of content, repetitive operations	Performs repetitive operations automatically.

	are not performed automatically.	
	The ability to present problems in different ways is not developed.	The ability to present problems in different ways is developed.
Pedagogical Knowledge	Pedagogical knowledge is poorly developed.	Pedagogical knowledge is highly developed.
	Behavior is relatively inflexible when teaching.	More opportunistic and flexible when teaching.
	Concise and important situations in the classroom can go unnoticed.	Concise and important situations in the classroom are noticed.
	Reflections after class are focused on the teacher's own performance and not on whether students have understood the topics covered during the lesson.	Reflections after class are concise and focused on certain aspects, especially on learning by the students.
	Tends to conform to established rules and procedures.	Has different strategies for responding to situations that occur in the classroom.
	Has problems relating the contents of the curriculum, the lesson and the teacher's own knowledge.	Has structured and easily accessible knowledge that allows making connections between the contents of the curriculum, the lesson and the teacher's own knowledge.
	Does not recognize the content areas in which students may have difficulties.	Recognizes problematic content areas.

Source: Prepared by the authors based on Auerbach et al. (2018), Berliner (1988; 2001), Borko and Livingston (1989), Borko et al. (2011), Rojas, Carrillo, and Flores (2012), Rojas (2014), and Schempp et al. (1998).

The Didactic-Mathematical Knowledge and Competences (DMKC) Model of Mathematics Teachers and Criteria of Didactic Suitability

Godino, Batanero, and Font (2008) mention that the factors that determine and improve the teaching and learning processes must be established, considering disciplines such as pedagogy, psychology, philosophy, and sociology (within the framework of research in didactics of mathematics). Therefore, the need arose to establish a theory that understands these characteristics, as well as the nature and development of the contents through an ontological and epistemological analysis. The Onto Semiotic Approach attempts to address this need.

The purpose of the Onto Semiotic Approach (Godino, Batanero, & Font, 2007) is to integrate different points of view and theoretical notions about mathematical knowledge with its learning and teaching (Godino, 2014). The OSA "provides theoretical tools to jointly analyze mathematical thinking, its ostensive aspects, and the situations and factors that affect its development" (Godino et al., 2008, p. 4). The Didactic-Mathematical Knowledge and Competencies (DMKC) model of mathematics teachers are based on the OSA. This model seeks to describe and understand the convergence between the mathematical didactic skills that teachers require and the knowledge necessary to put these competences into action. The relationship involves practice (such as the ability to perform a task competently) and objects (that arise from practice and are interconnected) (Godino et al., 2017).

In this model, competence is understood as the full action that demonstrates the ability to use knowledge in explicit actions. Thus, knowledge nourishes the different competences. In our case at least

four sub-competences are considered. Didactic and meta-didactic mathematical knowledge is part of the DMKC model and is based on the understanding of knowledge linked to the way mathematics is taught and the evaluation of the didactic appropriateness of the instructional process (the fourth sub-competency included in this model).

On the other hand, one of the main tools of the OSA is the didactic suitability model which provides a way to analyze whether a process has certain characteristics which allow it to be classified as adequate (Breda, Font, & Pino-Fan, 2018; Godino et al., 2017) and is a "tool to establish a bridge between descriptive didactics and normative or technical didactics" (Godino, Bencomo et al., 2006, p. 4). Godino (2013) mentions that the suitability model can be applied to a particular study of a class session, or a curricular proposal, and can even be applied in the analysis of didactic materials and tasks (see for example, Céspedes et al, 2022; Morales-López & Font, 2019). In this case, it will also be used to learn more about the concept of expertise.

Assessing didactic suitability within the DMKC is a competence and a tool which can be used in a comprehensive consideration of didactic practice. It has been divided into six criteria, which, in turn, subdivided into components and indicators (DSC). The criteria are:

1. Epistemic suitability, "refers to the extent of representativeness of the institutional meanings implemented (or intended), with respect to a reference meaning" (Godino, 2013, p. 116).
2. Interactional suitability refers to the actions carried out during the instruction process, when "potential semiotic conflicts are identified, and on the other hand it permits the resolution of conflicts that occur" (Godino, 2013, p. 116). In addition, it covers the interactions between teacher and students, and between students.
3. Mediational suitability, "degree of availability and adequacy of the material and time resources necessary for the development of the teaching-learning process" (Godino, 2013, p. 124).
4. Ecological suitability, the "degree to which the study process adjusts to the educational project of the center, the school and society, and to the conditions of the environment in which it takes place" (Godino, 2013, p. 116).
5. Affective suitability, the "degree of engagement of students in the study process" (Godino, 2013, p. 116).
6. Cognitive suitability, the "degree of understanding of the intended meanings in education, and the correspondence they have with what is learned by the students" (Godino, Contreras et al., 2006, p. 28).

The Use of Videos to Enhance Reflection of Teachers

Reflection is a process of development and testing of ideas stemming from teaching actions (Dewey, 1989; Schön, 1983). This process allows teachers to learn by themselves and to improve their teaching practices (Shulman, 1986; 1987). On the other hand, reflection on the practice of peers favors both the person who carries out the reflection and the teaching staff that is being analyzed, allowing them to learn about different points of view, interpretations, and questions about their way of teaching (Climent & Carrillo, 2007, p. 25). This type of reflection improves teachers' capacity to interpret different classroom situations, helping to develop the ability to determine relevant aspects and to improve knowledge about to interpret and participate in activities, as well as improving understanding the rationale for these actions (Climent & Carrillo, 2007; Climent et al., 2013).

When considering reflection in this light, the OSA's DSCs can be used as a guide for analyzing

teaching practice, since the DSCs help to describe, criticize, and justify the choice of means and ends, in order to improve the didactics of mathematics (Posadas & Godino, 2013; Seckel & Font, 2020). Thus, the use of the construct of didactic suitability in teaching processes "contributes to motivating and giving meaning to the systematic search for specialized knowledge about mathematical content" (Posadas & Godino, 2013, p. 19), representing a tool that can encourage teachers to reflect on their own practice and help to improve it (Posadas & Godino, 2013).

However, there are different ways and means to analyze or reflect on classroom situations. Authors such as Kaiser et al. (2015) highlight the importance of selecting short segments of video, focusing on the most relevant aspects of a class. On the other hand, Climent and Carrillo (2007) emphasize that it is important to divide the videos into segments to facilitate more detailed analysis. When reflecting on video clips of practice, Coles (2014) states that the teaching staff carrying out the analysis should focus on discussion and reflection on the video, rather than making value judgments. One of the great advantages of this approach, according to Borko et al. (2011) is that it allows teachers to consider aspects that they may not be able to visualize while observing a lesson. In short, reflecting with the use of properly selected short videos of classes provides the opportunity to observe and listen carefully to each of the situations presented, and makes it possible to understand, analyze and explain what was observed in a more detailed way.

METHODS

This research is framed within a qualitative methodology, as it seeks to understand a social phenomenon from the point of view of the participants. It also seeks to understand and interpret the information obtained in the didactic analysis carried out by a novice and an expert teacher. In this way, the analyses carried out by an expert teacher and a novice teacher were studied, based on the DSC and its components; to achieve this, the judgements, explanations, and reflections issued by the teachers were analyzed, and an attempt was made to understand and interpret them.

Research Subjects and Selection

The subjects that participated in the present investigation were two teachers who work in secondary school, selected based on convenience. In this case one of the teachers is classified as an expert and the other as a novice. The intention of the study is to compare the way in which they manage to make their justifications, their descriptions, and their analyses of what happens in the classroom. The characteristics used as a basis for the selection of the expert teacher were those developed by Rojas et al. (2012): a) A practicing teacher, with five or more years of professional experience in classrooms, b) with at least one institutional evaluation of "excellent" in the last five years (information provided by the teacher), c) who has taught mathematics content related to the object of study of interest more than once in the last five years of teaching, d) has participated in professional training activities in his discipline. Also considered were, e) the teacher's interest in participating in the research, and f) that the teach has a bachelor's or licentiate's degree in Mathematics Teaching.

To select the novice teacher, characteristics obtained from previous investigations were used, such as those of Berliner (1988), Bozu (2010), Rojas et al. (2012), and Schempp et al. (1998): a) Interest of the teacher in participating in the research, b) bachelor's degree or licentiate's degree in Mathematics Teaching, and c) Practicing teacher with at most two years of professional experience, regardless of whether or not he has taught the content being considered.

It is important to clarify that the gender of the participants was not relevant. Age also had no bearing on the selection, but there is a significant difference as this is linked to the experience of each teacher.

Research Protocol

Video recordings of secondary school math classes of the SIA0127-13 project were used for the study: Mathematical knowledge for training teachers of the subject of functions in the Diversified Cycle of Mathematics Education in Costa Rica. Three short sequences from complete class filming's were selected and extracted based on the expert opinion of the authors of this article. Subsequently, potential participants were classified using questionnaire 1 (Annex 1). The information obtained was synthesized in an information sheet (Annexes 2 and 3). Based on the data collected, teachers were classified as novice or expert. Final selection was carried out using these instruments.

The guide for didactic analysis (Annex 4) was applied to each teacher, and the teachers analyzed each of the selected video segments using this guide. Afterwards, a semi-structured online interview was conducted with each participant individually (Annex 5) to understand and broaden the reflections obtained using the previous questionnaire (Annex 4). Finally, analysis of the data collected was carried out.

To guide this analysis, the DSC criteria proposed by Godino (2013) were used, with their respective components. These criteria and components were used, respectively, as the categories and subcategories of the study, which are summarized in Table 2.

Table 2. Criteria (categories) and components (subcategories)

Categories	Subcategories
ES. Epistemic suitability	ESa. Problem situations, ESb. Languages, ESc. Rules, ESd. Arguments, ESe. Relations
IS. Interactional suitability	ISa. Teacher-student interaction, ISb. Interaction between students, ISc. Autonomy, ISd. Teacher evaluation
MS. Mediational suitability.	MSa. Material resources, MSb. Number of students, schedule and classroom conditions, MSc. Duration
EcS. Ecological suitability.	EcSa. Adaptation to the curriculum, ESb. Potential for didactic innovation, ESc. Socio-professional and cultural adaptation, ESd. Education in values, ESe. Intra- and interdisciplinary connections
AS. Affective suitability	ASa. Interests and needs, ASb. Attitudes, ASc. Emotions
CS. Cognitive suitability.	CSa. Previous knowledge, CSb: Curricular adaptations to individual differences, CSc. Learning

Source: Prepared by the authors based on Godino (2013) and Godino et al. (2017). **Note**: The categories, subcategories, and indicators of each of these are presented in greater detail in Annexes (6 -12), with their corresponding coding and category map. See https://doi.org/10.5281/zenodo.7682498

Selection and Content of Short Video Clips

The criteria used for the selection of the secondary school mathematics class to be recorded are detailed below, as well as the characteristics that the teacher to be recorded should meet: 1) Two to three lessons of 40 minutes each; 2) A curriculum topic is introduced or an activity is developed involving elements of problem solving, modelling or other processes of interest for didactic analysis; 3) The teacher must meet

the following criteria: a) Willingness to be filmed. b) Interest in participating in the study.

In the first video segment, the initial stage of a class is presented in which the teacher (the teacher who taught the class that was videotaped) presents the topic of functions in a tenth-grade class, using a problem that students must solve in groups. During the process of solving the problem, the teacher interacts with each group to clarify doubts. The second sequence presents the process in which the teacher explains the symbols related to the topic of functions and introduces the concepts of domain, codomain, and Image set. In the third sequence, a review and reconceptualization of the concepts related to the topic of functions is carried out, such as relation, domain, codomain, image, preimage, dependent variable, independent variable, Image set and a graph of the function, through examples in which he encourages student participation.

RESULTS AND DISCUSSION

In the present research, content analysis was used to analyze the information, according to Bardin (1996), this technique is defined as: the set of techniques of communication analysis aimed at obtaining indicators by systematic and objective procedures of description of the content of the messages, allowing the inference of knowledge relative to the conditions of production/reception of these messages (p. 32). This allows the qualitative data obtained in the research to be analyzed systematically and reliably, in order to make generalizations based on them, which are related to the categories, subcategories and indicators proposed by the researcher (Haggarty, 1996).

Content analysis was used to study the information obtained from the guide and the semi-structured interview applied to the teachers, which was analyzed in the light of the suitability criteria. In addition, by using content analysis it is possible to determine which indicators are present in each of the responses and which aspects are observed by experts and novices. It is also possible to determine which of the suitability criteria they refer to most.

The following is a summary of the main characteristics identified in the reflections of participants. It is first organized by each suitability criterion, and then by each instrument (the 3 video clips and interview).

Epistemic Suitability (ES)

Novice Teacher

With respect to epistemic suitability, the novice teacher identified the opportunities offered by use of the introductory problem in the video 1 (ESb3) and placed a stronger emphasis on them. In addition, the novice teacher emphasized that this allows his students to work independently; the teacher in the video is helping his groups to generate the necessary knowledge to solve the problem and, at the same time, introduce the subject (ESc3).

In video 2, the novice teacher mentioned that the mathematics teaching is of high quality since the approach to concepts and definitions are compatible with the content to be taught and the students to which it is addressed. (ESc1). In addition, ESe2 is observed because it refers to the way in which the teacher who delivers the class presents the contents and the clear compatibility and relationships between elements of these contents. In video 3, the novice teacher focuses on aspects related to the situations that are presented to students which promote argumentation and generation of their own definitions (ESb1, ESc3).

In the interview, the novice teacher noted that it is usual in the class to create situations in which

the students must present arguments and mentioned that in the first video in each subgroup, students must generate a formula to solve the problem assigned to them (ESd2). To do so, they must share ideas and build the definition of the topic to be studied (ESc3). He also mentioned that this can be observed in video 3 when the teacher assumes the role of a guide and builds definitions together with students, based on situations that students have faced in previous lessons. He did not identify major errors:

Novice teacher: *He was going to present the topic of functions; the introductory problem was very easy, since it is something that students learn how to solve in eighth grade. When it came to presenting definitions, I think it was well done. I liked how he explained pre-images and images. I think the content was good*.

Expert Teacher

In video 1, expert teacher described the activity, referring to the organization and instructions, and believes that the introductory activity was well planned (ESc1, ESc2 and ESd1). Regarding video 2, he mentioned that the symbols used are appropriate and consistent with the formal mathematical concept.

The expert focused on three areas: language (IBb), the arguments used (ESd) and the relationships between mathematical objects addressed in the class (ESe). In video 3, the expert teacher thought that general concepts of the subject matter were managed acceptably, which is related to the ESc1 and ESd1 indicators. However, he also indicated that there were some conceptual errors, with the notion of ordered pairs, graphs (as drawing) and graph (as a set), and abscissas and ordinates, since they were incorrectly presented as if they were synonyms (ESb1, ESb2, ESc1 and ESd1). Despite the errors detected by the expert, he believed that the definitions and procedures used were consistent and well planned, which is related to the ESc1 indicator.

Expert: *In video 3 there were three concepts that the teacher confuses. It's not the teacher's fault, I think it's because of the books that are used. For example, when he talks about graph (as drawing) and sometimes says graph (as a set), they are not the same thing […]. Later on, the teacher talks about an ordered pair, but does not say explicitly why say why the array (x, y) is called an ordered pair but rather does so implicitly.*

In the response to the first question of the interview, "Which aspect(s) do you consider strengthened the teaching and learning processes in the videos observed?", it was possible to detect topics that were related to indicators such as ESa2, ESb1 and the indicators of the subcategory of relationships ESe1 and ESe2, since the participant referred to the way in which the teaching process, the relationships between mathematical objects, the symbols used, and the proposed activities were strengthened.

The expert teacher did not agree with the order in which the subject matter was taught in classes and believed that it would have been better to reverse the order in which the activities were carried out (ESc1). He emphasized the abstractness of the concepts presented during the lesson and the difficulty that students would have in understanding them by themselves.

Interactional Suitability (IS)

The information collected on interactional suitability came from observations about whether the interaction in the class was adequate and contributed to solving students' difficulties.

Novice Teacher

The novice teacher mentioned the organization of the introductory activity shown in the video, indicating that instructions to the group are clear and concise, and highlighted the teacher's interventions during this video segment (ISa1, ISa2). In addition, he commented on the importance of the teacher during group work, when he encouraged the inclusion of each of the participants in the class dynamics through dialogue and discussion between them (ISa1, ISb3).

On the other hand, in the third video the novice teacher pointed out several important details related to subcategory ISa. He pointed out that there was no participation of the entire group of students, with only a few of them answering the teacher's questions (ISa2 and ISa5), and he thought that the rest of the group should have been encouraged to participate. This made it difficult for the teacher to recognize and resolve difficulties that might have arisen during the presentation of this subject matter (ISd1).

In the interview, highlights regarding interactional suitability focused on the importance of promoting dialogue and communication between students and generation of situations in which they must convince their classmates of the assumptions and answers that they obtained (ISb1, ISb2). The novice teacher also indicated that more autonomy should have been given to students: they received a great deal of support, and perhaps they should have been given more tools (ISc1). Likewise, the teacher should have directed questions to specific students, rather than addressing the group in general, making it impossible to verify if students had been learning.

Expert Teacher

In video 1, the expert teacher focused on the organization of the introductory activity (Ia1, Ia5). On the other hand, in video 2, the answers he gave focused on the fact that there was no direct engagement of the students, but he also referred to a positive attitude on the students' part, which is related to the ISa5 indicator.

Likewise, in video 3, the expert teacher emphasized that there was no teacher-student interaction, because students were not engaged in classroom dynamics through questions which could have been included during the explanation of the subject matter (ISa2, ISa4, ISa5)

Question 10 of the interview asks, "What do you consider to be the most interesting thing you observed in the video sequences?" The expert teacher focused on the interaction between teacher and students, since empathy could be observed between them, as well as respect of the students for the teacher. Regarding empathy, he mentions that:

Expert teacher: *"If it doesn't exist, there's nothing; you must "click" with the students."*

According to the participant, one of the important things that should exist in a class is a good teacher-student relationship, which is related to the indicators of subcategory ISa.

Mediational Suitability (MS)

To obtain the reference to the mediational indicator, teachers were asked if they consider that temporary resources, materials, ICTs, etc., have been properly used.

Novice Teacher

In video 1, the novice teacher referred more to material resources, mentioning that the teacher in the video made appropriate use of the resources in class. In addition, he considered that the activities were adequate, contextualized and contributed to students' development (MSa1 and MSa2), and that time management of the activities was appropriate, i.e., more time was dedicated to the most relevant topics

(MSc1). In related comments about video 3, he stated that although resources were used in the class, they could have been used better and more extensively, taking advantage of technological resources which could increase students' motivation and make the class more dynamic (MSa2).

In general, he emphasized allowing students to use resources for investigation as a complement to the book they are using (which facilitates class activities). In this regard, he commented:

Novice teacher: *"I didn't like that the teacher only used the blackboard, because there are so many tools for the students now. For me, ICTs were not used well."*

Expert Teacher

Commenting on videos 1 and 2, the expert teacher thought that no resources of any kind had been used, which is related to the MSa1 and MSa2 indicators. However, in video 2 the teacher states:

Expert teacher: *"There was no use of resources. However, the most important resource, i.e., the teacher, stood firm, had a good attitude, and managed well."*

The expert teacher believed that the teacher is one of the most important resources in the teaching and learning process, and he thought that this resource was appropriately used (MSa1, MSa2, MSc1, MSc2).

Regarding video 3 and question 9 of the interview, "What aspects do you emphasize about physical resources and student distribution, and why?", The expert teacher focused on the fact that technological resources were not used, and on their importance as support for the student teaching process. In the interview, he mentioned that:

Expert (teacher): *"One can teach a good class with few resources, but we live in different times and in one way or another we have to use technology."*

The teacher's comments were related to the indicators of the subcategory of material resources, in particular to MSa1 and MSa2. The teacher also made it clear that the use of technology is not unique to mathematics, but that it must be comprehensively implemented to provide better opportunities in the future.

Ecological Suitability (EcS)

In the case of Ecological suitability, information was obtained by asking the teachers if they considered that the contents are in accordance with the curriculum and are useful for students' social and labor insertion.

Novice Teacher

Both in video segments 1, 2 and 3, as well as in the interview, the novice teacher focused on the fact that the contents, their order, and the way they were presented, clearly correspond to what the official curriculum (EcSa1) suggests and considered that they will be useful for students in their higher education and professional lives (EcSc1). However, the videos show that the use of technology in the class was very limited, with students never using a calculator, computer, or other tools. (EcSb2).

Expert Teacher

In the three video sections, the expert teacher considered that the classroom teacher's approach to the subject complied with current curricular guidelines (EcSa1). Furthermore, in videos 2 and 3, the expert

teacher focused on the applications that the topic has in students' daily lives, and the importance of contextualizing the subject matter that is addressed in class, to make students aware of the usefulness of mathematics (EcSc1, EcSe1). He mentioned that:

Expert: *"The applications are not immediately obvious, but in the medium term the students will appreciate their applicability more."*

In the interview, the answer given by the teacher in/to the second question ("Which aspect(s) do you consider weakened the teaching and learning processes in the videos that you watched?"), focused on the implementation of new methodologies by the Ministry of Public Education of Costa Rica (MEP), which he does not consider to be appropriate (IGa1, IGc1).

Expert teacher: *"I do not agree with that [the change in the teachers' role by authorities in the new study program], I believe that the teacher is the number one factor, a learning asset, because the teacher has the knowledge that students do not; this cannot be overlooked. In addition, the methodology that the MEP is implementing is applicable in a postgraduate studies system."*

Affective Suitability (AS)

A question on whether tasks and management promoted student participation was used to obtain information on affective suitability.

Novice Teacher

The answer of the novice teacher focused on the interest that can be generated in students. In general, in the first video, he mentioned the importance of the introductory activity, which is intended to stimulate interest in the students so that they become really engaged in the topic (IAa1). According to the novice teacher, the classroom teacher presented this activity in a way that made the students the main actors in the class, so that they shared their ideas and created a formula that allowed them to solve the problem (ASb1).

In the interview, the theme that the novice teacher rescued from the three videos was the usefulness of creating activities that promote students' interest so that they become engaged, while maintaining an attitude of tolerance towards the ideas of their peers and achieving the inclusion of all students. This response emphasizes ASa1 and ASb1.

Expert Teacher

Regarding video 1, the expert teacher made further reference to the attitude displayed by students during the activity. He mentions that they were willing to work and participate in the class; the indicators present are: ASa1 and ASb1. Likewise, in video 2 and video 3, the expert teacher considered that the activities promoted student engagement (ASa1 and ASb1), but in the second video, he stated that:

Expert teacher: *"It is necessary to directly ask the students with specific examples about that type of relationship. That is, to return to what was presented at the beginning of the activity, illustrating the applications of the concept of functions in daily life."*

In summary, even though the expert teacher considered that the proposed activities promoted engagement and participation of students in class, it is still necessary to implement activities promoting

awareness about the usefulness of mathematics in real life. This is related to the indicators ASa2, ASb1, ASc1 and ASd1.

In the case of motivation, the answer to question 1 of the interviews indicates the presence of the ASa1 indicator, since the participant referred to the way in which students were motivated and its fundamental role. He also mentions that:

Expert teacher: *"I have the idea that no one can motivate anyone. In addition, there are things that we must learn whether we are motivated or not; they are simply things that we have to do because they are our responsibility."*

On the other hand, the ASb1 and ASb2 indicators were also present, since students' group work and their participation in the activities were mentioned.

In question 6, "What aspects do you highlight about the teacher's mediation strategies and why?", The expert teacher mentioned that the attitudes of both the students and the teacher in class were good, as shown by the empathy that the teacher has for the students and how they participate in the lesson and show interest in the subject; this is related to the ASa1, ASb1 and ASc1 indicators of affective adequacy.

Cognitive suitability (CS)

To obtain information on cognitive suitability, each teacher was asked if they considered that the students had learned from the proposed tasks.

Novice teacher

In videos 1 and 3, the novice teacher considered that students had acquired necessary prior knowledge in previous classes or years, and believed that, for this reason, students were more participative, as was observed in the introductory activity or when the teacher in charge asked a question and they answered (CSa1, CSa2). On the other hand, when observing the second video he indicated that there was a lack of activities other than the teacher's explanations, and that the class should be therefore have been reformulated to make it more dynamic, to reinforce students' development and learning (CSb2).

In the interview, the indicators most emphasized by the novice teacher were related to the importance of planning the topics; he detected a consistent order and a connection between them (based on the order and connections that the official program suggests). According to the novice teacher, this provided students with the prior knowledge necessary for the activities that were going to be carried out every day during the lessons (CSa1), allowing them to manage the difficulty of the content and, at the same time, offering them an interesting challenge.

Expert Teacher

In videos 1 and 2, the expert teacher focused on the fact that there was no way to analyze whether the students learned the content presented in class, because there has not been any form of formative evaluation of the content (CSb1 and CSb2). However, the expert also stated:

Expert teacher: *"You can see that the young people are paying attention to the class work, which allows me to infer that there was good internalization."*

This is important, because even though no activities were carried out to verify that the students had acquired knowledge, the expert teacher inferred that students were learning the content based on

the attitudes they displayed (CSa2, CSc1). In Video 3, the expert teacher stated that:

Expert teacher: *"Based on the previous videos, students are aware of what the teacher presented, so they handled the concepts very well, as their participation demonstrates."*

The expert teacher mentioned that students have the prior knowledge to study the subject matter and that they handled the topic well, which was shown by their participation in class (CSa1, CSa2, CSc1).

Concerning epistemic suitability, the expert teacher focused on problem-generating situations, as was the case with the introductory activity, and emphasized the language used by the teacher, both verbal and symbolic. In addition, he mentioned that the topic of functions is highly abstract, and that students could therefore have difficulties in creating their own definitions. Likewise, he identified some errors present in certain concepts: parabolas and quadratic equations, graphs, and ordered pairs.

On the other hand, the novice teacher did not refer to problems. Unlike the expert, he emphasized moments in which students must argue and share their ideas to create their own definitions. In addition, he focused on the mathematical language used by students. It should be noted that the novice teacher did not identify errors in the concepts presented by the teacher.

In general, both agree that the mathematical objects presented in class are related to each other and that the teaching and learning process of the topic of functions was carried out in a clear, consistent, and appropriate way. This is mainly derived from the analysis and comparison of the epistemic and cognitive criteria, where, according to the teachers, it is evident that mathematics and its learning have a certain concordance.

In the case of interactional suitability, both teachers focused on the importance and organization of the introductory activity to engage students in the lesson. Regarding the teacher-student interaction in all video segments, the expert teacher recognized the empathy and respect that existed between them and the willingness of the students to participate in the lesson. On the other hand, the novice teacher briefly mentioned the teacher's role in group work and the interaction between students during the proposed activities.

Finally, both teachers agreed in their comments about formative evaluation in each of the video segments; they believed that questions should be asked directly to students, to verify the learning obtained and thus improve student participation and attention during the classes.

When evaluating mediational suitability, both teachers agreed that there was an appropriate use of resources, but they mentioned that technological resources should have been used to a greater extent, because they are an excellent complement to the teaching process.

Regarding ecological suitability, the teachers considered that the topics addressed in the video segments were related to the current mathematics curriculum and that they contributed to the socio-professional training of students. Despite these similarities in their responses, the expert teacher emphasized that contextualization and application of the subject matter was left out, which affects student awareness of the usefulness of mathematics.

One of the most relevant differences between the teachers on adaptation of lessons to the curriculum is that the expert teacher does not agree with the methodologies currently in use, believing that these methodologies the teacher do not sufficiently recognize the key role of the teacher, who becomes only a facilitator of the teaching and learning process. He also believes that the teacher has knowledge that should not be ignored, and that the current methodologies do not recognize this importance.

Regarding affective suitability, the novice teacher emphasized that more activities should be

implemented to promote students' interest in the class and in the topic addressed to increase their participation in the proposed activities, while showing tolerance towards the ideas and contributions from other students. The expert teacher also considered that the motivation observed in the videos was important; however, he mentioned that while activities should be designed to increase students' interest, there are some topics that must be learned regardless of whether the students are motivated or not. Regarding students' attitudes during the class, the expert teacher mentioned that the proposed activities promoted student engagement, which was observed in their willingness to work and participate in classes, as well as the empathy they have for the teacher.

Finally, in the case of cognitive suitability there were several similarities between observations made by both teachers: they considered that students had the necessary prior knowledge to study the subject, and both indicated that activities to evaluate student development were not explicitly carried out in the videos. These results contrast with research such as Cai et al. (2022), where experts and novices succeed in interpreting students' actions and there is some feedback. Still, the expert has a guiding teacher role while the novice tries to give the direct answers. This coincides with the results of Moodliar et al. (2021) where the teacher tries to generate questions regarding emerging knowledge.

This investigation attempts to characterize the didactic analyses carried out by an expert teacher and a novice high school teacher, based on the use of didactic suitability criteria (DSC) when analyzing episodes of a high school mathematics class in Costa Rica.

The results indicate that, when the novice teacher mentioned some aspect or situation that took place in the video segments analyzed, he focused on describing the complete situation in detail (in agreement with findings of research carried out by Borko and Livingston (1989)), while the expert teacher not only describes the situation, but goes more deeply into in trying to understand and reflect on the situation when making observations on what happens in the video segments (see Auerbach et al. (2018), Borko et al. (2011), and Schempp et al. (1998)).

The expert teacher was able to recognize errors in mathematical concepts and proposed reformulating the order of the activities used in the class. In addition, he mentioned that it is vital that the teacher has empathy for students and connects with them well to carry out a class that is as dynamic and active as possible; this agrees with the findings of Borko and Livingston (1989), Rojas et al. (2012), and Berliner (2001, 2004). In contrast, the novice teacher focused on the students and how they performed in the activities of each class, without noting mathematical errors, which agrees with the findings of Borko and Livingston (1989), Chi (2011), Li and Kaiser (2011), and Schemp et al. (1998).

Both teachers recognized that the system used for class implementation is clearly consistent with what is proposed in the official curriculum. This coincides with the findings of Berliner (1988), who states that novice teachers tend to accept existing procedures when imparting classes that are defined in the curriculum without questioning them. However, the expert teacher described and evaluated the guidelines in the curriculum and discussed his view of the advantages and disadvantages of the current mathematics curriculum. This is related to the pedagogical knowledge and knowledge of the plans of the programs of study that the expert possesses, which coincides with the results of studies by Berliner (2001), Borko and Livingston (1989), and Rojas et al. (2012).

CONCLUSIONS

In summary, the novice teacher made descriptive comments while the expert teacher generally reflected on and analyzed each of the situations that occur in the classroom. This may support the view that it is

necessary to create or strengthen spaces in early or continuing education in which reflection on teaching skills during mathematics lessons is stimulated, to improve the process of instruction in the classroom and increase the capacity to recognize appropriate practices in teaching. For example, the DSCs themselves can be used as a methodological tool to organize teachers' reflections, as has been done in different training processes in various countries such as Spain (Esqué & Breda, 2021) and Costa Rica (Morales-López & Araya-Román, 2020).

The results of this investigation have shown that the use of the didactic suitability criteria of the OntoSemiotic Approach could eventually contribute to study of, and reflection on, the characteristics that different types of teachers (experts and novices) have. This is important, since, as previously indicated, it is highly desirable to have information about the characteristics of true expertise, to promote them in the initial stages of the education and training of teachers. Alongside this, other factors should be included in future studies as, for example, Bastian et al. (2022) show that cultural elements can be a determining factor in the development of expertise. Finally, it is clarified that this study had an approach which does not intend to generalize the results.

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