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Perspectives on the problem-posing activity by prospective teachers: A cross-national study

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Abstract

There is an ongoing research interest to disclose factors that influence problem-posing performance by involving cross-national backgrounds. This research extends that effort by conducting a comparative analysis on the performance of Indonesian and Hungarian prospective teachers in a problem-posing task. A total of eighty-three prospective teachers from Indonesia and Hungary were asked to pose a problem based on the current calendar. For more in-depth insights, an interview was conducted with a representative participant. The obtained data were analysed quantitatively using Fisher's exact test and qualitatively in nature. Their mathematical background seems to influence the characteristic of their proposed problem and the solution approach they utilized. The typical tasks proposed by Indonesian prospective teachers are exercise and mostly related to arithmetic operations, while those by Hungarian prospective teachers are challenging problems and generally connected to arithmetic sequences. Moreover, in solving their problems, Indonesians tend to show arithmetic reasoning while Hungarians often denote algebraic reasoning. These disparities might be attributed to the types of problems that each group typically encounters during their mathematics lessons.

Keywords: Cross-National Study, Problem-Posing, Problem-Solving, Prospective Teacher

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Problem-posing has been emphasized explicitly as a companion to problem-solving (Kilpatrick, 1987; Pólya, 1945; Pólya, 1962), which occupies its role before, during, or after a problem-solving process (Silver, 1994). The roles encompass generating new problems of a situation and reformulating the given problems (Papadopoulos et al., 2021). Generating new problems refers to the construction of personal interpretations of concrete situations and the formulation of the concrete situations as meaningful mathematical problems (Stoyanova & Ellerton, 1996). Whereas, reformulating the given problems reflects suggestions of devising a plan by Pólya (1945) to restate the original problem and solve a related problem that is more accessible and analogous. Thus, the second role can be meant as a series of transformations of the original problem.

Although the problem-posing aspect of problem-solving has been considered significant (Pólya, 1954; Freudenthal, 1973), attention to the problem-posing has not been proportionate to the importance of the problem-solving (Leung, 2013; Singer et al., 2013). Freudenthal (1973) emphasized that not only problem-solving but also problem-posing constitute mathematical activity: "The problem should grow out of the situation, and the child should learn to recognize the problem in the situation" (pp. 134-135). Nevertheless, in most mathematical classes, students are asked to solve rather than generalize or modify a problem. This means that the textbook and teachers dictate the problems to be solved (Crespo & Sinclair, 2008; Kilpatrick, 1987). The doubt lies in whether teachers are prepared properly to provide a wider learning opportunity, such as giving challenging and meaningful mathematical problems to their pupils. Thus, paying attention to the prospective teachers' performance in posing problems would be worthwhile since problem-posing skill can be a means to provide their prospective students with valuable mathematical experience.

Several preceding studies revealed a lack of problem-posing skills among prospective teachers in different countries, indicated by the proposed problems that are inadequately stated (Crespo, 2003; Ellerton, 2013; Leavy & Hourigan, 2020; Leavy & Hourigan, 2021; Silver & Cai, 1996), insufficient information (Kadir et al., 2020), lack in mathematical structure (Chapman, 2012), logically flawed (Ellerton, 2013; Leavy & Hourigan, 2020; Leavy & Hourigan, 2021), or not mathematically solvable (Crespo, 2003; Ellerton, 2013; Kadir et al., 2020; Silver & Cai, 1996). Therefore, several attempts have been made to introduce and improve prospective teachers' problem-posing skills through teacher preparation programs, such as Leavy and Hourigan (2020) in Ireland and Ellerton (2013) in the USA. In Hungary, the formulation of problems belongs to an essential element of mathematical competence, which is explicitly stated in the Hungarian National core curriculum. "The learner can solve concrete situations using visual and symbolic models and strategies, thereby developing problem-solving and problem-creating skills" (Government Decree, 2020). In Indonesia, the main indispensable mathematical competence remains only problem-solving. "…applying procedural knowledge in a specific field of study according to their talents and interests to solve problems. Processing, reasoning, presenting and creating in the realm of concrete and abstract contain problem-solving related to the topics learned" (Indonesian Ministry of Education and culture, 2018). Problem-posing, although closely related to problem-solving, it has not been explicitly stated in the National curriculum. Regardless of that difference, according to the authors' experiences, the emergence of problem-posing in both countries is sporadic in the classroom.

Given the problem-posing underperformance in several countries, cross-national study becomes a promising effort to provide insights for student performance in the context of different learning traditions. It helps to identify the factors that do and do not promote problem-posing. Notably, the cross-national study expands the possibility of investigating the nature of differences in students' mathematical performance and cultural and educational differences to understand why cross-national differences in mathematics exist (Cai & Lester, 2007). This study aims to contribute, through a cross-national comparative study, to the investigation of the sixth unanswered question proposed by Cai et al. (2015), "How do students in different countries and regions pose mathematical problems?". Particularly, this study aims to identify prospective teachers' problem-posing and problem-solving performance through guiding questions: What is the characteristic of the problem proposed by Indonesian and Hungarian prospective teachers? What is the approach used by Indonesian and Hungarian prospective teachers to solve their proposed problem? To answer these research questions, two groups of prospective teachers consisting of Indonesians and Hungarians were asked to respond to a problem-posing task and solve their proposed problem.

Examining the connection of mathematical problem-solving and problem-posing, two preceding studies by Ellerton (1986) and Silver and Cai (1996) utilized unrelated problem-solving and problem-posing tasks. Cai (1998) carried out the same method to see the relationship between those two phenomena by considering a cross-national perspective. In the current study, related problem-posing and problem-solving tasks were intentionally designed to take a closer look between those two. The given task allows us to examine the level of understanding by looking at the reasoning tendency of prospective teachers in solving their problems, whether they employ algebraic or arithmetic reasoning. Algebraic reasoning refers to the process of generalizing mathematical ideas from a particular set of examples (Kaput & Blanton, 2001) while arithmetic reasoning performs calculations with known quantities.

Some Perspectives on Problem-Posing

The concept of problem-posing has been explored by several researchers. For instance, Duncker (1945) and Silver (1994) defined problem-posing as generating new problems and reformulating the given problem, while Stoyanova and Ellerton (1996) interpreted problem-posing as the process of constructing personal interpretations of concrete situations and formulating concrete situations as meaningful mathematical problems. Examining the existing definitions, Papadopoulos et al. (2021) expanded the problem-posing definition into five categories: only generating new problems, only reformulating existing or given problems, both generating and/or reformulating problems, raising questions, and modelling. Problem-posing in this study refers to the first category, the construction of problems that demand a solution.

As an essential aspect of mathematics education, several researchers have considered problem-posing as a companion for problem-solving (Bonotto & Santo, 2015; Brown & Walter, 2004; Pólya, 1945; Silver, 1994). It can be supported by problem-solving (Bonotto & Santo, 2015) and might occur prior to, throughout, or after the solution of a problem (Silver, 1994). Problem-posing comes prior to problem-solving in the case of formulating problems based on a given or realistic situation. Problem-posing takes place during a problem-solving process if the proposed problem represents a subgoal of the larger problem. As Pólya (1945) suggested, we should think of a related and more attainable problem if we could not find the source of the original problem. In this case, problem-posing indicates successive reformulation of the original problem. Lastly, problem-posing occurs after problem-solving when the conditions of the original problem are explored to generate a related problem. This process is closely connected to the "Looking back" step by Pólya (1945) with guiding advice to investigate whether the result or the method can be used for some other problems. Moreover, it is associated with the idea of Brown and Walter (2004), "what-if" and "what-if-not" strategy in which the initial conditions or the initial goals are modified to obtain a new problem.

The "What-if-not" strategy leads to the construction of reasonable arguments and formal evidence to refute or validate the conjectures. The conjectures that reject the test gain credibility, stimulating evidence that yields mathematical validity once the evidence is achieved (da Ponte & Henriques, 2013). This activity emphasizes that investigation provides a stimulus for students to justify and prove their conjectures and explain mathematical arguments to their peers and teachers (Ernest, 1991), which then, generates considerable benefits for the posers (English, 2020; Leikin & Elgrably, 2020). Thus, problem-posing should be viewed as a goal and as a means of instruction (Kilpatrick, 1987). Moreover, in everyday life, instead of solving the problem right away, we have to discover the problem by ourselves first, accept it, and interpret it as our own. Through this notion, both teachers and students can be the source of good problems.

To provide a holistic view of problem-posing, Stoyanova and Ellerton (1996) introduced three problem-posing situations: free, semi-structured, and structured. Problem-posing is considered free when the students are asked to formulate problems based on contrived or realistic situations. Semi-structured problem-posing refers to the condition when the students are given an open situation. By applying knowledge, skills, concepts, and relationships from their previous mathematical experiences, they are invited to explore the structure and complete the situation. While in structured problem-posing activities, the students are asked to pose problems based on the previous specific problem. Those three situations correspond to what Pehkonen (1997) referred to as open problems, in which the starting point and/or the goal are open. Free problem-posing implies that both the starting point and the goal are open. Semi-structured problem-posing denotes an open starting point and a closed goal. Structured problem-posing means the starting point is closed while the goal is open.

The mathematics curriculum documents from several countries show that problem-posing in mathematics teaching has been recognized and holds current interest. In the United States, it is stated in the Principles and Standards for School Mathematics (NCTM, 2000) that students should be given the opportunity to formulate interesting problems based on a wide variety of situations, make and investigate mathematical conjectures, and learn how to generalize and extend problems. Also, in Australia, the inclusion of problem-posing, divergent thinking, reflection, and persistence receives strong support emphasized in the National Statement on Mathematics for Australian Schools (Stoyanova & Ellerton, 1996). Students are expected to look for alternative strategies, pose, and try to answer their mathematical questions. Silver (2013) compares China and England as countries with different mathematics teaching traditions but similar approaches to problem-solving in the classroom. Specifically, problem-posing is a goal and a means of improving students' mathematical reasoning. Interest in problem-posing on a global scale is also evidenced by the inclusion of problem-posing in several studies, i.e., Brown and Walter (2004), Ellerton (1986; 2013), Kilpatrick (1987), and Stoyanova (1997).

Prospective Teachers' Problem-Posing Performance

In a meta-analysis by Osana & Pelczer (2015), prospective teachers' problem-posing has been stated as a primary skill in mathematics professional development. To know the existing conditions, a number of previous studies attempted to reveal prospective teachers' problem-posing performance.

Chapman (2012) found that posing word problems was immensely challenging for prospective teachers who had no instruction or exposure to formal theory on problem-solving or problem-posing prior to the study. Although many of them were able to create interesting problem situations, in general, their understanding of posing word problems often overrides conscious considerations of the mathematical structure or context of the problem or its relationship to the problem situation. Particularly, some of the problems they posed were ill-formulated, not mathematical, lacked sufficient information, or closed question with yes/no/don't know possible responses when asked to pose open-ended word problems.

Another insight comes from Kadir et al. (2020), who revealed some challenges faced by Indonesian prospective teachers participating in their study when posing problems based on the given situation. Difficulties in posing problems are indicated by a problem with insufficient information, not suitable to the given situation, or the context inappropriate to their students. Meanwhile, based on the research by Masriyah et al. (2018), all problems posed by Indonesian prospective teachers who have obtained exposure to problem-posing were solvable and varied in terms of difficulty level, but only a few were open problems. In Hungary, starting out from a textbook problem, a group of Hungarian prospective teachers were asked to pose a problem. Of 70 proposed problems, most were analogous with the initial problem, simplified problem, exercise type, and unsolvable task (Kovács, 2020).

In Ireland, the initially proposed problems by prospective teachers were generally arithmetic, only required one step to solve, and only had one correct solution. But the quality of the proposed problems improved after the organized instruction (Leavy & Hourigan, 2020). In the early part of her class, Ellerton (2013) discovered that the routine problems posed by USA prospective teachers were rarely polished and often imperfect in wording and logic.

Tichá and Hošpesová (2013) examined the problems posed by Czech prospective teachers who attended a seminar on Mathematics Didactics. In the seminar, problem-posing is often used to improve students' initial ideas about mathematics and mathematics education. The authors found that students proposed stereotyped context and the proposed problems were monotonous in nature. The prospective teachers could not realize that small changes in wording can play an important role.

Realizing the lack of problem-posing performance among prospective teachers in various countries, cross-national studies hold special contributions in the aspect of internationalization perspective regarding some factors behind the performance. Cross-national studies allow mathematics educators to recognize effective teaching and learning problem-posing in a broader cultural context. Through examination of what is happening in the problem-posing learning in other societies, researchers and educators will grasp how it is implemented by teachers, learned, and done by students in different cultures, and how the effects are generated. It is intended to provide an understanding of how problem-solving and problem-posing are taught in each country as well as what lessons can be learned from cross-national studies to improve students' learning (Cai & Lester, 2007).

Specifically, in efforts to enhance problem-posing skills, Osana and Pelczer (2015) suggested considering various factors, such as focus on content and curricular knowledge, pedagogical criteria, and metacognition. Crespo (2003) made two instructional moves to improve prospective teachers' problem-posing skills, introduction and in-class explorations of non-traditional mathematical problems and collaborative problem-posing, which the prospective teachers consider as particularly profound experiences. In her study, the prospective teachers were asked to exchange weekly mathematics letters with their pupils. The letter written by the prospective teachers contains a mathematical problem, while the pupils wrote the letter based on the format their class had generated including the feedback on the problem they received and their advice on how to teach mathematics. The interactive work with students was supported in a variety of ways throughout the course, such as question and answer sessions, reading and writing letters in collaborative groups, and weekly journal reflections. The results, instead of traditional single-step and computational problems, the problems proposed by the prospective teachers were shifted into problems with multiple approaches and solutions, open-ended, exploratory, and cognitively more complex. Another attempt to incorporate problem-posing in the teacher training program was carried out by Kovács (2017), who asked the prospective teachers to explore a textbook problem more deeply by using "what-if-not" strategy (Brown & Walter, 2004). In the early stage of problem-posing practice, the prospective teachers got frustrated due to unfamiliar activity, but they were greatly open-minded and cooperative toward novelties.

Evaluation on Problem-Posing Activity

Several authors classified the problem-posing activity according to the task given (Cai, 1998; Crespo, 2015; Kovács, 2020; Tabach & Friedlander, 2013). Two of the references are as follows (see Table 1).

Table 1. References for problem-posing activity

Crespo (2015)	
• Disempowered/ disempowering problem: - Closed - Simplified - Blind	Closed problems are quick-translation story problems or computational exercises. In simplified problems, adaptations narrow the mathematical scope of the original version of the problem. In blind problems, the mathematical complexity is underestimated.
• Empowered/ empowering problem: - Open - Mathematically challenging - Mathematically interesting - Socially relevant	Open problems require solvers to explain their work and communicate their ideas. Mathematically challenging problems introduce new ideas, push the solvers' thinking, or challenge their understanding. Mathematically interesting problems explore and mathematize situations to generate interesting problems. Socially relevant problems explore and mathematize real-world situations to engage in understanding and addressing social issues with mathematics.
Kovács (2020)	
• Blind • Inadequate • Exercise • Problem: - Simplified - Analogous - Flexible	In this study, the task was to pose a problem based on the original problem. Blind problems are posed without being solved. Inadequate problems refer to reformulated problems with one correct answer or with a clear goal. Exercise leads directly to an arithmetic solution, or it can be handled with a simple numeric example. In the simplified problem, the mathematical scope is narrowed down. Analogous problems represent problems with the same methods and strategies used to solve the original problem. The mathematical structure in a flexible problem differs from that in the original problem.

Arithmetic-Algebraic Approaches

Arithmetic deals with procedural operations involving specific numbers (Carraher et al., 2006; Pillay et al., 1998). It leads to using the averaging algorithm to work backward through a series of inverse operations (Cai, 1998). In line with those notions, arithmetic should meet one or more of the following characteristics: it concentrates on the four fundamental arithmetic operations (Hiebert, 1999; Mason, 2011), it entails manipulating numbers in order to generate a numerical solution (Filloy & Rojano 1989; Schliemann et al., 2006), and it focuses on computational fluency (Kaput, 2008).

On the contrary, algebra is concerned with generalized numbers, variables, expressions, equations, and functions (Carraher et al., 2006; Friedlander & Hershkowitz, 1997). This approach is characterized by working forward, employing the averaging algorithm to directly describe the problem situation (Cai, 1998). Using the term algebraic thinking, Freudenthal (1977) defined it as an ability to depict relations and solving procedures in a general way. It belongs to a particular form of mathematical sense making related to symbolization (Schoenfeld, 2008). Thus, thinking algebraically means utilizing symbols effectively, purposefully, logically, and meaningfully. Such an activity is considered algebraic reasoning because it includes not only ways of thinking but also ways of doing and talking about mathematics as a fundamental entity.

Furthermore, Kaput (2008) defined two core aspects of algebraic reasoning. The first core aspect includes generalization and expression of generalizations in conventional symbol systems which are increasingly systematic (Core Aspect A). Initially, students use their natural language as their resource but then they increasingly turn to conventional forms of representation. The second core aspect denotes the action on symbols which is syntactically guided within organized symbol systems (Core Aspect B). Both core aspects are embodied in three strands, namely: (1) algebra as the study of abstracted structures and systems from computations and relations, which includes algebra as generalized arithmetic and those arising in quantitative reasoning, (2) algebra as the study about functions, relations, and joint variation, and (3) algebra as a modeling activity both inside and outside mathematics.

Some researchers highlight the transition from arithmetic towards algebra (Filloy & Rojano, 1989; Friedlander & Hershkowitz, 1997; Hohensee, 2017; Powell et al., 2016) and consider algebra as generalized arithmetic (Carraher et al, 2006; Hohensee, 2017; Kaput, 2008). In arithmetic, one focuses on numbers and numerical procedures (Carraher et al, 2006; Pillay et al., 1998) required for competence with algebra (Boulton-Lewis et al., 2000; Powell et al., 2016; Schliemann et al., 2006). According to the result of their study, arithmetic skill influences prealgebra performance, and without strong arithmetic skill, one is likely to have difficulty with algebra (Powell et al., 2016). In short, algebra employs arithmetic and belongs to the extension of arithmetic (Britt & Irwin, 2008).

Cross-National Aspects and Teaching Traditions

Of the education sector in Indonesia, two ministries are responsible for managing education institutions (either public or private), i.e., the ministry of education and culture and the ministry of religious affairs (OECD, 2015; Rosser, 2018). Since 1994, the Indonesian government has launched a nine-year basic education program mandatory for every citizen (Mukminin et al., 2019). The analogous condition has been documented in Hungary. Although schools may belong to several administrators, the state is responsible for legal oversight. The states' public service task is to ensure the right to participate in primary and secondary educations and acquire the first vocational qualification.

The framework of the education system in both countries is set out in the National Education Act. Concerning the teaching implementation, although each class in the primary grades handled by about three teachers prevails in both countries, Indonesian teachers are only responsible for one subject. In contrast, Hungarian teachers are generally responsible for two subjects in higher grades. Indonesia and Hungary share similar challenges in education, such as the existence of students' fear of mathematics (Győri et al., 2020; Tanujaya et al., 2017) and the persistence of conventional teaching traditions (Fauzan, 2002; Győri et al., 2020).

The Indonesian curriculum has undergone several changes since 1947 and most recently in 2013. Analyzing curriculum changes with a focus on mathematics, Mailizar et al. (2014) classified several curriculums implemented before 1975 as pre-modern mathematics curriculum since students were trained to memorize mathematical concepts without understanding them. As awareness grows among Indonesian scholars of the need to improve mathematics teaching in schools, more focus has been placed on developing understanding rather than memorization and calculation skills and emphasizing student-centered learning since the implementation of the 1975 curriculum (Mukminin et al., 2019). These efforts were continuously promoted through the 1984 curriculum by recommending the implementation of active learning in all schools (Bjork, 2005; Mailizar et al., 2014; Wahyudin & Suwirta, 2017). In 1994, more attention started to lead towards problem-solving as a means to develop students' reasoning skills which were not clearly stated in the previous curriculum (Mailizar et al., 2014). Along with curriculum reform, some challenges were still encountered, such as the unsatisfactory implementation of the active learning principles. Teachers tended to dictate formulas and procedures to their students (Fauzan, 2002) and narrate a mathematical problem instead of asking questions (Kuipers, 2011).

In response to several criticisms on the gap between expectations and implementation, the 2013 curriculum as the latest curriculum modified in 2018 has been arranged based on some key principles, such as assigning problem-solving as an increasingly intended skill to be mastered and focusing on scientific approaches.

The official textbook issued by the ministry of education and culture has been arranged with some important features related to scientific approaches, such as let's observe, let's ask, let's dig up information, let's try, and let's share. In the let's observe part, a daily life situation or problem related to the topic is intentionally brought up to bolster the enhancement of problem-solving skills. The content of the book includes modelling, proving, estimating, and reinventing mathematical concepts developed based on international standard frameworks, such as PISA and TIMSS (Ekawati et al., 2018). Although problem-solving is closely related to problem-posing, problem-posing has not been explicitly stated in the curriculum. Nevertheless, there has been increasing interest to incorporate problem-posing both in schools and in teacher training programs (Christidamayani & Kristanto, 2020; Hasanah et al., 2017; Masriyah et al., 2018).

In Hungary, authors who have provided overviews on the traditions and values of Hungarian mathematics teaching agree that the cornerstones of the traditions are a problem-oriented approach, exploratory or guided discovery mathematics teaching, and talent management (Gosztonyi, 2016; Győri et al., 2020). Among the traditions still alive today, two of them are highlighted. The first is the oeuvre of György Pólya, especially his works related to problem-solving, heuristic strategies, and mathematics teaching (Pólya, 1962). The other is the Complex Mathematics Teaching Experiment led by Tamás Varga (Halmos & Varga, 1978; Varga, 1988). According to Hungarian teaching traditions, the prominent roles of problem-solving and problem-posing are essential elements of mathematical competence, explicitly stated in the Hungarian National Core Curriculum. The attitude is well reflected in the words of Tamás Varga,

> It is an important recognition that does not affect todays' school practice enough: there is also a need for open situations in which students recognize and formulate the mathematical task, often of several kinds. It also helps to apply the acquired mathematical knowledge. Life does not present us with mathematical tasks formulated in textbook language but with situations. It adds the raw material to the tasks. (Varga, 1987, pp. 28-31)

Since problem-solving and problem-posing are closely related actions, problem-posing or problem sensitivity, often referred to in Hungarian literature, becomes an essential aspect of the Hungarian mathematics teaching tradition. It has been included in the Hungarian National Core curriculum since 1978. Even in the mathematics textbook by Gallai and Péter (1949), there have been problem variations in the geometry chapter.

Although problem-posing is familiar to Hungarian mathematics teachers, the emergence of problem-posing is still sporadic in Hungarian school practice. It is commonly implemented in special mathematics classes with extra time for mathematics lessons and in mathematics camps such as Pósa's camps, which plays a prominent role in talent management (Bóra, 2020; Juhász, 2019). Fazekas and Hraskó (2006) collected several problems that also contained problem variations generated by seventh-grade students in a special mathematics class.

METHODS

Participants

The participants of this study were 83 prospective teachers consisting of 47 Indonesians and 36 Hungarians. Particularly, the Indonesian sample comprised 25 prospective teachers (5 males, 20 females) from a public university and 22 female prospective teachers from a private university in Surabaya, East Java. Surabaya belongs to the second largest city in Indonesia with a developed economy. Generally, Indonesian public universities are highly selective universities concerning the entrance rate, which offer excellent educational programs (Rosser, 2018). To enter public universities, prospective students need to pass a competitive admission test. The Hungarian sample was from two public universities located in Nyíregyháza and Eger, 24 (5 males, 19 females) and 12 (6 males, 6 females) prospective teachers respectively. Not quite different from Surabaya, Nyíregyhaza and Eger are the seventh and the nineteenth largest city in Hungary with a developed economy. The inclusion of sample only from public universities follows the condition in Hungary in which only public universities provide mathematics teacher training programs at the time of the research. Due to resource and time constraints, only relatively small and localized samples were selected in this cross-national study. According to Bradburn and Gilford (1990), a relatively small sample that is localized in a small number of sites is acceptable for small in-depth cross-national studies.

The universities mentioned above are typical of other public and private universities in their regions. Involving Indonesian prospective teachers from public and private universities as well as all Hungarian prospective teachers only from public universities was intended to ensure an approximately equivalent national sample, since no sample was representative for each country and the prospective teachers were not randomly selected. The comparison of prospective teachers' performance by type of university was not the focus of this study but a cross-national comparison. The data analysis was carried out based on the total Indonesian and Hungarian samples.

It should be noted that Indonesian prospective teachers were in the second year of a 4-year teacher training program for grades 7-12 while Hungarian prospective teachers were in the third year of a 5-year program for grades 5-8. Indonesian prospective teachers were around 19-20 years old and Hungarian prospective teachers were between the ages of 21 and 22. In contrast to Indonesian prospective teachers from private university who were not taught the concept of problem-posing prior to this study, the Hungarian sample and Indonesian prospective teachers from public university had been introduced to problem-posing activity, including using situations as the source of the new problem, varying problem by changing the attributes of the initial problem, proposing new problem through generalization, etc. In both countries, the introduction to problem-posing activities was not through a specific problem-posing course but integrated into a problem-solving course.

The Task

In this study, all prospective teachers were assigned the calendar task (see Figure 1). Indonesian prospective teachers from the public university were given the task in English since they were attending an international program, but they were allowed to pose a task in either Indonesian or English. Meanwhile, Indonesian prospective teachers from the private university and Hungarian prospective teachers were given the task in Indonesian and Hungarian respectively. After completing the task, one prospective teacher with a typical case was selected as a representative to be interviewed.

The task fitted the school curricula, including arithmetic-algebraic reasoning, sequences, and recreational mathematics. The structure of the required problem was not determined in advance, so prospective teachers were placed in an open situation in which they could generate problems based on their interpretation (Stoyanova & Ellerton, 1996).

By observing the geometric layout of the calendar, prospective teachers were encouraged to recognize some arithmetic sequences, which then could be expressed in algebraic form. The task was arranged to follow up the suggestion by Cai (1998) that the mathematics curriculum should include explorations of algebraic ideas and processes so that students could use algebraic thinking to solve a variety of real-world problems. Furthermore, the task referred to some features included in English (2020), namely, it was open-ended, provided students with multiple entry points indicated by the found patterns, led to multiple opportunities to display intellectual competence, and deals with discipline-based and cognitively important content. It gives the prospective teachers an opportunity to create a task with high cognitive demand involving problem interpretation satisfying one of the important task attributes proposed by Crespo and Harper (2020).

(1) Discover at least three patterns in this months' calendar!
(2) Explain those patterns!
(3) Create a problem based on one of the above patterns and solve it!

March 2022						
Mon	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Figure 1. The calendar task

Data Coding and Inter-Rater Agreement

Prospective teachers' responses to the task were analyzed from three perspectives: (1) the category of the proposed problem; (2) the correctness of the solution to the problem they posed; and (3) the arithmetic-algebraic approach they used to solve their proposed problem. The categorization of the proposed problem was intended to examine the solvability and the mathematical complexity. It was carried out based on the coding chart flow as shown in Figure 2.

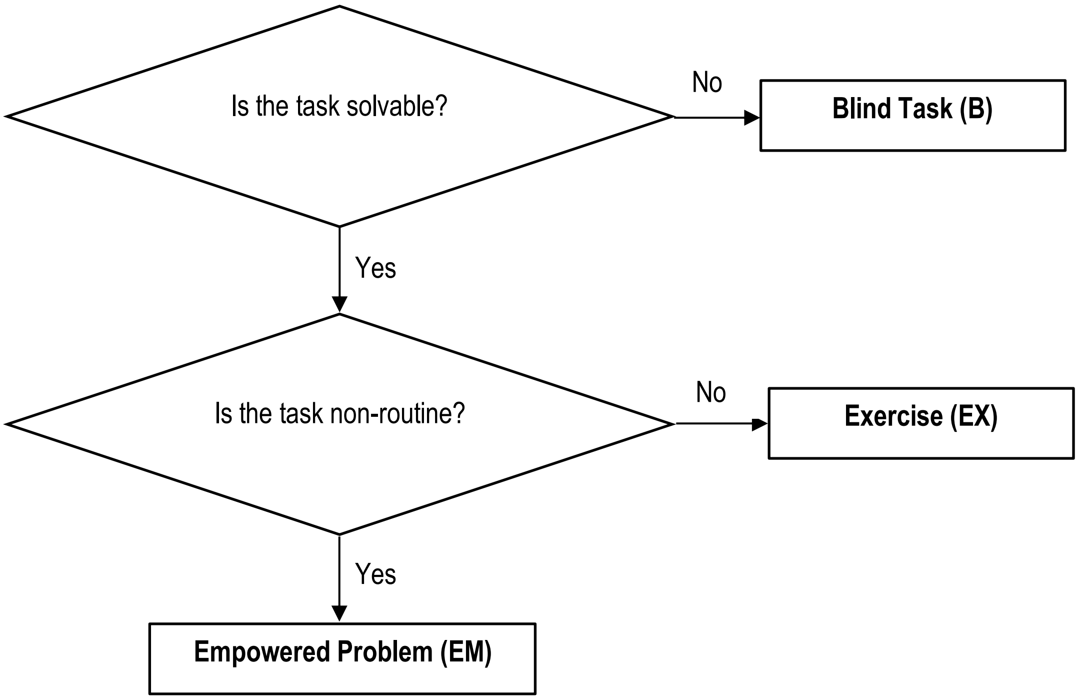


Figure 2. Coding chart flow

There are three categories, namely blind, exercise, and empowered problem that can be closed or open. The task is blind when it cannot be solved, or there is not enough data to solve. The task is considered as an exercise if it is a routine task or a simple word problem that can be solved using basic calculations. The task refers to an empowered problem when it is mathematically challenging, stimulates creativity, or gives an opportunity for the solvers to explore as many as possible ways and communicate their ideas.

To ensure a high rate of reliability, two raters (the authors of this paper) coded the entire corpus independently. The proportion of cases agreed upon by the raters was $p_0 = .90$, representing 75 out of 83 cases, and the Cohens' kappa value was $\kappa = .83$ (Cohen, 1960). Both p_0 and κ denoted a high inter-rater agreement. Later, the raters reached a consensus in open points used in the rest of this paper.

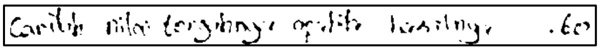
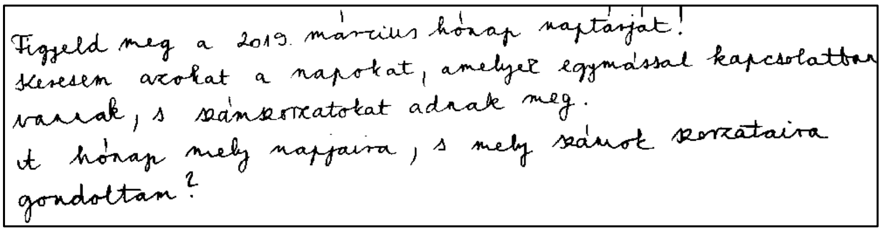
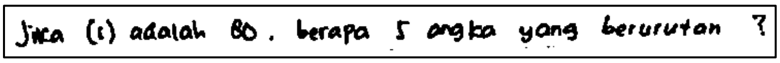
RESULTS AND DISCUSSION

The results are presented in four sections. The first section reveals the result of categorization along with the mathematical background of the proposed problem. The second section shows the correctness of the solution to the problem they posed. The third section identifies the approach used by prospective teachers in solving their proposed problems. The final section presents the findings of an interview with one representative participant.

The Result of Categorization

Table 2 represents the example of the problem in each category. HU and ID refer to the task proposed by Hungarian and Indonesian prospective teachers, respectively. The overall result of categorization is presented in Table 3.

Table 2. Example of the proposed problem

B	ID-3  Carilah nilai tengahnya apabila hasilnya 60 Translation: Find the middle value if the result is 60! *Commentary: The formulation of the task is not clear. There is no information about what middle number to look for.* HU-16  Figyeld meg a 2019. március hónap naptárját! Keresem azokat a napokat, amelyek egymással kapcsolatban vannak, s számsorozatokat adnak meg. A hónap mely napjaira, s mely számok sorozataira gondoltam? Translation: I'm looking for days that are related to each other and give a sequence of numbers. Which days of the month and which sequence of numbers did I think of? Which sequences of numbers? *Commentary: It is confusing unless there is an additional condition such as the sum of the numbers or asking for the underlying pattern.*
EX	ID-26  Jika (Σ) adalah 80. berapa 5 angka yang berurutan ? Translation: If the sum is 80, then what are the 5 consecutive numbers? *Commentary: The task can be solved by using a simple calculation. For example, divide 80 by 5 to get the middle number, then put the previous 2 consecutive numbers before and the next 2 consecutive numbers after the middle number.* HU-6

Mi lenne ha 10 napos lenne a hét? Akkor k lehetne, hogy a vízszintes helyett függőleges d=10 lenne a differencia.

What if one week was 10 days? What would be the difference in vertical sequences?

Commentary: The task can be solved by using a simple calculation. To know the new difference, the solver only needs to find out the difference between two numbers arranged vertically.

EM

Open

ID-22

The original version was typed by the poser:

> If the last number + first number, the second last + second number added, it will have same sum, in April it will equal to 31. (ex: 30+1, 29+2, 28+3)
> About the pattern number 3, can u tell me that it will possible in every month or only in a several months? Explain and prove it.

Commentary: The task requires solvers to investigate several possibilities and communicate their thought. Diversity of thoughts accompanied by various reasons can be obtained through this task.

HU-5

> FELADAT: VÁLASSZ EGY SZÁMOT, AMINEK MINDEN IRÁNYBÓL VAN SZOMSZÉDJA! ADD ÖSSZE EZEKET ÉS ÉN MEGMONDOM MELYIK SZÁMRA GONDOLTÁL.

Translation: Choose a number that has a neighbor from each direction. Sum those numbers, and I'll tell you which number you choose. Then, guess my strategy!

Commentary: The task is challenging, which gives solvers an opportunity to spur their thinking and propose a number of possibilities. More than one group of numbers are possible to find. Algebraic reasoning seems more appropriate to guess the numbers.

Closed

ID-15

> Tentukan 9 angka apa saja yang terdapat pada pola persegi 3×3 sedemikian hingga setiap 2 angka yang berseberangan secara horizontal, vertikal, dan diagonal berjumlah 26 (S=26)!

Translation: Determine 9 numbers in the 3x3 square pattern so that the sum of each 2 opposite numbers horizontally, vertically, and diagonally is 26 (S=26)!

Commentary: The task encourages the investigation of multiple numbers, but it only has a unique solution with 13 as the middle number.

HU-15

> Lehetséges-e bejárni 3, illetve 5 többszöröseit lólépésben úgy, hogy az utolsó mező a vasárnapi piros betűs 18, illetve 25 legyen, ha bármelyik mezőről indulhatunk?

The multiples of 3 and 5 are positioned in such a way that the knight piece of the chess game can move into them, the last field is Sunday 18 or 25 if you start from either field. Is it possible to move in such a way that the end of the path is on a Sunday, and it passes all the multiple numbers?

Commentary: The task requires exploration but only provides the answer, either possible or impossible.

Based on the result of categorization, Indonesians' typical task was exercise (63,8%) while Hungarians' typical tasks were empowered (47,2%), as presented in Table 3. Comparing the problem-type tasks (i.e., empowered problem) and non-problem-type tasks (i.e., blind and exercise), we get a contingency table (see Table 4) which shows that problem-type became Hungarian typical task.

Table 3. The result of categorization

Group	ID (n=47)	HU (n=36)
Blind	6	3
Exercise	30	16
Empowered	11	17

In contrast, non-problem-type appeared to be Indonesian typical task. Fisher's exact test with $p = .035 < .05$ indicates a significant correlation between nationality and the problem type (Sprent, 2011).

Table 4. The performance of Indonesian (ID) and Hungarian (HU) prospective teachers in a 2x2-type contingency table

	ID	HU	Total
Non-problem type	36	19	55
Problem type	11	17	28
Total	47	36	83

Another characteristic of the two groups could be identified from the mathematical background of the posed problems (see Table 5). Indonesian prospective teachers formulated tasks that are predominantly based on the concept of arithmetic operation (46,8%) and arithmetic sequence (23,4%) while Hungarian prospective teachers mostly proposed tasks related to arithmetic sequence (36,1%) and modular arithmetic (16,7%).

Table 5. The mathematical background of the proposed tasks

Mathematical backgrounds	ID (n=47)	HU (n=36)
Modular arithmetic	4	6
Arithmetic sequence	11	13
Arithmetic operation	22	4
Patterning	1	4
Algebraic expression	1	3
Geometry coordinate	1	1
System of linear equations	-	1
Graph theory	-	1
Least common multiple	1	-
Uncategorized (from blind task)	6	3

Table 6 depicts the examples of mostly submitted tasks by each group.

Table 6. Tasks with the most typical-mathematical background submitted by each group

Task by Indonesians (Arithmetic operation and arithmetic sequence)

ID-8

Andi ingin menanam pohon dipekarangan rumahnya, dimana pekarangan tersebut berbentuk seperti kalender bulan agustus di atas, dan memberikan penomoran sesuai kalender yang sebelumnya sudah dibagi sama besar. apabila andi ingin meletakkan pohonnya disalah satu angka tengah pada pola horizontal 5 angka, dimana jumlah salah satu polanya adalah 75, maka tentukan angka tengah pola tersebut yang merupakan tempat andi menanam pohon, dan tuliskan angkanya secara berurutan

Translation: Andi wants to plant a tree in his yard. The yard is shaped like the August calendar above, and he gives numbers like those in the calendar that is previously split equally. If Andi wants to place the tree in the middle number of the 5-digit horizontal pattern and the sum of the numbers is 75, then determine the middle number where Andi plants the tree and write the numbers in sequence!

ID-25

Find the 3rd term in the following sequence
5, 12, ..., 26

Task by Hungarians (Arithmetic sequence and modular arithmetic)

HU-3

Határozza meg a következő számsor differenciáját, illetve még egy összefüggést 6,12,18,24,30.

Translation: Determine the difference of this arithmetic sequence (6, 12, 18, 24, 30) and find out another relation!

HU-8

Ha 2018. októberben elseje hétfőre esik, anélkül, hogy egyenként kiszámolnád, meg tudod-e mondani, hogy 26.-dika milyen napra esik?

If the first day of October 2018 is on Monday, can you, without looking at the calendar, tell me what the 26th day of the month is?

The Correctness of the Solution

The solution of the proposed problem that prospective teachers provided was judged correct or incorrect. If prospective teachers skipped solving their problems or submitted blind tasks, it was considered as others type. Table 7 shows the percentages of both Indonesian and Hungarian prospective teachers' correctness in solving their proposed problem. Both Indonesians and Hungarians mostly provide the right solution, but Indonesians hold a larger percentage. The percentages of correct solutions provided by Indonesian and Hungarian prospective teachers are 80,9% and 77,8% respectively. However, the difference is not significant indicated by Fischer's exact test with $p = .79$.

Table 7. The correctness of the solution

Group	ID (n=47)	HU (n=36)
Correct	38	28
Incorrect	3	1
Others (Posing without solving & blind task)	6	7

Arithmetic-Algebraic Approach to Problem-Solving

Each solution was analyzed for its approach. Two categories were used, namely arithmetic and algebraic. In this case, only prospective teachers who proposed exercise or empowered problems along with the solution were analyzed. At the same time, those who posed a blind task or posed a task without a solution were not considered. Table 8 shows the number of Indonesian and Hungarian prospective teachers using each approach on their solution. In both groups, the arithmetic approach was most frequently used than the algebraic approach. The result reveals a higher percentage of Hungarian prospective teachers (37%) in using the algebraic approach than Indonesian prospective teachers (4,9%). Considering the interrelationships between groups of prospective teachers and the use of approaches, Fisher's exact test with $p = .001 < .05$ discloses a close relationship between Hungarian prospective teachers and the use of algebraic approach.

Table 8. Number of Indonesians and Hungarians using each approach

Group	Number of prospective teachers using arithmetic approach	Number of prospective teachers using algebraic approach	Total
Indonesia (n=41)			
Posing exercise	28	2	**30**
Posing empowered problem	11	0	**11**
Total	**39**	**2**	**41**
Hungary (n=27)			
Posing exercise	9	5	**14**
Posing empowered problem	8	5	**13**
Total	**17**	**10**	**27**

Table 9 portrays the examples of each approach used by Indonesian and Hungarian prospective teachers.

Table 9. Examples of each approach used by Indonesians and Hungarians

Arithmetic approach

ID-15

Tentukan 9 angka apa saja yang terdapat pada pola persegi 3×3 sedemikian hingga setiap 2 angka yang berseberangan secara horizontal, vertikal, dan diagonal berjumlah 26 (S=26)!

Penyelesaiannya : Karena S adalah jumlah dari 2 angka berseberangan pada garis horizontal, vertical dan diagonal dalam persegi 3 × 3 . Maka yang harus kita cari dulu yaitu bilangan yang di tengah. Bilangan yang di tengah = S : 2 = 26 : 2 = 13, berarti bilangan yang di tengah persegi 3×3 adalah 13. Selanjutnya kita harus mencari 1 angka sebelum dan sesudah 13 pada garis horizontal, vertical dan diagonal di dalam kalender untuk mencari bilangan apa saja yang terdapat pada pola persegi 3×3. Berarti 9 angka yang terdapat pada pola persegi 3×3 di dalam kalender, yaitu 5, 6, 7, 12, 13, 14, 19, 20, dan 21.

5	6	7
12	13	14
19	20	21

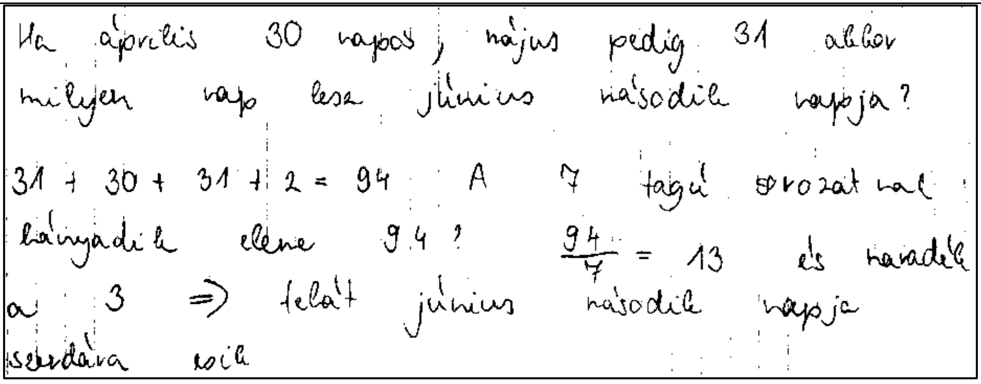
Proposed task:

Determine 9 numbers in the 3x3 square pattern so that the sum of each 2 opposite numbers horizontally, vertically, and diagonally is 26 (S=26)!

Solution:

Since S is the sum of 2 opposite numbers horizontally, vertically, and diagonally in a 3x3 square, the first step is to find the middle number. The middle number = S : 2 = 26 : 2 = 13. It means the middle number in the 3x3 square is 13. The second step is to find 1 number before and after 13 on the horizontal, vertical, and diagonal directions in the calendar to find out all the numbers in the 3x3 square pattern. It comes to the result that 9 numbers in the 3x3 square pattern on the calendar are 5, 6, 7, 12, 13, 14, 19, 20, and 21.

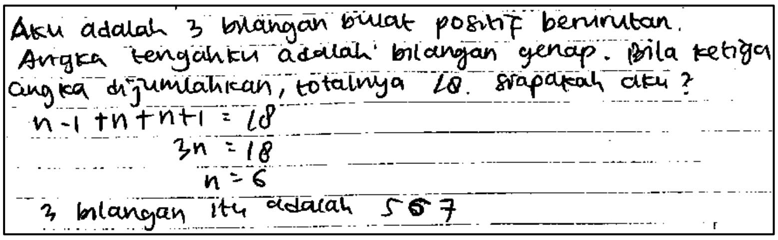
HU-4


Ha április 30 napos, május pedig 31 akkor milyen nap lesz június második napja?
31 + 30 + 31 + 2 = 94 A 7 tagú sorozat hal hányadik eleme 94? $\frac{94}{7}$ = 13 és maradék a 3 => felét június második napja szerdára esik

Proposed task:
If there are 30 days in April and 31 in May, what day is the 2nd of June? [The March calendar is available]
Solution:
31+30+31+2=94. 94 divided by 7 gives 3 remainders. So, the day in question is Wednesday.

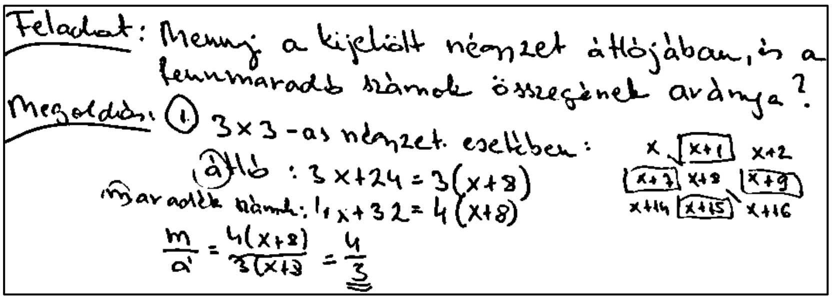
Algebraic approach

ID-2


Aku adalah 3 bilangan bulat positif berurutan. Angka tengahku adalah bilangan genap. Bila ketiga angka dijumlahkan, totalnya 18. Siapakah aku?
n-1 + n + n+1 = 18
3n = 18
n = 6
3 bilangan itu adalah 5 6 7

Proposed task:
I am a 3-consecutive positive integer. My middle number is an even number. If the three numbers are summed up, the sum is 18. Who am I?
Solution:
$n - 1 + n + n + 1 = 18$
$3n = 18$
$n = 6$
The 3 numbers are 5 6 7.

HU-6


Feladat: Mennyi a kijelölt négyzet átlójában, és a fennmaradó számok összegének aránya?
Megoldás: (1) 3×3-as négyzet esetében:
átló: 3x+24 = 3(x+8)
maradék szám: 4x+32 = 4(x+8)
$\frac{m}{á} = \frac{4(x+8)}{3(x+8)} = \frac{4}{3}$
x, x+1, x+2, x+7, x+8, x+9, x+14, x+15, x+16

Proposed task:
Referring to the calendar, think carefully about the 9 numbers that make up a 3x3 square pattern. What is the ratio of the sum of the numbers in a selected diagonal to the sum of remaining non-diagonal numbers?
Solution:
The numbers on the selected diagonal: $3x + 24 = 3(x + 8)$
The remaining non-diagonal numbers: $4x + 32 = 4(x + 8)$
Ratio = 4 : 3

Interview Result

To gain more in-depth insights into how the prospective teacher poses and solves the problem, an interview was conducted by considering the typical case. Given that arithmetic was the most commonly used approach in both groups, the prospective teacher ID-20 was selected as a representative to be interviewed because she utilized the arithmetic approach while utilizing the algebraic approach is also possible and makes it easier for her to solve her problem. Her proposed problem was: The sum of five numbers arranged in an "L" pattern (4 numbers in the vertical direction and 1 number on the bottom right side) is 84. What are the numbers?

Interviewer	:	How did you come up with this problem?
ID-20	:	Firstly, I determined the pattern first, which consists of 5 numbers. I chose L. I did trial and error. I added up the five numbers, then I divided them by 5. I got a decimal number, let's say 17,8. Then, I rounded it up to 18. I hold this result. Then, I saw the middle number of the numbers in the L pattern that I chose earlier, which is 19. It turns out, to get to that middle number, I have to add the result I have by one.
Interviewer	:	Why did you divide it by 5?
ID-20	:	Because there are 5 numbers, and I want to find the middle number.
Interviewer	:	After getting the result, which is a decimal number. Why did you round it up?
ID-20	:	To make it easy. I don't want any decimal numbers. So, after rounding it up, I add the result with 1. The last result will be the middle number. Then, we can find the other numbers. Two numbers vertically and 2 numbers horizontally.
Interviewer	:	But, how did you know and be sure, does your rule apply to all numbers with the L pattern?
ID-20	:	I tried it on numbers in another L pattern and it works.
Interviewer	:	How many times did you check it?
ID-20	:	I tried it three times, it still works. So, I use the problem.

The problem-posing activity by prospective teacher ID-20 started from her curiosity to a certain pattern. She tried to find the connection between the pattern, the numbers, and the sum. If she determined an "L" pattern consisting of 5 numbers but only told the sum, was it possible for someone else to find the numbers? This question led her to find the answer by doing trial and error. Her main intention was to find the middle number which will lead her to find the other numbers (see Figure 3). Because there were five numbers, she divided the sum by five and considered how to connect the result to the middle number. The result was then rounded up and added by one for no apparent reason other than to avoid seeing the decimal number. Moreover, she did not check whether her own rule applies to all cases or not. She only checked three of the eight possible options.

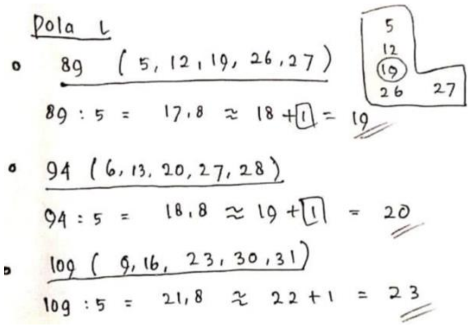


Figure 3. ID-20 strategy

The cross-national study offers considerable benefits, especially for the countries involved. Through cross-country study, it is possible to gain information about the similarities and differences of mathematics performance along with teaching and learning traditions in the respective countries. Furthermore, a cross-country study can be used to evaluate current traditions by identifying the positive and negative effects of each tradition. Then, to improve mathematics learning, the good traditions of the countries involved can be adopted.

This cross-national study reveals the performance of Indonesian and Hungarian prospective teachers in posing a problem and solving their own problem. In the problem-posing task, both groups were able to formulate problems with diverse mathematical backgrounds built upon the given situation. Among the apparent mathematical backgrounds, the Indonesian group preferred the arithmetic operation, while the arithmetic sequence appeared to be the most popular topic in the Hungarian group. An examination of the category of the problems posed yields the result that the Hungarians' problem-posing activity is characterized with the problem-type task, while the Indonesians with the non-problem-type task. Particularly, the typical task by Indonesians was exercise, while the typical task by Hungarians was empowered problem.

One reasonable interpretation of this result could be the type of problems regularly faced by each group during their mathematics lessons, rather than the novelty of the problem-posing activity, given that problem-posing activity is sporadic in both countries. In Indonesia, problem-posing activity has not been implemented widely, although it is starting to pique the interest of educators (Christidamayani & Kristanto, 2020; Hasanah et al., 2017; Masriyah et al., 2018). Likewise, it has not been stated explicitly in the curriculum. In Hungary, although the problem-posing activity has been stated explicitly in the curriculum and well reflected in the words of Varga (1987), it is commonly implemented only in special mathematics classes which have an extra time scale of mathematics lessons and in mathematics camps which play an important role in talent management.

In the problem-solving part, most Indonesian and Hungarian prospective teachers provided the correct solution to the problem they posed. The percentages of correct solutions by both groups were almost the same. Of arithmetic and algebraic approaches, arithmetic was the most frequent approach in both groups. This exploration follows up on the statement by Cai (2000) that a more important point than focusing on whether one can identify the correct solution or not is information about how one approaches the solution of a problem.

The above-mentioned interview with the prospective teacher demonstrates the use of an arithmetic approach to solving her problem. Instead of treating the middle number as an unknown, she did trial and error and concentrated on fundamental arithmetic operations (Hiebert, 1999; Mason, 2011). Despite the fact that the problem is classified as an empowered problem, a flaw is discovered in the solution in which she took "rounding trick" without any proof, believing it to be true only by her own experience. Given that algebra can be used as an effective tool for analyzing, reasoning, and justifying the general patterns we discovered (Friedlander & Hershkowitz, 1997), a better understanding of the algebraic approach will be more advantageous to the prospective teacher. Furthermore, a better understanding of the application of algebra can be beneficial in solving arithmetic problems that are too difficult to solve using pure arithmetic methods.

CONCLUSION

Problem-posing as a part of Hungarian teaching tradition might give an impact but not robust, indicated by less than half of the Hungarian sample submitted a problem-type task. A long experience in commonly encountered mathematical tasks might be another factor influencing the problem-posing performance. The findings also suggest that algebraic instruction in both Indonesian and Hungarian classrooms needs to be improved. The less frequently used algebraic approach to solving problems might require teachers in both countries to adopt a different teaching approach and provide more encouragement to apply the algebraic approach. Taking lessons from the Chinese teaching tradition in which students are more likely to use an algebraic approach than arithmetic, the reference book presents how problems are solved using arithmetic and algebraic approaches. The teacher consistently encourages students to solve a problem both arithmetically and algebraically. More lessons are needed on the comparison of the two approaches. A discussion of the similarities and differences between the two approaches provides an opportunity for students to experience the advantages of using an algebraic approach to solve a problem.

Kilpatrick (1987) provides a theoretical argument that the quality of problems posed by subjects can serve as an index of how well they can solve problems. In this study, problem-posing and problem-solving performances appear to be related. The Hungarian group has a higher percentage in posing a problem-type task and using an algebraic approach, while the Indonesian group has a higher percentage in posing non-problem-type task and using an arithmetic approach.

The result of this study should be used carefully because of the limitation. The data comes primarily from the performance of narrow sample scope in both countries. Although the result provided such rich data, the coverage content was limited to one integrated task, including problem-posing and problem-solving. Further research involving more prospective teachers will be more worthwhile to strengthen the results.

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Declarations

Author Contribution	:	LDF: Conceptualization, Methodology, Formal analysis, Investigation, Writing - Original Draft, Writing – Review and Editing, Visualization, Project Administration. RE: Validation, Methodology, Investigation, Review, Supervision. ZK: Conceptualization, Methodology, Formal Analysis, Investigation, Writing – Review and Editing, Visualization, Supervision.
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Conflict of Interest	:	The authors declare no conflict of interest.

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