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Exploring students' proportional reasoning in solving guided-unguided area conservation problem: A case of Indonesian students

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Abstract

Proportional reasoning has been greatly influencing the development of students' mathematical abilities. Along with the area conservation ability, it helps elementary students comprehend area measurement. This exploratory study aimed to produce qualitative-descriptive data on elementary students' proportional reasoning in solving the conservation of plane figures. The study used guided-unguided area conservation problems using a proportional reasoning level as the analysis framework. Data were collected from 4 primary school students in Sidoarjo, Indonesia, who were in fifth-grade class. The students' strategies used were identified to analyze the students' proportional reasoning in solving area conservation. Results show that the level of proportional reasoning varies from zero to two. Regarding the students' proportional reasoning levels, most of the students' strategies use visual clues and cute paste strategies. Only one student can reach the level of quantitative reasoning by using a formula to compare both area measurements. Interestingly, the problem of the conservation of plane figures failed to reveal students' formal proportional reasoning due to their insufficient knowledge of fractions, division, multiplication, and decimals. Some implications regarding students' proportional reasoning and interventions in the area conservation problem are discussed.

Keywords: Area Conservation, Guided-Unguided Problems, Part-Whole Relation, Proportional Reasoning

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The development of students' mathematical abilities is greatly influenced by proportional reasoning, which is identified as a standard for students' mathematical proficiency and as an essential idea in both elementary and advanced mathematics (Boyer & Levine, 2012; Johar et al., 2018; Kaitera & Harmoinen, 2022; Kilpatrick et al., 2001; Möhring et al., 2015; 2016). Hence, proportional reasoning is a fundamental mathematical idea that underpins many other fields of mathematics, including algebra, geometry, and statistics. Consequently, many countries' national teaching and learning standards have strongly emphasized the need for pupils to develop their proportional thinking (ACARA, 2019; Boyer & Levine, 2012; He et al., 2018; Johar et al., 2018; NCTM, 2000).

Several studies on proportional reasoning primarily focus on elementary and early-age children because proportional reasoning is a vital skill in various mathematical topics at an early age. Moreover, some of them focus on scaled drawings (Möhring et al., 2015, 2016), measurement (Sari & Fong, 2022), and problem-solving (Aktaș, 2022; Boyer & Levine, 2015; He et al., 2018; Jacob et al., 2012). Its significance also extends to aspects of science and everyday life (Beswick, 2011; Cramer & Post, 1993). Because of its broad application, proportional reasoning is considered a key concept and a fundamental building block in elementary mathematics (Lesh et al., 1988), leading to higher math and success.

Some researchers have examined the significance of reasoning in the learning of mathematics. Students' mathematical proficiency and the characteristics of their proportional thinking could be determined (Johar et al., 2018). In mathematics learning, proportional reasoning, also known as logical reasoning in proportional settings, is a mental activity that comprehends the relationship of one quantity changing to another via a multiplicative relationship and is utilized to solve proportion problems (Aktaș, 2022; Boyer & Levine, 2015; He et al., 2018; Möhring et al., 2016). The comparable circumstance is related to the ratio and proportion concepts (Boyer & Levine, 2012; He et al., 2018; Jacob et al., 2012; Kaitera & Harmoinen, 2022). Numerous subjects covered in elementary and middle school education curricula are directly or indirectly associated with proportional reasoning and its related concepts, including ratios.

Children in Grades 3, 4, and 5 might show a grasp of the notion of proportion (Vanluydt et al., 2020). However, it starts as a fragmented skill that depends on the context of the problems (Françoise, 1986; He et al., 2018; Möhring et al., 2015). Research on revealing the best strategies for imparting proportional reasoning is still lacking. It needs context to understand well the meaning of proportionality for children in those grades (Ayan & Isiksal-Bostan, 2018). Elementary students barely understand structured problems involving ratios and proportions when it employs small numbers and integer ratios paired with the appropriate manipulatives (Françoise, 1986; Karplus et al., 1983; Möhring et al., 2016). Students' understanding of ratio and proportion can also be improved when these manipulatives are used with high-quality instruction and critical thinking-based curricula.

Proportional reasoning may be symbolic or non-symbolic. It may interface with other branches of mathematics, such as arithmetic and geometry (Gouet et al., 2020). Geometry mathematical abilities may be built from non-symbolic foundations (Lourenco et al., 2012). Research conducted by Sophian (2000) provides strong evidence that young children, as well as adults, are sensitive to the proportional properties of visual stimuli. Furthermore, Ayan and Isiksal-Bostan (2018) suggest that the use of proportional reasoning in various relationships such as between the length and the area can enhance the students' understanding of proportional situations, related concepts, and different relationships by investigating in real-life contexts. In this way, students could develop alternative strategies for proportional reasoning problems in which the use of area conservation in area measurement problem.

Most elementary children aged 6 to 13 years in Indonesia have reached the concrete operational stage (Saleh et al., 2018). One of the characteristics of concrete operational children is thinking and reasoning through conservation problems. Based on the results of research conducted by Piaget and his colleagues, six types of conservation developed in children's cognitive structures, namely number conservation appearing at 5 or 6 years old, quantity conservation appearing around the age of 7 or 8, long conservation occurring at 7 or 8 years old, extensive conservation also occurring at the age of 7 or 8, weight conservation appearing at the age of 9 or 10, and volume conservation appearing at 11 or 12 years old (Piaget et al., 1960).

Elementary students must study area conservation before they learn about area measurement (Ekawati et al., 2019; Funny, 2014). Conservation of area is a modification in geometric shapes where the shape's area does not change (Ekawati et al., 2019; Funny, 2014; Kordaki & Balomenou, 2006; Sari & Fong, 2022). In addition, conservation entails the idea that the redistribution of material does not alter its mass, number, or volume (Ekawati et al., 2019; Smith et al., 2011). The term conservation may not be well-known among students in Indonesia (Funny, 2014). However, it has been around since 1965 when a psychologist and inventor, Piaget, created a broad conservation assignment for children in elementary schools to investigate their cognitive development (Funny, 2013). Furthermore, the conservation of area has been previously investigated in isolation from the concept of area measurement and formula (Kospentaris et al., 2011).

Area conservation is one of the basic concepts of geometry that is important to learn because it is used as a prerequisite for studying the area of plane geometry (Clements & Stephan, 2004; Funny, 2014; Piaget et al., 1960; Sari & Fong, 2022). Extensive conservation materials facilitate students in developing reasoning skills because they can separate, draw, model, find, measure, and construct geometric relationships. For example, in combining and consolidating their ideas in proving the conservation of the area of a plane figure, students take several steps such as finding and constructing a plane figure, comparing its attributes, classifying it, and developing and considering the definition of the properties of a plane figure. Furthermore, they are capable to carry out a thinking process in which they try to relate predetermined facts or evidence to a conclusion. The relevant research in the broad field of conservation has focused on uncovering the strategies used by students (Kospentaris et al., 2011); students' difficulties with area conservation using tangram (Fiangga, 2014); students problem solving (Sari et al., 2017); and quantitative relationship through area conservation hands-on activity (Sari & Fong, 2022). It can be seen that research regarding proportional reasoning in area conservation is still limited to focus on quantitative reasoning and specific strategies. Based on these ideas, the present study investigated students' performances in area conservation-proof situations presented in tangram patterns and irregular patterns.

There are two approaches in the introduction of area conservation namely qualitative and quantitative. The first example is in solving broad conservation problems using computer microworld software (C.AR.ME). The function of the C.AR.ME software, which uses a qualitative approach, is to copy, cut, paste, rotate, and symmetrize. At the same time, the square and iteration units are the functions of C.AR.ME with a quantitative approach (Kordaki & Potari, 2002). Another research conducted by Funny (2013) in designing broad conservation learning uses a qualitative approach, namely a cut-paste strategy using the concept of compensation and the relationship between parts and whole comparisons. Fiangga (2013a; 2013b; 2014) also uses a qualitative approach, namely the tangram strategy using compensation and comparing parts and whole relations. The present study uses two approaches to solving the area conservation problems: a qualitative approach (e.g., cut-paste and relation concepts) and a quantitative approach (e.g., using grid lines or a ruler and the formula for the area of plane figures).

Langrall and Swaford (2000) suggested a draft of the proportional reasoning category based on the four levels of students' proportional reasoning in solving an area conservation problem, especially for elementary school students. The four levels of proportional reasoning used in this study include level 0 to level 3. In level 0 (non-proportional reasoning), students have not demonstrated proportional reasoning abilities in solving proportional problems. Usually, they solve problems of one unknown value using the difference or an unpatterned count, i.e., using any number or operation. In level 1 (informal reasoning about proportional situations), students demonstrate proportional reasoning skills by using pictures, models, or manipulations in solving problems of an unknown value. In level 2 (quantitative reasoning), students show proportional reasoning skills using composite units, finding, and using unit rates, identifying, or using scalar factors or tables, utilizing equivalent fractions, and building up both measures. Last, in level 3 (formal proportional reasoning), students demonstrate proportional reasoning skills by setting up proportions using variables and solving problems using the cross-product rule or equivalent fractions. They also fully understand the invariant and covariant relationships. The draft of the proportional reasoning levels is based on students' proportional reasoning characteristics in solving area conservation problems (see Table 1).

Table 1. Level of proportional reasoning in solving area conservation problem

Level	Proportional Reasoning	Strategy for Solving Area Conservation Problem
0	Non-proportional reasoning	• Guesses or uses visual clues ("It looks like") • Is unable to recognize multiplicative relationships • Randomly uses numbers, operations, or strategies • Is unable to link the two measures
1	Informal reasoning about proportional situations	• Uses tangram, models, or manipulatives to make sense of situations • Makes qualitative comparisons
2	Quantitative reasoning	• Uses grid lines or ruler to find the number of units of the area • Uses a formula to compare both area measurements
3	Formal proportional reasoning	Sets up proportions using variables and solves using the cross-product rule or equivalent fractions

(Adapted from Langrall & Swafford, 2000)

As the mathematics activities involved comparing magnitudes of area, the student began to establish essential notions in area conservation. The concrete materials and geometrical representations were discovered to be useful in stimulating the learner's thinking, with each succeeding activity challenging the students to a higher degree of notions related to geometrical figures, such as guided-unguided compensation and part-whole relation. Furthermore, recent studies on proportional reasoning suggest that non-symbolic proportional reasoning (Abreu-Mendoza et al., 2021), represented as guided-unguided problems in area conservation, can help students better understand the application of proportional reasoning. Thus, the present study aimed to address the research question of how to describe students' level of proportional reasoning in solving guided-unguided area conservation problems.

METHODS

This study used an exploratory research design using a qualitative research approach to produce descriptive data in the form of a description of the proportional reasoning profile of elementary students in proving the conservation of plane figures. Exploratory research aimed to explore a broader phenomenon. Data were collected from students' responses and interviews. The present study used an in-depth investigation of specific subjects or events (Cohen et al., 2007). It was a comprehensive analysis of a bounded system (i.e., activity, event, process, or individuals) based on significant data collection. A bounded system denotes that the case has been divided for research purposes based on time, place, or some physical boundaries (Creswell, 2013).

Participants

The research participants were 30 students in the 5th grade of elementary school. The participants were chosen based on the following considerations: 1) the fifth-grade elementary students were included in the concrete operational stage where they could prove broad problems by using extensive conservation; 2) based on the cognitive development stage of proof, they had entered the verbal description stage, could relate and represent visual objects or symbols, and began to think about specific properties and relationships, and 3) they obtained fraction material as a prerequisite material in solving proportional problems. Then, the selected subjects were chosen based on pupils' comparable communication and math skills. Furthermore, communication skills were designed so that pupils do not struggle when asked to communicate verbally and can convey their thoughts. The equivalent mathematical abilities referred to in this study were represented by mathematical scores, with a math score differential of 5 on a scale of 1-100. This information was gathered from the class's mathematics teacher. Their answer was analyzed further with the solution strategy and the level of proportional reasoning. So, in this study, there were four subjects whose proportional reasoning was explored.

Instruments

The data collected in this study were obtained directly by the researcher because the characteristic of this qualitative research was that the researcher became the main instrument. Meanwhile, the supporting instrument was in the form of an assignment sheet that contained problems regarding proving the area of a plane figure with an area conservation approach (see Figure 1). There were four types of area conservation questions given (see Table 2), which consisted of:

1. A problem was proving the area of plane shapes where the two planes had the same pattern and arrangement (guided part-whole relation).
2. Only one of the plane shapes was known for the pattern, and the two plane shapes had the same arrangement (unguided part-whole relation).
3. Only one of the plane shapes was known for its pattern, but the arrangement was not known directly (unguided compensation).
4. The shape is similar, but the arrangement and pattern of the two plane shapes were unknown (guided compensation).

Table 2. Matrix of area conservation problems

	The pattern and arrangement are given	**The pattern and arrangement are not given**
Shape similar	guided part-whole relation	guided compensation
Shape not similar	unguided part-whole relation	unguided compensation

Three lecturers who had taken the doctoral program, especially in realistic mathematics education, had previously validated this supporting instrument. The experts reviewed the instrument to assess its content validity of the instrument. Then, the instrument's inter-rater reliability was evaluated to assess the consistency of judgments among them. Finally, the data obtained from the content validity and inter-rater reliability analysis was statistically analyzed using intra-class correlation coefficients. The instrument met the validity and reliability requirements by obtaining the feasibility of content validity and the inter-rater reliability result of 0.84. Then, the test was administered to 30 5th-grade students to obtain the instrument's reliability. Then, the instrument obtained Cronbach's alpha reliability of 0.96.

Now, you'll need to play with a ruler, transparent graph paper, a picture of a shape, and scissors. Then you can just solve the given problem using the method you understand.

Could you take a look at the two figures below? Do Figure A and Figure B have the same area? Explain your reason!

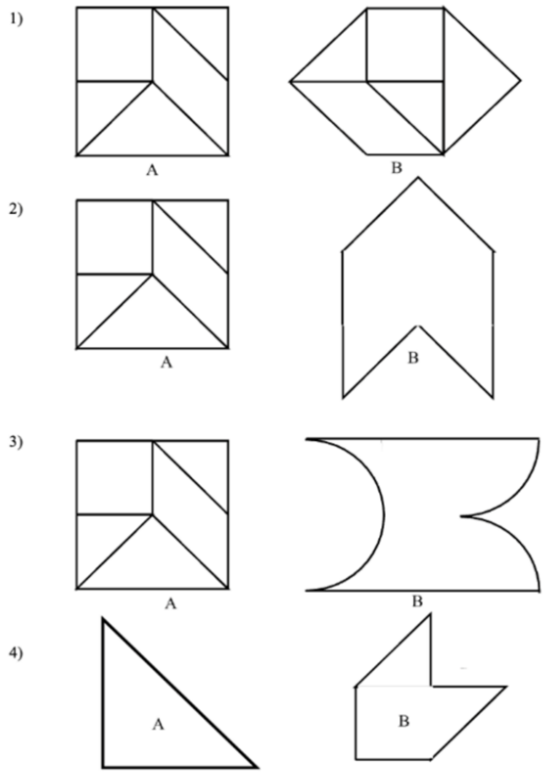


Figure 1. Area conservation proofing task

Data Analysis

The phases of qualitative data analysis included data reduction, data presentation, and verification/conclusion (Miles & Huberman, 1994). In the reduction process, the data on the answers of the selected participants were simplified and then grouped with data according to the need to describe the student's proportional reasoning profile in solving the problem of proving area conservation. The responses were coded according to their level of proportional reasoning and its strategies. Two coders independently review and transcribe the video recordings of the interviews. After completing their transcriptions, the coders compared their work and reconciled discrepancies through discussion and consensus. This process is called intercoder reliability, which measures the level of agreement between the two coders. Intercoder reliability was calculated using Cohen's kappa, which resulted in 0.71. The results are consistent and can be used for further analysis (Hsu & Field, 2003).

The coding schemes for the level of proportional reasoning were non-proportional reasoning (L0), informal reasoning about proportional situations (L1), quantitative reasoning (L2), and formal proportional reasoning (L3). Meanwhile, the strategy consisted of: using visual clues (VC), cut paste strategy (CP), reticle or ruler (RR), using a plane figure formula (FP), and using symbolic proportion between the number of tangrams and the area measurement (SPTA). The interviews data with the selected students in each topic were transcripted using the prepared criteria corresponding to student action (see Table 3).

Table 3. Strategy for Solving Area Conservation Problem

Level	Proportional Reasoning	Strategy for Solving Area Conservation Problem
0	Non-proportional reasoning	• Guesses or uses visual clues ("It looks like") (VC)
1	Informal reasoning about proportional situations	• Uses tangram, models, or manipulatives to make sense of situations (CP) • Makes qualitative comparisons (CP)
2	Quantitative reasoning	• Uses grid lines or ruler to find the number of units of the area (RR) • Uses a formula to compare both area measurement (FP)
3	Formal proportional reasoning	Sets up proportions using variables and solves using the cross-product rule or equivalent fractions (SPTA)

Finally, the data were presented by summarizing the students' proportional reasoning and strategy for each task (guided and unguided area conservation problem) based on the answers and interviews. The researcher concluded by observing patterns in student responses and comparing them to pertinent interviews. Afterward, verification was acquired by comparing the results of student responses and interviews using method triangulation.

RESULTS AND DISCUSSION

Distribution of Proportional Reasoning in Each Task

Four types of questions in the proof task in area conservation on plane figures were given to 30 students enrolled in the fifth grade of elementary school. The first type of question, namely proving whether the area of shape A was equal to the area of shape B by knowing the patterns (guided part-whole relation) was mainly answered correctly by 28 students. While the third type of question, which was proving the area of A with an area of B with no known pattern of the building blocks (unguided compensation), was the least correct answer; only 7 out of 30 students answered correctly.

Table 4. Distribution of the students' proportional reasoning in each level

	Non-Proportional Reasoning (L0)	Informal Reasoning about Proportional Situations (L1)	Quantitative Reasoning (L2)	Formal Proportional Reasoning (L3)
Guided part-whole relation	5 (17%)	18 (60%)	7 (23%)	0 (0%)
Unguided part-whole relation	6 (20%)	16 (53%)	8 (27%)	0 (0%)
Unguided compensation	10 (33%)	17 (57%)	3 (10%)	0 (0%)
Guided compensation	7 (23%)	19 (63%)	4 (14%)	0 (0%)

Table 4 indicates the distribution of solvable tasks regarding the type of task and level of proportional reasoning. In addition, Table 4 also points out that most students' answers could not hit the top level of proportional reasoning regarding the level of proportional reasoning, which was formal proportional reasoning. Meanwhile, most of the student's answers used informal reasoning about proportional situations using a cut-paste strategy.

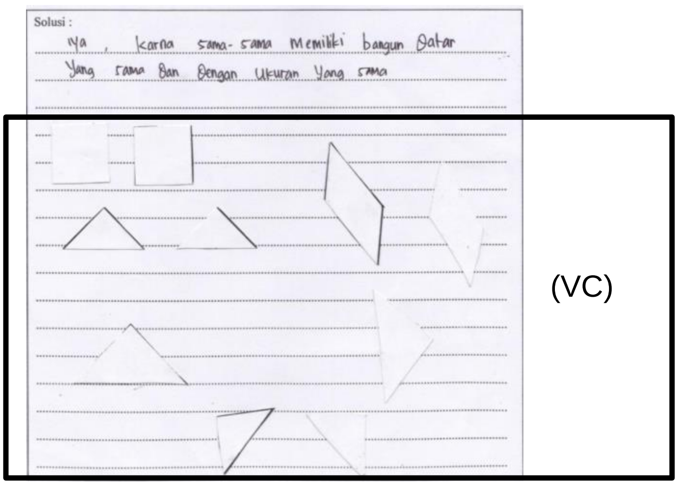
Table 5. Strategies employed by students in solving area conservation problems

Type of Problem	Proportional Reasoning	Strategy for Solving Area Conservation Problem
Guided part-whole relation	Non-proportional reasoning (L0)	Visual Clues (VC)
Unguided part-whole relation	Informal reasoning (L1)	Cut Paste (CP)
Unguided compensation	Quantitative reasoning (L2)	Cut Paste (CP), Plane Figure Formula (FP)
Guided compensation	Informal reasoning (L1)	Cut Paste (CP), Reticle or Ruler (RR)

Table 5 summarizes the proportional reasoning outcomes and strategies used by each student when solving each type of conservation area problem. Table 5 shows that students' proportional reasoning varies from non-proportional to quantitative reasoning. The most utilized strategy is the cut-paste strategy, while the visual clue strategy is the only strategy used in the guided part-whole relation problem type. For a more detailed explanation of the results of student work, it is presented in the following description.

Student's Proportional Reasoning in Proving Area Conservation Problem Type a Guided Part Whole Relation

Here, the examples of solutions created by 5th-grade students in solving guided part-whole relation problems regarding their level of proportional reasoning use are provided in Figure 2.



Translation: Yes, because both plan figures have the exact arrangement shape with the same size

Figure 2. Student's answer to guided part-whole relation problem

Transcript 1

R : Please, try to explain the steps that you take to solve this problem.

S : From here (Figure 1), it is cut one by one continuously because it is the same as this, then this is the same as and so on, so this area (plan figure A) is the same as this area (plan figure B). (VC)

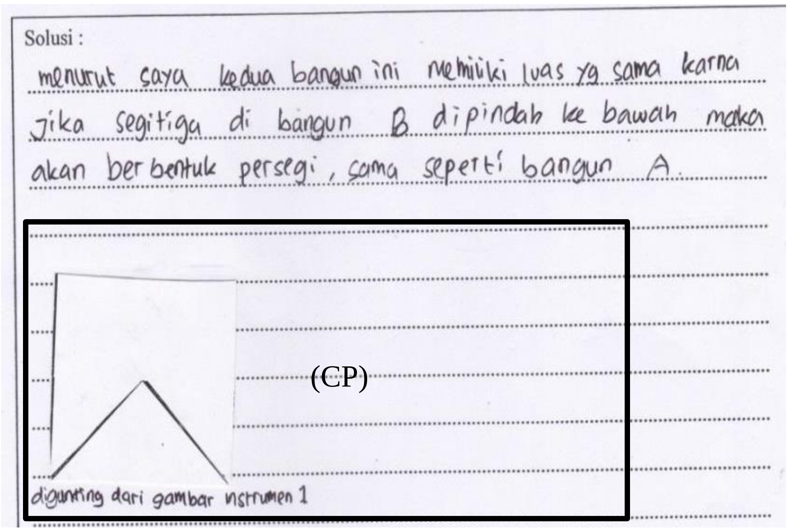
R : Are there any other strategies you use besides this one?

S : Nothing, that is all.

From the results of the interviews (Transcript 1), students used the strategy of cutting out each arrangement of shapes from the two given shapes and then seeing that the arrangement of shapes had the same size as seen in solving the conservation area of the guided part-whole relation type. Then, students reasoned that the strategy could show that the area of A and B was similar if the arrangement of the shapes and the constituent shapes' sizes were the same. Based on the triangulation method from the results of the work and interviews, the things expressed by the students and their arguments tended to be similar. From these results, it was found that students used visual clues to prove strategy that the area of plan figure A was equal to plan figure B. Therefore, these students were included in L0, namely, non-proportional reasoning with the characteristics of: (1) using visual clues, and (2) randomly using scissors to cut the building elements and only attaching them to show the same size without making a qualitative comparison.

Student's Proportional Reasoning in Proving Area Conservation Problem Type Unguided Part Whole Relation

In solving the unguided part-whole relation problem, students' responses are provided in Figure 3.



Translation: I think these two plane figures have the same area because if the triangle in shape B is moved down, it will form a square like A.

Figure 3. Student's answer to unguided part whole relation problem

Transcript 2

R : How do you show that the area of these two shapes is the same?

S : This one has a triangle (the subject points to a large triangle made of shape A and shape B), but this one (shape B) does not have any other shapes (building blocks). So, I took this triangle (the subject pointed to the large triangle in shape A). Let's say this triangle (the big triangle in shape A) did not originally exist. If it is placed here (the subject points to a large triangle in shape A and then points to the top side of shape A), it means that it is the same shape (shape B). But rather than that, it's better if I put this here so it looks square (the subject points to a large triangle in shape B and then moves it down to cover the bottom side of shape B, which has a hole), so if I calculate it, it's the same as this. (CP)

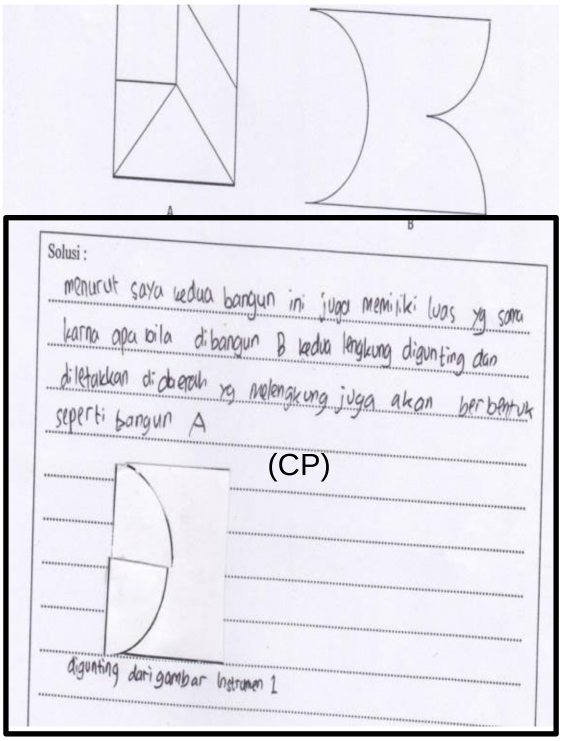
R : So, your strategy is based on the relationship between these two shapes.

S : Yes, so I cut out this shape (shape B) and moved it around to form a shape like this one (shape A).

The above interview (Transcript 2) excerpts and results revealed that in proving the conservation area of the unguided part-whole relation type, the student used a strategy of cutting out one part of shape B. Moreover, the student thought that it had the same characteristics as shape A, namely a large triangle, and placed the part in another part, namely form B, so that it made form B. The strategy used by the student was a cut-paste strategy. It was consistently seen from the results of the work and the results of interviews. Therefore, this student belonged to L1, which was informal reasoning with the characteristics of: (1) using tangrams, models, or manipulatives to make sense of situations, and (2) making qualitative comparisons.

Student's Proportional Reasoning in Proving Area Conservation Problem Type Unguided Compensation

In solving the unguided compensation problem, students' responses are provided in Figure 4.



Translation: I think these two shapes also have the same area because if the curves in shape B are cut out and placed in the curved area, they will also be formed like shape A.

Figure 4. Student's answer to unguided compensation problem

Transcript 3

R : Please explain your strategy in answering this question.

S : So, this part is cut (the curved side of shape B), rotated and reversed its position, and then placed here (the right side of shape B). It can be a square shape, so the result is like this (build A), and the size is the same. (CP)

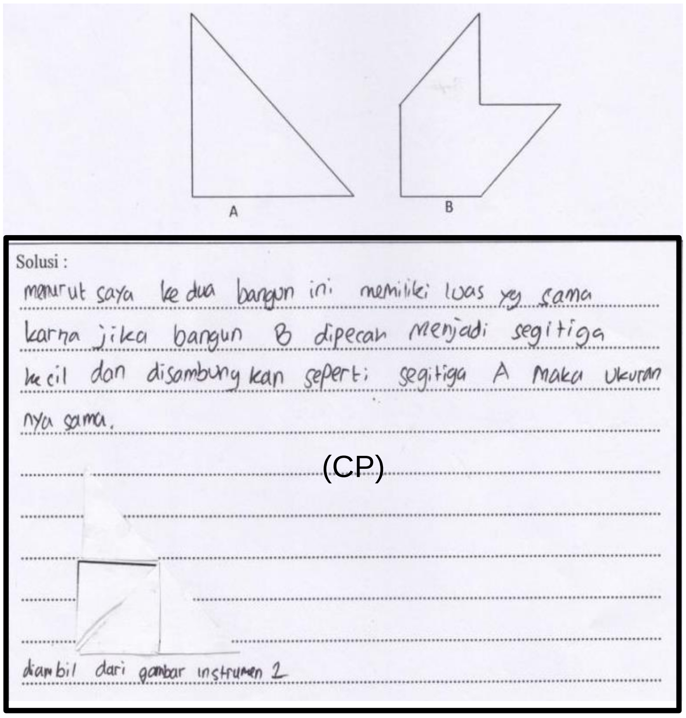
R : What is the size of the area?

S : (the subject measures the sides of the shape made up of the modified form B, then proceeds to measure the sides of the shape A using a ruler) (RR). The length of the side is 4, so the area is 16 cm^2 too. (FP)

From the above interview excerpts and results (Transcript 3), in proving an unguided compensation type of conservation area, the things expressed by the student and the arguments tended to be the same. So, it could be said that the strategy used by the student was a cut-paste strategy to show that the shape of shape B, when rearranged, had the same shape and area as shape A. The calculation supported it in finding the area of shape A and shape B carried out by the student, which resulted in 16 cm^2. In the question of the unguided compensation type, the student applied to level 2 proportional reasoning, namely quantitative reasoning (L2), with the characteristics of: (1) using the ruler to find the number of units of the area, and (2) using a formula to compare both area measurement was valid.

Student's Proportional Reasoning in Proving Area Conservation Problem in Guided Compensation Type

In solving the guided compensation problem, students' responses are provided in Figure 5.



Translation: I think these two shapes have the same area because if shape B is broken up into small triangles and hidden like a triangle in shape A, the size is the same.

Figure 5. Students Answer on Guided Compensation Problem

Transcript 4

R : Can you explain the steps you used to solve this problem?

S : Right, it doesn't look like a triangle. Suppose you break it down first and cut it into small triangles. After that, it is assembled again. For example, it has reversed (a small right triangle); it has also reversed (a small right triangle), right? So, it looks like a right triangle (shape A). (CP)

R : Where did you get that strategy?

S : Hmmm, I was looking at it (shape A and shape B), and it turns out that it can be assembled again using scissors.

R : Are there other strategies?

S : Yes, you can use this (subject takes a transparent grid lines) (RR)

R : Did you try that before?

S : Yes, but it wasn't written, so earlier, I found that the number of squares for each shape is the same (shape A and shape B).

From the above interview excerpts and results (Transcript 4), there was consistency in the students' opinions and their arguments in proving the area conservation in the guided compensation type. So, it could be said that the strategy used by the student was a cut-paste strategy to show that the shape B, when rearranged, had the same shape and area as shape A. It was also supported by identifying the number of boxes with transparent grid lines in each shape, which was the same as 16 cm^2. In guided compensation questions, the student applied proportional reasoning level 1, namely informal reasoning (L1), with the characteristics of: (1) using grid lines to find the number of units of the area, and (2) making qualitative reasoning valid.

Based on the study results, each student had a different level of proportional reasoning, starting from level 0 to level 2. Moreover, elementary students could not reach level 3, namely formal proportional reasoning, with a set-up of a proportion using variables and solving using cross-product rule or equivalent fractions. The results of this study about elementary school students in Indonesia shared similar findings. Several studies had shown that primary school students could do proportional reasoning (Boyer & Levine, 2015; Toluk-Ucar & Bozkus, 2018; Vanluydt et al., 2022b). In addition, their proportional reasoning level can be determined by students' solution strategies (Huang & Witz, 2013; Lim, 2009). Thus, various strategies also enabled students to engage in different types of proportional reasoning concerning the area conservation problem (Vanluydt et al., 2022a; Zacharos, 2006).

The lowest level was level 0 or non-proportional reasoning, where the subject proved the area conservation of the plan figure by only paying attention to the constituent shapes, especially in working on guided part-whole relation type questions. This level most often appeared in the first type of problem, the guided part-whole relation problems. The strategy often used was to look at the constructors of the two shapes in question. The conclusions were drawn only based on visual looks without further proof that involved quantitative reasoning, such as calculating the area of each constructor. Non-proportional reasoning based on visual looks often occurred in children, especially elementary school students, because they were still dominant in their concrete operational thinking (Nelson et al., 2022; Saleh et al., 2018). It was also supported by previous studies (Sari & Fong, 2022; Schoevers et al., 2020), where children could use appropriate language for length measurements using both paired words ("shorter" and "higher") and comparative words ("longer" and "shorter") based on the appearance of the constituent shapes. Similarly, Möhring et al. (2016) showed that students at the elementary level tended to rely primarily on visual cues rather than numerical information.

Informal reasoning was a level of proportional reasoning characterized by solving the conservation area problems using manipulative tools such as scissors, ruler, or grid lines to prove that the area of A was the same as B, supported by qualitative comparisons. This level of reasoning appeared in all types of conservation area questions in this study. The strategies used by students affected their way of reasoning. The results of this study were supported by van de Walle et al. (2021), where elementary students' measurement showed the number derived by comparing an object's property (situation/incident involving a unit with the same property). It also appeared in Ergül and Artan (2017) research that due to environmental circumstances, children only did activities such as counting, recalling the names of shapes, and writing numbers in measurement objects within the context of mathematics.

Quantitative reasoning in the level of proportional reasoning was the highest category that the students in this study achieved in this study. The reasoning was under formal proportional reasoning, the highest level of proportional reasoning. Quantitative reasoning emerged from students who had perfect memorization of the area of the plane. The strategy of using the formula for the area of a plane figure was used by students in comparing the two areas of the plane figure in question. If the two figures had the same area, then they validated the answer that shape A and shape B had the same area. Using a ruler and then formulas made students easier to understand the algorithm process in proving the conservation of plane figure area (Zacharos, 2006). It was also reinforced in the research of Christodoulou (2022), where they mainly depended on standard tactics such as applying formulas in solving problems related to proportional reasoning.

Level 3, formal proportional reasoning with the characteristics of setting up a proportion using variables and solving using cross-product rule or equivalent fractions, did not appear in every type of problem to prove the area conservation of the plane figure. One of the reasons students did not use formal proportional reasoning was their low ability to understand the concepts of multiplication, division, fractions, and decimals. Numerous investigations conducted in Indonesia focused on proportional reasoning among junior high school students, as they have typically been exposed to concepts of ratio and proportion in their curriculum (Nugraha et al., 2016; Yuliani et al., 2021). It helped the researchers easier to display students' formal proportional reasoning. In addition, Lo and Watanabe (1997) stated that developing students' understanding of ratios and proportions was difficult because their concepts of multiplication, division, fractions, and decimals were generally poor. Toluk-Ucar and Bozkus (2018) portray that elementary school students did proportional reasoning concerning numbers rather than applying formal techniques to solve mathematical problems. However, student reasoning proportionally using qualitative and quantitative methods defined a better grasp between the relations, which were the half-mark of formal operation based on Piaget's Theory (Nasution & Lukito, 2015).

For a long time, proportional reasoning was associated with the competence to solve proportional problems directly, such as questions of finance and fairness (Sawatzki et al., 2019), proportional missing-value problems (Cramer et al., 1993), and multiplicative problems related to many mathematical concepts such as scale, pictures, recipe ingredients, price comparisons, value for money, number of children, weight, and speed (Beckmann & Izsák, 2015). So, these questions can easily describe students' proportional abilities. However, in this study, the guided-unguided area conservation problem was able to describe the proportional abilities of elementary school students from level 0 to level 2.

The questions presented in this study were geometric problems that did not directly mention ratio and proportion. However, these guided-unguided problems served as non-symbolic proportional reasoning that occur earlier in the students' reasoning development. It is in line with research by Fernández-Verdú et al. (2010), which suggested a new model of proportional reasoning where proportional reasoning not only implies success in solving a range of proportional problems, such as routine missing-value and comparison problems, but it also involves handling verbal and arithmetic analogies. Moreover, Gouet et al. (2020) found that fourth-grade students contribute to an understanding of how proportional reasoning (especially non-symbolic proportional reasoning) may interface with other branches of mathematics, namely geometry.

Seah and Horne (2020) stated that grasping the notion of area measurement, especially area conservation, could contribute to success in reasoning about volume and surface area issues and spatial thinking. Meanwhile, Sari and Fong (2022) suggest that the area conservation exercise can develop the concept of quantitative relationships in early algebra. These studies align with this study's finding that the area conservation tasks encourage the students to investigate proportional reasoning in area measurement.

To sum up, these activities were strongly related to the proving process and, as such, could be seen as meeting the challenge that policymakers, curriculum designers, and researchers had thrown about the need for student engagement in the proving process not only at the high school level, where it had typically played an important role but also for better continuity at earlier grade levels (Maschietto & Bartolini Bussi, 2009; Saleh et al., 2018; Stylianides, 2007). Moreover, this study could enhance the understanding of proportional reasoning through proof tasks. Various types of tasks are recommended, along with guidance on incorporating them into instruction. The study also provides the necessary theoretical knowledge to serve as a conceptual framework for professional development (Ben-Chaim et al., 2012). This framework could be beneficial in improving students' proportional reasoning skills in real-world scenarios involving area conservation.

CONCLUSION

Most of the conservation area proofing problems depicted in this study were answered by the students who were free from invalid assumptions and insufficient information. Meanwhile, the students used informal reasoning in solving the conservation task area using the cut-paste strategy combined with the reticle and ruler strategy. Regarding the type of problems, the unguided compensation problem is a question with few students with the highest level of reasoning, namely quantitative reasoning. In addition, the problem of proving the conservation of plane figure area cannot reveal formal proportional reasoning involving proportion using variables and solving using cross product rule or equivalent fractions due to students' lack of knowledge of the concepts of multiplication, division, fraction, and decimals. Therefore, further research needs to be done on the types of evidentiary questions that can reveal the formal proportional reasoning of elementary school students.

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