Journal on Mathematics Education
Volume 16, No. 1, 2025, pp. 197-224

Javanese folklore with moral values: An impactful context in learning relations and functions

Rully Charitas Indra Prahmana[1,2,*], Irma Risdiyanti[3], Nur Robiah Nofikusumawati Peni[1,2], Novita Ristiana[1,2], Rahmi Ramadhani[2,4]

[1]Mathematics Education Department, Universitas Ahmad Dahlan, Yogyakarta, Indonesia
[2]Ethno-Realistic Mathematics Education Research Center, Universitas Ahmad Dahlan, Yogyakarta, Indonesia
[3]Mathematics Education Department, Universitas Sriwijaya, Palembang, Indonesia
[4]Informatics Department, Universitas Potensi Utama, Medan, Indonesia
*Correspondence: rully.indra@mpmat.uad.ac.id

Received: 30 September 2024 | Revised: 3 February 2025 | Accepted: 7 February 2025 | Published Online: 20 February 2025

Abstract

Indonesia, particularly the Java region, is home to a wealth of folklore rich in moral teachings. Among these, the story of Rama and Sinta is one of the most prominent, and upon initial investigation, it reveals an underlying presence of mathematical concepts, particularly relations and functions. Despite this potential, there is a lack of research on integrating such cultural elements into the teaching of mathematics. This study aims to fill this gap by exploring the use of the Rama and Sinta narrative as a contextual tool for teaching relations and functions. Following the design research method within the Ethno-Realistic Mathematics Education (Ethno-RME) framework, we developed instructional materials for seventh-grade students at a public school in Magelang, Central Java, Indonesia. These materials, consisting of both student and teacher books, were designed to contextualize the mathematical concepts of relations and functions within the cultural narrative. The resulting learning trajectory, consisting of five interconnected activities, not only deepened students' understanding of the mathematical concepts but also reinforced the moral lessons embedded in the folklore. This paper details the development process, implementation, and outcomes of this culturally responsive approach, contributing valuable insights into the integration of local cultural narratives with core mathematical concepts to enhance the learning experience.

Keywords: Ethno-RME, Learning Trajectory, Rama and Sinta Story, Relations and Functions, Shadow Puppets

How to Cite: Prahmana, R. C. I., Risdiyanti, I., Peni, N. R. N., Ristiana, N., & Ramadhani, R. (2025). Javanese folklore with moral values: An impactful context in learning relations and functions. *Journal on Mathematics Education, 16*(1), 197–224. https://doi.org/10.22342/jme.v16i1.pp197-224

The development of mathematical concepts is influenced by various factors arising from human endeavors to explain, comprehend, and respond to phenomena and experiences encountered in their environment (D'Ambrosio, 2007; Freudenthal, 2006). Mathematics does not evolve in isolation; rather, it is shaped by cultural influences, which include historical, geographical, and social dimensions that individuals encounter throughout their lives (Risdiyanti & Prahmana, 2018; Utami et al., 2019). Despite its close ties to human experience, mathematics is often transformed into an abstract and formalized discipline within the confines of formal educational systems (Alangui, 2017). This detachment often results in the subject being taught in a mechanical manner, disconnected from students' cultural backgrounds and lived experiences. As a result, many students struggle to comprehend mathematical concepts, experience anxiety, and fail to recognize the relevance of mathematics in everyday life (Diponegoro et al., 2024). Consequently, there is an evident need for an instructional approach that bridges the gap between formal mathematics education, students' cultural contexts, and their daily experiences.

One promising approach for fostering this connection is Ethno-Realistic Mathematics Education (Ethno-RME). This approach aims to facilitate a deeper and more accurate understanding of mathematical concepts by aligning them with the process of mathematization and the cognitive development of students (Prahmana, 2022). Additionally, Ethno-RME encourages the internalization of socio-cultural values that support ethical growth and character development. A fundamental aspect of this approach is the use of real-world, particularly cultural, contexts as the starting point for mathematical learning (Prahmana et al., 2023). By integrating real-world contexts into mathematics instruction, students are better able to connect abstract mathematical concepts with practical, everyday situations, thereby facilitating the process of mathematization (Freudenthal, 1991; Zulkardi et al., 2020). Through mathematization, students' progress from intuitive reasoning to formal mathematical thought, bridging the gap between their lived experiences and the formal structures of mathematics. This process not only enhances their understanding but also enables them to apply mathematical concepts effectively in diverse contexts. Balacheff (1987) highlights that recognizing these connections significantly improves students' comprehension of mathematical ideas.

Indonesia, with its vast and diverse cultural heritage, possesses a rich repository of ideas, methods, and techniques that collectively form a complex system of knowledge. The Ethno-RME approach utilizes this cultural diversity as a context for learning, thereby enhancing students' understanding of mathematical concepts while also facilitating the internalization of the socio-cultural values embedded within these traditions (Risdiyanti & Prahmana, 2021; Shahidayanti et al., 2024; Leton et al., 2025). By incorporating various cultural contexts from Indonesia into mathematics instruction, educators are able to provide opportunities for students to explore and reconstruct mathematical knowledge within familiar cultural settings (Muhtadi et al., 2017; Alghiffari et al., 2024). This approach makes mathematics more relatable, counteracting its common perception as an abstract discipline that is disconnected from students' everyday lives.

A notable element of Indonesian culture, recognized globally and inscribed by UNESCO as a world cultural heritage, is *Wayang* (Vidani, 2014). *Wayang* refers to traditional Indonesian puppet theater, which includes two main types: *Wayang Kulit* (shadow puppets made from leather) and *Wayang Golek* (wooden rod puppets). *Wayang* performances often dramatize stories from the Mahabharata, Ramayana, or local folklore, combining moral lessons with entertainment. In addition to its artistic significance, *Wayang* embodies a profound philosophical framework (Prahmana & Istiandaru, 2021). Beyond its role as a form of entertainment, *Wayang* serves as a medium for communicating life lessons to the public (Ramli & Lugiman, 2012). Furthermore, *Wayang* incorporates mathematical concepts that have naturally evolved within the community (Prahmana & Istiandaru, 2021; Risdiyanti & Prahmana, 2020).

Among the concepts embedded in *Wayang Kulit* are those of sets and related notions, such as relations and functions, which are evident in the strategic arrangement of puppets on the *kelir*, the screen used during shadow puppet performances. This strategic placement mirrors the way communities position puppets during performances, offering an opportunity for educators to integrate *Wayang* as a cultural context for teaching mathematics through the Ethno-RME approach (Prahmana & Istiandaru, 2021).

The *kelir* itself, a white screen integral to *Wayang Kulit* performances, holds symbolic significance beyond its functional role, enriching the cultural depth and artistry of the performance. This context provides educators with a unique medium through which mathematical concepts can be connected to local traditions, thereby facilitating the integration of *Wayang* into mathematics instruction within the Ethno-RME framework.

Incorporating folklore into mathematics instruction within the scope of ethnomathematics is a recognized practice. However, Javanese folklore presents a particularly unique context. As van der Kroef (1955) observes, Javanese warrior culture, as depicted in *Wayang*, is deeply intertwined with the agrarian lifestyle. *Wayang* and traditional Javanese dance not only teach proper facial expressions, posture, and etiquette but also reflect the cultural ideals of Javanese society. These elements convey moral values, which can be leveraged to enhance character development in students, particularly in the context of mathematics education.

The effectiveness of utilizing *Wayang* contexts to facilitate the comprehension of mathematical concepts has been demonstrated in various studies. However, these studies predominantly focus on geometric concepts, such as similarity and magnification (DeJarnette et al., 2015) and patterns and geometric shapes (Putri et al., 2021). Furthermore, Ethno-RME-based approaches have successfully employed Mahabharata shadow puppets to teach set theory in grade 7 (Risdiyanti & Prahmana, 2021; Yuliani et al., 2023). While these studies affirm the efficacy of puppet contexts in mathematics instruction, no research has specifically examined the use of *Wayang* puppets to aid the understanding of relations and functions.

Furthermore, Utari and Gustiningsi (2024) identified several challenges students face when learning about relations and functions. These challenges include correctly expressing relations and functions, distinguishing between functions and non-functions, appropriately applying formulas, and utilizing these concepts to solve real-world problems. Contributing factors include the prevalence of conventional, mechanistic teaching methods, a lack of engaging and motivating learning experiences, insufficient interactive teaching materials, and limited opportunities for discussion and independent thinking (Jariyah et al., 2024; Utari & Gustiningsi, 2024). Therefore, there is a pressing need for more effective teaching strategies to address these challenges. Previous research has shown promising results with the use of *Wayang* in teaching mathematical concepts, such as number patterns (Risdiyanti & Prahmana, 2020) and set theory (Risdiyanti & Prahmana, 2021; Prahmana & Istiandaru, 2021). Additionally, Prahmana (2022) developed an Ethno-RME framework, and Prahmana et al. (2023) further delineated the stages and syntax of Ethno-RME, which serves as the foundation for this research.

METHODS

This study adopts a design research methodology, specifically the validation studies type, which is aimed at developing and validating theoretical frameworks related to educational interventions, including programs, processes, strategies, or learning materials intended to address educational challenges (Plomp, 2013; Prahmana, 2017). The design research methodology allows for the creation of learning trajectories that enhance educational quality (Bakker, 2019; Gravemeijer & van Eerde, 2009). The choice of the validation studies approach is warranted by its proven effectiveness in developing and validating theories surrounding educational interventions (Akker, 1991; Akker et al., 2006).

This method distinguishes itself from others by emphasizing not only the design of learning processes but also their validation (Plomp, 2013). Furthermore, it is particularly relevant in developing learning designs based on the Ethno-RME approach, as both design research and Ethno-RME share their roots in Realistic Mathematics Education (RME) principles (Risdiyanti et al., 2024). Accordingly, the principles and procedures of RME underpin the development of learning materials through the Ethno-RME framework (Plomp, 2013; Prahmana, 2022). The Hypothetical Learning Trajectory (HLT) was formulated using the four levels of the mathematization process: the situational-concrete level, the mode-of level, the mode-for level, and formal mathematics (Gravemeijer, 1994). Hence, this research approach is considered suitable and effective for the development of learning designs incorporating the Ethno-RME framework.

The design research methodology, specifically using the validation studies type, is implemented in three stages: preliminary design, design experiment, and retrospective analysis (Akker et al., 2006). The detailed description of each stage provides as follows.

Preliminary Design

During the preliminary design phase, the research team conducted an extensive literature review, observed classroom settings, and developed the Hypothetical Learning Trajectory (HLT). The primary goal of this phase is to establish a learning trajectory that will be refined through subsequent design experiments (Akker et al., 2006; diSessa & Cobb, 2004). Initially, the researchers reviewed pertinent literature related to mathematics education in junior high schools, learning activities, the Ethno-RME approach, and the Rama Sinta *Wayang* narrative. Field observations were then carried out in schools to evaluate the learning environment, as well as students' prior knowledge and experiences. Based on these observations, the HLT was formulated, serving as a hypothesis about the progression of students' thinking and understanding during the learning process (Risdiyanti et al., 2019; Widjaja, 2008).

Design Experiment

The design experiment phase focuses on testing and refining the learning trajectory developed in the preliminary design stage (Akker, 1991; Akker et al., 2006; Plomp, 2013). This phase is divided into two sub-phases: pilot experiments and teaching experiments. In the pilot experiment, the HLT is applied to a small group of six students from an experimental class. The primary objective of this phase is to explore students' prior knowledge and collect data to inform necessary revisions to the learning trajectory. Following the pilot experiment, the teaching experiment phase involves the implementation of the refined HLT in a larger class, with the goal of further adjusting the learning trajectory and collecting data for subsequent analysis in the retrospective analysis phase.

Retrospective Analysis

The final phase, retrospective analysis, aims to provide a thorough understanding of the implementation process and its impact on the development of the learning trajectory (Akker, 1991). During this phase, data collected from various sources—such as lesson descriptions, documentation (e.g., photographs and videos), transcripts, and student work—are systematically analyzed (Akker, 1991; Plomp, 2013). The analysis process involves comparing the outcomes of the learning design implementation with the original HLT proposed in the preliminary design phase (Prahmana & Kusumah, 2016).

The steps involved in retrospective analysis include data coding, reflection, interpretation of results, and the formulation of recommendations for future research (Pujiastuti et al., 2025; Prahmana & Kusumah, 2016). These steps facilitate the integration of data from multiple sources and allow for the identification of emerging patterns across the collected materials. Additionally, the participants in this study will be seventh-grade students from a junior high school in Yogyakarta or Magelang during the 2024/2025 academic year. Specifically, during the pilot and teaching experiments, multiple classes will be involved. Data collection will include lesson descriptions and documentation, with qualitative analyses conducted on the gathered materials, including photographs, videos, transcripts, and student work.

RESULTS AND DISCUSSION

This study developed a learning trajectory centered on relations and functions, employing the Ethno-RME approach, with contextualization based on the Rama Sinta shadow puppet narrative. The trajectory consists of five distinct activities, which progressively transition from concrete, situational experiences to formal mathematical knowledge. These activities are as follows:

1. Engagement with the Rama Sinta Puppet Performance
   Students observe the puppet show, thereby immersing themselves in its cultural context.
2. Participation in the Rama Sinta Puppet Relation Map Game
   This activity serves as an introduction to relational concepts, facilitated through interactive gameplay.
3. Participation in the Rama Sinta Puppet Shadow Function Map Game
   In this phase, students investigate the concept of functions within the context of an engaging game.
4. Exploration of Number Relations and Functions
   This activity deepens understanding by providing numerical examples that reinforce the mathematical concepts.
5. Reflection on the Learning of Relations and Functions through the Rama Sinta Puppet Shadow
   The final activity allows students to reflect on their learning experiences, consolidating the connections made throughout the preceding activities.

The development of this learning trajectory, which integrates the Ethno-RME approach and contextualizes the mathematical concepts within the Rama Sinta shadow puppet story, is elaborated upon in the following stages.

Preliminary Design

The preliminary design phase commenced with a comprehensive review of relevant literature, which provided the foundational guidelines for the development of the learning trajectory. These guidelines informed the subsequent school observations, aimed at understanding how mathematics was taught and perceived by students. Additionally, interviews were conducted with both mathematics teachers and students to gather further insights. The observations revealed a gap in the integration of cultural contexts within the mathematics teaching approach, particularly in facilitating the mathematization process. Interviews with the teachers corroborated these findings, indicating that teaching practices primarily focused on abstract mathematical concepts, rather than connecting them to cultural or real-life contexts. As a result, students were not encouraged to engage with problems linked to their cultural experiences, hindering their ability to naturally progress from concrete to abstract understanding.

Further insights obtained through student interviews aligned with these observations. Students indicated that they were accustomed to memorizing mathematical formulas and applying them in problem-solving without a deep understanding of the underlying concepts. Consequently, they encountered difficulties when faced with mathematical problems that deviated from the typical examples they had encountered. These findings, derived from both school observations and teacher interviews, underscored critical issues that shaped the design of the mathematics learning trajectory. One of the primary concerns was the absence of a mathematization process in students' learning, highlighting the necessity of incorporating the Ethno-RME approach to ensure the effectiveness of the learning trajectory.

In the course of the interviews, the researcher also explored students' familiarity with cultural activities. A significant finding was that students were aware of several cultural activities, notably the Rama Sinta puppet story, with all students reporting prior exposure to the puppet show. However, they also expressed that their mathematics lessons had never incorporated cultural narratives such as the Rama Sinta story. The results from the literature review, school observations, and interviews provided a solid foundation for the development of learning activities, particularly for teaching the concepts of relations and functions. These activities were meticulously designed, taking into account both students' cognitive development and the process of mathematization, as depicted in the iceberg model presented in Figure 1.

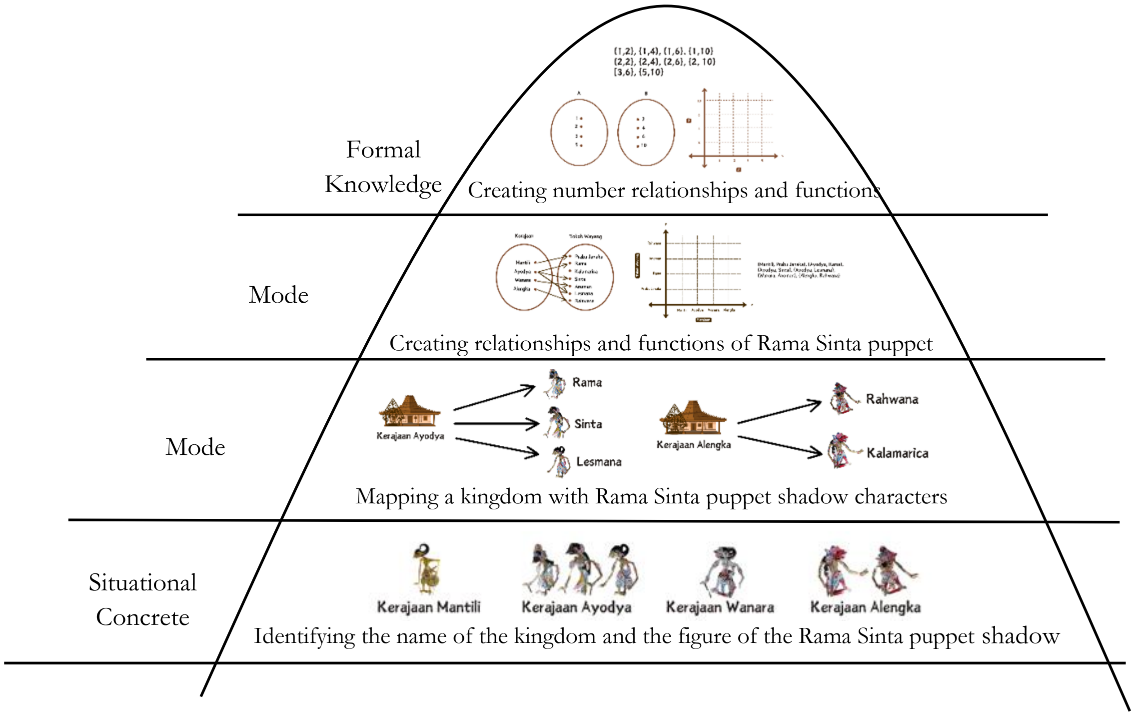


Figure 1. Learning trajectory on relation and function using Ethno-RME approach

The learning trajectory for teaching relations and functions begins with immersing students in the cultural context of the Rama Sinta puppet story. The initial activity involves the teacher narrating the story, thereby engaging students and facilitating meaningful connections to the material. Following this, students are guided into a concrete situational context in which they identify the names of kingdoms and characters within the story. This activity culminates in the creation of a set containing the names of the kingdoms and royal figures, which serves as a precursor to understanding relations and functions.

Subsequently, students' progress to the next stage, where they map each kingdom to its corresponding royal puppet character. This mapping exercise facilitates an understanding of the geographical origins, social connections, and the relationships and functions between the kingdoms and their respective royal characters.

As students move toward more formal stages of learning, they begin to express their understanding of relationships and functions in mathematical terms. This includes the use of arrow diagrams, Cartesian diagrams, and sets of ordered pairs, enabling students to transition from concrete experiences to formal mathematical representations.

Finally, at the formal knowledge stage, students attain a comprehensive understanding of number relations and functions, extending beyond the specific context of the Rama Sinta puppet shadow. They are then able to apply these concepts to broader mathematical contexts. This learning trajectory is designed to ensure that students develop both conceptual understanding and practical skills in recognizing and utilizing relations and functions.

The trajectory is visually represented in the iceberg model (see Figure 1), which serves as a metaphor for the structure of learning in this approach. The visible, formal mathematical concepts (the "tip of the iceberg") are supported by a deep foundation of informal, context-rich learning experiences (the "underwater part"). This model emphasizes the importance of grounding abstract mathematical concepts in concrete, relatable contexts to foster meaningful learning. The use of the iceberg metaphor in Realistic Mathematics Education (RME) facilitates the transition from informal to formal mathematics (Gravemeijer, 1994; Van Den Heuvel-Panhuizen, 2003).

Design Experiment

Following the design of the learning trajectory using the Ethno-RME approach, contextualized within the Rama Sinta shadow puppet narrative, the research progressed to the design experiment phase. This phase commenced with a pilot experiment involving a small cohort of six students, selected to represent a range of initial mathematical abilities, with two students from each ability level. These students engaged with the learning trajectory developed during the preliminary design phase.

The findings from the pilot experiment indicated the need for adjustments to the initial design, particularly concerning the activities at the concrete situational level, which involved understanding the puppet groups featured in the Rama Sinta puppet story. Initially, the activities were supported by the use of original puppet media, including a *kelir*, in alignment with the story's narrative. However, students encountered difficulties handling the original puppet media, which hindered the effectiveness of the activities. In response to this challenge, the researcher modified the approach by developing smaller puppet media and constructing a curtain using materials easily accessible to the students. These adjustments were made to optimize the concrete level activities, ensuring that the learning trajectory could be effectively implemented and achieve the desired outcomes.

Following the revisions based on the pilot experiment's findings, the next phase involved conducting a teaching experiment with a larger group of students. The learning trajectory, designed using the Ethno-RME approach, was implemented in a class of 32 seventh-grade students. The implementation of the trajectory proved to be effective in enhancing students' understanding of relations and functions. The integration of cultural context not only made the learning experience more engaging and relevant but also facilitated students' ability to link mathematical concepts to real-world applications. The outcomes of this implementation can be categorized into several key aspects.

Activity 1: Watching the Rama Sinta Puppet Story

The first activity begins with the teacher preparing the necessary teaching materials for the lesson. To initiate the session, the teacher greets the class and leads the students in a brief prayer before commencing the learning activities. The class, consisting of 32 eighth-grade students, is divided into 8 groups, each comprising 4 students. The teacher provides an explanation of the structured guide contained in the student activity book, clarifying how the book will be used throughout the lesson. Upon ensuring that the instructions are understood, the teacher distributes the student activity books and miniature puppets to each group for the upcoming tasks.

Figure 2 illustrates the teacher's approach to engaging and assessing students' prior knowledge and understanding of the Rama Sinta puppet shadow story. On the left side of Figure 2, the teacher is shown asking students about their familiarity with the story while holding one of the puppets from the Rama Sinta narrative. This visual prompt serves to stimulate student engagement and encourages participation in the ensuing discussion. On the right side of Figure 2, students are depicted responding to the teacher's questions, sharing their knowledge and experiences related to the puppet shadow story.

Figure 2. Teacher's engagement with students on the Rama and Sinta story

The interaction between the teacher and students reflects an effective method of eliciting prior knowledge, as demonstrated in the following dialogue:

Teacher : Let me see what I'm holding?
Student : Wayang (shadow puppet), Ma'am.
Teacher : Yes, this is a shadow puppet. So today, we will learn math from the shadow puppet story of Rama Sinta. Does anyone know the story of Rama Sinta?
Student : Yes, I know, Ma'am.
Teacher : If you know, try explaining it to your friends!
Student : It's about Sinta being kidnapped and then searched for by Rama.
Teacher : Yes, that's correct. The puppet shadow story of Rama Sinta tells the tale of Sinta, who was kidnapped by a giant named Rahwana and later sought by her husband Rama, until they were reunited.

This dialogue highlights the teacher's use of cultural narratives to contextualize learning, engaging students' thinking through familiar frameworks. Once the students recognized the cultural context for learning mathematics, the teacher proceeded by encouraging them to recall the story of Rama Sinta. The teacher then narrated the story at the front of the class, utilizing miniature shadow puppets, while simultaneously referencing the Student Activity Book, which contained both the text and illustrations of the Rama Sinta story. As the teacher narrated the story, the students listened attentively, engaging with the accompanying illustrations in their books, as depicted in Figure 3, where students are shown to be highly involved in the storytelling and the visual representations of the puppet shadow characters.

Following this, the teacher invited students to further explore the characters in the Rama Sinta puppet shadow story. Each group was provided with miniature puppets and access to character illustrations within their activity books. The students used these resources to identify the puppet characters and their corresponding kingdoms. They then grouped the characters by their respective kingdoms and recorded the results in the form of sets in their activity books. Figure 4 displays the outcomes of this identification process.

Figure 3. Students enthusiastically observing puppet illustrations of the *Rama* and *Sinta* story

On the left side of Figure 4, it is evident that students successfully identified the names of puppet characters, associating them with their respective illustrations from the Rama Sinta puppet story, demonstrating their comprehension of the characters. On the right side of Figure 4, students are shown having accurately identified the origins of the characters' kingdoms and organized them into sets.

Activity 2
Identifying the Rama Sinta Puppet Characters

Following the viewing of the story of *Wayang* Rama Sinta narrated by the teacher, the next task involves identifying the characters depicted in the shadow puppets. Students are instructed to carefully examine the puppet images and record the names of the characters in the designated columns provided within the activity book.

Activity 3
Grouping the Rama Sinta Puppet Characters

After knowing the characters in the story of *Wayang* Rama Sinta. Try to group the characters based on the origin of the kingdom.

Mantili Kingdom
Prabu Janaka

Ayodya Kingdom
Rama, Lesmana, Sinta

Wanara Kingdom
Rahwana, Kalamarica

Alengka Kingdom
Anoman

Figure 4. Results of identifying Rama and Sinta puppet characters and their kingdom origins

Understanding these puppet characters and their kingdom origins is crucial for students at the concrete situational level, as it aids in visualizing and connecting these elements to real-world relationships and functions. By documenting the sets of puppet characters and their regions, students gain an early understanding of sets and mappings, which lays the groundwork for later discussions on relations and functions in mathematics.

Activity 2: Mapping the Rama and Sinta Relationships

Building upon students' understanding of the historical context of the kingdoms and characters within the Rama and Sinta shadow puppet play, the second activity involved an interactive exercise focused on mapping the relationships among the puppet characters and their respective kingdoms. Before initiating the mapping task, the instructor introduced key terminologies, specifically "home region" and "friend region." It was clarified that the set of puppet kingdoms recorded by the students during the initial activity constituted the home region, whereas the set of Rama and Sinta puppet characters formed the friend region.

Figure 5. The instructor outlines the game instructions (left) while distributing the puppet character cards to students (right)

As depicted in Figure 5, the instructor provided a detailed explanation of the game instructions and distributed cards to each group member. These cards contained the names of various kingdoms, including Mantili, Ayodya, Wanara, and Alengka, as well as the corresponding characters from the Rama and Sinta puppet play, such as Prabu Janaka, Rama, Sinta, Lesmana, Anoman, Kalamarica, and Rahwana. Additionally, each member of the kingdom groups was given a rope, which served as a tool for establishing connections between the home region (the kingdom group) and the friend region (the character group).

After receiving and understanding the instructions, the members of each kingdom group sought out counterparts from the puppet character group who held cards corresponding to characters from their respective kingdoms. Each student used the rope to connect with the character whose name was listed on their puppet card. The procedural dynamics of this activity are illustrated in Figure 6.

After successfully mapping the members of the home region (the royal group) with those of the friend region (the character group), the instructor elaborated on the mathematical implications of the mapping process. The instructor explained that a mapping in which a member of the home region is linked to multiple members of the friend region is referred to as a relation in mathematical terms. Furthermore, the concept of the "result area" was introduced, referring to the output of the mapping between the home region and the friend region. Once the students grasped these concepts, they proceeded to represent their constructed Rama and Sinta shadow puppet relationships using an arrow diagram, which was recorded in their activity books.

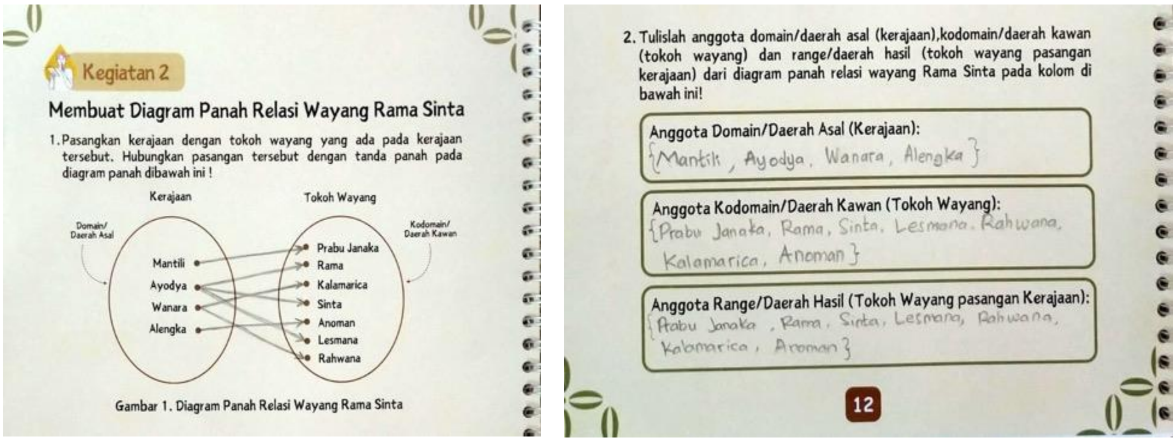
Figure 6. Students actively engage in the Rama and Sinta relationship mapping activity

The presentation process of the Rama and Sinta relation map, represented as an arrow diagram, is illustrated in Figure 7.

Figure 7. Students present the Rama and Sinta relation using an arrow diagram representation

As shown in Figure 7, students are depicted drawing arrows with rulers to connect the home region with the friend region, thereby illustrating the Rama and Sinta shadow puppet relation. Additionally, the image captures a group of students discussing the mapping process between the home and friend regions. As a result, the students successfully established accurate connections between the members of the home region (kingdom group) and the friend region (character group). The outcomes of the Rama and Sinta relation presentation, represented as an arrow diagram, are displayed in Figure 8.

Figure 8 illustrates the students' success in correctly mapping the friend region to the result area. Specifically, the Mantili Kingdom is associated with Prabu Janaka, the Ayodya Kingdom with Rama, Sinta, and Lesmana, the Wanara Kingdom with Anoman, and the Alengka Kingdom with Kalamarica and Rahwana. Moreover, the students effectively recorded the members of the home region, the friend region, and the result area in their activity books. The members of the home region included Mantili, Ayodya, Wanara, and Alengka, while the friend region comprised Prabu Janaka, Rama, Sinta, Lesmana, Anoman, Kalamarica, and Rahwana. The result area was composed of Prabu Janaka, Rama, Sinta, Lesmana, Anoman, Kalamarica, and Rahwana.



Activity 2
Creating a *Wayang* Rama Sinta Relationship Arrow Diagram
1. Pair the kingdom with the *Wayang* characters in the kingdom. Connect the pairs with the arrows on the arrow diagram below!

Kingdom *Wayang* Characters

Domain Codomain

Figure 1. Arrow Diagram of *Wayang* Rama Sinta Relation

2. Write down the domain (kingdom), codomain (*Wayang* characters), and range (Kingdom pair *Wayang* characters) members of the *Wayang* Rama Sinta relationship arrow diagram in the columns below!

Domain Members (Kingdom):

Codomain Members (*Wayang* Characters):

Range Members (Kingdom pair *Wayang* Characters):

Figure 8. The outcome of the presentation of the Rama and Sinta relation, represented as an arrow diagram

Each member of the home region exhibited varying pairings within the friend region; some members of the royal home region had one pairing, while others had two or three. The instructor prompted the students to analyze the characteristics of these relations as reflected in the mapping results. Several students observed that, based on the game and the arrow diagram presentation, certain members of the result area were paired with a single character, while others were linked to multiple characters. The instructor acknowledged these insights and further elaborated on the concepts in front of the class, as shown in Figure 9. This figure depicts the instructor explaining the presentation of the Rama and Sinta puppet shadow relation, as illustrated in the arrow diagram.

Figure 9. The instructor elucidates the presentation of the Rama and Sinta relation as illustrated in the arrow diagram

In the final segment of the Rama and Sinta relation mapping activity, students were further engaged in presenting the relationships using a Cartesian diagram. Two groups of students, after completing the mapping, reconvened with their respective teams. The instructor then distributed props, including a three-dimensional Cartesian diagram representing the Rama and Sinta relations and miniature figures of the puppet characters, to each group.

In this phase of the activity, students were tasked with illustrating the Rama and Sinta relations on a Cartesian diagram. In this diagram, the x-axis represented the home region (the royal group), while the y-axis represented the result area of the friend group. Students were required to identify the intersection points on the y-axis and x-axis after pairing each member of the x-axis with the corresponding members of the friend region.

Figure 10. The process of illustrating the Rama Sinta puppet shadow relation using a three-dimensional Cartesian diagram

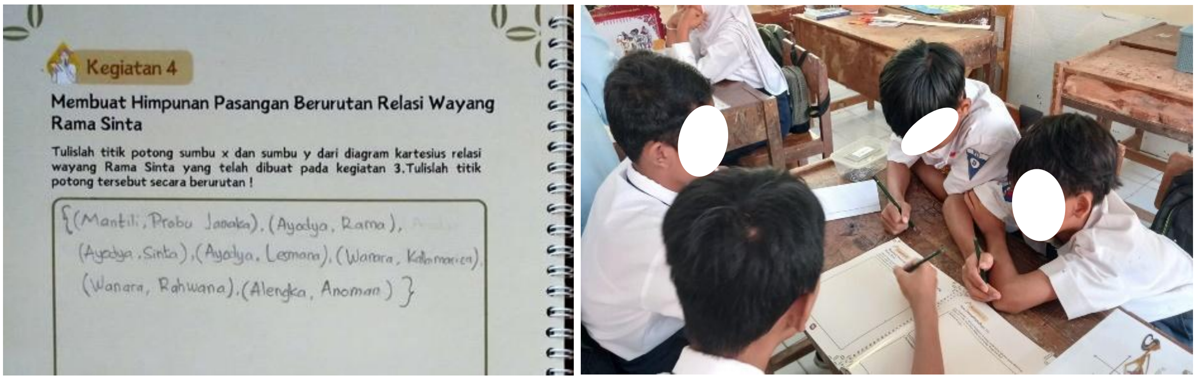
As shown in Figure 10, students demonstrate considerable enthusiasm when presenting the Rama Sinta puppet shadow relation within a three-dimensional Cartesian diagram. The image illustrates how students successfully position the Rama Sinta puppet characters at the intersection points between the x-axis, representing the home region (comprising the names of the kingdoms), and the y-axis, representing the friend region (comprising the names of the Rama Sinta puppet characters).

During this segment of the activity, the instructor prompts the students by asking for the definition of the result area (or range). In response, students articulate that the result area corresponds to the puppets positioned at the intersection points between the result area and the friend region. The instructor acknowledges the students' responses with verbal affirmation and a thumbs-up.

Figure 11. The process and outcomes of illustrating the Rama and Sinta relation in a two-dimensional Cartesian diagram

After successfully completing the three-dimensional representation, the instructor instructs the students to replicate the presentation in a two-dimensional Cartesian diagram, as shown in Figure 11. In this representation, students substitute the physical puppet characters with points at the intersection points. Consequently, students accurately represent the Rama Sinta puppet shadow relation within their activity books, transitioning from a three-dimensional to a two-dimensional format.

Once students have completed the two-dimensional representation, they proceed to translate the same relation into a set of ordered pairs derived from the intersection points of the x-axis (origin area) and the y-axis (result area). These ordered pairs are recorded in the designated columns of the student activity books, as illustrated in Figure 12.



Activity 4
Creating the set of ordered pairs of *Wayang* Rama Sinta relation

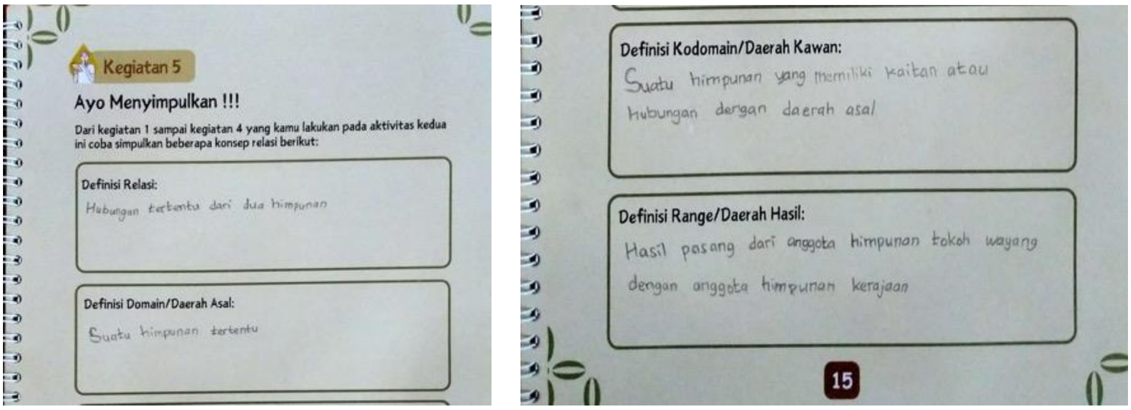
Write down the intersection points of the x-axis and y-axis of the Cartesian diagram of the *Wayang* Rama Sinta relation created in activity 3. Write down the intersection points in order!

Figure 12. The process and results of illustrating the Rama and Sinta relation as a set of ordered pairs

In Figure 12, students can be seen collaboratively discussing and documenting their representation of the Rama Sinta puppet shadow relation as ordered pairs. Throughout this process, students occasionally refer to the previously presented Cartesian diagram for guidance. As a result, the students accurately write the set of ordered pairs for the Rama Sinta puppet shadow relation, which includes: {Mantili, Prabu Janaka}, {Ayodya, Rama}, {Ayodya, Sinta}, {Ayodya, Lesmana}, {Wanara, Anoman}, {Alengka, Kalamarica}, {Alengka, Rahwana}.

At the conclusion of this task, the instructor prompts the students to reflect on the nature of the relation by asking whether any member of the origin region has more than one corresponding pair in the friend region. In response, the students observe that the kingdoms of Ayodya and Alengka each have multiple counterparts in the friend region. The instructor acknowledges these observations and guides the students toward a conclusion, emphasizing that the relation exhibits the property where certain members of the origin region correspond to multiple pairs in the friend region.

Following the completion of all activities in the second session, the instructor invites the students to articulate their understanding of key concepts, including the definitions of the origin (or domain), the friend area (or codomain), the range (or result area), and the overall concept of relations, based on their learning experiences. The students provide a range of definitions, some grounded in the context of the Rama Sinta puppet shadow play, while others generalize these concepts. The students' responses are illustrated in Figure 13.



Activity 5 Let's summarize!!! From activity 1 to activity 4 that you did in this second project, try to summarize some of the following relation concepts: Definition of Relation: A particular relationship of two sets Domain Definition: A particular set	Codomain definition: A set that has a relationship with the domain. Range definition: A pairing of members of the set of *Wayang* characters with members of the set of Kingdoms.

Figure 13. Student responses regarding the definitions of the origin, friend area, and result area

In Figure 13, various student responses are documented. Some students define a relation as "the relationship between members of the royal set and members of the puppet shadow characters," thereby connecting their understanding to the specific context of the activity. Others provide a more generalized definition, describing a relation as "a certain relationship between two sets." When defining the origin region, some students refer to it as "all members of the royal set," while others describe it as "a certain set." The friend area is characterized by students as "all members of the set of puppet shadows," with some elaborating that it is "a set that has a link or relationship with the origin area." Lastly, the range or result area is defined by some students as "the outcome of pairing members of the set of puppet shadows with members of the kingdom set," while others define it as "members of the set of friend regions resulting from the mapping of the relationship between the origin and friend regions." Overall, the students demonstrated a strong understanding of these concepts, with some definitions maintaining contextual relevance, while others adopted a more generalized perspective.

Activity 3: Playing the Wayang Rama and Sinta Function Map

Building on students' understanding of the concept of relations through the Rama Sinta puppet shadow relation map game, Activity 3 introduces the concept of functions using the Rama Sinta puppet shadow function map game. This activity mirrors the previous one, but with distinct differences in its underlying concepts. The teacher organizes the students into two groups: the "kingdom" group and the "puppet character" group, with the latter performing in front of the class. The teacher begins by revisiting the definitions of the origin, friend, and result areas, along with the properties of relations. This foundational knowledge is essential for helping students grasp the concept of functions and to distinguish between the properties of relations and functions effectively.

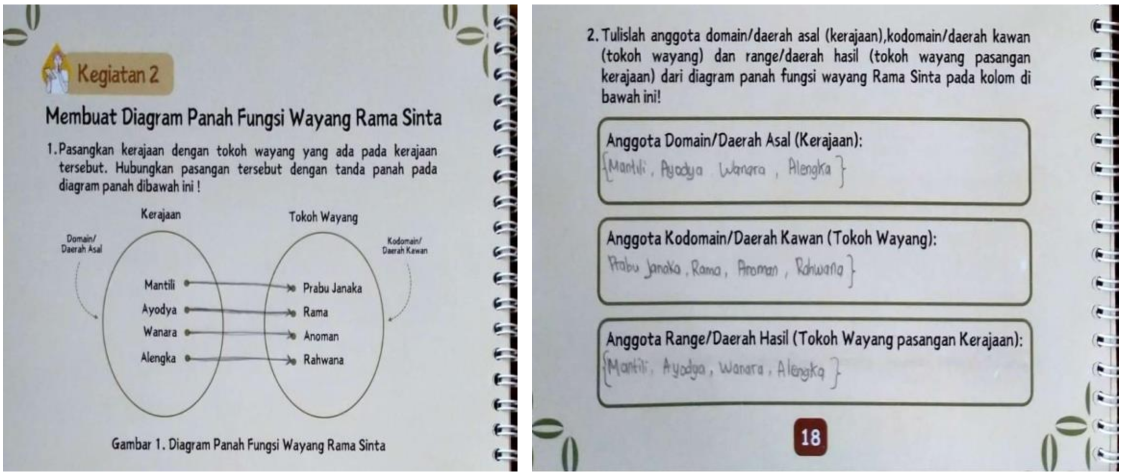
Following this, the teacher clarifies that, while the activity shares similarities with the Rama Sinta puppet shadow relation map game, there are key differences in how members of the origin area (the kingdom group) and the friend area (the puppet character group) are treated. Once the students are familiar with the game's instructions, each group retrieves and displays name cards provided by the teacher. The teacher then distributes arrows to the kingdom group, instructing them to aim their arrows at the corresponding members of the puppet character group representing their respective kingdoms. The process of the Rama Sinta function map game is depicted in Figure 14.

Additionally, students in the kingdom group actively engage in the Rama Sinta function map game by aiming arrows toward the members of the puppet character group that correspond to their respective kingdoms. This process visually demonstrates the establishment of functional relationships between the origin and friend areas. Finally, students successfully map the origin area, represented by the kingdom group, to the friend area, consisting of the puppet character group, as shown in Figure 14.

Figure 14. Students participating in the Rama and Sinta function map activity

Notably, each member of the kingdom group is paired with exactly one corresponding member from the puppet character group. To stimulate critical thinking, the teacher poses the question, "Are there any members of the origin region, or the kingdom group, that have more than one pair?" The students collectively respond with "no," prompting the teacher to guide them toward the conclusion that, in the context of a function, each element of the origin set can be associated with only one corresponding element in the friend set. The teacher further initiates a discussion to contrast the defining features of a function and a relation, referencing prior activities. Students articulate that, unlike a relation, where elements of the origin area may be paired with multiple elements of the friend area, a function stipulates that each element in the origin area must be paired with exactly one element in the friend area.

Upon the students' successful comprehension of the function concept and its distinguishing properties from relations, they proceeded to represent the Rama Sinta puppet shadow function in the form of an arrow diagram. The students successfully constructed the arrow diagram, illustrating the following pairs: the Mantili kingdom paired with Prabu Janaka, the Ayodya kingdom paired with Rama, the Wanara kingdom paired with Anoman, and the Alengka kingdom paired with Rahwana. The results of this presentation are depicted in Figure 15.



Activity 2 Creating an Arrow Diagram of the *Wayang* Rama Sinta Function 1. Pair the Kingdom with the *Wayang* characters in the Kingdom. Connect the pairs with the arrows on the arrow diagram below! Kingdom *Wayang* Characters Domain Codomain Figure 1. Arrow diagram of *Wayang* Rama Sinta function	2. Write the domain (Kingdoms), codomain (*Wayang* Characters), and range (Kingdoms' pair *Wayang* Characters) members of the *Wayang* Rama Sinta function arrow diagram in the columns below! Domain Members (Kingdoms): Codomain Members (*Wayang* Characters) Range Members (Kingdoms' pair *Wayang* Characters):

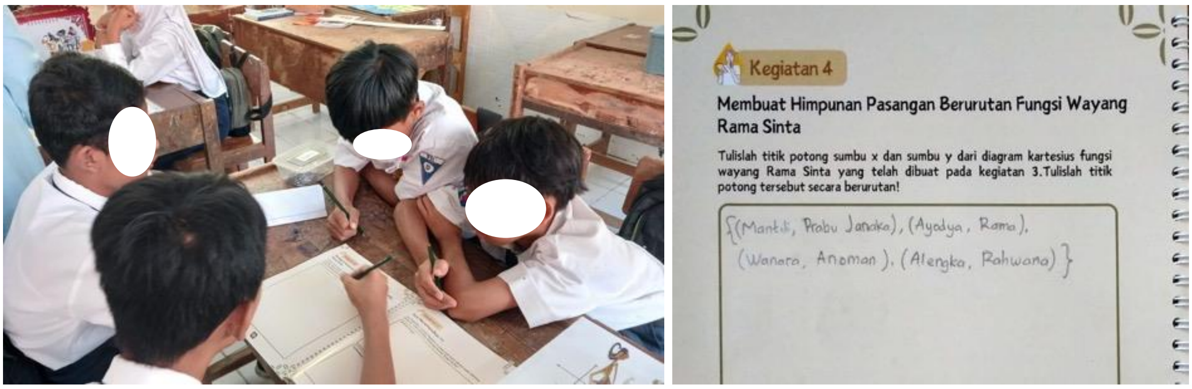
Figure 15. The process and results of presenting the Rama Sinta puppet shadow function in the form of an arrow diagram

Next, the students demonstrated the function of the Rama Sinta puppet shadow using a three-dimensional Cartesian diagram, where the x-axis represents the origin, and the y-axis represents the friend area. The students accurately positioned the puppet characters at the intersection points between the x-axis and y-axis, representing their respective relationships.

Figure 16. The process and results of presenting the Rama Sinta function in the form of a Cartesian diagram

After successfully placing the puppets on the three-dimensional Cartesian diagram, the teacher posed a critical inquiry: "Is there an x-axis, or a kingdom, where more than one puppet is positioned if a vertical line is drawn?" The students unanimously responded with "no," confirming that each kingdom has exactly one puppet positioned on the vertical axis. This response further demonstrates the students' correct understanding of the concept and properties of functions illustrated in Figure 16.

After the students successfully completed the presentation of the Rama Sinta puppet function on the Cartesian diagram, they proceeded to formalize their understanding by documenting the set of ordered pairs that emerged from the intersection points of the x-axis (representing the origin area) and the y-axis (representing the friend area). This step was crucial for reinforcing their comprehension of how functions are represented in mathematical terms. As shown in Figure 17, the students accurately recorded the ordered pairs, which corresponded to the functional relationships between the elements of the origin and friend areas. The recorded pairs were as follows: {mantili, prabu janaka}, {ayodya, rama}, {wanara, anoman}, and {alengka, rahwana}. These pairs clearly reflect the intended functional mappings from each kingdom (origin) to its associated puppet character (friend), consistent with the concept that each element in the origin area is paired with exactly one element in the friend area, a key property of functions.



Activity 4
Creating the set of consecutive pairs of the *Wayang* Rama Sinta function

Write down the intersection points of the x-axis and y-axis of the Cartesian diagram of the *Wayang* Rama Sinta function created in the project 3. Write the intersection points in order!

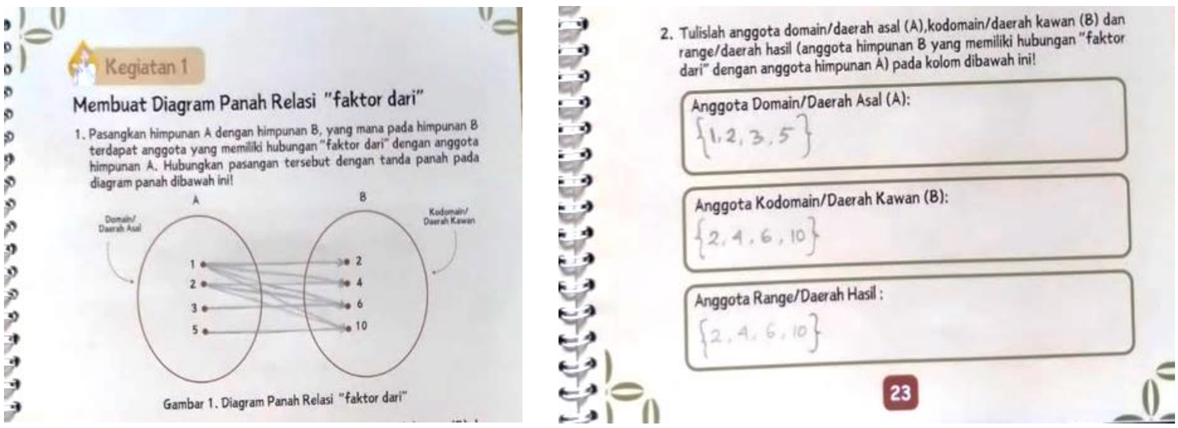
Figure 17. The process and results of presenting the Rama Sinta function in the form of a set of ordered pairs

The process of documenting the ordered pairs was accompanied by a critical validation step, where students engaged in collaborative discussions to verify the accuracy and consistency of the recorded pairs. This peer review process allowed students to assess each other's work, thereby enhancing their understanding of the function concept through active engagement and cooperative learning. The figure also highlights a group of students participating in this discussion, emphasizing the importance of collective reasoning in the mathematical learning process. By successfully identifying and recording the set of ordered pairs, the students demonstrated not only their ability to apply the concept of functions in a practical context but also their capacity for critical thinking and collaborative validation, which are essential skills in higher-level mathematical reasoning and inquiry.

Activity 4: Number Relations and Functions

Upon achieving an understanding of the concepts of relations and functions within the context of the Rama Sinta puppet, Activity 4 shifts students' focus towards more formal mathematical knowledge. In this phase, students engage with number-related relations and functions independent of the prior contextual framework. The activity comprises two primary components: the relation of "factor of" and the function of "root of."

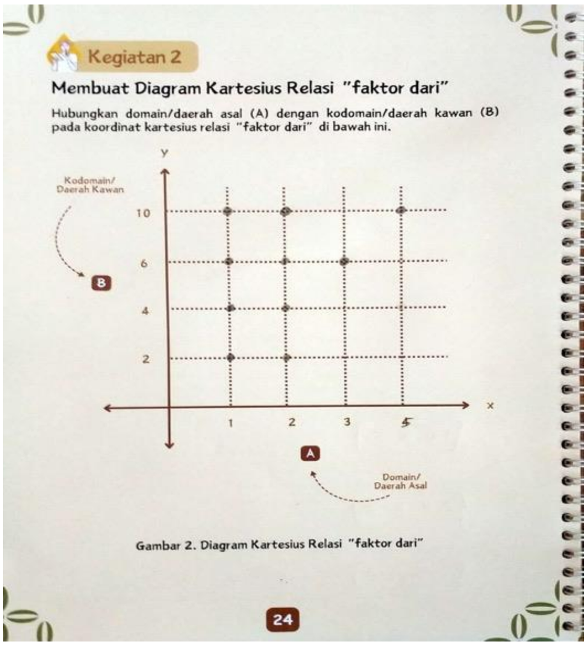
The teacher begins the session by guiding students through Activity 1 in their student books, where they are tasked with constructing an arrow diagram to represent the "factor of" relation. The outcomes of this exercise are presented in Figure 18. It illustrates both the process and results of students' presentations of the "factor of" relation as an arrow diagram. The findings demonstrate that students recognize the "factor of" relation as a correspondence between two sets, where elements of the origin set (set A) are factors of elements in the "friend" set (set B). For example, students connect the number 1 from the origin set to the numbers 2, 4, 6, and 10 in the friend set, indicating that 1 is a factor of these numbers. Similarly, they connect the number 2 from the origin set to the same numbers in the friend set, affirming that 2 is also a factor of these values. Further connections include linking number 3 from the origin set to 6 in the friend set, as 3 is a factor of 6, and number 5 to 10, as 5 is a factor of 10.



Activity 1 Create an arrow diagram of the relation 'factor of' 1. Pair set A with set B, in which set B has a member that has a 'factor of' relationship with a member of set A. Connect the pair with an arrow on the arrow diagram below! A Domain B Codomain Figure 1. Arrow Diagram of 'factor of' Relationship	2. Write the members of domain (A), codomain (B), and range (a member of set B that has a 'factor of' relationship with a member of set A) in the columns below! Domain Members (A): {1, 2, 3, 5} Codomain Members (B): {2, 4, 6, 10} Range Members: {2, 4, 6, 10}

Figure 18. Student process and outcomes in presenting the "factor of" relation through an arrow diagram

Subsequently, students present the "factor of" relation as a Cartesian diagram, as shown in Figure 19. In this representation, students accurately map the numbers on the x-axis to those on the y-axis that exhibit a "factor of" relationship. The range or result area of the "factor of" relation is identified by marking the points on the Cartesian diagram that group the numbers accordingly.



Activity 2
Create a cartesian diagram of the relation 'factor of'

Connect the domain (A) with the codomain (B) in the cartesian coordinate of the relation 'factor of' below.

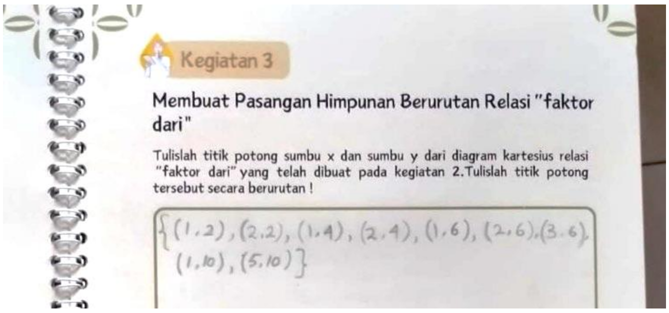
Codomain (B)

Domain (A)

Figure 2. Cartesian diagram of 'factor of' relation

Figure 19. Student process and outcomes in presenting the "factor of" relation as a Cartesian diagram

Next, students represent the "factor of" relation as a set of ordered pairs, as depicted in Figure 20. From the intersection points on the Cartesian diagram, students derive the following set of ordered pairs representing the relation: {(1,2),(1,4),(1,6),(1,10),(2,2),(2,4),(2,6),(2,10),(3,6),(5,10)}.



Activity 3
Create pairs of consecutive sets of the relation 'factor of'

Write down the intersection points of the x-axis and y-axis of the Cartesian diagram of the relation 'factor of' created in the project 2. Write the intersection points in order!

{(1,2),(1,4),(1,6),(1,10),(2,2),(2,4),(2,6),(2,10),(3,6),(5,10)}

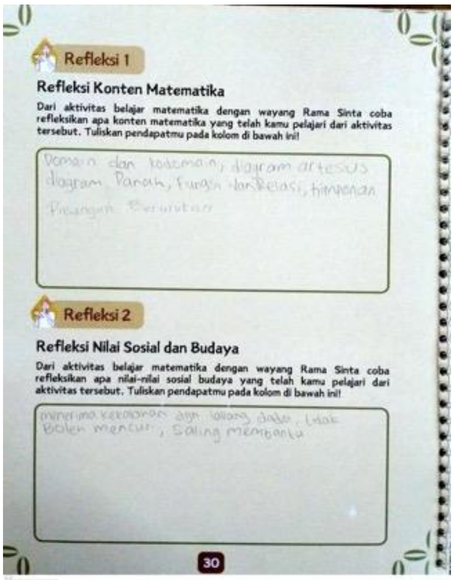
Figure 20. Presentation of the "factor of" relation as a set of ordered pairs

Once students have established an understanding of relations in formal terms, they proceed to explore functions, specifically the "root of" function. The activities for this concept mirror those of previous lessons. The teacher introduces the "root of" function, which establishes a correspondence between the numbers in the origin set {2, 3, 4, 5} and those in the friend set {4, 9, 16, 25}. Furthermore, students link the number 2 from the origin set to 4 in the friend set, as 2 is the root of 4. Similarly, they connect 3 to 9, 4 to 16, and 5 to 25, since 3 is the root of 9, 4 is the root of 16, and 5 is the root of 25, respectively.

After successfully constructing the "root of" function as an arrow diagram, students then represent it using a Cartesian diagram and a set of ordered pairs. Furthermore, students illustrate the intersection points on the Cartesian diagram, connecting the numbers from the origin set on the x-axis to the corresponding numbers from the friend set on the y-axis. Students list all identified pairs, resulting in the set {(2,4), (3,9), (4,16), (5,25)}. Finally, students verify that each element of the origin set corresponds to exactly one pair in the friend set, thus confirming that the relationship qualifies as a function.

Activity 5: Reflection on Learning the Relationship and Function of Rama and Sinta

The final activity in the learning sequence regarding the relations and functions in the context of the Rama Sinta puppet shadow involves a reflective analysis of both the mathematical content and the socio-cultural values that emerged throughout the learning process. Through this reflection, students were able to articulate their understanding of the mathematical concepts explored, which included relations, functions, domains, codomains, arrow diagrams, Cartesian diagrams, and sets of ordered pairs. In addition, students identified several socio-cultural values, such as the importance of graciously accepting defeat, avoiding theft, offering assistance to others, and respecting others' property. The results of students' reflections are presented in Figure 21.



Reflection 1
Reflection on Mathematics Content
From the activity of learning mathematics with *Wayang* Rama Sinta, reflect on what mathematical content you have learnt from the activity. Write your thoughts in the column below!

"Domain and codomain, cartesian diagram, arrow diagram, function and relation, set of consecutive pairs"

Reflection 2
Reflection on Social and Cultural Values
From the activity of learning mathematics with *Wayang* Rama Sinta, reflect on what social and cultural values you have learnt from the activity. Write your opinion in the column below!

"Accepting defeat gracefully, not stealing, helping each other"

Figure 21. Outcomes of students' reflections on learning relations and functions within the context of the Rama and Sinta puppet shadow

Retrospective Analysis

The implementation of the Ethno-RME approach, particularly through the context of the Rama Sinta puppet shadow, demonstrated significant effectiveness in enhancing students' understanding of mathematical concepts related to relations and functions. By incorporating culturally relevant materials, students engaged more deeply with the mathematical content, fostering a stronger connection between abstract mathematical concepts and their practical applications. This aligns with previous studies, such as those by Kolovou (2023) and Mark and Id-Deen (2022), which emphasize the importance of culturally relevant pedagogy in improving student engagement and comprehension within the domain of mathematics education.

In the introductory activity, students were introduced to the concept of relations through the puppet shadow mapping game. The diverse range of definitions presented by students, as depicted in Figure 13, reflects varying levels of comprehension, with some students providing contextualized definitions and others offering more generalized interpretations. This variation highlights the efficacy of using culturally rich contexts to bridge students' prior knowledge with new mathematical concepts, promoting critical thinking and practical application (Clements et al., 2023; Rosa & Orey, 2020; Alghiffari et al., 2024). These findings are consistent with Polman et al. (2021), who emphasized that contextualized learning enhances students' conceptual understanding in mathematics.

In Activity 3, as students advanced to the concept of functions, the integration of puppet characters facilitated an interactive and engaging learning environment. The mapping activity demonstrated that students grasped the key properties distinguishing functions from general relations, particularly the one-to-one correspondence characteristic. The teacher's strategic questioning effectively prompted students to reflect on and articulate these differences, thereby deepening their understanding of the concept of functions. This observation is in line with the work of Ran et al. (2022), who found that structured questioning significantly enhances students' comprehension of mathematical functions.

The subsequent activities, particularly the formal representations of relations and functions, further reinforced students' understanding. The arrow diagrams and Cartesian diagrams, as shown in Figures 15 and 16, provided evidence of students' ability to transition from concrete representations to more abstract mathematical frameworks. This progression is a crucial aspect of their mathematical development, enabling students to visualize and systematically represent relationships. These findings are consistent with the perspectives of Goldin (2020) and Mainali (2020), who emphasized the vital role of visual representations in deepening students' understanding of mathematical concepts.

In Activity 4, students' exploration of formal mathematical relations, such as "factor of," and functions, like "root of," demonstrated their ability to abstract concepts from the cultural context of the puppet shadows to broader mathematical principles. The students' presentations of these concepts in various formats, including Cartesian diagrams and sets of ordered pairs, showcased their proficiency in expressing mathematical relationships coherently and accurately. This finding supports the work of Krawitz et al. (2022), who advocate for the integration of real-world contexts into mathematics instruction to foster deeper conceptual understanding.

The reflection activity provided insight into the integration of socio-cultural values within the mathematics learning process. Students' articulation of values such as cooperation, integrity, and respect suggest that the Ethno-RME approach not only promoted cognitive development but also encouraged personal growth and ethical awareness (Yuliani et al., 2023; Ristiana et al., 2024; Pujiastuti et al., 2025). This dual emphasis on mathematical content and socio-cultural values underscores the holistic nature of the Ethno-RME framework, aligning with contemporary educational goals of nurturing well-rounded individuals (Prahmana, 2022; Prahmana et al., 2023). This perspective is supported by Sutomo (2014) and Meeran and Van Wyk (2022), who argue that embedding socio-cultural values within mathematics education can significantly enhance student engagement and academic achievement.

Finally, the results of this study affirm the effectiveness of integrating Javanese folklore as an ethnomathematical contexts into mathematics education, particularly through the use of the Rama and Sinta story. The findings suggest that culturally embedded learning experiences can significantly enhance students' understanding of complex mathematical concepts while simultaneously fostering essential socio-cultural values.

CONCLUSION

This study successfully designed a learning trajectory for the concepts of relations and functions using the Ethno-RME approach, contextualized within the cultural framework of the Rama and Sinta story. The learning trajectory consists of five sequential activities that guide students from concrete, situational experiences to the formalization of mathematical concepts. These activities include engaging with the Rama and Sinta puppet performance, participating in relation and function mapping games, learning about number relations and functions, and reflecting on the connections between the mathematical concepts and the narrative. The design of this trajectory is carefully aligned with students' cognitive development and mathematization processes, demonstrating the potential of integrating cultural contexts into mathematical instruction to enhance student engagement and understanding.

However, despite the positive outcomes of the implemented trajectory, several limitations of this study warrant attention. A key limitation is the absence of a comprehensive evaluation of students' understanding of relations and functions following the implementation of the Ethno-RME learning trajectory. Future research should include both quantitative and qualitative assessments to gauge the effectiveness of the activities in improving students' comprehension of these mathematical concepts. Additionally, there is a need for broader studies that extend the application of the learning trajectory to other educational settings, which would facilitate the establishment of this approach as a Local Instructional Theory. Such studies would contribute significantly to the growing body of knowledge on how cultural contexts, particularly those derived from ethnomathematics, can be systematically integrated into mathematics education.

This research is also limited by its focus on designing a learning trajectory specifically for relations and functions, utilizing the Ethno-RME approach within the context of the Rama and Sinta puppet story. Future research could expand upon this framework by developing learning trajectories for other mathematical topics and exploring the use of different cultural contexts. For instance, incorporating other puppet stories or cultural narratives could offer diverse insights into how cultural knowledge can enhance students' mathematical learning. In conclusion, this study provides valuable insights into the integration of cultural narratives with mathematical instruction, highlighting how the Ethno-RME approach can promote deeper understanding and socio-cultural awareness in mathematics education. Building upon these findings, future research should continue to explore and validate the effectiveness of the Ethno-RME approach across diverse educational contexts, further enriching the field of culturally relevant mathematics education.

Acknowledgments

The authors would like to formally acknowledge and express their sincere gratitude to Universitas Ahmad Dahlan, Universitas Potensi Utama, Universitas Sriwijaya, and SMP N 2 Secang for their invaluable support and collaboration throughout the course of this research. Furthermore, the authors extend their heartfelt appreciation to Laila Nurnaningsih for her role as a teacher model. Special thanks are also due to Hafizh Naufal Azmi, Eka Kevin Alghiffari, Arga Bagus Pratama, Dyah Aan Firmansyah, and Khairullah for their significant contributions to the collection and development of the media employed in this study.

Declarations

Author Contribution : RCIP: Conceptualization, Supervision, Validation, Writing - Original Draft, Methodology, and Writing - Review & Editing.

		IR: Investigation, Data Curation, Formal Analysis, Writing - Original Draft, and Resources. NRNP: Investigation, Formal Analysis, and Methodology. NR: Investigation, Data Curation, and Resources. RR: Formal Analysis and Writing - Review & Editing
Funding Statement	:	The authors express their gratitude to the Directorate of Research, Technology, and Community Service, Ministry of Education, Culture, Research and Technology, Indonesia for their support through the Penelitian Fundamental grant (Contract Number: 0459/E5/PG.02.00/2024; 107/E5/PG.02.00.PL/2024; 0609.12/LL5-INT/AL.04/2024, and 105/PFR/LPPM-UAD/VI/2024).
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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