Journal on Mathematics Education
Volume 14, No. 1, 2023, pp. 129-148

Integral (antiderivative) learning with APOS perspective: A case study

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Received: 22 December 2022 | Revised: 1 March 2023 | Accepted: 15 March 2023 | Published Online: 17 March 2023

Abstract

The objective of this study is to determine students' learning barriers in understanding integral concepts through their thinking processes with the perspective of APOS theory (Action, Process, Object-Scheme). This research applied qualitative research with a case study method. The samples of this research were 19 civil engineering students who had contracted calculus courses and who had been given a written test. The results of the written test were divided into three different categories – they are comprehension ability is high (score ≥ 75), medium ($60 \leq$ Score <75) and low (score < 60). Deep interviews were conducted with three representative students who took the written test and met the criteria for each group. The results of the interviews showed that students in the high category still had conceptual ontogenic learning obstacle despite passing the APOS path. Students in the moderate category be able to reach the encapsulation stage but they had not been able to de-encapsulate it to the process. They had conceptual and epistemological ontogenical learning obstacles. Whereas the low category students had the tendency to only reach the action stage and had difficulty in doing initial de-encapsulation due to lack of learning experience in the prerequisite materials. The learning obstacles they experienced were psychological and conceptual-ontogenical learning obstacles. Aggregately, the students tended to experience conceptual ontogenic learning obstacle. The result of this study is expected to be used as a basis for designing a hypothetical learning trajectory in future research.

Keywords: APOS Theory, Case study, Integral, Learning Obstacle

How to Cite: Nurhayati, L., Suryadi, D., Dasari, D., & Herman, T. (2023). Integral (antiderivative) learning with APOS perspective: A case study. *Journal on Mathematics Education, 14*(1), 129-148. http://doi.org/10.22342/jme.v14i1.pp129-148

Integral, as antiderivative integral or indefinite integral, is part of the integral developed by Gottfried Wilhelm Leibniz. Integral is a material that is closely related to derivatives, as explained in Anaya et al.'s research (2022). Integrals as antiderivatives and derivatives have an important role as the main operations in calculus. Integral as the antiderivative is further called the integral. Integral is a part of calculus, which is a universal science that serves as the foundation for modern technological development. Integral has an important role in various disciplines. It also takes part in the development of human thinking power. Many studies are related to the use of integrals in technological developments to overcome various problems in the world, as can be seen in the research results of Nedelcu et al. (2020), where integrals were used to signal model in earthquake structures; then Albertini (2021), who used integrals in formulating digital wave structures; and Wahba (2022), who studied integrals in fluid mechanics.

In line with the previous elaboration, the integral is a very urgent material. Integral is a subject that must be studied by all engineering students in Indonesia, including civil engineering students. Civil engineering students use the integral to calculate the moment of inertia, volume, area, center of gravity, all of which are used as tools in designing the strength and durability of a building. Students must master the comprehension of integral concepts to later be applied in solving problems encountered in the real world (Johnson et al., 2022). However, learning integrals is not easy. Many students experience difficulties in calculating integrals, especially in distinguishing variables and constants in the integration process (Nguyen & Rebello, 2011). This failure provides evidence that the students do not understand fractional terms in integrals and do not understand the concept of the accumulation of fractional quantities. This study examines what makes it difficult for students to study the integral further from their thinking processes and learning experiences in the case of engineering students.

Student learning obstacle in integral learning can be identified using questions that measure the understanding of concepts with the APOS framework. Kilpatrick (2001) stated that the indicators of understanding concepts are (1) reviewing the concepts learned; (2) identifying the events and non-occurrences; and (3) classifying objects according to certain characteristics based on the concept by introducing the concept and algorithmically applying the concept, (4) viewing how the concepts and methods relate to each other, and (5) explaining the facts about the concept features. The comprehension of mathematical concepts is important for enhancing students' mathematical thinking ability, especially in integral learning. The understanding of integral concepts would be empirically established from student learning experiences. Students' understanding of integral concepts would be seen from their learning process. This is in line with what Maulida et al. (2020) stated about the basis of learning is the teaching and learning process, which is not only focused on student achievement, but also on the learning process. Students' experience becomes the basis of understanding the meaning of the concepts in integral learning. Students' processes in learning integrals would be influenced by their previous experience in studying derivatives. The process of establishing knowledge about integral understanding is hierarchical because it is closely related to previous derived knowledge. Students must study integrals deeply and systematically to understand them. This is reciprocally to what Ernes (2016) said, particularly that students must understand mathematical concepts in an organized and deep manner.

Several studies have applied the APOS theory (Action, Process, Object-Scheme), including Baker et al., who stated that this research is the development of a scheme in the context of APOS theory which is used to study students' responses to analyze students' understanding of complex calculus graphic problems (2000) The process of mental coordination of learning with previous learning experiences is closely related to the APOS theory (Dubinsky, 2014). Edyta Nowi (2014) and Afgani et al. (2019) conveyed that mental objects in the APOS theory of mathematical thinking are used to support the understanding of mathematical concepts. Tokgoz (2022) examined the ability of students to answer calculus-based questions based on their ability to apply algebra to calculus concepts by classifying student responses using APOS theory. Student understanding of exponentials and how to construct exponential and logarithmic functions using the APOS theory approach was researched by Berrioz & Planell (2022). APOS theory is used as a framework to find out students' understanding of the concept of circles (Kemp & Vidakovic, 2023). We used the APOS theory as a framework to find out the learning obstacles of students in understanding integral concepts which is the novelty of this research.

The learning process through the perspective approach of APOS Theory can establish new knowledge. Integral learning will likewise become new knowledge for engineering students. New established knowledge must be understood and believed to be true by the students. The thinking process of students in understanding the integral as a process of forming new knowledge can be seen through the perspective of APOS theory, with the flow seen in Figure 1.

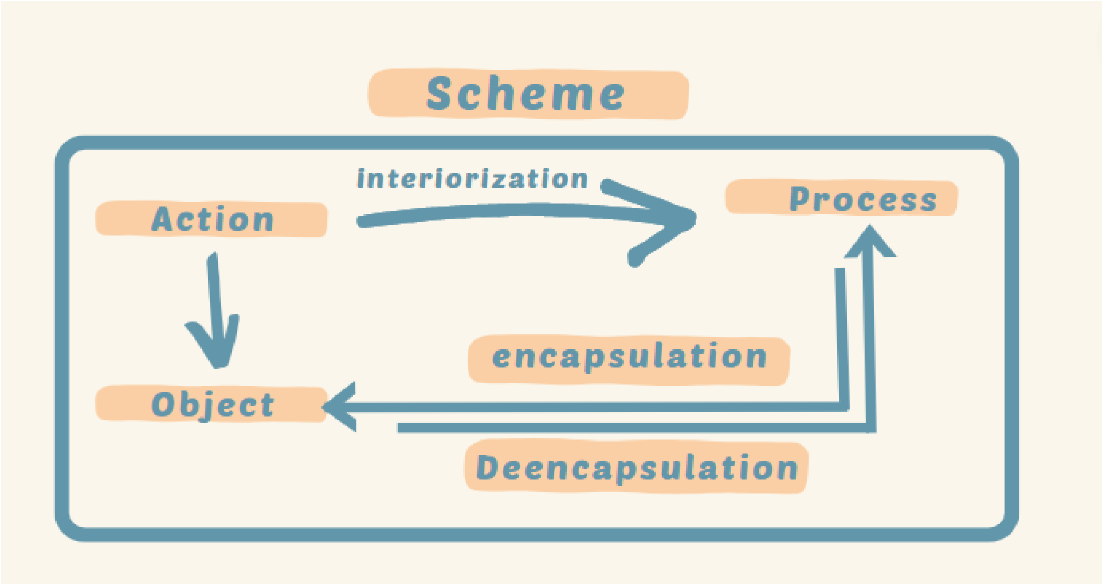

Figure 1. APOS theory flow

A student starts an action when external stimulus occurs. An action in integral learning starts with the presence of external stimuli through perception. An action is a mental reaction existing within students by interiorizing mental objects through their perceptual and memorial knowledge. The action takes place by de-encapsulating the mental objects in the memorial knowledge in response to these external stimuli. De-encapsulation means that the students describe mental objects from the mental processes they go through (Suriyah et al., 2022). A process is a mental construction within students because of repeated and independent interiorization. The interiorization in students is mentally coordinated with mental objects, which are initially random and later rearranged in the encapsulation process to form a new object. Objects are the result of encapsulation of processes. Newly formed objects can be de-encapsulated again into a process where this can happen several reversions, resulting in the formation of a new schema (Jojo, 2013). A scheme is a collection of actions, processes, and objects as a whole to form a new object in the student's memorial knowledge. In line with the research findings of Bajo-Benito et al. (2021), the students' ability to understand integral concepts is built through mental structures comprising actions, processes, objects, and previous schemes. Referring to Adeniji and Baker (2022), the conceptual understanding is seen from cognitive constructions related to integral concepts.

The interiorization involves mental actions that randomly occur in memory knowledge with the experience someone has been through. This mental action is one way of thinking and understanding integral matter. This is in line with Harel's answer to the philosophy of "What is mathematics?" through a triadic model, the process of which is described in Figure 2.

This triadic relationship explains how the originally random mental actions occurring within students, such as interpreting, guessing, searching, solving problems, and so on, would be rearranged to form a flow of thought that intervenes the formation of Way of Thinking (WoT). This leads to the mathematical mental objects referred to in the APOS framework. Through this flow of thinking, a process happens that brings someone to an understanding of Way of Understanding (WoU). This flow of thinking becomes the rationale for this research which aims to find out what learning obstacles students have in understanding concepts in integral learning through their thinking processes, as viewed through the perspective of APOS theory and assisted by the triadic model.

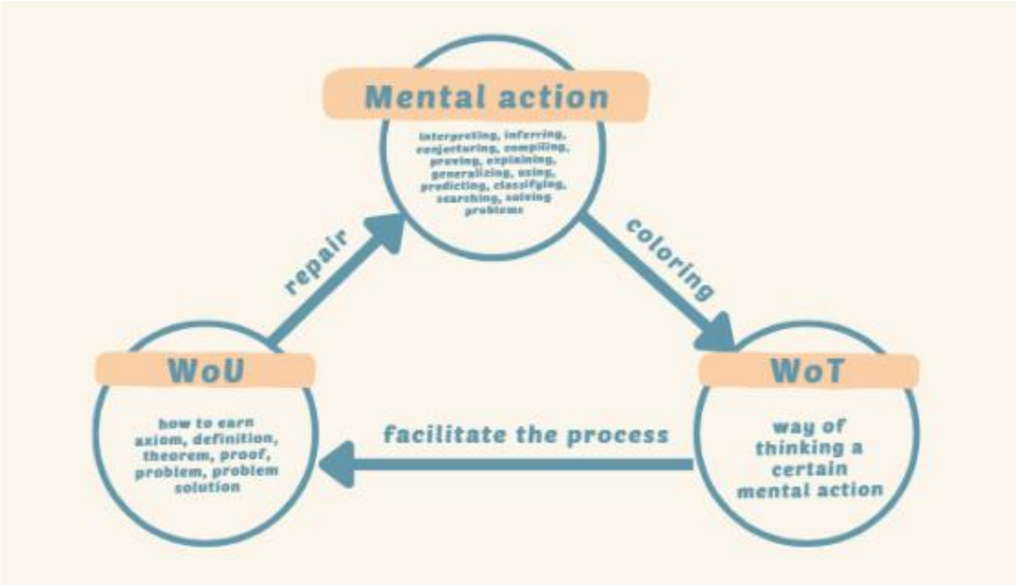

Figure 2. M Triadic model

METHODS

Research Design

This research is qualitative research with a case study (Feagin et al., 1991; Flyvbjerg, 2011; Yin, 2012; Creswell, 2018). This case study belongs to an intensive study of one unit with the objective to be able to represent a series of units for larger cases (Thomas, 2021). Case studies are heuristics that are continuously functioning to focus one's attention to a phenomenon by collecting evidence that involves a careful description of the phenomena that happens based on facts (Feagin et al., 1991). A case study of a phenomenon cannot fully comprehend all phenomena without giving meaning to the experience of the research subject. This study employed cased studies as its primary method for identifying student learning barriers by analyzing civil engineering students' learning experiences while studying integral concepts from an APOS (Action, Process, Object-Schema) theoretical perspective approach.

Participants

This research was conducted for approximately two months from September 1 to October 28 in 2022 at Sangga Buana University, Indonesia. The research sample selection applied a purposive sampling technique (Etikan, 2016). Purposive sampling technique is sampling with certain considerations. This qualitative research does not generalize to the population from the selected sample Pre-research procedures conducted by the researchers are as follows:

1. Data collection for regular class of Civil Engineering students in semester 3,
2. Data collection of students who passed basic Mathematics courses as prerequisite courses,
3. Data collection of students who have passed calculus,
4. Data collection of students who are willing to attend on September 9, 2022, to take the written test courses.

Based on the pre-research results, the researchers obtained the 19 research participants of the third-semester students majoring Civil Engineering. The research participants consisted of 6 female students and 13 male students. The ages of the study participants ranged from 19 to 22 years old. Students with this age range already have sufficient experience in learning and are more prepared to be interviewed in research. This study was divided into five stages which are explained as follow.

Stage 1

As shown in Figure 3, participants were given written test questions about their comprehension of integral concepts. The concept comprehension test questions were given to the students as part of research conducted by Antonio Rivera-Figueroa (2019). Four essay questions were given to the participants in order to assess their understanding of the integral concept. The instrument validation of the concept comprehension test questions was carried out by expert judgements. The legibility of the questions, whether the test questions are appropriate can be seen from the results of the participants' answers. The results of the participants' answers were scored using the Holistic Scoring Rubrics. Moskal (2000) stated that the Holistic Scoring Rubric is an aggregate assessment without separately dividing the components. The rubric was adjusted to reflect indicators of understanding the integral concept.

Stage 2

After carrying out the written test, the evaluation to the test results of the participants were done in the second stage. The results of the participants' work were grouped into three categories. Using the Holistic Scoring Rubrics, three categories for integral concept comprehension test results were defined, which can be seen in Table 1 by referring to the Holistic Scoring Rubrics.

Table 1. The criteria of test results

Score	Criteria
Score ≥ 75	Comprehension ability is high
$60 \leq$ Score <75	Comprehension ability is moderate
Score < 60	Comprehension ability is low

Stage 3

In the third stage, an in-depth interview was conducted with representative from each group based on the mentioned criteria. Participants who will be interviewed are selected based on the results of a purposive written test from each group. The selection of one participant for in-depth interviews from each group is sufficient for this case study, because this qualitative research does not generalize to the population of the selected sample. The interview was done to confirm the responses of their written answers. The questions given in the interview process were adapted to the APOS framework. The researchers analyzed the participants' answers to whether they were already at the stage of action, process, object, or scheme. Participants used their experiences during integral learning to reveal the reasons. Here, the participants' perceptual knowledge and memory were visible. Posit to the interview result, the researcher analyzed the participants' learning barriers in integral learning.

Stage 4

Stage four is research documentation, which includes collecting the written test results to be on a soft file. The researchers also documented the progress of the written test with photographs, as well as the outcome of the interview in a form of a recording of the participants.

Stage 5

The final stage of this study was to analyze the data based on the research flow as seen in Figure 3.

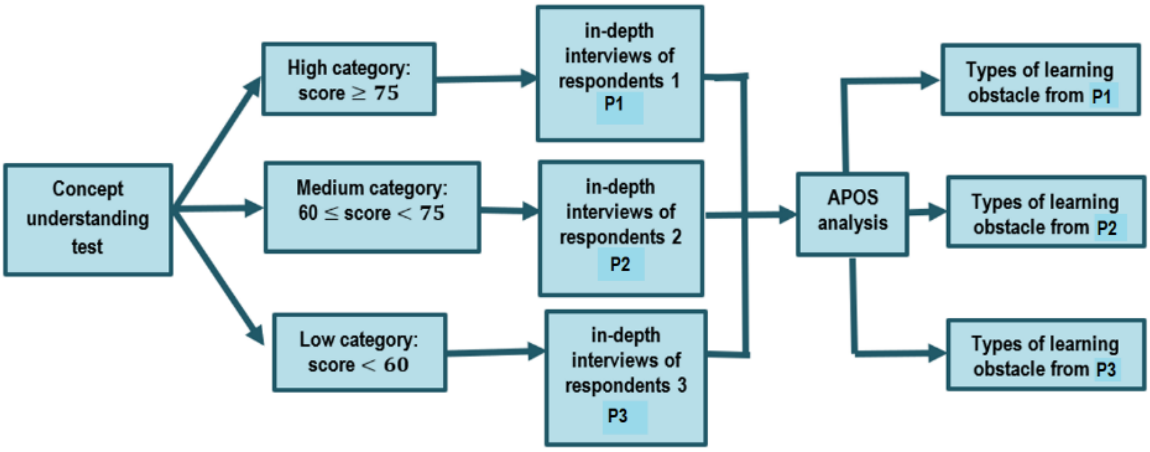

Figure 3. Data analysis flow

The data analysis of this research was conducted with the reference to Huberman and Miles (2002). In the stage of qualitative data analysis, data reduction happened in the data presentation stage to draw the conclusion and to verify the data. Validation of the test question about concept comprehension was performed by the participant themselves. The result of the participants' response reflected 'whether the questions were ambiguous or not, whether the questions could be understood or not' through the justification of the participants. The validation of the data was required in order to be accounted for as justification for the data taken. The researchers checked the validity of the data to minimize the errors prior to further analysis of the data. The data validation in this study was conducted through triangulation. In-depth interviews (IDI) served as triangulation of data sources for validation testing in qualitative inquiry (Carter et al., 2014). This IDI revealed a broader understanding of the participants through their experiences. The justification of the test results was revealed based on the results of the IDI interviews, thus obtaining the true epistemic information and the intact description of the participants' information in thinking based on the APOS theory. APOS was categorized based on its characteristics in Table 2, which is the modification of research by Tokgoz (2022). Based on the analysis of IDI interviews with the APOS framework, the integral learning of the participants was uncovered. The characteristics in each of the APOS categories below are the basis of the instrument for in-depth interviews. In this case the expert judgement becomes a validator of the validity of each question asked.

Table 2. APOS characteristics

Category	Characteristics
Action	• Providing responses to questions from the interviewer • Conducting initial de-encapsulation of perceptual and memorial knowledge • Rewriting questions • Not answering the test question • Being able to substitute the exponential value of the integral in the given integral formula • Write down answers even if they are not correct • Calculating the derivative of a linear function

Category	Characteristics
Process	• Being able to explain procedural steps in integral calculations • Being able to calculate integral constants and reduce them back to the original function • De-encapsulating material derived from linear functions • Being able to calculate the result of substitution of the exponential value of the integral in the given integral formula • Being able to write the integral form of the derivative of a function • Encapsulating of prior knowledge to calculate integrals
Object	• Being able to explain the properties of integral linearity • Being able to clearly distinguish variables and constants in the integrant • De-encapsulating algebraic knowledge and integer operations • Using algebraic procedures and calculating integrals using integer operations
Schema	• Being able to use integral formulas to solve problems • Being able to correctly determine the original function using integrals • Being able to link materials needed to calculate integrals • Being able to prove that they understand the results of the integral calculation if it is reduced to an integral • Being able to calculate integrals using the linearity properties of integrals • Being able to calculate integrals using formulas $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ correctly • Being able to formulate the right integral solution by implementing all actions, processes, and objects

RESULTS AND DISCUSSION

Participant Written Test Results

The results of the participants' written test in the form of a description of the answers related to the indicators of the linkage of concepts and procedures can be seen in Table 3.

Table 3. Test results for indicators of the relationship between concepts and methods

Question	Description of Participants' answer	Participants
Calculate: (i) If $y = ax + b$, for $a, b \in R$ then $\frac{dy}{dx} =$ (ii) $\int adx =$ (iii) What do you understand about	• No answer	• P3, P4, P11, P14
	• Rewrite questions	• P1, P2, P5, P6, P7, P8, P13
	• (i) $\frac{dy}{dx} = 0$ (ii) $\int adx = ax + C$ • (iii) No answer	• P12
	• (i) $\frac{dy}{dx} = a$	• P15, P16, P17, P18, P19

Question	Description of Participants' answer	Participants
the relationship of the questions (i) and (ii)	(ii) $\int a dx = ax + C$ (iii) Statement (i) is a function derivative and if it is integrated it becomes statement (ii), because integral is the opposite of derivative	

The results of the participant's written test in the form of a description related to the indicators of applying the concept algorithmically to question number 2 can be seen in Table 4.

Table 4. Test results for indicators of the relationship between concepts and methods

Question	Description of Participants' Answer	Participants
Pay attention to the following integral formula: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$ Based on the above formula, calculate! (i) $\int x^{-2} dx$ and (ii) Can you calculate $\int x^{-1} dx$ Elaborate!	• Unable to answer (only rewrite questions)	• P2, P3, P4, P5, P6, P8, P13, P14
	• Only answered part (i) with an inaccurate calculation (i) $\int x^{-2} dx = -\frac{1}{x} + C$	• P1, P7, P9
	• (ii) $\int x^{-1} dx = \frac{1}{0} x^0$ it cannot be calculated, because it is divided by zero.	• P10, P11, P12, P16, P18, P19
	• (i) $\int x^{-2} dx = -\frac{1}{x} + C$ (ii) $\int x^{-1} dx = \ln \lvert x \rvert + C$	• P15, P17

The next indicator of understanding the concept was classifying the objects according to certain characteristics based on the concept. The results of the participants' written test related to these indicators for question number 3 can be seen in Table 5.

Table 5. The indicator test results of classifying objects according to certain characteristics based on the concepts

Question	Description of Participants' Answers	Participants
Calculate $\int (x+y)^2 dx$	• No answer	• P1, P2, P3, P4, P11, P14, P17, P18, P19
	• Up to the stage of describing the quadratic form and still quite incorrect	• P5, P6, P8, P9, P10, P13
	$\int (x+y)^2 dx = \frac{1}{3}(x+y)^3 + C$	• P7
	• $\int (x+y)^2 dx = \frac{1}{3}x^3 + x^2 y + \frac{1}{3}y^3 + C$	• P12
	• $\int (x+y)^2 dx = \frac{1}{3}x^3 + yx^2 + y^2 x + C$	• P15, P16

The next indicator of understanding the concept was applying the concept with an algorithm. The results of the participants' written test related to these indicators for question number 4 can be seen in Table 6.

Table 6. The indicator of the test results in the application of algorithmically concepts

Questions	Description of Participants' Answers	Participants
If the derivative of a function says $\frac{dy}{dx}=2x+3$ Functions score 1 to $x=0$ Determine the formula of the function.	• No answer and stated to forgot how to do the question • Answer up to stage $y=\int(2x+3)\,dx=x^2+3x+C$ • $y=\int(2x+3)\,dx=x^2+3x+C$, substitute $x=0$ to the obtained function $C=1$, then the obtained function is $y=x^2+3x+1$	• P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P19, P13, P14 • P16, P17 • P12, P15, P18

The written test results of the 19 participants revealed a minimum score of 10 and a maximum of 95. The descriptive statistical values had a mean of 35.16, a median value of 20, and a mode value of 20. The results of the participants' integral concept understanding test were grouped into three categories. There was a total of 14 participants in the low category, three participants in the medium category, and two participants in the high category. The percentages of these groups can be seen in Figure 4.

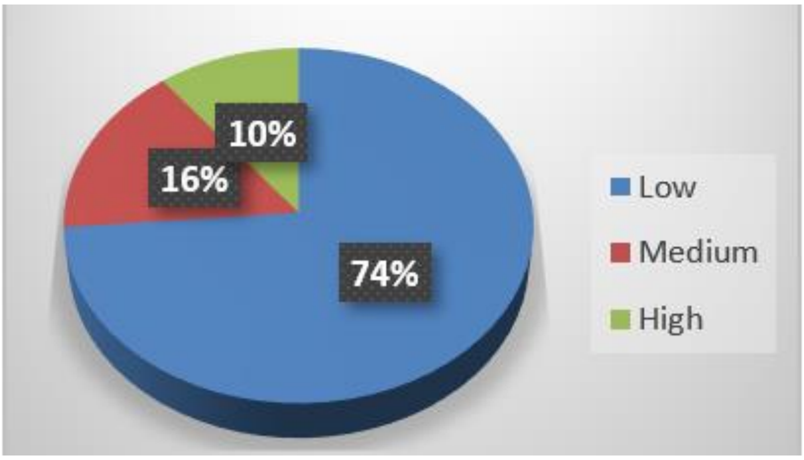

Figure 4. The percentage of category score group

The participants who had the ability to understand the concept of high integral was only 10%, while 16% had a moderate understanding of concepts, and those with low abilities were 74%. Based on these results, it can be assumed that the participants' ability to understand the integral concept still tends to be low. Based on the results of the written test, the researcher selected three students with unique answers to represent each of the three categories. The results of the written test did not completely reveal the participants' knowledge of understanding the integral concept. The researchers used epistemic virtue to be more careful in drawing conclusions, rather than relying solely on the results of written tests to gain epistemic knowledge (Greco & Turri, 2017) . Participants' inability to answer the test did not imply that they did not understand the integral, as we do not know what is in the participant's mind. It is possible that participants who fall into the low category could explain integrals when interviewed by IDI for clarification, and vice versa. To see further confirmation of the answers to the participant's written test questions, it can be seen from the results of the in-depth interviews in the next section.

The Test Results of Participant IDI Interview and Learning Obstacles

Based on the results of the written test above, the researcher took one participant from each category group. First participant P1, second participant P2, and third participant P3 were interviewed by IDI, along with interviewer I. The following are the results of the interview with P1 as a representative of the high category group. The interview process was conducted using the APOS perspective to find out whether learning obstacles existed and, if so, what types of learning obstacles were included. Learning obstacles experienced by the students can be grouped into three categories: ontogenic obstacles, epistemological obstacles, and didactical obstacles (Brousseau, 2002). According to Brousseau, ontogenic is a barrier related to ontogenetic. There are three types of ontogenic learning obstacle, in particular, psychological, instrumental and conceptual. Psychological learning obstacle means that the students are not mentally ready either in the form of enthusiasm for learning or interest in the subject matter. Instrumental obstacles involve technical obstacles that prevent students from participating in teaching and learning activities properly. Conceptual ontogenic obstacle relates to the conceptual level being taught that is not in accordance with the student's condition, which can be too high or low. According to Duroux (Brousseau, 1997), epistemology obstacle is a learning obstacle primarily caused by errors in understanding the concept of early learning.

The results of the interviews with the three respondents are presented below. The results of the R1 written test can be seen in Figure 5.

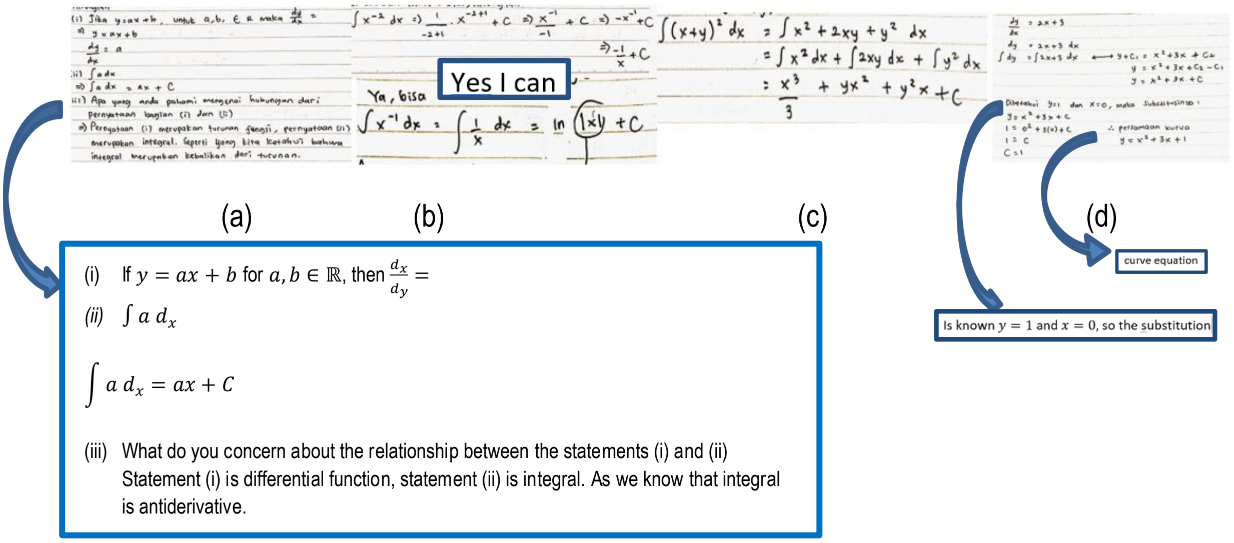

Figure 5. P1's answer

The results of the interview with P1 confirmed that the written answer to question number 1 in Figure 5(a) showed no learning barriers. P1 acted in responding to questions quickly and precisely by doing the initial de-encapsulation of perceptual knowledge and its memorization. P1 could calculate the constant integral and reduce it back to the original function, as well as properly de-encapsulate the material derived from the linear function. P1 performed the encapsulation of his prior knowledge to correctly calculate the integral of a constant function, thus forming a schematic to solve problem number

1. P1 could solve problem number 2 (see Figure 5(b)) and number 4 (see Figure 5(d)) properly. All actions, object, and processes could be passed by P1, thus forming a new scheme that could calculate integrals using formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ correctly and could use integral formulas to solve problems. P1 showed learning barrier when confirming answer number 3 (see Figure 5(c)). The following is an excerpt of the conversation with P1.

I	:	P1, can you calculate $\int (x+y)^2\, dx$?
P1	:	Yes, ma'am. It is done by elaborating inside the parentheses first, which becomes $\int \left(x^2 + 2xy + y^2\right) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{3}y^3 + C$
I	:	What is the variable here?
P1	:	x, mam. Because it is *dx*
I	:	What is *y*?
P1	:	It is a variable, ma'am. But it is... no, no. Err, well, I think *y* is the constant.
I	:	Previously, you said that *y* was not a variable, but it can still be calculated using the previous formula, isn't it?
P1	:	Well, I think it cannot be calculated, ma'am. It is wrong. It should be $\frac{1}{3}x^3 + \frac{1}{2}.2x^2y + y^2x + C$
I	:	Why so?
P1	:	Ma'am, I think the previous one is wrong. Because *y* is the constant. So, it is not $\frac{1}{3}y^3$ but y^2x. I am sorry, ma'am. I think I've been so inattentive that I was not aware of the mistake. So, it is the constant.

From the conversation, it can be seen that P1 had a problem when encapsulating the calculation process of $\int \left(x^2 + 2xy + y^2\right) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{3}y^3 + C$ by using the properties of integral linearity to be a new mental object, it is the integral result. P1 is challenged by a lack of understanding of algebraic concepts for variables and constants. Therefore, P1's learning obstacle is ontogenic (Sulistiawati, 2019). Based on Table 2, to facilitate the process of analyzing the results of in-depth interviews, the researchers put them in the form of Table 7. The rubric in the instrument has been previously tested and validated with the help of expert judgement. Table 7 summarizes P1's interview with the APOS flow and the obstacles he experienced (remarks: ✓ was done, and ✗ was not done).

Table 7. The result of P1's interview

Question number	Action	Interiorization	Initial de-encapsulation	Process	Encapsulation	object	De-Encapsulation	Scheme	Learning Obstacle
1	✓	✓	✓	✓	✓	✓	✓	✓	✗
2	✓	✓	✓	✓	✓	✓	✓	✓	✗
3	✓	✓	✓	✓	✗	✗	✗	✗	Conceptual ontogenic

Question number	Action	Interior ization	Initial de-encapsula tion	Process	Encaps ulation	object	De-Encapsula tion	Scheme	Learning Obstacle
4	✓	✓	✓	✓	✓	✓	✓	✓	×

Furthermore, the results of the conversation excerpt with P2 to confirm his written answer can be seen as follow.

I : Suppose one function $y = ax + b$, for $a, b \in R$, can $\frac{dy}{dx}$ be calculated?
P2 : $\frac{dy}{dx} = b$
I : why *b*?
P2 : Because we are going to find the x, ma'am.
I : R2, do you know what $\frac{dy}{dx}$ means?
P2 : I do, mam. It means the derivation of *y* to *x.*
I : Do you still remember about derivation materials, R2?
P2 : I do mam, I still remember about it. The derivation is not *b*, but the derivation of $\frac{dy}{dx} = a$ because there is *x,* ma'am. I used to be taught that way.
I : How about you, R2? Do you know what is $\int a\, dx$?
P2 : It is the derivative of integral a. If I am not mistaken it becomes $ax + C$
I : What happens if it is derived again?
P2 : Mmm, well .... I am confused, ma'am. I don't know. I've forgotten about the derivative material.

From the conversation, it can be seen that P2 took action by answering the derivative even though it was not correct. P2 could not perform the initial de-encapsulation of derived and integral material. The process occurred by calculating the constant integral but failing in deriving it back to the original function, resulting in no encapsulation did not occur, as well as no formation of an object that is the result of the integral and reducing it back to a function. As a result, the scheme was not formed. P2 experienced conceptual ontological learning obstacles (Suryadi, 2013) and epistemological obstacles (Brousseau, 2002). Furthermore, the learning obstacle was found during the confirmation interview for question number 2 as follows.

I : How can we find $-x^{-1} + C$?
P2 : The rank is stored forward, then the power follows the formula, ma'am. The x power is increased by 1, the n is -2 plus 1, put forward
I : How about *b*?
P2 : $\frac{1}{-1+1} x^{-1+1} + C$ equals 1 + C
I : How if 1 is divided by 0?
P2 : 0
I : x power 0?
P2 : 1
I : If 0 multiplied by 1?

P2 : 0, ma'am…
I : So, the answer is?
P2 : 0 plus C, ma'am.

Based on the excerpt of the conversation, the learning obstacle of P2 was in the calculation of numbers and algebra. This is included in the conceptual ontogenic learning obstacle. P2 confirmed to the answer of question number 3; however, no learning obstacle was found in the confirmation of question number 4.

I : P2, how did you obtain $\int (x+y)^2\, dx = \int x^2 + 2xy + y^2\, dx$
P2 : The internal is multiplied $x+y$ with $x+y$
I : Which one is the variable in the integral?
P2 : X, mam.
I : Why did you obtain $\frac{1}{3}y^3$? P2's answer is $\int x^2 + 2xy + y^2\, dx = \frac{1}{3}x^3 + 2xy + \frac{1}{3}y^3 + C$
P2 : Because it has power, we can use the formula of integral, mam.
I : P2, did you find any obstacles in learning integral?
P2 : I have some trouble in memorizing the formula. Since senior high school, I have gotten bad grades, ma'am, and I don't understand it. I found that the teacher was not good at explaining it. I can understand it if someone explains it to me. I also learn from YouTube. If it's just from writing, I would get confused when entering it into the formula and would wonder why it was the answer.

In the excerpt of conversation with P2 to confirm question number 3, it can be seen that P2 had difficulty in distinguishing integrant variables and constants. P2 had difficulty in remembering the integral formula so he couldn't do integral calculations properly. The learning obstacle experienced by P2 is a conceptual learning obstacle. More vivid vision of the learning obstacles experienced by P2 can be seen in Table 8.

Table 8. The interview results of P2

Question Number	Action	Interiori-zation	Initial De-Encapsulation	Process	Encapsulation	Object	De-Encapsulation	Schema	Learning Obstacle
1	✓	✓	✓	×	×	×	×	×	Ontogenic and conceptual epistemology
2	✓	✓	✓	✓	×	×	×	×	Conceptual ontogenic
3	✓	✓	✓	✓	✓	✓	✓	×	Conceptual ontogenic
4	✓	✓	✓	✓	✓	✓	✓	✓	×

We confirmed with the third respondent, P3, who belonged to the respondents with low comprehension abilities. Interviews were conducted to confirm the answers from P3. The following is an excerpt of an interview conducted with P3.

I : P3, do you know $\frac{dy}{dx}$?
3 : I am sorry, I forgot it, ma'am.
I : How about $\int a\ dx$?
P3 : I also forgot it, ma'am.
I : Have you learned about derivatives and integrals
P3 : Yes, ma'am. I have.
I : If I have y = 2x + 3 so $\frac{dy}{dx} = 2$,and y = 3x -1, so $\frac{dy}{dx} = 3$. Now, please calculate P3 if $y = ax + b$ so $\frac{dy}{dx} = ?$
P3 : 1, ma'am.
I : Why 1?
P3 : mmmmm……1 ma'am
I : How about question 2. Can you calculate the integral, P3?
P3 : Is this right, ma'am? $\frac{1}{n+(-2)} x^{n+(-2)}$
I : If adjusted to the formula, how much is n?
P3 : -2, ma'am.
I : Please change n withs -2, can you calculate -2 +1?
P3 : 1, ma'am.
I : Do you somehow face some problems in calculation, P3?
P3 : You are right, ma'am. I am slow in calculation and even more if negative exists.
I : Can you calculate this?
P3 : Yes, ma'am. So, $\frac{1}{-1+1} x^{-1+1} + C = \frac{1}{0} x^0 + C = 1x^0 + C = 1.0 + C = 1 + C$. Oh, no, this is wrong. Wait, it should be 0 plus C
I : How about questions number 3 and 4?
P3 : I do forget ma'am what should be done.
I : Do you like math?
P3 : I used to like it when I was in primary school. When I found that *x* does come a lot, I found myself confused and I could not comprehend it.

Based on the interview excerpt, P3 carried out the action for the whole while interiorizing only the initial part. De-encapsulation only occurred once at the time of calculating $\frac{1}{-1+1} x^{-1+1} + C = \frac{1}{0} x^0 + C = 1x^0 + C = 1.0 + C = 1 + C$ although the mental object summoned wasn't quite right yet. P3 struggled to interiorize mental objects that take a long time to de-encapsulate, such as derived material, integer calculations, substitution, and elaboration of algebraic forms. This identified P3 as experiencing conceptual ontogenical learning barriers. In addition, P3 tended to have difficulty finding memorial knowledge about the prerequisite material; therefore, encapsulation from process to object and vice versa did not occur. P3 tended to lack motivation and interest in studying integrals, which suggests that P3 was experiencing psychological ontogenic barriers. Table 9 shows the learning obstacles as well as the APOS flow that P3 went through in greater detail.

Table 9. The result of P3's interview

Question number	Action	Interiorization	Initial De-Encapsulation	Process	Encapsulation	Object	De-Encapsulation	Schema	Learning Obstacle
1	✓	✓	×	×	×	×	×	×	Conceptual ontogenic
2	✓	✓	✓	✓	×	×	×	×	Conceptual ontogenic
3	✓	×	×	×	×	×	×	×	Ontogenic conceptual and psychology
4	✓	×	×	×	×	×	×	×	Ontogenic psychology and conceptual

In APOS, an action can be considered as a transformation of a mathematical object from the individual as an external factor (Díaz-Berrios & Martínez-Planell, 2022). All respondents with their perceptual knowledge could act. It differs from processes, objects, and schemes that occur within individuals and involve mental action, which not all individuals can perform. These limitations can be in the form of psychological limitations and knowledge possessed by students (Brousseau, 2002; Brown, 2008; Aebi & Linde, 2015). The learning barriers experienced by most of the participants were conceptual ontogenic obstacles, which could be interpreted as a type of learning difficulty related to students' low initial math skills or prerequisite material, namely operations on numbers and algebraic operations. This is due to a lack of previous students' mathematics learning experience (Moru, 2007; Ferdianto & Hartinah, 2020). The group with high understanding ability experienced obstacles when encapsulating previous knowledge to calculate integrals; it is the translation of algebraic forms, so when calculating $\int\left(x^2+2xy+y^2\right)dx=\frac{1}{3}x^3+\frac{1}{2}x^2+\frac{1}{3}y^3+C$, x and y are both subjects to integral formula rules. The obstacle P1 experienced was not being able to clearly distinguish the variables and constants in the integrant, which were indicators of the object (see Table 2). The percentage of APOS passed by P1, who is in the high understanding group, can be seen in Figure 6.

The action was carried out perfectly, while processes, objects and schemes reached 75% with conceptual ontogenic learning obstacles. The second group of understanding the integral concept was represented by P2, who experienced obstacles during the initial de-encapsulation to form the process of calling derivative material, due to their incorrect understanding of the derivative of linear functions taught by educators, such as when looking for derivatives $y=ax+b$, P2 with high certainty answered $\frac{dy}{dx}=b$. As a result, P2 experienced epistemological obstacles. Like P1, P2 experienced the problem of not being able to distinguish between variables and constants in the integrand, which is an indicator of the object. In addition, the problem of integers is also that P2 tended to be unable to calculate such integers $\frac{1}{-1+1}x^{-1+1}+C$, where the result becomes 1 + C. P2 experienced a conceptual ontogenic learning obstacle. Perfectly executed action. Process reached 75%, object 50% and schema 25%. Finally, the group with low understanding of integral concepts experienced obstacles from the initial de-encapsulation of derived material. Prerequisite materials, such as integer operations and the translation of algebraic forms were the main obstacles experienced by P3, so conceptual ontogenic becomes the main obstacle for P3. Furthermore, psychological factors such as motivation and interest in learning were also the main causes of P3's difficulty understanding this integral concept. The APOS flow that P3 went through (see Figure 6) is as follows: the action is 100%, the process is 25% while there are no objects and schemes. P3 had difficulty to comprehend integrals, so understanding integrals as mental objects could not be achieved by P3. Then, knowledge is not formed.

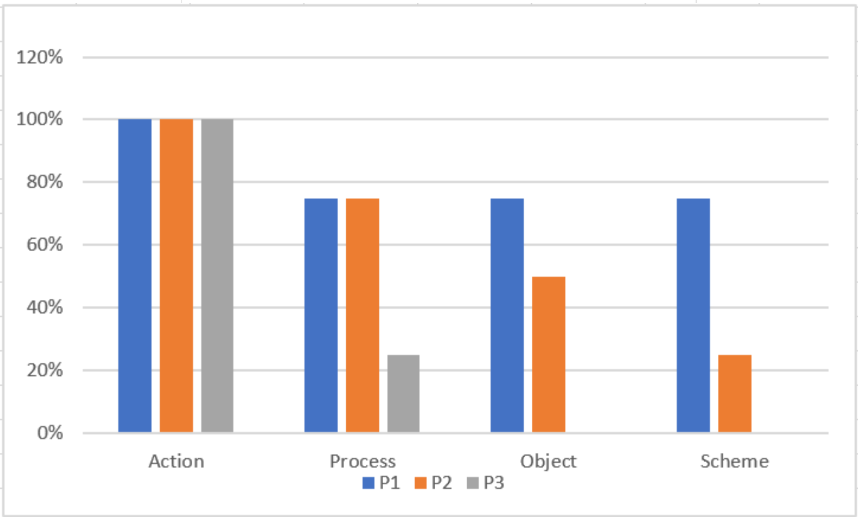

Figure 6. The result of APOS' participants

CONCLUSION

Learning obstacles found from the results of this study are dominated by ontological learning obstacles conceptual and psychology. The mental actions carried out by students become a reference to find out their obstacles in the integral learning process. All students acted because there are external factors or external stimuli. Due to conceptual ontogenic learning obstacles however, not all students can perform mental actions within themselves to carry out initial de-encapsulation. Obstacle to learning psychology can be seen from the initial processes of interiorization, the encapsulation of processes into objects as well as the de-encapsulation of objects into processes. Because mental objects coordination and reversion (object-process loop repetition) do not occur, students are unable to form a good scheme for this integral material due to psychological learning obstacle. The scheme refers to a new mental object, in particular, the understanding of integral concepts as knowledge. The limitation of this research is to find learning obstacles in integral learning. Suggestions for further research is expected to be able to make appropriate didactic designs to overcome these learning obstacles by making hypothetical learning trajectory designs.

Acknowledgments

We thank the civil engineering students who had agreed to participate in this research. We would like to thank the Ministry of Education and Culture, LPDP and Puslapdik for providing BPI scholarships.

Declarations

Author Contribution : LN: Conceptualization, Writing - Original Draft, Editing and Visualization.
DS: Writing - Review & Editing, Formal analysis, and Methodology.
DD &TH: Validation and Supervision.

Funding Statement : This research was funded by the Indonesian Education Scholarship provider at the Ministry of Education, Culture, Research and Technology (Kemdikbudristek) based on the mandate of the LPDP Board of Trustees (Educational Fund Management Agency) implemented by the Ministry of Education and Culture through the Education Financing Service Center (Puslapdik).

Conflict of Interest : The authors declare no conflict of interest.

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