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A proposed constructivism-based instructional model to enhance metacognition and mathematical problem-solving skills in Bhutanese grade nine students

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Abstract

The enhancement of metacognitive abilities and problem-solving skills is essential for effective mathematics instruction. However, these critical components are frequently overlooked in traditional teaching practices. This study addresses the challenges and requirements faced by mathematics educators and explores the integration of constructivist activities in classroom settings. It aims to develop and evaluate the suitability of an instructional model designed to address these issues. Employing a mixed-method approach within a research and development framework, the study gathered data through semi-structured interviews with seven mathematics teachers in Bhutan to identify their instructional challenges. Additionally, two experts from Bhutan and one from Thailand were consulted to provide insights into constructivist teaching methodologies. The content analysis of teacher interviews revealed a predominant reliance on structured, teacher-centered instructional methods, with limited emphasis on fostering higher-order cognitive skills. To bridge this gap, an instructional model emphasizing the development of higher-order thinking was designed. This model incorporates active learning, problem-solving, collaboration, scaffolding, reflection, and self-monitoring, organized into six steps: prior knowledge activation, mediation, internalization, generalization, transfer, and evaluation. The model was evaluated using a 5-point Likert scale, achieving a mean score of 4.33 (SD = 0.70), indicating high levels of appropriateness and acceptability. Furthermore, a pilot test yielded an effective index (E.I. = 0.51), demonstrating the model's efficacy in fostering metacognitive and problem-solving skills.

Keywords: Constructivism, Instructional Model, Metacognition, Monitoring, Problem-Solving

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Mathematics plays a critical role in driving technological advancement and industrialization. However, a growing concern is the underperformance of students in international mathematics assessments (Bermejo et al., 2021). This issue is not limited to specific countries but extends to Bhutan, where students consistently face challenges in the subject. Mathematics is a mandatory component of the Bhutanese education system; however, despite its compulsory nature, students continue to underperform in national examinations, a trend that has raised significant concern among educators, researchers, and policymakers.

Research conducted by Bhutanese scholars has highlighted two primary issues contributing to this underperformance: difficulties in understanding mathematical content and the challenges of applying mathematical concepts to real-world scenarios (Dorji et al., 2021; MoE, 2014). Correspondingly, the results of the PISA-D assessment revealed that Bhutanese students exhibited low performance, with a solution rate of 38.8% in Mathematics, 45.3% in English, and 45.1% in Science (BCSEA, 2019). Further analysis indicated that Bhutanese students performed better on tasks that required lower cognitive engagement but displayed a significant performance gap in more complex tasks that necessitate higher-order cognitive skills such as critical and creative thinking. Low-cognitive tasks typically involve skills such as recall and the application of basic computational methods, often without a full understanding of underlying principles (Azid et al., 2022).

This pattern is indicative of the prevailing pedagogical approach, which is often characterized by teacher-centered methods, such as "show-and-tell" and "teach-then-solve" techniques. These methods tend to emphasize rote memorization and computational skills while limiting the development of higher-order cognitive abilities (Van de Walle, 2007). Reigeluth (2013) further argues that while current instructional practices may ensure short-term success in specific contexts, they fail to foster essential skills like initiative, critical thinking, and the ability to solve open-ended problems. Consequently, the predominant issue in the teaching of mathematics in Bhutan lies in the traditional, teacher-centered approach, which prevents students from gaining a deep conceptual understanding and inhibits the cultivation of advanced cognitive skills. As a result, students often resort to memorization as their primary learning strategy, which impedes their ability to perform effectively when confronted with unfamiliar or contextually novel problems.

Instruction and learning are inextricably linked, with students' learning experiences being profoundly influenced by the nature of teachers' instructional practices (Yu & Singh, 2018). To enhance mathematical performance, there is an urgent need for instructional approaches that equip students to tackle complex mathematical challenges, including problem-solving, mathematical reasoning, and establishing connections between concepts (Bai et al., 2023). As Bhutan continues to advance technologically and integrate into a globalized economy, the demand for a mathematically proficient workforce has increased, as mathematical competence is critical for both national and individual economic stability (Andrade-Molina, 2021).

Learning theories underscore the importance of constructivism, which emphasizes the deductive construction of knowledge (Bermejo et al., 2021). While the current instructional methods should not be discarded, they could be enhanced by integrating elements of constructivist teaching approaches to facilitate more effective learning (Reguith, 2013). Constructivism posits that learners actively construct meaning and knowledge through interactions with their environment. This process fosters critical and creative thinking, especially when students engage in problem-solving tasks that challenge their existing understanding. As learners work through such challenges, they are encouraged to reflect on their strategies, assess the effectiveness of their methods, and adjust their approaches as necessary—key components of metacognitive awareness that support the development of metacognitive skills (Van de Walle, 2007). Schoenfeld (2016) asserts that the awareness and continuous monitoring of one's cognitive processes significantly improve mathematical performance. Thus, the constructivist teaching method nurtures both metacognitive skills and problem-solving abilities, underscoring the integral relationship between constructivism, problem-solving, and metacognition.

Building on these insights, this study aims to propose and develop an instructional model grounded in constructivism to enhance metacognitive awareness and problem-solving skills among Grade 9 students in Bhutan. A previous study that implemented the IMPROVE model (Mevarech & Kramarski, 1997), which stands for Introducing the new concept, Metacognitive questioning, Practicing, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment, demonstrated significant improvements in the mathematical performance of Grade 7 students. While the IMPROVE model was effective, it is not employed in this study, as the focus of the two models differs. The constructivist-based model emphasized in this research prioritizes learning through varied activities, whereas the IMPROVE model centers on learning through metacognitive questioning and reflection.

This study specifically targets Grade 9 students, as this stage marks the transition to high school and serves as the foundation for subsequent higher secondary education. The development of mathematical abilities at this stage is crucial for students' future academic success, their choice of college majors, and their career paths (Yu & Singh, 2018). The problem-solving and metacognitive skills acquired during this period are likely to shape students' self-perceptions of their mathematical abilities and influence their decision to pursue mathematics as a subject for higher studies.

The following section outlines the study's theoretical framework and conceptual foundation. This framework incorporates constructivism and its implications for mathematics learning, problem-solving, and metacognition, as well as the significance of these elements in enhancing mathematics performance. Constructivism offers numerous benefits for learning, with the core principle of active engagement requiring students to reconstruct knowledge rather than passively accepting information. Knowledge acquired through this process is more deeply internalized and is believed to have a lasting impact on students' success (Semerci & Batdi, 2015).

Cognitive psychologists, including Piaget, Vygotsky, and Bruner, have emphasized the importance of activities in which, with the teacher's support, students' thought processes and procedures facilitate the internalization of concepts and skills (Kozel et al., 2023). Vygotsky's (1978) theory underscores that individual cognition develops from internalized social interactions, primarily through language. These interactions serve as vehicles for higher-order thinking through the use of language signs and symbols (Sriraman & English, 2010). Social constructivism, therefore, interprets knowledge as a product of development through interaction.

Numerous studies (e.g., Kozel et al., 2023) have demonstrated that knowledge acquired through active engagement is more effective and enduring than knowledge gained through direct instruction, as it enables learners to apply concepts to new and unfamiliar situations. In contrast, students who rely solely on rote learning of mathematical concepts often struggle when confronted with novel challenges (Kozel et al., 2023; Lithner, 2015).

While constructivism offers significant advantages for learning, research indicates that educators face difficulties in fostering metacognition and problem-solving skills. Dignath and Büttner (2018) found that teachers primarily focus on teaching cognitive strategies, with few explicitly incorporating metacognitive strategies, highlighting the need for targeted support in metacognitive instruction. Similarly, a study conducted in Turkey revealed that teachers were unsuccessful in regularly prompting students to engage in reflective thinking or problem-solving, and failed to summarize the content being learned, which led to a diminished focus on metacognition (Temur et al., 2019). These challenges in fostering metacognition and problem-solving are not unique to Bhutan, indicating a global need for more effective instructional strategies.

Research has shown that constructivism offers a practical approach to overcoming these challenges by promoting metacognition and problem-solving skills. Constructivism emphasizes the active role of learners in constructing knowledge through interactions with their environment (Bermejo et al., 2021; Ngussa & Makewa, 2014; Von Glasersfeld, 2012). In the process of building knowledge, learners engage in continuous monitoring and regulation of their thinking, a key aspect of metacognition. This approach encourages learners to reflect on and evaluate their learning process, further reinforcing metacognitive development. According to Ngussa and Makewa (2014), constructivism also fosters problem-solving by promoting collaborative and problem-based learning. By engaging in group activities and addressing real-world problems, students can enhance their critical thinking, decision-making, and problem-solving abilities (Seibert, 2021).

In conclusion, constructivism provides a valuable framework for developing metacognition and problem-solving skills by emphasizing active learning, self-reflection, and collaborative problem-solving. By nurturing critical thinking and self-regulation, constructivist methods facilitate the development of essential cognitive skills. The following sections will further explore the concepts of metacognition and problem-solving and highlight their significance in cognitive tasks.

Problem-solving ability is a crucial life skill encompassing processes such as problem analysis, interpretation, reasoning, prediction, solution development, implementation, evaluation, and reflection (Chinofunga et al., 2024; Schoenfeld, 1980). Metacognition, defined as "thinking about thinking" (Flavell, 1985), refers to the skills involved in thinking and learning, including critical thinking, problem-solving, and decision-making (Lan & Thi, 2020). The Singapore School Mathematics Curriculum characterizes metacognition as the "monitoring of one’s thinking," emphasizing the ability to control one's cognitive processes during problem-solving (Lee et al., 2019). Metacognition consists of two key components: knowledge of cognition and regulation of cognition. Knowledge of cognition involves awareness of concepts, procedures, applications, and strategies (Özsoy, 2011), while regulation of cognition refers to the skills and processes used to guide, monitor, control, and regulate cognition and learning (DeJaeghere et al., 2023, p. 241). Both components are vital for successful problem-solving. A collaborative study involving researchers from Japan, Singapore, and Turkey found that when students are aware of their metacognitive processes, the quality of their individual learning improves (Kesici et al., 2021).

Research suggests that focusing solely on cognitive teaching, without incorporating metacognitive thinking, results in an incomplete teaching approach (Schoenfeld, 2016). In a similar vein, Garofalo and Lester (1985) argue that while a rich knowledge base is necessary for solving mathematical problems, it is insufficient without the ability to monitor and regulate the learning process. Metacognitive processes, such as awareness of one's cognitive abilities and the capacity to monitor and control performance, are fundamental to effective problem-solving (Schoenfeld, 2016). Lan and Thi (2020) further note that students often struggle with problem-solving because they fail to attend to critical information in the problem, lack the skills to select appropriate strategies, and are unable to monitor and adjust their learning process.

The instructional model developed in this study aligns with the principles outlined by the National Council of Teachers of Mathematics (NCTM). One of the core principles of effective teaching, as stated in the NCTM (2014), is the "acquisition of conceptual knowledge as well as procedural knowledge, enabling students to meaningfully organize their knowledge, acquire new insights, and transfer and apply their knowledge to new situations" (p. 20). Similarly, teaching metacognition and problem-solving skills appears to be a key avenue for achieving the goals of the Bhutanese mathematics curriculum, which aims to "empower learners with the mathematical competencies necessary to apply mathematical concepts, fundamentals, and principles in real-life workplaces and life" (Department of Curriculum and Professional Development, 2022, p. 11).

The proposed instructional model addresses the first two phases of development, with this study focusing on two primary objectives, such as to investigate the challenges and needs of teachers in mathematics instruction and the implementation of constructivist teaching practices and to develop and assess the appropriateness and acceptance of the instructional model. As the instructional model was developed in phases, the methodology, results, and discussion are presented sequentially under Phase 1 and Phase 2.

METHODS

The study adopts a mixed-methods approach within the framework of Research and Development (R&D). The methodology comprises four distinct phases: examining the challenges and needs of mathematics teachers, along with the implementation of constructivist activities in the classroom; designing, developing, and evaluating the instructional model; implementing the model and assessing its outcomes; and investigating students' satisfaction with the instructional model. However, as the primary aim of this paper is to contribute to the development of the proposed instructional model, only the first two phases were undertaken in the current study.

Phase 1: Problem and Need Analysis

Phase 1 focused on identifying the challenges and needs faced by mathematics teachers in Bhutan and understanding how constructivist activities were being implemented in their teaching practices. To investigate the teachers' challenges and needs, semi-structured online interviews were conducted via Zoom with seven Grade 9 mathematics teachers in Bhutan. Additionally, expert interviews were held to gather insights on the issues faced by teachers and to examine how constructivist activities were structured and enacted in the classroom. One expert from Thailand and two from Bhutan, selected for their expertise in constructivism, subject knowledge, and metacognitive strategies, participated in the interviews.

Research Instrumentation

The primary research instruments used in this phase were semi-structured interview questions designed for both the teachers and the experts. To ensure content validity, the item-objective congruency (IOC) for the interview questions was assessed by expert evaluators. Items receiving an average score of 0.5 or higher were considered acceptable. Feedback from the evaluators was used to refine and improve the interview questions to enhance clarity and relevance.

Participants

In Phase 1, data was collected through interviews with seven mathematics teachers and three experts, two of whom were based in Bhutan and one in Thailand. Participants were selected using purposive sampling. The teachers were chosen based on their experience teaching Grade 9 mathematics and having a minimum of five years of teaching experience. The experts, comprising three for interviews and five for evaluation, were selected for their academic qualifications (PhD), expertise in constructivism, knowledge of metacognition, and teaching experience in university-level education, particularly within education faculties.

Data Collection

Data collection in this phase was conducted through semi-structured interviews. Teacher interviews were conducted first, followed by expert interviews. All interviews were conducted via Zoom, each lasting approximately one hour. Participants were briefed on the purpose of the study and assured of the confidentiality of their responses. Interviews were recorded with the participants' consent.

Data Analysis

The interview data from both the teachers and experts was analyzed using content analysis, following Bengtsson's (2016) four-stage qualitative content analysis process. The stages included: (1) decontextualization, where the transcribed data was reviewed, segmented into smaller meaning units (e.g., explanation, demonstration), and coded accordingly; (2) recontextualization, where the alignment of the codes with the research objectives was verified; (3) categorization, wherein codes were grouped into categories, themes, and sub-themes (e.g., 'passive learning' categorized under 'low-level cognitive tasks'); and (4) compilation and analysis, wherein the results were organized based on the identified themes and categories.

To enhance the validity of the findings, the raw transcriptions were sent back to the interview participants for verification. Participants were asked to confirm the accuracy of the transcriptions and to clarify or correct any discrepancies. Additionally, the transcribed data and coding were reviewed by the research team for peer validation to address any potential misinterpretations.

Ethical Considerations

The study received ethical approval from the Graduate School of Naresuan University (Approval Letter No. 083/2023). Prior to data collection, consent was obtained from individual participants via official letters sent by the Graduate School. A formal request was also submitted to the Ministry of Education in Bhutan (RE # 0603.02/3492), outlining the study's objectives, participating teachers and experts, and the schools and classes selected for the pilot phase. Permission was subsequently granted by the Ministry through letters addressed to District Education Officers and school principals.

All participants provided voluntary informed consent after receiving detailed explanations of the study's purpose and procedures. Consent was also obtained from school principals to involve students, in adherence to the ethical guidelines established within the Bhutanese education system. Furthermore, participants were assured that all data collected would be kept confidential and used exclusively for research purposes. To maintain anonymity, participants were assigned identification codes in the study's reporting.

Phase 2: Development and Assessment of the Instructional Model

Phase 2 aimed at developing and evaluating the quality of a constructivism-based instructional model designed to enhance metacognitive abilities and problem-solving skills. This phase was carried out in two key steps:

1. The data collected from the interviews with teachers were analyzed and subsequently used to formulate questions for gathering additional insights from experts. The responses from both the teachers and the experts were critically examined, providing the foundation for the development of a constructivist-based instructional model intended to improve metacognitive skills and problem-solving capabilities among Grade 9 students in Bhutan.
2. The quality and validity of the instructional model were assessed by five experts. The research instruments included the instructional model, which consisted of five components: (1) principles, (2) objectives, (3) learning content, (4) learning instructions and activities, and (5) evaluation. The evaluation form, along with the model, was distributed to the experts to assess the model's appropriateness and acceptance using a 5-point Likert scale ranging from 1 to 5. Additionally, the Item-Objective Congruency (IOC) for the mathematical problem-solving questions and the semi-structured interview questions regarding student satisfaction were developed and sent to the experts for evaluation.

Furthermore, pilot tests were conducted using pre- and post-test designs to assess the effectiveness of the instructional model in enhancing metacognitive and problem-solving skills. A pilot test was carried out in Bhutan with a sample of 27 Grade 9 students, representative of the target population. The pre- and post-test consisted of four open-ended mathematical problem-solving questions. Following the test, a focused group semi-structured interview was conducted to gauge the students' satisfaction with the instructional model used in the teaching and learning process.

Research Participants

The study population consisted of Grade 9 students from Bhutan, grouped by district. Each district served as a cluster, given that the teaching, learning, and assessment systems are standardized across all 20 districts. Consequently, each district was considered representative of the entire population. A random selection process was employed, with Samtse district being chosen for this study. Within this district, a school was randomly selected for the pilot study, and a class was chosen at random from five available sections.

Data Analysis

The data collected from the evaluators were analyzed using mean scores ($\overline{X}$) and standard deviation (SD) to assess the appropriateness and acceptability of the instructional model. The mean scores were interpreted using an adapted validity scale proposed by Terano (2015). To assess the effectiveness of the instructional model, the pre- and post-test scores from the pilot study were analyzed. The Effectiveness Index (E.I.) was calculated using the formula proposed by Goodman et al. (1980).

RESULTS AND DISCUSSION

Phase 1: Problem and Need Analysis

The results and discussion of the themes identified in Phase 1, derived from the analysis of teachers' interviews concerning the problems and needs in mathematics instruction, along with expert recommendations, are presented below:

Structured Teaching and Low-Cognitive Level Activities

The teacher interviews revealed that the predominant instructional activity involved solving textbook questions either individually, in pairs, or in groups. For instance, Teachers 3 and 4 stated: "The activities we typically conduct revolve around solving questions from the textbook." Additionally, teachers commonly introduced concepts by providing an explanation, followed by examples, and then assigned similar questions for practice. Teachers 3 and 6 elaborated: "I introduce the problem and concept to the students, then explain the concept, attempting to relate it to real-life scenarios, and afterward, I assign a similar question for students to solve." According to Teachers 4 and 5, the main challenge identified was time management. They noted: "The syllabus is so extensive that we simply do not have sufficient time. Due to time constraints, we are forced to expedite our teaching."

However, the expert interview analysis underscored the necessity for teachers to design instructional activities that foster higher-order thinking skills and deeper understanding through active student engagement in concept exploration. Expert 2 commented: "Rather than merely demonstrating the process of solving problems, it is more aligned with constructivist teaching when teachers engage students in exploring and constructing the meaning of concepts within the classroom."

Challenges in Engaging All Group Members in Collaborative Work

The results indicated that teachers encountered significant challenges in ensuring full participation from all students during group work activities. Several teachers reported: “In group activities, some students are not engaged at all; only one or two students actively participate in the tasks” (Teachers 1, 2, 4, and 7).

According to the expert interviews, three primary factors influence the effectiveness of group work: the conditions established by the teacher, the nature of the task, and the availability of appropriate materials for group work. Experts suggested the creation of a supportive and motivating learning environment, one that encourages students to view mistakes as part of the learning process and minimizes group competition. Furthermore, experts recommended assigning tasks that require collaboration and the use of sufficient materials for group activities. The selection of appropriate tasks for group work was also highlighted as a crucial skill for teachers. Expert 2 emphasized: “Teachers must be able to discern whether an activity is suitable for group work, paired work, or individual effort. If they are unable to make these distinctions in advance, they often encounter difficulties when implementing the activity.”

Need for Teaching Strategies to Enhance Mathematical Problem-Solving and Metacognition

The teacher interviews revealed that while real-world word problems are frequently incorporated into instruction, educators expressed uncertainty regarding problem-solving strategies and effective instructional methods. For instance, Teacher 3 stated, “We, as educators, struggle to effectively guide students through problem-solving processes and provide clear instructions on how to approach these tasks.” Additionally, the interviews revealed that many teachers lacked awareness of metacognitive processes and their role in learning.

Experts emphasized the importance of thorough preparation for constructivist teaching and creating an engaging classroom environment that stimulates curiosity and excitement, ensuring sustained student participation (Expert 2). The analysis of teacher interviews identified three major themes: (i) structured teaching and low-cognitive level tasks, (ii) challenges in involving all students in group work, and (iii) the need for appropriate strategies for teaching mathematical problem-solving and metacognition. These themes highlight critical areas of concern that necessitate immediate intervention.

The first theme, “structured teaching and low-level cognitive tasks,” underscores the predominant use of teacher-centered instructional approaches, which limit opportunities for students to actively engage in learning activities and construct a deep understanding of mathematical concepts. Consequently, students tend to learn through memorization and recall, which restricts their creativity and the flexibility with which they can approach mathematical ideas. Research underscores that mathematics instruction should prioritize conceptual understanding and meaning-making over rote learning (Newton & Sword, 2018). Routine problem-solving tasks fail to challenge students, as they primarily require lower-order thinking skills (Masingila et al., 2018). Thus, non-routine problem-solving tasks are recommended to foster higher-order thinking and mastery of fundamental mathematical skills (Schoenfeld, 1992). In line with these findings, there is a pressing need to equip Bhutanese mathematics teachers with innovative, constructivist instructional strategies.

The second theme, “challenges in involving all students in group work,” highlights the difficulties teachers face in ensuring equitable participation during group activities. While group work has been shown to motivate students, encourage peer teaching, facilitate the exchange of diverse perspectives, and stimulate creativity, teachers report that equal participation remains a significant challenge (Rezaei, 2018). Rezaei's study identified free-riding and unequal contributions as common drawbacks of group work. To address these issues, teachers need effective strategies for implementing group activities, such as assigning divergent tasks, keeping group sizes small, establishing clear expectations and ground rules, and ensuring effective communication between the teacher and students regarding group tasks. Additionally, thorough planning and preparation of group work by the teacher are essential to its success (Rezaei, 2018).

The third theme, "the need for instructional strategies to teach mathematical problem-solving and metacognition," points to a critical area for improvement in mathematics education. Numerous studies have highlighted students' low problem-solving skills and the necessity for teachers to enhance their teaching of problem-solving strategies (Cahyaningsih et al., 2021). Polya's (1945) problem-solving techniques have been widely adopted to address these challenges; however, Schoenfeld (2016) argues that fostering metacognitive thinking is an essential component of effective cognitive teaching, an aspect not explicitly emphasized in Polya's framework (Garofalo & Lester, 1985). As a result, the instructional model used in this study incorporate Polya's steps alongside Schoenfeld's metacognitive framework, which includes four key components of problem-solving: cognitive resources, heuristics, control, and belief systems.

The three themes identified in this study reveal substantial challenges faced by Bhutanese mathematics teachers and underscore the need for robust support. These challenges resonate with those identified in Principles to Actions, which highlight factors contributing to low student performance in the USA and Canada on the PISA assessments (OECD, 2013). These factors include: (i) an overemphasis on procedural learning without meaningful application, (ii) limited access to instructional materials and technology, and (iii) insufficient professional development opportunities for mathematics educators. To address these issues, the National Council of Teachers of Mathematics (NCTM, 2014) draws on Lester's (2007) research, emphasizing the importance of active learning, the construction of mathematical knowledge through personal experience and peer feedback, and the development of metacognitive awareness to support students in monitoring their cognitive performance.

In response to these findings, an innovative instructional model grounded in constructivism has been developed. This model emphasizes active engagement, collaborative learning strategies, and the integration of problem-solving and metacognitive instruction. It is designed to address the challenges faced in teaching mathematics in Bhutan and has the potential for application in other contexts as well.

The Experts' Interviews on Organizing and Implementing Constructivist Teaching Practices

The results and discussion of expert interviews on the principles, roles of teachers and students, and the organization and implementation of constructivist activities in the classroom are divided into three sections: the principles of constructivism, the roles of the teacher and students in constructivism-based teaching and learning, and the organization and implementation of constructivist activities in the classroom. All findings are discussed below.

Based on expert interviews, the key principles of constructivism consist of two major parts. Firstly, teaching should prioritize students' active engagement in the learning process, fostering a student-centered learning environment. Students should be encouraged to build new knowledge based on their prior experiences, through exploration and collaboration. Instruction should focus on collaborative learning, which enhances students' cognitive skills by engaging them with real-world, contextual problems. The learning process should involve reflecting on what has been learned, while scaffolding the students' learning experiences. These principles align with the views of Zadja (2021) and Brook and Brook (1999), who advocate for learning through social interaction and self-regulated metacognition. They emphasize contextualized learning, which takes into account students' perspectives and prior knowledge. Secondly, the instructional model's learning process should be built around the core principles of active engagement, experiential learning, collaborative learning, real-world problem solving, reflection, and scaffolding. These principles should be embedded in the design of constructivist teaching practices.

Furthermore, experts highlight that teachers play a critical role in facilitating student learning by creating opportunities for students to engage with experiences that promote reflection and deep thinking. According to Expert 1, “The teacher’s role should be that of a supporter, facilitator, and coach, guiding students to reflect on their learning by posing higher-order thinking questions, rather than just asking yes-or-no questions.” On the other hand, the active involvement of students is equally crucial. Students should actively participate in their learning, reflect on their actions, ask questions, contribute to teamwork, collaborate effectively, and take ownership of their learning. Thus, the role of the learner is to engage with and contribute to the evolving classroom dynamic (Gredler, 2009). A clear understanding of both the teacher’s and student’s roles is essential in developing an instructional model that fosters metacognitive skills and problem-solving abilities rooted in constructivist principles.

Finally, experts emphasized that constructivist activities should be designed to promote critical and reflective thinking. Such activities should include questions that not only require recall but also encourage higher-level thinking, such as reflecting on what has been learned and assessing its value. Additionally, collaborative learning, concept exploration, posing questions, and verbalizing thought processes are essential components of these activities. Suggested activities include scenario-based tasks, photographs, videos, story problems (Expert 3), and real-world problems (Expert 1). Schunk et al. (2014) asserts that organizing collaborative learning and group discussions enhances cognitive growth and improves mental function. The use of tools and language plays a significant role in facilitating the thinking process and mediating better understanding. Similarly, the integration of technology ensures active student engagement and promotes effective learning (Expert 2).

Insights from expert interviews and relevant literature have informed the development of an instructional model to address teachers' challenges with structured teaching. The constructivist-based instructional model aims to tackle three primary concerns that emerged from field observations. The model is designed to address the following critical needs of Bhutanese mathematics teachers: (i) the incorporation of constructivist activities that actively engage students in the learning process, (ii) strategies to ensure equitable participation in group activities, and (iii) instructional strategies to teach problem-solving skills and foster metacognitive awareness.

Phase 2: Development and Assessment of the Instructional Model

Developing and Assessing the Proposed Instructional Model

The second objective of the study is to develop an instructional model and assess its appropriateness. The instructional model, as developed, is based on suggestions and information collected from experts and literature, following the constructivist framework.

1. Cognitive Constructivist Theory: Learning involves the construction of knowledge through active interaction with the environment, followed by reflection. New knowledge is built on the learner's prior knowledge.
2. Social Constructivist Theory: Learning occurs through social interaction, mediated by cultural tools such as language, manipulatives, symbols, and formulas. This social interaction enhances cognitive growth and higher mental functions through communication, discussion, justification, and problem-solving.

The results of instructional model development revealed the following components, such as

Principles, Objectives, Learning Content, Learning Instruction, and Evaluation, elaborated as follows:

1. Principles: The principles of constructivism are considered as the foundation of instructional model development. They include learners constructing meaning based on prior knowledge and experiences by actively interacting with the environment, engaging in collaborative learning, and providing scaffolding to help learners understand better. Learning is successful when students can explain the problem, analyze it, use appropriate strategies, reflect, and monitor the process of solving any cognitive task.
2. Objectives: To enhance the metacognition and problem-solving skills of grade 9 students, the following competencies are considered: i) Students should demonstrate declarative and procedural knowledge in solving problems on linear relationships, ii) decide on the strategy and verbalize the reason for selecting the strategy to solve the problem, iii) represent the problem with visual representation, iv) Monitor the cognitive performance and change the strategy if necessary, and v) evaluate the reasonableness of the solution.
3. Learning Content: The following content is covered in Unit 3, such as patterns and relation, linear and non-linear relations, discrete and continuous data, scatter plot, meaning of slope and y-intercept, frame the equation for slope and y-intercept from, linear relation in a standard form, and determine the slope and y-intercept from the standard form of an equation.
4. Learning Process: This section illustrates the sequence of the learning process, supported by constructivist philosophy and theoretical assumptions. It includes six learning steps developed by the researcher after an in-depth study of constructivist learning theories, metacognition, and problem-solving, summarized in Table 1.

Table 1. Summary of six steps of the learning process

Steps	Implementation strategies	Resources used	Role of the teacher	Role of the student
Activate Prior knowledge	Use metacognitive questioning, brainstorming, and graphic organizers. Share experiences	Multimedia, visuals, diagrams, situations	Pose open-ended questions Use prompts and visuals	Recall and share experiences. Brainstorm ideas.
Mediation	Organize mixed-ability groups and problem-solving tasks. Facilitate peer discussions.	Handmade or locally available materials for activities	Facilitate collaborative learning. Provide tools like charts	Work collaboratively in groups. Engage in problem-solving
Internalization	Encourage reflection through verbal, written, or graphic. presentations	Graphic organizers, PowerPoint	Clarify misconceptions. Scaffold learning as needed.	Reflect and present learning. Seek clarification
Generalization	Facilitate identification of patterns and general rules.	Blackboards, graphing tools, real-life examples.	Guide discussions to identify patterns. Highlight rules.	Analyze data. Deduce patterns. Formulate general rules.

Steps	Implementation strategies	Resources used	Role of the teacher	Role of the student
Transfer	Design tasks that apply concepts to real-world problems.	Word problems, Desmos, GeoGebra	Provide contextual problems. Guide problem-solving tasks.	Apply knowledge to solve problems. Justify solutions.
Checking /Evaluation	Encourage self-reflection and peer assessments. Use rubrics for evaluation	Checklists, rubrics, apps like Desmos/GeoGebra	Guide reflection and monitoring. Provide constructive feedback.	Reflect on strategies and solutions. Identify improvements.

These steps are arranged chronologically to foster metacognition and problem-solving skills. An example is provided for each learning step to enhance understanding.

a. Activate prior knowledge: This step is characterized by activities that help learners connect previous knowledge and new knowledge. According to constructivist theories, learning occurs when the student can interpret from memory. Moreover, pre-existing knowledge and personal experiences help build meaningful links between unfamiliar and familiar knowledge (Mevarech & Kramarski, 1997). Example: Before starting a lesson on linear equations, the teacher asks students to recall what they know about 3, 3x, and 5+2x. Then, the teacher introduces 3x+5=20 and connects to the new topic.
b. Mediation: This step is characterized by grouping learners (mixed ability grouping) and the active involvement of learners in learning activities and constructing new knowledge. Mediation involves meaning making. According to Vygotsky's theory, mediation involves an object or symbol (i.e. material or psychological tool: symbolic cultural artefacts, including signs, symbols, texts, formulae, and most fundamentally, language) used to represent a particular behavior or another object in the environment. Example: The teacher provides counters to represent 2x - 5 =15. Students collaborate to represent it using counters and find the value of x. The teacher visits the groups, providing guidance and answering questions.
c. Internalization: According to Shvarts (2022), when learners can describe the method verbally, in writing, and picture form, the concept formation is achieved at a higher level. This is similar to the iconic and symbolic representation proposed by Bruner. The teacher will ask learners to present the knowledge constructed through explanation, writing, or graphic organizer. The teacher poses metacognitive questions, such as why you did that way or why you think so, provides additional information, and clarifies misconceptions and scaffolds. This step helps learners deepen their understanding and retain knowledge in long-term memory. Example: Students articulate (verbally or in writing) how they used counters to represent the equation and solve for the value of x. They reflect on the steps taken and explain their reasoning. The teacher addresses misconceptions, clarifies, and ensures all students understand
d. Generalization: In this step, learners deduce the general understanding or formulate the conjectures based on the characteristics of the concept learned or the pattern observed. For Vygotsky, it is this "transition from one structure of generalization to another" that is at the heart of concept development (Albert et al., 2012). Example: Facilitate discussions to draw out the general principle/rule of the equation 2x - 5 =15.

e. Transfer: In this step, learners are required to apply the knowledge and skills learned to solve the problem. The problems include real-world applications. As per the metacognitive theory, it involves conditional knowledge, which is the student's awareness of the task and strategy and deciding which strategy to use and why. Example: Students apply their understanding to solve a real-world problem. For example: "There are 36 students in a class. How many students will be in each group if the students are grouped into four equal groups?" Students write the equation $4x=36$ to represent the scenario, solve it, and explain their solution.

f. Checking or Evaluation: At the end of the task, students reflect on the procedure and strategy used and check if the solution is correct. Evaluating includes testing the knowledge transformation and seeing whether the analysis and generalization of knowledge are appropriate and whether the operation is correct (Wen, 2018). In metacognitive theory, evaluation pertains to checking whether the strategy and procedure used to solve the problem are correct and whether the task's goal is achieved. Example: Students check their solution by reflecting on whether the procedure is correct, explaining how they solved the equation step by step, and justifying their result ($x = 9$).

5. Evaluation: Evaluation of the effectiveness of the instructional model to enhance metacognition and problem-solving skills will be carried out using both formative and summative assessment. Formative assessment includes authentic assessment, which is carried out to assess the process of problem-solving and thinking in metacognition. Polya's problem-solving stages, combined with Schoenfeld's framework, are used for problem-solving. Tools used are observation sheets, worksheets, reflective writing, and self-assessment. To assess the product of the instructional model, pre and post-tests of a problem-solving test and self-report questionnaire will be employed.

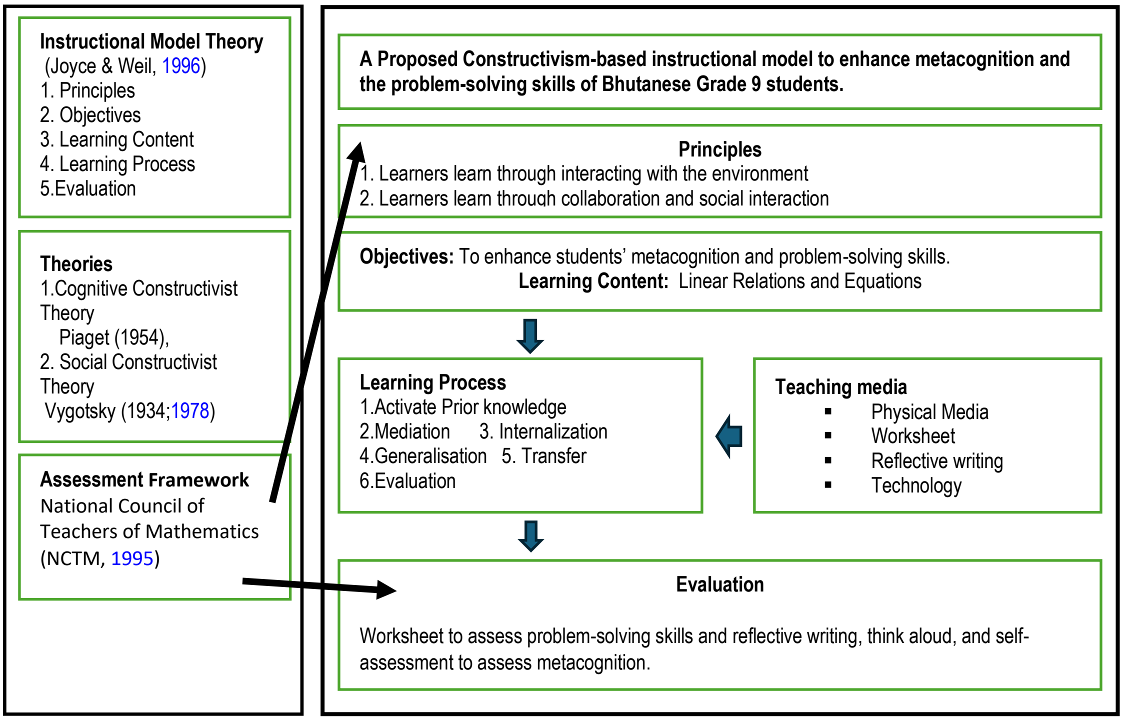


Figure 1. The Proposed Instructional Model

In conclusion, the proposed instructional model, as depicted in Figure 1, offers considerable advantages over traditional teaching methods by emphasizing active learning and fostering deeper engagement. Unlike conventional teacher-centered approaches that rely on passive learning, the new model prioritizes conceptual understanding, critical thinking, and real-world application. Furthermore, it encourages active interaction, reflection, and task monitoring, thereby promoting higher-order thinking and contextual problem-solving—key components of metacognition and problem-solving. In contrast to passive learners who depend on external motivation and effort, a constructivist learner develops and adapts through active construction, resolving challenges as they arise (Bermejo et al., 2021).

The appropriateness and acceptance of the instructional model for enhancing metacognition and problem-solving skills were evaluated. The evaluators assessed the consistency and relevance of the model based on its five components: (1) Principles, (2) Learning Objectives, (3) Learning Content, (4) Learning Instruction/Activities, (5) Instructional Materials, and (6) Evaluation of Learning Outcomes. A total of five evaluators rated the model using a 5-point scale ranging from 1 to 5, as shown in Table 2.

Table 2. Results of the Instructional Model

Components of the Instructional Model	Mean	SD	Level of appropriateness
Principles	4.47	0.78	Appropriate and Acceptable
Learning Objectives	4.45	0.52	Appropriate and Acceptable
Learning Content	4.28	0.75	Appropriate and Acceptable
Learning Instruction and activities	4.53	0.55	Highly Appropriate and Acceptable
Learning Materials	4.0	0.69	Appropriate and Acceptable
Evaluation of Learning Outcomes	4.1	0.86	Appropriate and Acceptable
Overall mean	4.33	0.70	Appropriate and Acceptable

As shown in Table 2, the overall evaluation of the instructional model indicates that it is both appropriate and acceptable, with an average rating of $\overline{X}$ = 4.33 (SD = 0.70). According to existing literature, a mean score greater than or equal to 3.50 suggests that the model is considered both appropriate and acceptable (Terano, 2015). The component "Learning Instruction and Activities" received the highest rating ($\overline{X}$ = 4.53), indicating that the activities designed align well with constructivist principles and facilitate the achievement of objectives aimed at fostering metacognition and problem-solving skills. Other components, including "Principles" ($\overline{X}$ = 4.47), "Learning Objectives" ($\overline{X}$ = 4.45), "Learning Content" ($\overline{X}$ = 4.28), "Learning Materials" ($\overline{X}$ = 4.00), and "Evaluation of Learning Outcomes" ($\overline{X}$ = 4.10), were also deemed appropriate and acceptable. However, "Learning Materials" and "Evaluation of Learning Outcomes" received comparatively lower mean scores. Feedback from expert evaluators indicated the necessity for more detailed information on how metacognition is integrated into the content and clearer guidelines for utilizing materials to enhance mathematical learning. For example, one evaluator remarked, “Add more details for learning materials; unable to make a decision” (Expert 3).

The instructional model was found to be both appropriate and acceptable for enhancing metacognition and problem-solving skills, with a mean score of $\overline{X}$ = 4.33 and SD = 0.07. This finding aligns with the results of a similar study on instructional model development to enhance critical thinking skills conducted in Cambodia (Vong & Kaewurai, 2017), which reported a mean score of $\overline{X}$ = 4.21 (SD = 0.23), indicating the model's high applicability and consistency with the needs of trainee students.

The evaluation of learning activities yielded the highest mean score ($\overline{X}$ = 4.53 and SD = 0.55), reflecting that the activities designed are well-aligned with constructivist principles and effectively support the development of metacognition and problem-solving skills. In contrast, "Learning Materials" and "Evaluation of Learning Outcomes" received lower ratings ($\overline{X}$ = 4.00 and SD = 0.69 for materials, $\overline{X}$ = 4.10 and SD = 0.86 for evaluation), indicating a need for more comprehensive materials and additional activities to assess the lesson's effectiveness. Based on evaluator feedback, the learning materials and evaluation methods were revised and improved. Additionally, feedback on the model's principles was incorporated, particularly drawing on the constructivist principles proposed by Brooks and Brooks (1999) and Zajda (2021).

The results of the instructional model evaluation contribute to the achievement of the second objective of this study, demonstrating that the proposed model is both appropriate and acceptable for enhancing metacognitive and problem-solving skills.

Pilot Study

The pilot study was conducted in a Grade 9 classroom, with the researcher serving as the instructor. The learning activities were carefully designed to align with the core principles of the proposed instructional model, which include active engagement, collaborative learning, and scaffolding. Activities involved the use of physical manipulatives, group work, think-aloud exercises, reflective writing, problem-solving worksheets, and technology for graphing tasks. These activities were intended to promote students' metacognitive awareness and problem-solving capabilities.

To ensure that the instructional model was implemented accurately, the researcher followed structured lesson plans developed based on the model, which had been validated by experts. Furthermore, the mathematics teacher of the pilot class observed the classroom teaching using an observation form designed to assess the model's feasibility and usability. Both pre-test and post-test assessments were conducted using word problems based on the Grade 9 mathematics curriculum on "Linear Algebra." The questions aimed to assess both problem-solving skills and metacognitive awareness. The validity of these questions was ensured through Index Objective Congruence (IOC), and the assessment rubric used to evaluate student performance was similarly validated by experts. The use of word problems and expert validation ensured that the tests effectively measured the intended learning outcomes of the instructional model.

The Pilot Test Results

Table 3 shows the results of the effective index (E.I. = 0.514), indicating that the instructional model is effective in enhancing metacognitive awareness and problem-solving skills. According to the literature, an effective index (E.I.) of ≥ 0.50 is considered effective (Goodman et al., 1980). The effective index was calculated using the formula outlined in the research methodology section. The pre-teat and post-test results of the mathematical problem-solving test align with the findings of a meta-analysis of constructivist models conducted by Xie et al. (2018) in mainland China, which demonstrated that approaches emphasizing exploration and collaboration significantly improved mathematics achievement compared to traditional methods. Collaborative learning had the highest effect size (+0.67), followed by problem-based learning (+0.58) and inquiry-based learning (+0.52), while autonomous learning had the lowest effect size (+0.43).

Table 3. Effectiveness of the Instructional Model

Pretest scores	Post-test score	(S)	(n)	Effective Index
857	1527	80	27	0.51

Classroom observations conducted by the pilot class subject teacher revealed that while the lesson components were well-structured and applicable, challenges related to student engagement in individual tasks and the additional time required for slower learners were noted. These findings highlight the need for the instructional model to accommodate the diverse ability levels of students.

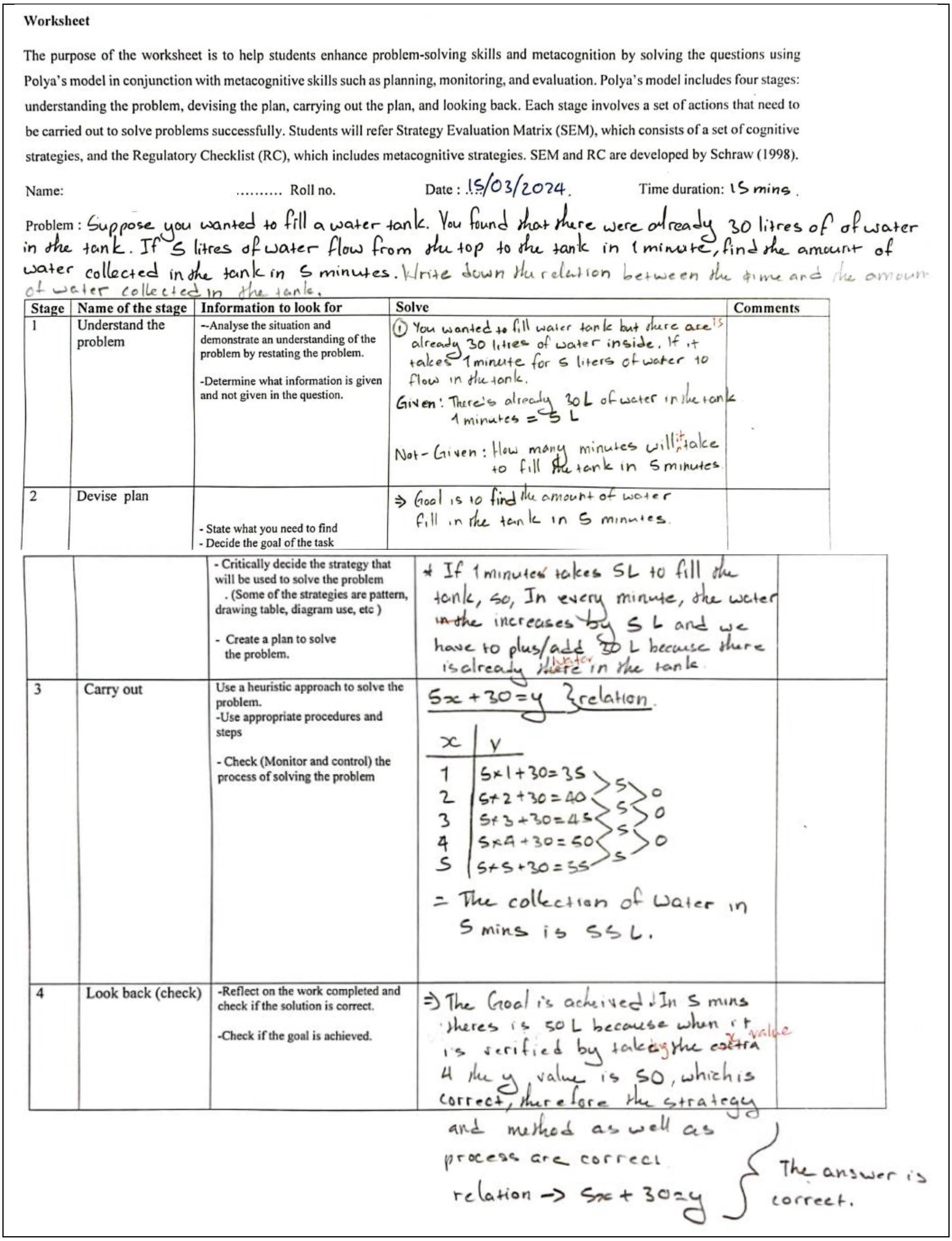
To gauge students' satisfaction with the instructional model, semi-structured interviews were conducted. A focus group of four boys and four girls from the pilot class was interviewed. The analysis of the interview data revealed that students found the teaching and learning process engaging and distinctly different from their previous experiences. One female student noted, "I find this learning very interesting and different because we have never been divided into groups before, where we clarified our doubts together." A male student added, "We had to respond to many questions, which made us think critically, be aware of what we were doing, and make decisions." Additionally, students indicated that breaking down problems into smaller steps facilitated their problem-solving process.

The results of the instructional model evaluation ($\overline{X}$ = 4.33, SD = 0.70) and pilot tests (E.I. = 0.514) indicate that the instructional model is both appropriate and effective in enhancing metacognition and problem-solving skills. Based on feedback from classroom observations, the instructional activities were revised to include more open-ended questions in group activities, such as, "Can you think of a different way to solve this problem?" and "How can you apply this concept in a real-world scenario?" Additionally, students with higher abilities were encouraged to support their peers who faced difficulties.

The interview data further highlighted that the instructional model effectively fostered problem-solving skills and metacognitive awareness. A sample of student work (Figure 2) from the 'Transfer' stage of the learning process demonstrated the integration of problem-solving and metacognitive components. The worksheet, structured around Polya's (1945) four-step problem-solving approach, provided prompts designed to encourage metacognitive thinking. The tasks required students to engage in both cognitive and metacognitive skills, as evidenced by the sample in Figure 2.

Students' interviews indicated that the model motivates them, promotes critical thinking, enhances awareness, and aids in decision-making. This is corroborated by the sample work shown in Figure 2, which reflects their problem-solving skills and metacognitive abilities, such as reflection, evaluation, and justification. The technique of breaking down problems helped students arrive at solutions. Therefore, the findings demonstrate that the instructional model successfully enhanced students' ability to solve problems involving metacognition and problem-solving skills. These results fulfill the second objective of the study.

Although the pilot study demonstrated positive outcomes in terms of metacognition and problem-solving skills, these results were based on short-term observations. Further research will investigate the long-term impact of the instructional model on students' metacognitive and problem-solving abilities through advanced statistical analyses and longitudinal studies, incorporating a control group for comparison.



Worksheet

The purpose of the worksheet is to help students enhance problem-solving skills and metacognition by solving the questions using Polya's model in conjunction with metacognitive skills such as planning, monitoring, and evaluation. Polya's model includes four stages: understanding the problem, devising the plan, carrying out the plan, and looking back. Each stage involves a set of actions that need to be carried out to solve problems successfully. Students will refer Strategy Evaluation Matrix (SEM), which consists of a set of cognitive strategies, and the Regulatory Checklist (RC), which includes metacognitive strategies. SEM and RC are developed by Schraw (1998).

Name: .... Roll no. Date : 15/03/2024. Time duration: 15 mins.

Problem : Suppose you wanted to fill a water tank. You found that there were already 30 litres of of water in the tank. If 5 litres of water flow from the top to the tank in 1 minute, find the amount of water collected in the tank in 5 minutes. Write down the relation between the time and the amount of water collected in the tank.

Stage	Name of the stage	Information to look for	Solve	Comments
1	Understand the problem	--Analyse the situation and demonstrate an understanding of the problem by restating the problem. -Determine what information is given and not given in the question.	(1) You wanted to fill water tank but there are already 30 litres of water inside. If it takes 1 minute for 5 liters of water to flow in the tank. Given: There's already 30 L of water in the tank 1 minutes = 5 L. Not-Given: How many minutes will it take to fill the tank in 5 minutes.	
2	Devise plan	- State what you need to find - Decide the goal of the task	⇒ Goal is to find the amount of water fill in the tank in 5 minutes.	
		- Critically decide the strategy that will be used to solve the problem . (Some of the strategies are pattern, drawing table, diagram use, etc) - Create a plan to solve the problem.	+ If 1 minutes takes 5L to fill the tank, so, In every minute, the water increases by 5 L and we have to plus/add 30 L because there is already water in the tank.	
3	Carry out	Use a heuristic approach to solve the problem. -Use appropriate procedures and steps - Check (Monitor and control) the process of solving the problem	$5x+30=y$ } relation. x: 1, 2, 3, 4, 5; y: $5\times1+30=35$, $5\times2+30=40$, $5\times3+30=45$, $5\times4+30=50$, $5\times5+30=55$ (differences 5, 5, 5, 5; 0, 0, 0). = The collection of Water in 5 mins is 55 L.	
4	Look back (check)	-Reflect on the work completed and check if the solution is correct. -Check if the goal is achieved.	⇒ The Goal is acheived. In 5 mins theres is 50 L because when it is verified by taking the extra x value 4 the y value is 50, which is correct, therefore the strategy and method as well as process are correct. relation → $5x+30=y$ } The answer is correct.	

Figure 2. Sample of student work on problem-solving and metacognitive activities

CONCLUSION

This study aimed to develop a constructivism-based instructional model to enhance the metacognition and problem-solving skills of Grade 9 Bhutanese students. The results from the teacher and expert interviews provided valuable insights into the current challenges faced by the educational system.

Teachers indicated concerns about the low cognitive demands of current teaching methods, limited student engagement, and the need for more effective instructional strategies in teaching problem-solving and metacognition. Expert interviews further emphasized the significance of key constructivist teaching elements, such as concept exploration, reflective questioning, and collaborative learning, which were integrated into the instructional model. The evaluation of the model revealed that it was both appropriate and acceptable, with an average score of 4.33 (SD = 0.70), and was effective in improving students' metacognitive and problem-solving skills (E.I. = 0.51). Students also found the model engaging, as it promoted critical thinking and provided clear guidance for problem-solving.

Despite these positive results, this study has several limitations that need to be addressed in future research. The sample size, consisting of only seven Grade 9 mathematics teachers, may not fully represent the broader range of issues and needs faced by Bhutanese teachers. To enhance the generalizability of the findings, future studies should involve a larger sample of teachers from various grades and schools. Additionally, this research focused on the design and evaluation of the instructional model's appropriateness, acceptance, and initial effectiveness. However, it did not include the implementation phase or more advanced statistical analyses of its impact on student performance, which are essential for providing a comprehensive understanding of the model's effectiveness in a real classroom setting.

For future research, it is recommended that a larger-scale implementation of the instructional model be conducted to assess its long-term impact on student learning outcomes. Detailed statistical analyses, including pre- and post-assessment comparisons of metacognition and problem-solving skills, would provide a more robust understanding of its effectiveness. Furthermore, it would be valuable to explore the professional development needs of teachers to ensure they are well-equipped to implement constructivist strategies effectively. Future studies could also investigate the model's applicability in other subject areas beyond mathematics to determine its broader relevance and impact.

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Declarations

Author Contribution	:	BHS: Conceptualization and writing of the manuscript. SC: Supervision, Validation, Writing - review and editing. WP: Supervision, Validation, Writing- review and editing.
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