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Exploring grade 11 learners' mathematical connections when solving two-dimensional trigonometric problems in an activity-based learning environment

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Abstract

In this paper, we explored the type of mathematical connections Grade 11 learners make when solving two-dimensional (2D) trigonometric problems in an Activity-Based Learning (ABL) environment. We followed a qualitative case study design within an interpretive paradigm. Convenience sampling was used to select a whole class of 45 Grade 11 learners from one of the public non-fee-paying secondary schools in Capricorn District, Limpopo Province of South Africa. Group work presentations and classroom interactions were used to collect data. Data were analyzed using deductive thematic analysis guided by the mathematical connections' framework. The findings indicated that learners managed to make procedural, meaning, reversibility, different representations, feature, and inclusion part whole as well as integrated connections as they worked on 2D trigonometric problems in an ABL environment. We established that learners did not make generalization part-whole connections. In addition, we found that some learners lacked mathematical connections skills and failed to solve the problems. Engaging learners in an ABL environment provided a fine-grained approach that allowed them to make mathematical connections. We, therefore, recommend that teachers should create an ABL environment to enable learners to make different types of mathematics connections during the teaching and learning of trigonometric concepts.

Keywords: Activity-Based Learning Environment, Deductive Thematic Analysis, Integrated Mathematical Connections, Mathematical Connection Skills, Two-Dimensional Trigonometry

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This paper explored the type of mathematical connections Grade 11 learners make when solving two-dimensional (2D) trigonometric problems in an Activity-Based Learning (ABL) environment. Trigonometry is one of the challenging topics in the South African high school curriculum (Spangenberg, 2021). As part of content areas, Grade 11 learners are expected to solve 2D trigonometric problems (Department of Basic Education (DBE), 2011). Solving 2D trigonometric problems requires learners to connect multiple concepts from different mathematical topics (Ngu & Phan, 2020; Wardhani & Argaswari, 2022). When learners connect multiple concepts, they can recognize and apply mathematical knowledge in contexts outside of mathematics (Kleden et al, 2021). In addition, Jackson et al. (2019) pointed out that connecting mathematical concepts enable learners to adapt to new and real-world problems by applying those skills and knowledge. Therefore, in this study, learners were solving a real-life 2D trigonometric problem. Our focus was to explore the mathematical connections they make when solving the problem.

Several studies have found that learners experience multiple challenges when solving trigonometric problems, particularly 2D problems. For example, Marufi et al. (2022) found out that learners experience challenges in comprehending trigonometric symbols and language, using correct algorithms, and retrieving appropriate prior knowledge. For Setiawan and Surahmat (2021), learners encounter the challenge of comprehending and extracting information from 2D trigonometric problems. This contributes to learners committing procedural errors (Nanmumpuni & Retnawati, 2021). Bayu et al. (2021) found that learners are not being careful when reading trigonometric questions and thus rush into solving problems by giving reasons that do not make sense. According to Ayunani and Indriati (2020), learners' difficulty in solving trigonometric problems is due to the interconnectedness nature of the concepts in the topic. Therefore, if a learner fails to identify and connect concepts and operations associated with the problem, then the learner will obtain a wrong solution (Pambudi et al., 2020). Putra et al. (2020) noted that learners encounter difficulties in making connections when solving trigonometric problems. This could be due to the traditional ways of teaching which still dominate trigonometry classes (Kamber & Takaci, 2018). The use of traditional approaches to mathematics has been criticized for being ineffective for many learners (Çelik, 2018; Iyamuremye et al., 2021; Tularam, 2018; Williams & Choudry, 2016). In these environments, mathematical knowledge is acquired in compartmentalized forms (DBE, 2022). On this basis, it is therefore important to consider alternative learning environments such as the Activity-Based Approach that provide learners with opportunities to acquire mathematical connection skills.

According to Khurana (2015), in an ABL environment learners are engaged actively in the learning process. Akkuş (2015) stated that an ABL environment provides learners with opportunities to work, discuss and share ideas with their peers, and their teachers, in their learning situations. ABL environment also fosters the development of skills such as critical thinking, communication, problem-solving, and creative thinking (Karapetian, 2020; Pokhrel, 2018). Posited by Maphutha et al. (2022), an ABL environment provides learners with an opportunity to acquire process skills such as connection, representation, communication, and reasoning when solving mathematics problems. Thus, ABL provides learners with an opportunity to construct new knowledge (Nwoke, 2021). The development of mathematics process skills in an ABL environment depends on the type of activity given to learners (Nwoke, 2021). The type of activity can be hands-on, use of manipulatives and/or real-world mathematics (Festus, 2013; Khurana, 2015). An activity is defined as "anything that learners are expected to do beyond reading and listening, to learn, practice, apply, evaluate, or in any other way to respond to curricular content" (Brophy & Alleman, 1991, p. 9). In mathematics, an activity involves solving problems and constructing models or displays (Huang et al., 2019). In an ABL environment, learners can interact with each other and experiment with information leading to increased engagement (Anwer, 2019). For this paper, participants engaged in an ABL environment to solve real–world trigonometric problems during group work presentations and classroom interactions.

The South African Mathematics Teaching and Learning Framework acknowledges that learners acquire new knowledge when classroom-based activities empower them to make connections among various mathematics topics and relate mathematics to real-life contexts (DBE, 2018). When mathematical ideas are connected to each other or to real-world phenomena, learners can begin to view mathematics as useful, relevant, and integrated (Zengin, 2019). However, Mabena et al. (2021) indicate that South African mathematics classrooms are still dominated by traditional teaching styles. In these teaching styles, mathematical knowledge is imparted in fragmented pieces and affects learners' development of connection skills. The South African mathematics curriculum encourages teachers to present mathematics in ways that develop connections within itself and other disciplines (DBE, 2018; González-Martín et al., 2021). Making connections to the experiences of learners is a powerful process for developing mathematical understanding. In this paper, we explored the type of mathematical connections that grade 11 learners make when solving 2D trigonometric problems in an ABL environment.

Mathematical connections are defined as networks and the associations that learners create between mathematical concepts and topics (intra- mathematical connections), between mathematics and other fields (extra-mathematical connections), and between mathematics and the real world or everyday life (extra-mathematical connections) (García-García & Dolores-Flores, 2018; Nabilah et al., 2019; Hasbi et al., 2019; Quilang & Lazaro, 2022; Rodríguez-Nieto et al., 2022). According to Jawad (2022), learners who possess mathematical connection skills can recognize and connect mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole. A learner who can make mathematical connections has a deep understanding and long-lasting knowledge of the subject (Saminanto & Kartono, 2015). The South African mathematics curriculum supports the need for teachers to equip learners with skills of making mathematical connections during problem-solving (DBE, 2018). Making appropriate connections between mathematics as a discipline and its application in real-world contexts is a key aim of mathematics in the Further Education and Training phase (DoE, 2003). Unless teachers create learning environments that promote the development of mathematical connection skills, learners will continue to view their learning of mathematics concepts as an accumulation of unrelated and discreet ideas (Davis & Maher, 2013). Teachers who encourage learners to make mathematical connections make mathematics more understandable and meaningful (Selvianiresa & Prabawanto, 2017).

García-García and Dolores-Flores (2021) cited procedural, meaning, feature, different representations, reversibility, and part-whole connections as the different types of connections learners can make during problem-solving. Kenedi et al. (2019) found that learners experience difficulties in making conceptual connections (meaning and feature) when solving mathematical problems. In their study, Saleh et al. (2018) found that most participants were able to make procedural connections with no realization of the real-life context embedded in the task item. They also found that learners were able to make different representation connections of the problem as well as connections within mathematics concepts. On the other hand, Khairunnisak et al. (2020) found that participants had challenges in connecting different mathematical concepts needed to solve a given problem. In particular, participants had difficulties in modelling contextual problems, and making different representations and procedural connections. On the contrary, Rohmah et al. (2020), found that learners were able to translate everyday problems into mathematical algebraic sentences, thus making different representations connections. However, the authors found that learners had a challenge of not understanding the next stage/ steps (procedures) of solving mathematical problems due to their lack of prerequisite knowledge or basic skills for making relationships. Therefore, learners' success in solving mathematical problems depends on the connections they can make within the topic or subject itself and in real-life situations.

Learners with mathematical connection skills can make relevant observations or link ideas during problem-solving (Hine, 2021). Learners solve a mathematical problem successfully when they can connect it to an existing network of ideas within their knowledge base (Cai & Ding, 2017). Networks of mathematical ideas are created when learners assimilate new ideas into their existing knowledge (Richards et al., 2020). Despite the important role of connections in the teaching and learning of mathematics, it has been highlighted that there is a dearth of literature which focuses on the mathematical connections learners make when solving mathematical problems (Garcia-Garcia & Dolores-Flores, 2021). In this paper, we, therefore, explored the type of mathematical connections learners make when solving 2D trigonometric problems in an ABL environment. The guiding question was: What type of mathematical connections do the Grade 11 learners make when solving 2D trigonometric problems in an ABL environment? According to Menanti and Sinaga (2018) mathematics teachers should engage learners in activities that enhance the development of connection skills. Thus, when teaching and learning mathematics concepts, learners should be taken through processes that would enable them to make mathematical connections.

Theoretical Framework

This paper is guided by the mathematical connections' framework (Garcia-Garcia & Dolores-Flores, 2018). This framework is an extension of the model of connections of Businskas (2008). According to García-García and Dolores-Flores (2018), mathematical connections emerge when learners can identify concepts, procedures, and representations either in written form or orally as they solve specific mathematics tasks. This framework categorized the connections that learners make when solving mathematical tasks as procedural; different representations; feature; reversibility; meaning and part whole.

(1) Procedural connections: These mathematical connections occur when learners apply rules, algorithms, or formulas as well as reasoning and argument when solving mathematical problems (García-García & Dolores-Flores, 2018, p. 21). For example, a learner may use Pythagoras theorem to find the length of a missing side of a 2D object.

(2) Different representations connections: These mathematical connections occur when a learner uses two or more representations to represent a mathematical idea or concept. Different representations are identified when a learner represents a mathematical concept by using its equivalent and/or alternate representations. An equivalent representation of a mathematical concept is the transformation of the representation made in the same representation system which can be from one symbolic form to another symbolic form. For example, using trigonometric identities $\sin\left(\theta+\frac{3\pi}{2}\right)$ and $-\cos\theta$ are equivalent. An alternate representation is the translation of representation made across representation systems, for example, using a diagram to define trigonometric ratios. Karnasih and Sinaga (2014) indicated that learners who possess the mathematical connection's ability are able to switch between equivalent representations of the same concept.

(3) Feature connections: These connections are identified when learners can define the characteristics or describe the properties of the mathematical concept that make them different or like others. For example, the learner may be able to recognize or identify and extract a right-angled triangle from the 2D diagram drawn. To solve such a problem, learners need to apply trigonometric ratios or Pythagoras theorem.

(4) Reversibility connections: These types of mathematical connections occur when learners can reverse mathematical processes. For example, they can start from a 'concept A to get to concept B and invert/ reverse the process starting from concept B to return to concept A' (García-García & Dolores-Flores, 2018, p. 21). For example, a learner may be able to realize that if the calculated size of an angle in a right-angled triangle is 30°, then the opposite side divided by the hypotenuse should be half and vice versa, that is, a function and its inverse.

(5) Meaning connections: These types of mathematical connections are identified when learners can assign a meaning to a mathematical concept in terms of what it means for them to remember it in

the future. This meaning may be different from what other learners assign to it and what it represents; it can include the definition that they have built for the concepts (García-García & Dolores-Flores, 2018).

(6) Part-whole connections: These types of mathematical connections occur when a learner establishes a logical relationship between mathematical concepts. Rodríguez-Nieto et al. (2022), part-whole connections are manifested in two ways: generalization and inclusion. Generalizations connections are identified when a learner realizes that concept A is a generalization of the concept of B, where concept B is a particular case of concept A) or inclusion (where a learner identifies that concept of A is part of the concept of B or concept B is contained in the concept of A) (García-García & Dolores-Flores, 2018). For example, a learner makes a generalization when s/he realizes that all vertical objects form an angle of 90° with the horizontal plane in a 2- dimensional object. Inclusion connection is observed when the learner realizes that when lines are perpendicular to each other, they form an angle of 90°.

METHODS

For this paper, we followed a qualitative approach. An interpretive case study research design was used (Stake, 1995). The case was the nature of Grade 11 learners' mathematical connections bounded within solving 2D trigonometric problems in an ABL environment. Convenience sampling was used to select participants for this study (Andrade, 2021). A conveniently selected class of 45 Grade 11 learners from one of the public non-fee-paying secondary schools in Capricorn District, Limpopo Province of South Africa constitutes the participants of this study. Participants' ages ranged from 16 to 18 years. Participants were chosen because of the proximity of their school to the researchers. Participants worked in seven mixed-ability groups consisting of five to seven learners during the teaching and learning process in an ABL environment. Groups were purposively selected to present in an ABL environment after they have solved the problems. Reporting groups were selected based on the nature of responses and connections that they made during problem-solving. For groups with similar connections and responses, only one of them presented their solution to the whole class. Therefore, only responses from groups two, five, six and seven were discussed during classroom interactions.

Data for this paper were drawn from group work presentations and classroom interactions, hence providing for data triangulation. Learners were grouped into seven groups. All the group work presentations and classroom interactions were captured using a video recorder and later transcribed. Learners gave their assent, and their parents gave their consent. Codes were used to identify learners as they participate in the interactions, for example, L3G5 represents learner 3 from group 5. Permission was sought from the Provincial Department of Education as well as the selected school.

The nature of mathematical connections which learners made when solving 2D trigonometric problems in an ABL environment were analyzed using deductive thematic analysis (Braun & Clarke, 2006). The findings from the data analysis were fitted into pre-existing themes from the theoretical framework. These themes were categorized as procedural; different representations; feature; reversibility; meaning and part-whole connections as outlined in the theoretical framework. The transcribed video recordings of the classroom interactions were read and reread to identify the extracts which fit into each theme. The extracts were coded using different colors for easier identification. For example, all the data extracts in the groups' presentations and the classroom interactions that fit into the procedural connections were highlighted using green color. The same color extracts were then grouped under one theme, reread, and reviewed to ensure that they fit properly into that identified theme. These extracts were written and discussed in this paper.

RESULTS AND DISCUSSION

Group work Presentations and Classroom Interactions

Classroom interactions were captured when learners were solving the following real-life 2D trigonometric problems in an ABL environment. One of the questions that were attempted in this classroom learning environment was presented as follows:

> *Bonolo standing at point S, sights an electric pylon in front of her. She observes that the angle of elevation from where she is standing to the top of the pylon is $35°$. She then walks 20 meters towards the pylon and realized that the angle of elevation from this new point to the top of the pylon is $45°$. Find the height of the pylon.*
>
> **Source:** Maphutha et al. (2022)

The question was pitched at the level of the complex procedure according to DBE (2011). The question has been set in a context familiar to learners' background. Learners were expected to draw a diagram to represent the given situation and then apply trigonometry to solve the problem. Figure 1 shows the groups' interactions during their discussions.

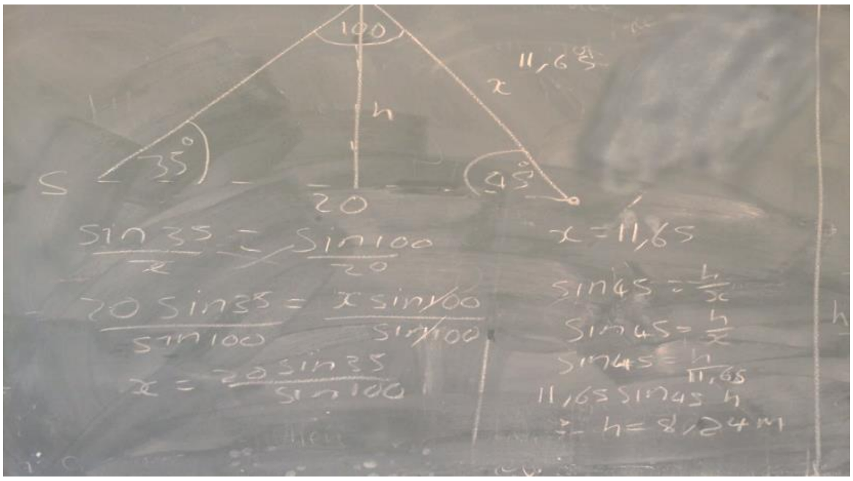

Figure 1. Group 2 Presentation

Feature Connections

Group 2' s presentation suggests that they failed to make alternate different representations connections because they failed to translate the word problem into a 2D diagram. However, their explanation suggests that they were able to make feature connections. Below is the explanation from group 2 learners:

L5G2: The angle formed by point S and the ground is 35°, the distance is 20 meters and the other angle formed is 45°. We used the properties of a triangle to get the missing angle as 100°. Here is our diagram (pointing at their diagram on the board). Because the triangle is not right-angled, then we apply the rules. In this case, it is a sine rule. From there we applied a sine ratio to get the height. Then we got our solution as 8, 2 meters.

From their explanation, though they have drawn an incorrect diagram, learners in Group 2 indicated that they had prior knowledge of the properties of triangles and were able to apply them to find the missing angle in the triangle. These results are contrary to Sarkam et al. (2019) who found that learners fail to use prerequisite knowledge and concepts during problem-solving. Learners were also able to apply the trigonometric rules since the triangle was not right-angled. This indicates that the learning environment provided learners with an opportunity to make feature connections during the 2D trigonometric problem-solving.

Meaning Connections

Meaning connections were evident from one of the questions asked by one learner from group seven. In trying to verify the triangle drawn by group 2 learners, the learners asked the questions:

L3G7: *From which direction was Bonolo coming? Was she at both sides of the pylon at the same time?*
L4G2: *No, she wasn't at both sides of the pylon. She moved 20 meters from point S and the angle formed was 45° at this new point.*

This question was critical as it required group 2 learners to reflect on their diagram and make sense of the connection between their diagram and the description in the question. Drawing an angle of $45°$ on the other side of the pylon by group 2 learners indicates a lack of meaning and different representations connections. These results are in line with Setiawan and Surahmat (2021) who found that participants find difficulty in solving trigonometric problems as they fail to pay attention to (interpret) the information in the question. Additionally, the results agree with the results of Bayu et al. (2021) who found that learners fail to comprehend questions and rush to solve problems by giving reasons that do not make sense. These findings are also consistent with Khairunnisak et al. (2020) who found that learners had challenges with modelling the contextual problems. The importance of reading and understanding the question carefully is also consistent with the work of Liljedahl et al. (2016) who suggested that learners must understand and develop a mathematical model when solving standard problems.

Reversibility and Different Representations Connections

Learners in group 2 failed to apply reversibility connections in their response to a question asked by a learner who scrutinized the height of 8, 24 meters.

L1G6: $Sine\ 45° = 0{,}707$. My question is: if $h = 8,24$ meters and $x = 11,65$ meters then we divide h by x. Why is it not giving us $0,707$?

L1G6: When we were doing trig ratios, we were saying everything we do in mathematics should make sense. Therefore, sine 45° should always give us 0,707. If not, then it means something is wrong.

L6G2: We realized our mistake immediately when sine 45° was not giving us 0,707 that our solution is not correct.

L1G6's asked a question to group 2 which indicated that the group was wrong if the process could not be reversible. Hence, the learner asked why the answer was not 0,707 after dividing h by x. The learner was aware that the process should be easily reversed for the answer to be correct. This indicates that the learner was able to connect concepts within trigonometry, that is, making intra-mathematical connections. The learner was able to switch between special angles and trigonometric ratios, hence making reversibility and equivalent different representations connections. The learner also was able to reflect and draw out the meaning of the presented solution. These results are contrary to Khairunnisak et al. (2020) who found that learners were unable to connect concepts in mathematics.

Feature and Meaning Connections

As the interactions above continued, one learner from group 2 had this to say:

L1G2: that one is taking us back to special angles and we are now solving 2D problems.

Learner L1G2 is making both meaning and feature connections by asking the two questions. Connecting this question to special angles is still reasonable as the angle of $45°$ allows for that. The second part of the question suggests that the learner was not aware that the knowledge of special angles can be applied when solving 2D problems.

L3G2: These things are frustrating ma'am. We always move back and forth.

This learner's reactions indicate the consequence of failing to make connections, the learner is frustrated because of not making a breakthrough to the solution of the problem. The learner is also not aware that moving backwards and forward is a normal practice during problem-solving (Hine, 2021). The technique of thinking backwards is called retrograde analysis and is a form of meaning connections.

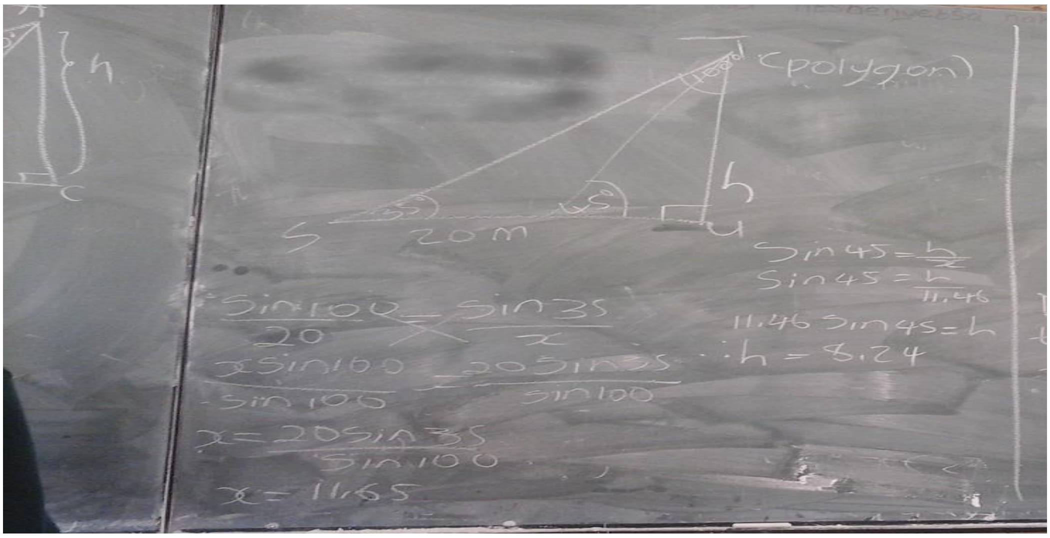

Figure 2. Group7 Presentation

Group 7 learners were able to make correct different representations connections by converting the word problem into a correct diagrammatic representation but incorrect procedural connections, presented in Figure 2. The group successfully interpreted the question and drew a correct diagram but failed to make procedural connections to solve the problem. These results concur with the study by Khairunnisak *et al.* (2020), who found that participants were unable to make procedural connections during problem-solving. However, the results are in sharp contrast with Saleh et al. (2018) who found that participants were able to make procedural connections but failed to contextualize their answers. The same conclusion was arrived at by Rohmah et al. (2020) who found out that learners were able to identify a given concept (meaning connections) and relate it to the procedures associated with the concept. Learners made correct procedural connections but made errors in determining the solution. Hence, they failed to make part-whole inclusion connections.

L3G7: *In our diagram, from point S where Bonolo was standing, the angle of elevation was 35°. This is our 35° (putting it in the diagram). After walking for 20m, the angle increased to 45°. That is why we have put 20m here (pointing 20m on the diagram) and 45° here (also pointing on the diagram). Then we used geometry to calculate the remaining angle and found that it is 100° because* $180° - 35° - 45° = 100°$*. Then we used the sine rule to calculate this side (pointing at the side between TS and TU) because the triangle is not right-angled. Then we got 11,65 and then calculated the height by using sine 45° and got* $h = 8{,}42$.

Procedural Connections

The procedural connections made were relevant to the task at hand as learners were able to connect or apply correct algorithms for calculating the missing angle using the properties of a triangle (features connections). This is consistent with Fujita et al (2020) who coined that students' spatial reasoning skills play an important role in solving problems involving 2D and 3-D geometrical shapes. The South African Department of Basic Education) also stressed that learners acquire new knowledge when classroom-based activities empower them to make connections among various mathematics topics and relate mathematics to real-life contexts (DBE, 2018).

L2G2: I think their diagram is correct, but their solution is wrong because they are mixing triangles. You cannot say sine 100° divided by 20 because 100° is opposite to the whole side SU not only 20 meters.

Teacher: Do you mean that everything in the diagram is correct?

L2G2: Yes, mam, the only problem is the substitution in the sine rule.

L5G4: Mam, I think L2G2 is not correct because even 100° is not correct.

(Learners started to murmur)

Teacher: Let us give him a chance to explain why?

L5G4: Because they did not use a 90° angle, so we should also add 90°.

Teacher: Do you mean we should add all the given angles?

L5G4: *Mam, bjale ya nthlakathlakantšha* (meaning: It is now confusing me), but mam, I think we should say $180° - 90° - 45° - 35° = 10°$. (Learners started to murmur and L4G3 said: "I have told them that and they never listened to me").

Meaning Connections

Due to classroom discussion learners were able to realize their mistakes and therefore make meaning connections. The results support Saleh et al. (2018) that participants were able to connect the given problem with suitable mathematical concepts. In this case, whole-class discussions encourage learners to learn from one another and to articulate course content in their own words.

L3G7: We realized that we made a mistake on 100°. We should have used triangles separately because 35° and 45° are not in the same triangle. Then that affected our solution.

L2G7: Yes mam, I think that even that one of 10° is not correct. If you can remove the middle line, then you also remove the angle of 45°. Then, you will have a triangle that is having 90°; 35° and angle STU. Then angle STU cannot be 10° because the angles will be less than 180°.

These findings agree with those of Akkuş (2015) and Nwoke (2021) who stated that an ABL environment provides learners with opportunities to work, discuss and share ideas with their peers, as well as their teachers, in their learning situations. Thus, the ABL environment provides learners with an opportunity to construct new knowledge, hence learners were able to make meaning connections, can be seen in Figure 3.

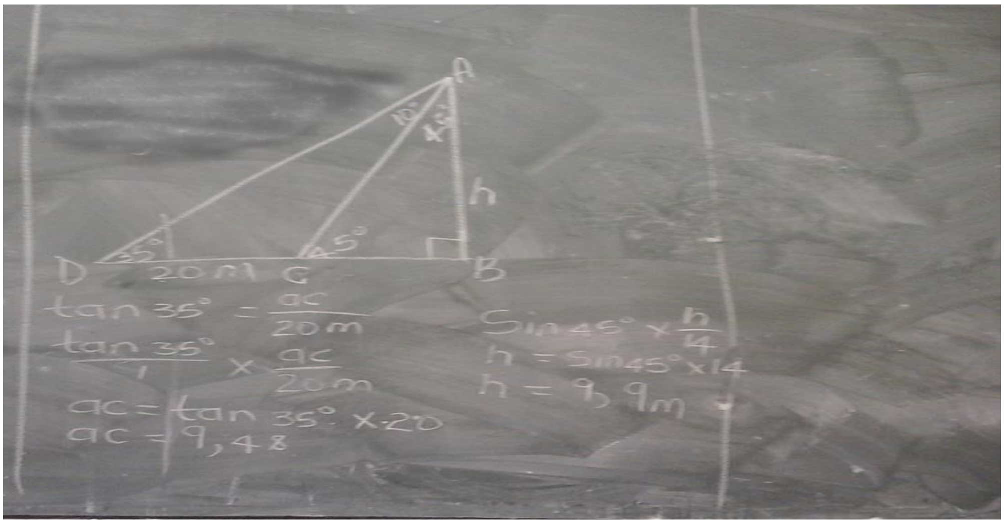

Figure 3. Group 5 Presentation

L1G5: Our solution is like this: In our case, Bonolo was at point D. She then walked 20 meters to point C. That is why we put 20 meters here (pointing it at their diagram on the board). Then the angles as stated are 35° and 45° (pointing at the angles at the board). AB represents the height of the pylon. Therefore, two triangles were formed, we have a right-angled triangle ABC and a scalene triangle ADC. In triangle ABC, $\hat{A} = 45°$ because the angles of a triangle add up to 180°. In triangle ADC, $\hat{A} = 10°$ because the angles of a triangle add up to 180°. We said $90° + 45° + 35° = 170°$, then we subtract 180° and the answer was 10°. Because we are given the adjacent side in this triangle (pointing at triangle ADC) and we want *h,* which is the opposite side, we started by using the tangent ratio to get side AC which is also the hypotenuse of triangle ABC. We got our answer as 9, 48 meters which is the same as 9, 5 meters and the nearest is 10 meters. We then calculated the length of AB which is *h* by applying the sine ratio and get the answer as 9, 9 meters which to the nearest is 10 meters. This means that triangle ABC is isosceles because two sides are equal, and two angles are equal.

Feature Connections

From the above explanation, Group 5 learners were able to make feature connections. L1G5 indicated that in triangle ADC, $\hat{A} = 10°$ because the angles of a triangle add up to 180°. In addition, this group was able to find the height of the triangle using its properties. This indicates that learners were able to make feature connections and solve the problem successfully. Thus, the chances of obtaining correct solutions are higher in learners who can make feature connections. These findings are contrary to Setiawan and Surahmat (2021) who found that participants fail to pay attention to the information given in the trigonometric problem. Participants were able to pay attention to every detail of the information given.

Procedural and Feature Connections (PF)

Learners in this group were able to make both procedural and feature connections (PC) as they were able to say:

"We said $90° + 45° + 35° = 170°$, then we subtract 180° and the answer was 10°".

Learners were able to use the properties of a triangle to find the third angle. In addition, they also identified that triangle ABC is right-angled and applied the sine ratio to calculate its height. Thus, learners were able to make multiple connections or integrate connections to solve the problems. This aligns well with the multiple mathematical connections that learners made in a study conducted by Hatisaru (2022).

Reversibility and Feature Connections

Learners in this group were able to make reversibility connections as they were able to say:

> L1G5: "We got our answer as 9, 48 meters which is the same as 9, 5 meters and the nearest is 10 meters. We then calculated the length of AB which is *h* by applying the sine ratio and get the answer as 9, 9 meters which to the nearest is 10 meters. This means that triangle ABC is isosceles because two sides are equal, and two angles are equal".

This is clearly visible in their diagram, where they were able to see that angle $\hat{A} = 45°$. This explanation indicates that learners in this group were able to make feature and reversibility connections by noting that when a triangle is right–angled, and one of the angles is 45°, then the other angle is 45°. Hence, they rounded off twice to make sure that the resulting sides meet the properties of a right-angled isosceles triangle. In this case, they were able to make both reversibility and feature connections. This is consistent with Dudung and Oktaviani (2020) who mentioned that mathematical ideas are interconnected and build on each other to produce a coherent and meaningful mathematical concept. Learners were able to make connections within mathematics (intra- mathematical connections).

Reversibility and Different Representations Connections

Learners were able to make reversibility and equivalent different representations connections as evidenced by the following utterances:

> L4G5: The correct value there is 14. The 9, 48 was written by a mistake. Even if you can use a calculator, $20 \times tan\, 35° = 14$.
>
> L3G1: *Mam, when we use our calculators, we found that* $tan\, 35° = 0,7$ *and 14 divided by 20 is 0, 7. This implies that they are correct because the answer on both sides is 0,7.*

Verifying the results by using the tangent ratio indicates that learners were able to apply reversibility connections as a way of authenticating their answers. Learners' realization that $tan\, 35° = 0,7$ *and 14 divided by 20 is 0,* 7 indicates that learners were able to make equivalent different representations connections. Learners were able to connect concepts within trigonometry (Sarkam et al., 2019).

Feature and Meaning Connections

L5G5 failed to make correct meaning connections while for learner L7G7 the learning environment was not supportive for him to make mathematical connections while L6G4 was supported by the learning situation to ask a question that was favorable for creating more connections. Thus, learner L5G5 was not able to connect various geometrical ideas and experienced failure in solving the problem. This is consistent with

Pambudi et al. (2020) who mentioned that if a learner fails to identify and connect concepts and operations associated with a problem, then the learner will not successfully solve the problem.

L6G4: I think the diagram is correct, but the calculations are incorrect. Are we not supposed to apply a tangent ratio when we have a right-angled triangle?

L5G5: Not always, because the tangent ratio does not involve the hypotenuse side. It involves only opposite and adjacent sides. So, you are free to use it in any triangle.

L7G7: You are right. Initially, I thought the tangent ratio is always defined when there is a right-angled triangle. Now I realize that the ratio could be applied in any triangle because it does not involve the hypotenuse side.

The findings are also in consonant with Ayunani and Indriati (2020) who found that the interconnectedness nature of the concepts in the topic of trigonometry presents learners with difficulties when solving trigonometric problems. The use of the ABL environment was in consonant with Noreen and Rana (2019) who confirmed that the learners benefit from the dynamic conversations it affords during problem-solving.

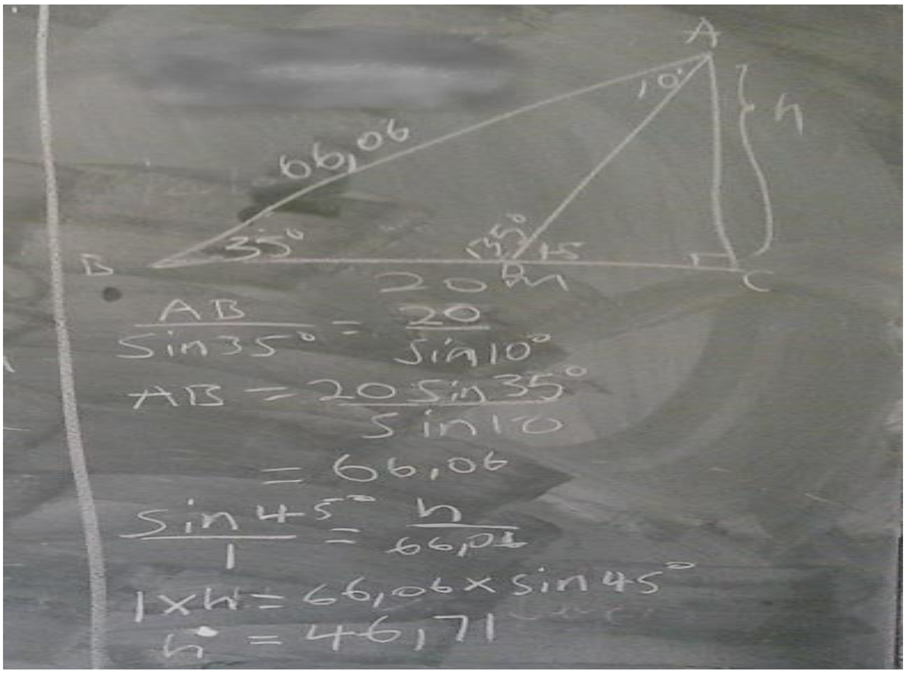

Figure 4. Group 6 Presentation

Feature and Different Representations Connections

Group 6 presentation in Figure 4 and class interactions indicate that learners were able to make feature and alternate different representations connections.

L7G6: Our diagram is the same as that of Group 5. Unlike them, we focused on this triangle (pointing to triangle BAD). We used the sine rule to calculate the length of AB then we got our answer as 66,06 meters. Because AB is the hypotenuse side of triangle ABC and we are looking for *h,* which is the height of the triangle, we then used the sine ratio and got the height as 46,71.

L4G4: They are wrong. What they have done is not a sine rule. I think they should have used sine 135°

not sine 35° because AB is opposite to the 135° angle.

L1G5: Yes, I agree with him. If they use sine 35°, they should divide it by side AD, not AB.

Teacher: Are they totally wrong?

(Learners kept quiet for a moment)

Ultimately L3G5 said: they should just erase 66,06 meters from AB and write it on AD, then the solution will be right.

L5G6: We are not wrong; it is just a writing error. Instead of writing AD, we wrote AB, otherwise, everything is correct.

(Learners shouted at her)

Teacher: What have you learned from item 2?

(At this point, it was 2 minutes towards the end of the period, and the teacher asked L3G6 to summaries the lesson by reflecting on what they learnt.)

Figure 4 shows that learners were able to make a diagrammatic representation of the given real–world problem. This indicates that learners were able to make alternate different representation connections. These results were inconsistent with Setiawan (2022) who found that learners were not able to comprehend and represent the information in each mathematics problem. Learners in group 6 managed to make feature connections as evidenced by using the phrase "hypotenuse side of triangle ABC" and the inclusion of the side labelled "h" in their drawing on the board. The group managed to identify the features of the right-angled triangle. This helped them to choose the correct formula and procedure to solve the problem. The results are consistent with Saleh et al. (2018) who found that the participants were able to use the correct procedure when solving problems. Learners were able to solve these problems concisely and appropriately using their ideas by writing down what is known from the problem that is not quite right and can determine what is asked of the problem. These results concur with Pambudi et al. (2020) who concluded that the correct use of mathematical connections plays an important role in enabling learners to solve mathematical problems. Thus, learners with good mathematical connection skills succeed in solving mathematical problems well, while poor mathematical connection skills cause students to fail in solving mathematical problems. This concurs with the findings of Hine (2021) and Jawad (2022).

CONCLUSION

This paper explored the types of mathematical connections grade 11 learners make when solving 2D trigonometric problems in an ABL environment. Group work presentations and classroom interactions were used to collect data during the teaching and learning process. Several findings emerged from the data set which provides insights into the type of mathematical connections learners make when solving 2D trigonometric problems in an ABL environment. The findings indicated that learners managed to make procedural, meaning, reversibility, different representations, feature, and inclusion part whole as well as integrated connections as they worked on 2D trigonometric problems in an ABL environment. For those groups that solved the problem correctly, the integration of mathematical connections emerged as a common feature. The most common combination of connections consists of feature, meaning, reversibility and different representations connections. We established that learners did not make generalization part-whole connections during group work presentations and classroom interactions. This can be attributed to the nature of the task which did not provide for such opportunities. During group presentations and classroom interactions, it was evident that some learners failed to solve the given activity. This implied, learners were unable to make mathematical connections. This difficulty could be attributed to their lack of prior knowledge of solving trigonometric problems.

We realize that to solve trigonometric activities successfully, learners should be able to integrate mathematical connection skills. Engaging learners in an ABL environment provided a fine-grained approach that allowed them to make mathematical connections. We, therefore, recommend that teachers should create an ABL environment to enable learners to make different types of mathematics connections during the teaching and learning of trigonometric concepts.

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