Journal on Mathematics Education
Volume 15, No. 1, 2024, pp. 295-316

Exploring default-interventionist interaction of thinking activity types on probability problem-solving

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Received: 1 March 2023 | Revised: 9 November 2023 | Accepted: 24 November 2023 | Published Online: 3 January 2024

Abstract

This research aims to determine the thinking activity types dominated by a mental process in producing answers characterized by automatic, unconscious, and subjective-empirical processes (system 1) in solving problems so that the default-interventionist interaction occurs. This research novelty is the formulation of the contents and thinking activity arrangement adapted to students' thinking when solving problems. The problem used in this research is a mathematical problem that triggers students to produce answers quickly with confidence that the answers are correct at a high level. Another problem is about probability because the mode of occurrence of students' learning difficulties at the secondary school level occurs when learning the concept of probability. This is qualitative research with a case study approach. The research subjects were students of Mathematics Education in semester 1. The results showed that thinking activity one could condition the occurrence of type 1 default-interventionist interaction. Thinking activity two could condition the occurrence of type 2 default-interventionist interaction. Thinking activity three could condition the occurrence of type 3 default-interventionist interaction. This research concluded that the default-interventionist interaction occurred because the content and arrangement of the thinking activity conditioned the subjects to pay attention to information gradually and change the subjects' beliefs. Lecturers were recommended to produce, develop, and research thinking activities on topics other than probability at various levels of education. The default-interventionist interaction was essential to be conditioned when system one dominated students' thinking, causing difficulties.

Keywords: Default-Interventionist Interaction, Dual-Process Theory, Probability, Problem-Solving, Thinking Activity Types

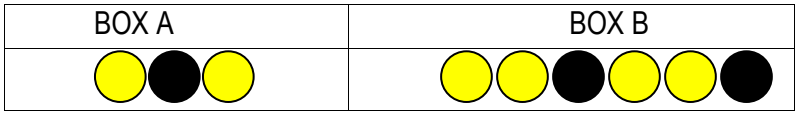
How to Cite: Susiswo, Darmawan, P., Murtafiah, W., & Osman, S. (2024). Exploring default-interventionist interaction of thinking activity types on probability problem-solving. *Journal on Mathematics Education, 15*(1), 295-316. http://doi.org/10.22342/jme.v15i1.pp295-316

Probability is taught at the secondary school level (Douven et al., 2022; Musser et al., 2011). Moreover, at the tertiary level, this concept is also taught to students (Susiswo, 2017). The concept of probability is not only taught to students majoring in mathematics but also to students in other majors, such as information systems, mechanical engineering, physics, and other fields (Douven et al., 2022). It shows how vital the position of the concept of probability is.

Learning the concept of probability at various levels has various problems. Students' learning difficulties at the secondary school level occur when learning the concept of probability (Douven et al., 2022; Musser et al., 2011; Pennycook & Thompson, 2012). Furthermore, the difficulty of learning to solve

the probability increases when the problem of probability is represented in a story problem (Darmawan et al., 2021; Dewolf et al., 2014; Pennycook & Thompson, 2012). The probability evidence of the learning difficulties of probability was also found in the preliminary study of this research. The difficulties experienced by students majoring in mathematics in solving simple probability problems are presented in Figure 1.

Musa and Ali are kindergarten students who are asked to take a black marble from a box stored in the school warehouse by their teacher. In the warehouse, they saw two boxes of marbles in different quantities. The following is an illustration of two boxes containing the marbles.



Musa said he would choose box A to get the black marble. Meanwhile, Ali said it was better to choose box B to get the black marbles. Help Musa and Ali get the black marble from one of the boxes in the warehouse! You can help them by choosing box A or B as the best choice. Explain your answers!

Figure 1. Simple Probability Problem
Source: Researcher's data (2022)

There are two versions of the answer to the simple probability problem produced by students: choosing box A and choosing box B. However, despite producing different answers, students experienced the same mental process, namely system 1. System 1 is one type of mental process in dual-process theory. System 1 is a mental process in producing answers characterized by the occurrence of automatic process, unconscious process, and subjective-empirical process (Darmawan et al., 2020, 2021). This mental process is indicated to cause students difficulties in solving the simple probability problem. The following are some of the students' answers.

Translation: The handwriting with strikethrough mentions, "I chose box B because it contained two black marbles."

Figure 2. Student's Answer
Source: Researcher's data (2022)

Figure 2 indicates that it was generated automatically because students only focus on specific attributes spontaneously. The automatic process is a mental process in generating answers spontaneously based on information that has been internalized (Darmawan et al., 2020, 2021). Internalized information is believed to be true after appreciation (Ackerman & Thompson, 2017; Darmawan et al., 2020, 2021; Trippas et al., 2016). The information that has been internalized by students who chose box A was that the occurrence probability value would be greater if the quantity of sample space elements was less than the quantity of other sample space elements. Meanwhile, the information

internalized by students who chose box B is that the occurrence probability value would be more excellent if the quantity of sample space elements were more significant than that of other sample space elements.

Furthermore, students ignored the relationship between the probability scores of drawing a black marble in each box—incidents like this need to be addressed because they are detrimental to students. Moreover, if students graduate and have entered the work field with a broad impact, then an incident like this is detrimental (Ackerman & Thompson, 2017). Thus, a solution needs to be generated involving system 2.

System 2 is a mental process in producing answers characterized by the conscious process and the empirical-accuracy process (Darmawan et al., 2020, 2021). The conscious process is a mental process of producing answers by matching information characteristics with a learning experience or other information. Meanwhile, the empirical-accuracy process is a mental process in producing answer accuracy through empirical steps, for example, measuring the sides of a triangle with a ruler. The conscious and empirical-accuracy processes can be used to control or intervene in system one results so that difficulties in solving the probability problem are avoided (Brocas & Carrillo, 2016; Dautov, 2021; Kruglanski, 2013; Trippas et al., 2016). The system's two events that interfere with the outcome of system one are referred to as a default-interventionist interaction field (Keren, 2013; Kruglanski, 2013; Stanovich & Evans, 2014).

The default-interventionist interaction is complicated when system 1 dominates in generating answers (Ackerman & Thompson, 2017; Bago & De Neys, 2017; De Neys, 2018; Keren, 2013; Kruglanski, 2013; Stanovich & Evans, 2014; Trippas et al., 2016). Someone who experiences system one activation in producing answers has a high feeling of rightness (FOR) (Bago & De Neys, 2017; Darmawan et al., 2020; Pennycook et al., 2017; Talat et al., 2017). Therefore, the resulting answer is believed to be correct without feeling the need to be re-examined (Bago & De Neys, 2017; Jang et al., 2016; Newman & Thompson, 2023; Stanovich & Toplak, 2023). So far, research results indicated that the default-interventionist interaction did not just happen. Treatment is needed for the default-interventionist interaction to occur (Darmawan et al., 2021). Previous studies conducted by experts showed that subjects were given additional time to evaluate the answers generated through System 1 (De Neys, 2018; Durning et al., 2015; Lem, 2015). The studies focused on the impact of giving extra time and the number of subjects on changing answers (De Neys, 2018; Durning et al., 2015; Lem, 2015). De Neys (2018) examines changes in the direction of students' thinking in the context of psychology. Durning et al. (2015) examined changes in the thinking of health workers in diagnosing patients. Lem (2015) examines changes in students' mathematical thinking when given a stimulus in large numbers. However, it could not explain how the process caused the subjects to change their answers. Some subjects changed their answers, but the answers were not as expected by the previous researchers and were not studied further. The studies were also unable to reveal the characteristics of the information that caused the subjects to change their answer.

In this research, the researchers provided a thinking activity to the subject so that the default-interventionist interaction occurred. A thinking activity is a stimulus that has the effect of system 2 interfering with the result of system 1. The thinking activity was given in various forms, including a picture of cardboard pieces with a fraction of a value. The construction of this thinking activity is adjusted to the results of previous studies conducted by Darmawan et al. (2021), and Thompson and Johnson (2014). These studies did not examine the default-interventionist interaction specifically. However, from the results of these studies, it was found that the characteristic of the stimulus that was most effective in activating system 2 was a stimulus that contradicted the subject's beliefs. Therefore, the thinking activity

of this research was constructed against the results of the subject's system 1. The thinking activities in this research were arranged in a different context from the given probability problem. In the probability problem, the context of the problem is picking black marbles.

Meanwhile, the thinking activity of this research used the context of taking red apples from the basket. This was done to avoid the subject's automatic process activation due to the similarity of the problem context. Similarity is a significant factor in activating automatic process Fields (Ackerman & Thompson, 2017; Bago & De Neys, 2017; De Neys, 2015, 2018). Furthermore, the following thinking activity was given to the student, produced in Figure 3, during the preliminary study.

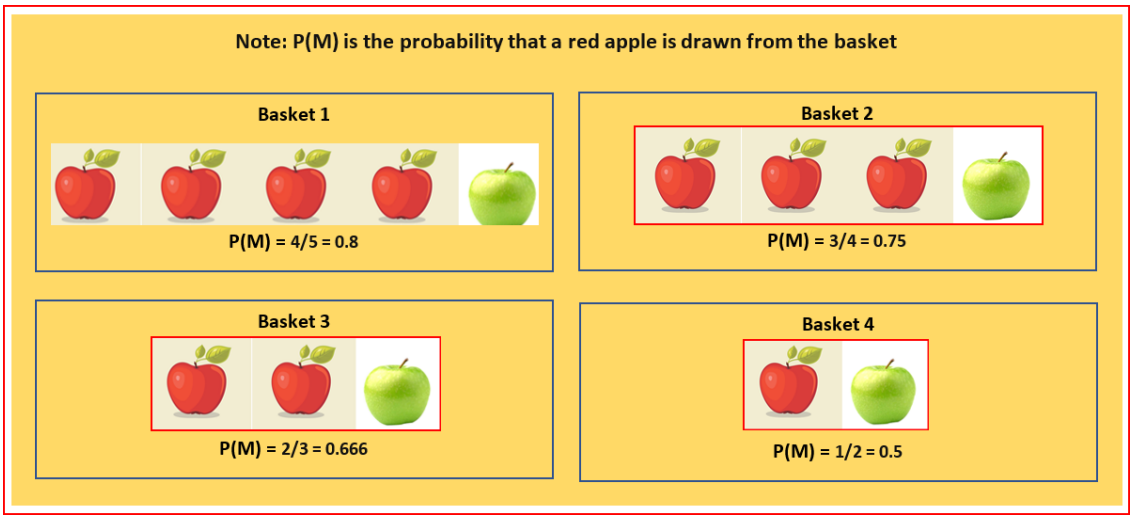


Figure 3. Default-interventionist Interaction Thinking Activity
Source: Researcher's data (2022)

The student who produced in Figure 3 experienced system 1 activation with the main trigger being the quantity of marbles. The student only focused on the quantity of sample space elements and ignored the definition of occurrence probability. After the thinking activity was given, students generated the answers below (see Figure 4).

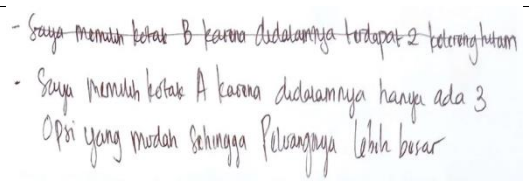
	- I chose box B because it contained 2 black marbles. Crossed out and changed to -I chose box A because there were only 3 easy options in it, so the probability was bigger.

Figure 4. Results of Thinking Activity Giving
Source: Researcher's data (2022)

Figure 4 shows that the student changed the answer. The student compared the probability of picking a black marble in boxes A and B. This is revealed through the interview below.

Researcher	:	Why did you change your answer?
Student	:	The first answer only looked at marbles at a glance.
Researcher	:	What is the matter if there were only black marbles?

Student	:	[I could not make sure that the probability was bigger than the apple example]
Researcher	:	Why did you change your mind about choosing box A?
Student	:	The probability was bigger, Sir.
Researcher	:	Why was it bigger?
Student	:	Because there were only three choices.
Researcher	:	Why was that so?
Student	:	The divisor was smaller.
Researcher	:	What was the probability that a black marble is drawn from each box?
Student	:	1/3 and 3/9

The statement in sign [ ] in the interview above reveals the results of activating the student's awareness process. The student stated that the quantity of event elements was the only determinant of the number of the probability value. The student stated that he was conscious after being given a thinking activity by the researcher. However, the student needed to realize that the probability value was 1/3 = 3/9. In other words, the conscious process is active in parallel with the unconscious process, which indicates the activeness of system 1. For this reason, other thinking activities need to be studied more deeply, formulated, and given back so that mental processes are categorized in system 2 so that the student's answer is correct.

This research aimed to formulate the thinking activity characteristics of the default-interventionist interaction. In addition, this research also aimed to construct a thinking activity adapted to the dominant processes of system 1 so that the processes of system 2 are expected to be active in solving the probability problem. Moreover, the research results complemented the dual-process theory in two respects. First, the research results explained a person's mental condition and information that acts as a thinking activity, so a default-interventionist interaction occurs. Second, the research results provided details of the types of default-interventionist interactions that could occur.

An example is an automatic process interacting with the conscious process, referred to as type 1 default-interventionist interaction. The results of this research can contribute to learning from the perspective of lecturers and students. Lecturers can use the results of this research as a reference in providing scaffolding at crucial moments that have the potential to cause difficulties for students. In addition, lecturers and students can use the results of this research to develop instruments that condition the occurrence of default-interventionist interactions in learning mathematics.

METHODS

Research Type

This research is qualitative research with a case study approach. This research examined collective cases consisting of several events. The event studied as a case was giving a thinking activity and its impact on several subjects whose mental processes were categorized into system 1 when solving the probability problem.

Subject Characteristics

The prospective subjects of this research were students of Mathematics Education in semester 1 because they have experience learning probability at the high school level. The learning experience is a necessary condition for the default-interventionist interaction to occur. Both systems 1 and 2 can be active when

processed information or stimulus is wholly or partially related to a person's learning experience (Darmawan et al., 2020; 2021). The characteristics of the subjects of this research were those with average cognitive abilities, state university students, and students aged 18 to 19 years.

Research Procedure

A class of 40 students is selected for a probability problem. The problem-solving duration was set to trigger the system 1 activation of the prospective subject. Prospective subjects who were given time pressure when solving problems were likelier to experience system 1 activation than when solving problems with time slots. Bago and De Neys (2017), Kahneman (2012), and Trippas et al. (2016) suggested giving a maximum duration of 5 minutes for solving math problems for system 1 activation based on the results of their research. In this study, prospective subjects were also given the opportunity to solve probability problems for 5 minutes.

Prospective subjects who experienced system 1 activation were classified according to their triggers. Analyzing the candidate's written answers and interviews revealed the group of triggers. After that, the prospective subject was given a thinking activity, so system 2 was active. The thinking activity was adjusted to the trigger group for system activation 1. Prospective subjects who experienced system 2 activation after being given a thinking activity were selected for this study. The subject-taking activity was repeated until the data was saturated. The procedural steps for this research are as shown in Figure 5.

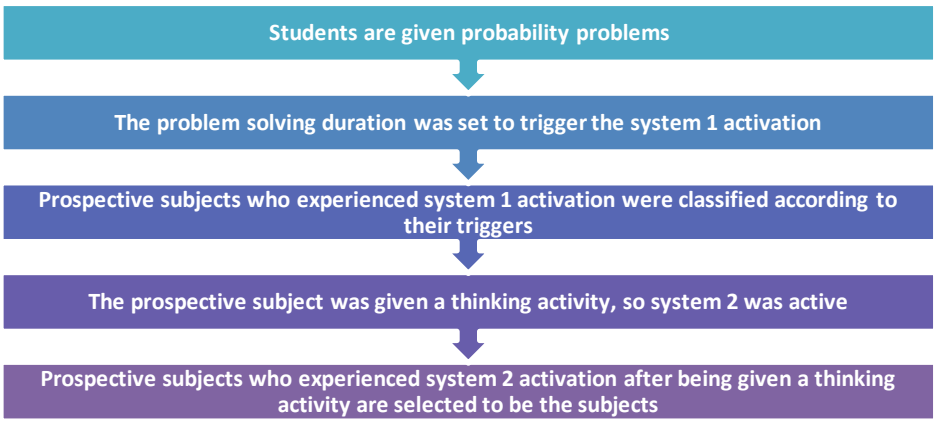


Figure 5. Research Procedure

Probability Problem Instrument

The problem of the probability instrument in this research was structured to trigger the active mental process categorized into system 1. These mental processes were the automatic process, subjective-empirical process, and unconscious process. Once these mental processes were activated, a thinking activity was provided to activate system 2 so that a default-interventionist interaction occurred. This probability problem instrument was structured thoroughly but had the potential to trigger spontaneity. Characteristics of problems that have the potential to activate system 1 are simple problems and contain prominent information because it makes accessibility to long-term memory easy when one produces the solution (Boissin et al., 2023; Darmawan et al., 2020; 2021; Kahneman, 2012). The simple probability problem used in this research is presented in Figure 1 above.

The marble quantity is prominent information in this research probability problem. The probability problem could potentially trigger the activation of the unconscious process if the subject did not calculate

the probability of drawing a black marble in each box. Meanwhile, the automatic process could be active if the subject only focused on the quantity of black marbles. Subjects would spontaneously choose black marbles in a small or large quantity depending on their learning experience.

Data Sources and Research Data

The data sources of this research were students who were the research subjects. The data source produced data in the form of written answers and recordings when the subject was given a thinking activity, so default-interventionist interaction occurred.

Default-Interventionist Interaction Thinking Activity

The thinking activity the researcher gives has unique characteristics, so a default-interventionist interaction occurs with a specific type. The following is the researchers' formulation of thinking activity 1 so that type 1 default-interventionist interaction occurred in Table 1.

Table 1. The Characteristics of Thinking Activity 1

Subject Condition	Thinking activity Characteristics	Thinking activity Effects
Subject spontaneously calculated the probability value only based on the quantity of incident elements considered more and ignored the quantity of universal elements or Subject only visually calculated the quantity of universe elements and chose a box considered to have more universal elements.	The contents of this thinking activity were the occurrence probability value that a red apple was drawn from the basket. The odds values were presented or arranged differently based on the apple's quantity in the basket. The quantity of all apples and red apples in the basket decreased successively from basket number 1 to basket number 4. However, the probability value of the occurrence of picking red apples successively increased from basket number 1 to basket number 4.	Subject realized that it is not only the quantity of occurrence elements that determined the size of the probability value or Subject accurately counted the quantity of black marbles and the quantity of all marbles in each box so that the probability value was known and realized that the determinant of the size of the probability value was not only the quantity of the universal elements.

The following is thinking activity 1 in Figure 6, for subjects who assumed that the occurrence probability value would be greater if the quantity of elements of the occurrence increased or the quantity of elements in the universe increased.

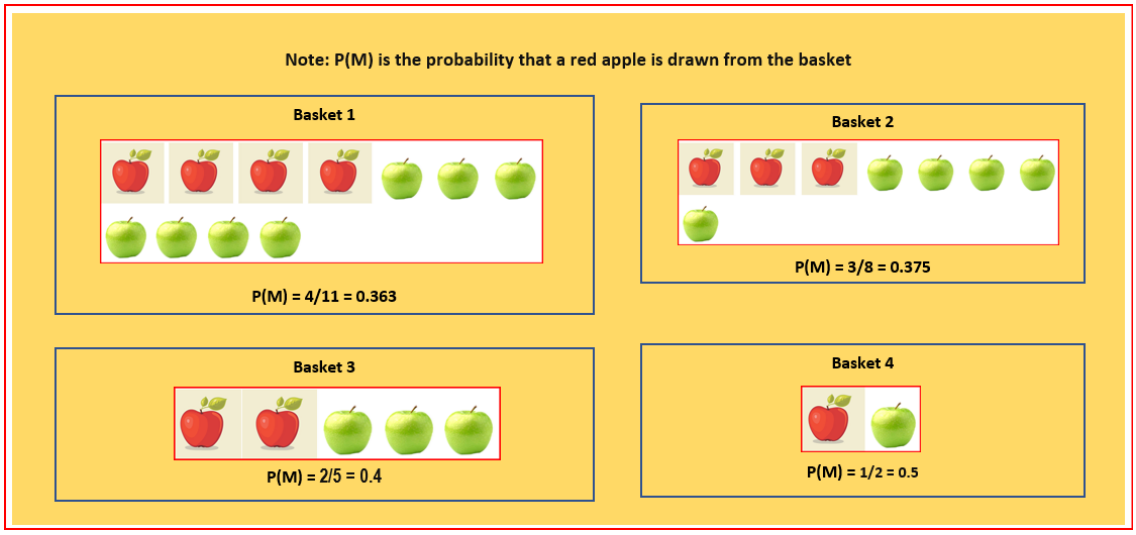


Figure 6. Thinking activity 1
Source: Researcher's data (2022)

The following are the characteristics of thinking activity 2, so the type 2 default-interventionist interaction occurs in Table 2.

Table 2. The Characteristics of Thinking Activity 2

Subject Condition	Thinking activity	Thinking activity Effects
Subject spontaneously calculated the probability value only based on the quantity of element events considered less and ignored the quantity of universal elements or Subject only visually calculated the quantity of universe elements and selected the box considered to have fewer universal elements.	The thinking activity contents were the occurrence probability value that a red apple was drawn from the basket. The odds values were presented or arranged differently based on the quantity of apples in the basket. The quantity of all apples and red apples in the basket decreased successively, but the quantity of green apples remained constant from basket 1 to basket 4.	Subject realized that it was not only the quantity of occurrence elements that determined the size of the probability value or Subject accurately counted the quantity of black marbles and the quantity of all marbles in each box so that the probability value was known and realized that the determinant of the size of the probability value was not only the quantity of the universal elements.

The following is thinking activity 2 in Figure 7 for subjects who assumed that the occurrence probability value would be more excellent if the number of elements of the occurrence decreased or the quantity of the elements of the universe decreased.

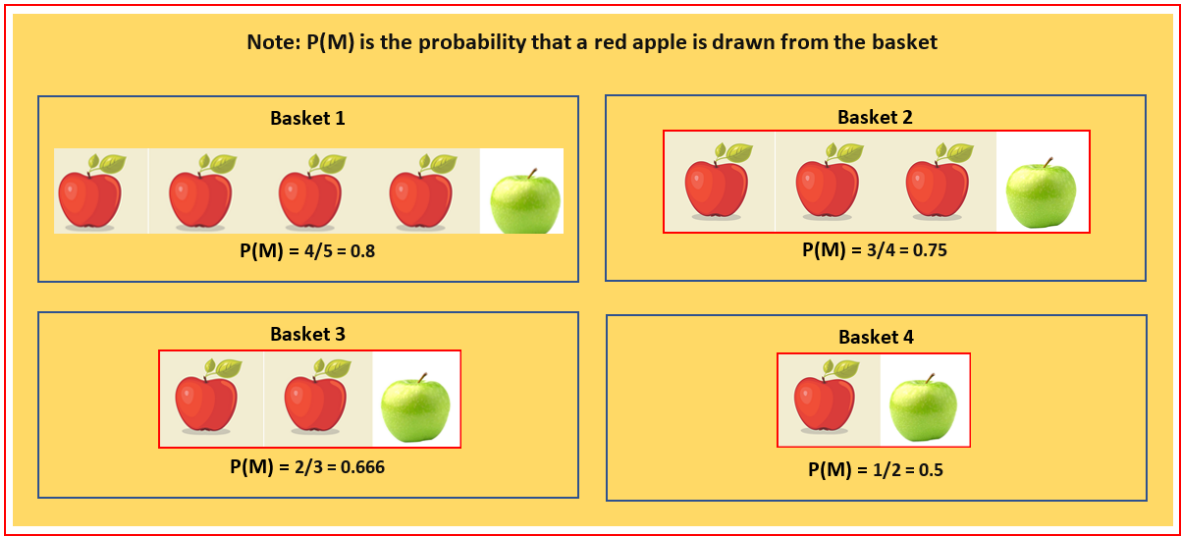


Figure 7. Thinking activity 2
Source: Researcher's data (2022)

The following are the characteristics of the thinking activity for type 3 default-interventionist interaction in Table 3 and Figure 8.

Table 3. The Characteristics of Thinking Activity 3

Subject Condition	Thinking activity	Thinking activity Effect
Subject did not realize that the probability value of drawing a black marble from box A was the same as the probability value of drawing a black marble from box B.	The contents of this thinking activity were equivalent fractions. Fractions of value were presented or arranged in a cardboard cutout image. Fractions 1/2 and fractions equal to 1/2 were in yellow. Meanwhile, the 1/3 fraction was in green. Certain colors were applied to attract the subject's attention so that system 2 was active. Focusing attention was the initial stage of activating system 2. Below the cardboard cutout image was value presented from 1/2 equal to 2/4 and 4/8. Meanwhile, the subject was asked to fill in a fraction equal to 1/3 based on a cardboard cutout picture.	Subject realized that the probability of getting a black marble from box A and box B was the same.

Here is thinking activity 3 for the subject thinking that 1/3 was not equal to 3/9.

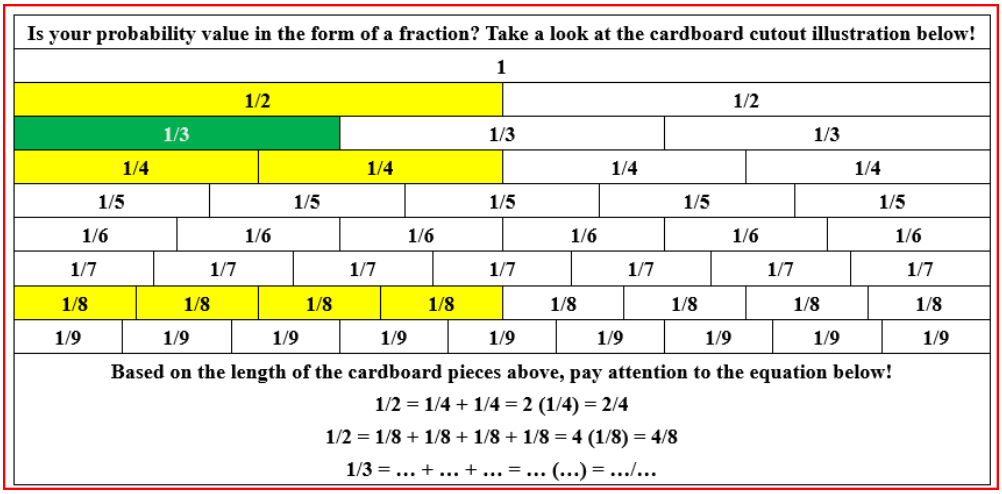


Figure 8. Thinking activity 3
Source: Researcher's data (2022)

Data Analysis Technique

The qualitative data analysis technique this research uses is interactive (Miles et al., 2014). Data analysis started from data collection by sorting out answers indicated to be generated through system 1 activation. Data reduction was carried out when the researcher confirmed the written answer through interviews and then classified system 1 activation based on the trigger. After that, the data were presented in detail according to the chronology of solving probability problems, and research conclusions were made.

RESULTS AND DISCUSSION

Research Results on Subject 1

Subject 1 was a subject who experienced type 1 default-interventionist interaction and also experienced type 3 default-interventionist interaction. The following explains subject 1's mental process in detail in solving the probability problem so that there was type 1 default-interventionist interaction and type 3 default-interventionist interaction. Subject 1 produced an answer based on the quantity of black marbles only. Subject 1 ignored the quantity of other colored marbles in each box. The following is the Subject 1's written answer.

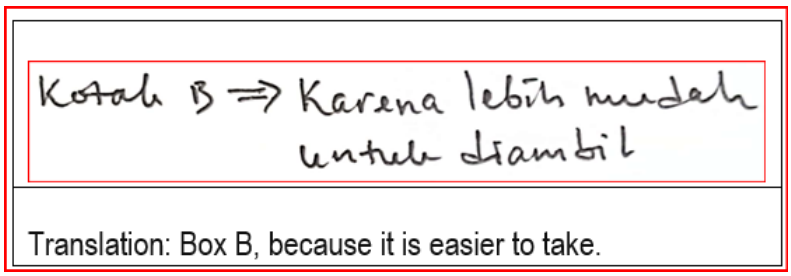


Figure 9. Subject 1's Written Answer
Source: Researcher's data (2022)

Subject 1 generated Figure 9 through an automatic process. The automatic process marked the activation of system 1. It is revealed through the following interview results.

Researcher : How did you come up with this answer?
Subject 1 : I chose the box with more black marbles.
Researcher : Why?
Subject 1 : Because the probability would be bigger. [Then, it would be easier to choose, more choices]
Researcher : Did you calculate the probability value
Subject 1 : [No, it was based on my experience]

The statement in sign [ ] in the interview results above reveals the automatic process of Subject 1. Subject 1 spontaneously chose the box containing more black marbles because it was considered to produce a greater probability value based on his learning experience. Subject 1 chose the box without calculating the probability value of the occurrence that one black marble was drawn from each box. Therefore, the researcher gave Figure 5 as thinking activity 1 so that type 1 default-interventionist interaction occurred. After being given thinking activity 1, Subject 1 gave the following statement to the researcher.

Subject 1 : Sir, my answer was wrong.
Researcher : Why?
Subject 1 : It's not necessarily a bigger probability even though there were more choices.
Researcher : How did you know?
Subject 1 : From the picture that you gave as an example.

The interview results above indicated that subject 1's conscious process was active after being given thinking activity 1. Subject 1 stated that the answer was wrong because the quantity of event elements not the only determinant of the probability value. After that, the researchers again conducted an in-depth search through interviews. Here is the result.

Researcher : How did you realize that?
Subject 1 : I saw each picture and the probability. I compared each picture along with the probability value in order
Researcher : What made you do that?
Subject 1 : [At first, I only briefly saw the number of apples in each basket and the probability values in column P (M) only. From that, it was known that the probability value was getting bigger from top to bottom. It intrigued me to notice the number of red apples. The probability value to go down was getting bigger, but there were fewer red apples]

Subject 1's statement, which is in sign [ ] in the interview results above, revealed the cause of the conscious process activation. Moreover, based on the italicized statement, the conscious process was active after the empirical-accuracy process. The empirical-accuracy process was active when subject 1 counted the quantity of red apples and then compared the occurrence probability value of picking the red apples from each basket in order from basket number 1 to basket number 4. Then, the results of the empirical-accuracy process activated the conscious process. The conscious process was active when subject 1 matched the characteristics of the answer with the results of comparing the quantity of red apples along with the occurrence probability value of picking the red apples from each basket. After that,

subject 1 produced the following written answers.

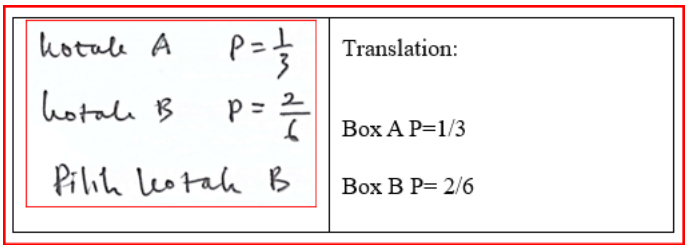


Figure 10. Subject 1's answers after being given Thinking Activity 1
Source: Researcher's data (2022)

Figure 10 shows the resulted of the empirical-accuracy process and the unconscious process. This is revealed through the interview results below.

Researcher : How did you come up with this answer?
Subject 1 : [I divided the black marbles with all the marbles in each box]
Researcher : Why did you choose box B?
Subject 1 : Because the probability was bigger

Subject 1's statement in sign [ ] in the interview above revealed the empirical-accuracy process. This process was active when subject 1 counted the quantity of black marbles, counted the quantity of all marbles, and then divided the quantity of black marbles by the quantity of all marbles in each box. Subject 1 has a 1/3 probability of getting a black marble from box A and a 2/6 probability of getting a black marble from box B. Based on the statement in the interview results above, subject 1 chose box B to take one black marble because it was considered a greater probability. It also marked the active unconsciousness process. Subject 1 did not realize that 1/3 was equal to 2/6. Therefore, the researcher gave Figure 7 as thinking activity 3. After being given thinking activity 3, subject 1 produced the following answers.

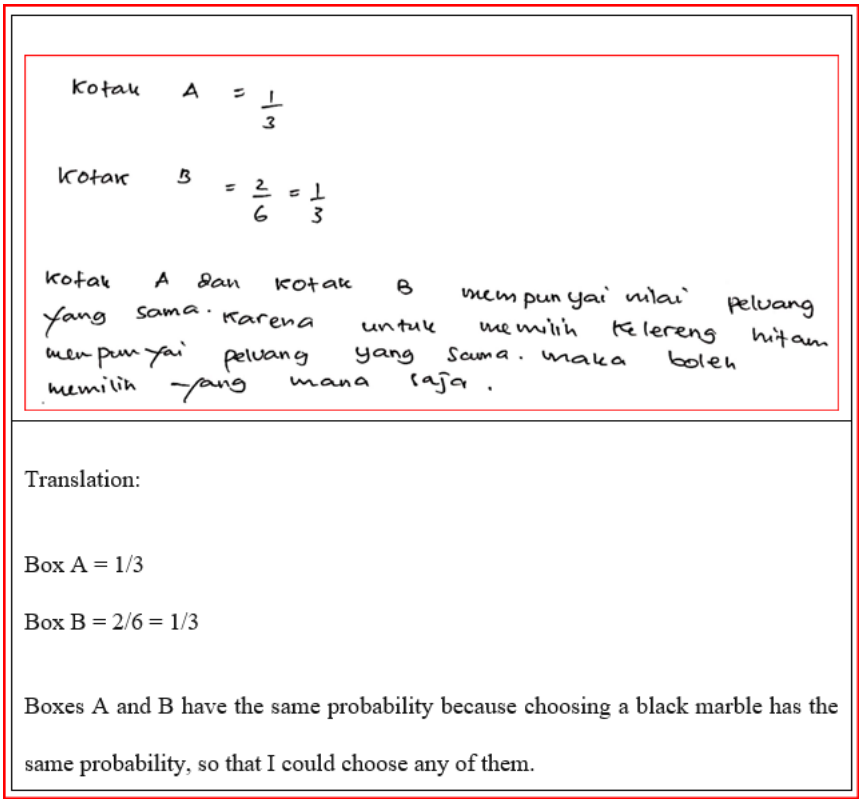


Figure 11. Subject 1's Final Answer
Source: Researcher's data (2022)

Figure 11 resulted from the empirical-accuracy process followed by the conscious process. It is revealed through the interview results below.

Researcher : How did you come up with this answer?
Subject 1 : First, I saw the fraction ½, which was marked yellow, and I filled in the part that had to be filled following the instructions in the picture. I've also seen the pattern at the bottom of the picture or in the filled part. Then, I noticed that it was colored green, apparently, I was not conscious.
Researcher : What do you mean by not being conscious of that?
Subject 1 : [I did not think 1/3 is equal to 2/6]
Researcher : How did you realize that?
Subject 1 : First, I only noticed the information at the beginning and the end of the cardboard cutout. After that, I just looked at the pictures.
Researcher : How did you examine the picture?
Subject 1 : The information below guided me to look at the pictures. 1/2 was served first then 1/3, so I saw the yellow picture. After that, I saw 1/3 which was colored green, from that I became more conscious that 1/3 had equal fractions, it turned out to be 2/6.
Researcher : How did you make that 2/6?
Subject 1 : I measured a cardboard cutout whose length was equal to 1/3, i.e., 1/6 plus 1/6 or 1/6 + 1/6 = 2 (1/6) = 2/6.

The interview results above revealed the process of subject 1 in producing Figure 11 while at the same time showing that thinking activity 3 has succeeded in causing type 3 default-interventionist

interaction. The empirical-accuracy process is indicated by subject 1's statement in the interview results above. Subject 1 stated that a fraction equal to 1/3 was generated after examining a fraction equal to 1/2. A fraction equal to 1/3 was produced by measuring a piece of cardboard that is the same length as the image of a piece of cardboard with a fraction of 1/3. The method of determining a fraction equal to 1/3 was known by subject 1 from determining a fraction equal to 1/2. The measurement results were two pictures' of 1/6 pieces of cardboard. Thus, subject 1 concluded that 1/3 = 1/6 + 1/6 = 2 (1/6) = 2/6. Then, the result of the empirical-accuracy process activated the conscious process. The conscious process was indicated by the statements of subject 1, which are in sign [ ] in the interview results above and Figure 11. Subject 1 stated that 1/3 equals 2/6, so boxes A and B have the same probability of getting a black marble.

Research Results on Subject 2

Subject 2 was a subject who experienced type 2 default-interventionist interaction after being given thinking activity 2 and also experienced type 3 default-interventionist interaction after being given thinking activity 3 by the researcher. The explanation of Subject 2's mental process in detail in solving the probability problem, such that there was type 2 default-interventionist interaction and type 3 default-interventionist interaction, is as follows. Subject 2 produced an answer by selecting the box considered to contain fewer marbles than the other boxes. The following is Subject 2's written answer.

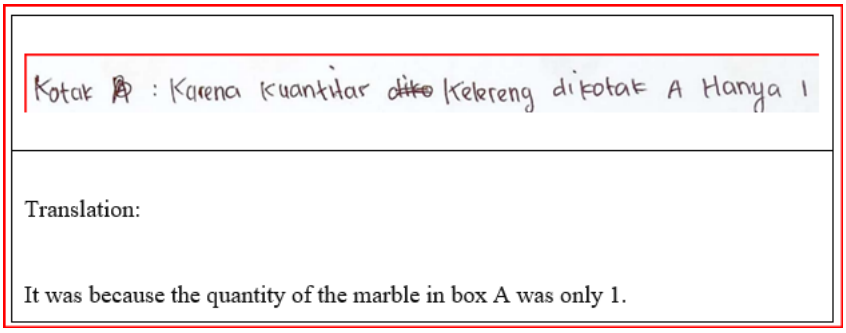


Figure 12. Subject's 2 Written Answers
Source: Researcher's data (2022)

Figure 12 was generated through a subjective-empirical process and an automatic process. Furthermore, the subjective-empirical process and the automatic process marked the activation of system 1. It is revealed through the interview results below.

Researcher : Why did you choose box A?
Subject 2 : The probability was bigger. [Because it will be easier to choose, fewer marbles]
Researcher : How do you mean easier that?
Subject 2 : [There are few choices that allow wrong taking]
Researcher : How many marbles were in each box?
Subject 2 : I have not counted accurately, but apparently, there was one black marble in box A.
Researcher : How did you know if box A had less?
Subject 2 : I glimpsed a little more, but I did not count the number of marbles for Sure.

The statement in bold in the interview above reveals the subjective-empirical process. Based on this statement, subject 2 only briefly glimpsed the contents of box A and box B to conclude that box A contained fewer marbles without accurately calculating the quantity. Meanwhile, the automatic process was revealed through the statement in sign [ ] in the interview results above. Based on this statement, subject 2 spontaneously chose box A because it was considered to contain fewer marbles. A box containing fewer marbles was considered to have a greater probability value because the potential for wrongly picking up black marbles was considered smaller. Furthermore, subject 2 chose the box without calculating the probability value of the occurrence of drawing a black marble from each box. Therefore, the researcher gave Figure 6 as thinking activity 2 so that the type 2 default-interventionist interaction occurred. After being given thinking activity 2, subject 2 gave the following statement to the researcher.

Subject 2	:	*Sir, how come the picture that you gave showed different results?*
Researcher	:	*What's the difference?*
Subject 2	:	*This had a lot of marbles and a big probability. The marbles were few and the probability was small. Different from my answer. I'll check my answer first.*

The statement of subject 2 above reveals the activation of system 2, namely the conscious process. The conscious process was active when subject 2 matched the characteristics of the information on thinking activity 2 with its response. The result of this conscious process was subject 2's decision to re-examine the answers. In other words, thinking activity 2 caused type 2 default-interventionist interaction to occur. Next, subject 2 generated the answer below.

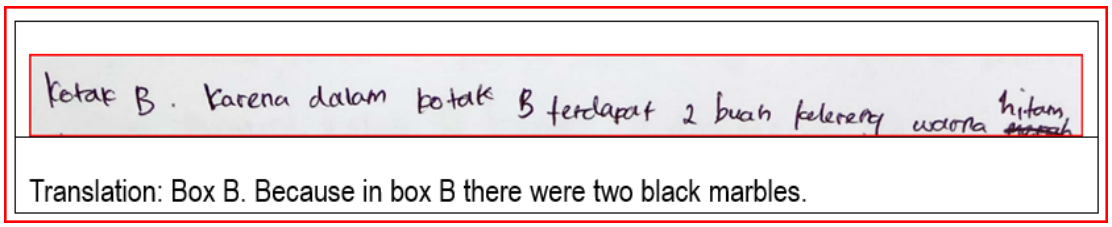


Figure 13. Subject 2's Answer after Given Thinking activity 2
Source: Researcher's data (2022)

Figure 13 was generated by subject 2 through a series of mental processes, namely the empirical-accuracy process, the conscious process, and the automatic process. It is revealed through the following interview results.

Researcher	*Why did you change your answer to box B?*
Subject 2	*The probability was bigger. Because there were more black marbles.*
Researcher	*Why would that result in a bigger probability?*
Subject 2	*[I think it is like the picture you gave earlier]*
Researcher	*Which information in the picture?*
Subject 2	*[The one with the most apples and the reddest apples]*

The statement in sign [ ] in the interview above reveals these mental processes. Based on this statement, the empirical-accuracy process was active when subject 2 observed and calculated the

quantity of apples accurately in each basket and observed the probability value. After that, the conscious process was active when subject 2 abstracted information on thinking activity 2 so that it can be concluded that the biggest probability value would occur in the condition of the highest occurrence elements quantity and the highest universe elements quantity. Then, the result of the conscious process triggered the activation of the automatic process such that Figure 14 above was generated. The automatic process was active when subject 2 spontaneously used the conclusions generated previously through the conscious process to change the answer to box B. Therefore, the researcher gave thinking activity 1 to Subject 2. After being given thinking activity 1, Subject 2 produced Figure 14.

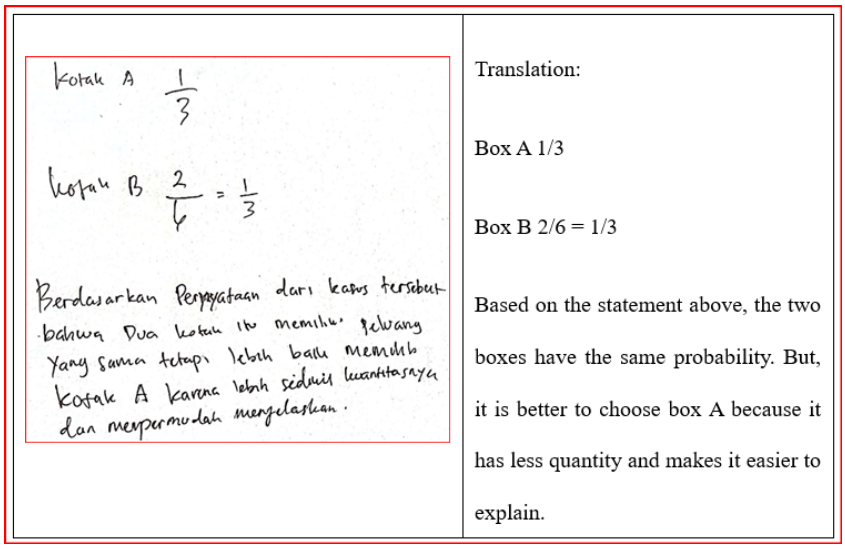


Figure 14. Subject 2's Answer After Being Given Thinking activity 1
Source: Researcher' data (2022)

Figure 14 was generated by Subject 2 through the conscious process and the empirical-accuracy process respectively. In other words, thinking activity 1 caused type 1 default-interventionist interaction to occur. This is revealed through the following interview results.

Researcher : How did you come up with this answer?
Subject 2 : I was inspired by the picture that you gave. I just remembered after you gave the pictures that the probability value was influenced by the universe and its occurrence.
Researcher : Why did you suggest choosing box A? Earlier you stated that the probability was the same in your writing.
Subject 2 : [Yes, I thought I must choose one, sir, so I chose box A. In my opinion, either box A or box B could be chosen because the probability was the same]

The statement of subject 2 which is bold in the interview results above reveals the results of the conscious process. Subject 2 stated that the probability of occurrence was determined by two factors, namely the quantity of universe elements and the quantity of occurrence elements. Subject 2 realized this after observing and studying the information on thinking activity 1 and thinking activity 2. In other words, thinking activity 1 and thinking activity 2 resulted in type 1 default-interventionist interaction.

Meanwhile, the empirical accuracy process is revealed in Figure 14, and the statement in sign [ ] is in the interview results above. Subject 2 produced an accurate probability value for each box by counting the number of occurrence elements and universe elements. Moreover, subject 2 also simplified the fraction 2/6 so that it was known to be equal to 1/3.

Discussion

This section discusses the research findings, namely the characteristics of the thinking activity that condition the default-interventionist interaction. The thinking activity could condition the default-interventionist interaction in this research because of its composition, content, and theoretical basis. The composition and content of the thinking activity were based on two main theories, namely the saliency effect theory (Lem, 2015) and the feeling of rightness (FOR) theory (Boissin et al., 2022; Darmawan et al., 2020; Reyna, 2015). The saliency effect theory was applied to the arrangement of thinking activity to give an interesting impression to the subject. Interesting information has the potential to be observed by the subject, so system 2 is an active (Alós-Ferrer & Strack, 2014; Reyna, 2015). FOR theory is used to underlie the thinking activity's contents to influence the subject's beliefs or learning experience. Thinking activity contents that have an impact on low FOR levels or low confidence in the correctness of the information can prevent automation of system 1 (Ackerman & Morsanyi, 2023; Ackerman & Thompson, 2017; Darmawan et al., 2020; Papa, 2016; Talat et al., 2017).

Based on the research results, thinking activity 1 could condition the occurrence of type 1 default-interventionist interaction, which began with activating the automatic process. The arrangement and content of thinking activity 1 could activate the empirical-accuracy process or the conscious process categorized in system 2. The type 1 default-interventionist interaction occurred in two different forms in this research. The first form involved an automatic process and an empirical-accuracy process. The second form involved an automatic process and a conscious process. One type of thinking activity resulted in two different forms of default-interventionist interaction. This happened because the thinking activity was passive. Meanwhile, the default-interventionist interaction may occur depending on the subject's response to a specific thinking activity or information (Bago & De Neys, 2017; Borodin, 2016; Darmawan et al., 2021; De Neys, 2018; Derous et al., 2015).

The focus of the subject attention, who experienced the first type 1 default-interventionist interaction on thinking activity 1, was the number of apples and the occurrence probability value of picking red apples in each basket. The subject counted and concluded that the fewer the occurrence elements, the greater the probability of that occurrence. This contradicts the subject's belief when generating answers to the probability problem. The subject believed the opposite: the less the quantity of the occurrence element, the smaller the occurrence probability. Low FOR level begins the system 2 activation (Darmawan et al., 2020; Dewolf et al., 2014; Handley & Trippas, 2015; Kruglanski, 2013; Thompson & Johnson, 2014). This resulted in the subject doing a more in-depth examination and then realizing that the occurrence probability was determined by the quantity of occurrence elements and the quantity of the universe elements.

The focus of attention of subjects who experienced the second default-interventionist interaction type 1 on thinking activity 1 was the quantity of red apples and the quantity of all apples in each basket. This subject was given thinking activity 2 before being given thinking activity 1. Thinking Activity 1 was given because the subject spontaneously used the results of information abstraction in thinking activity 2. The abstraction result was that the more elements of the event and the more elements of the universe, the greater the occurrence probability. This spontaneity occurred because of the quantity similarity factor

(Handley & Trippas, 2015; Jang et al., 2016; Janssen et al., 2021; Thaneerananon, Wannapong, & Nokkaew, 2016).

Meanwhile, information in Thinking Activity 1, which is the focus of the subject's attention, contradicts the results of the subject's abstraction of information in Thinking Activity 2. This conflict resulted in a low FOR level of the subject. Furthermore, the subject was conscious that the occurrence and universe elements determined the occurrence probability.

Subjects assigned thinking activity 2 experienced type 2 default-interventionist interaction involving automatic and conscious processes. The subject's attention on thinking activity 2 was the quantity of apples in each basket and the probability of picking a red apple. Thinking activity 2 was arranged so that the less the number of universal elements or the number of apples in the basket, the greater the probability of getting a red apple. This information was contrary to the subject's belief in producing an answer to the probability problem. The subject believed that the fewer elements of the universe, the greater the occurrence probability. Information on thinking activity 2 contradicting this subject's beliefs resulted in a low FOR level. Therefore, the subject examined the answer again, and the conscious process was active.

Subjects were given thinking activity three and experienced type 3 default-interventionist interaction involving the unconscious and empirical-accuracy processes. The subject's focus of attention who experienced type 3 default-interventionist interaction on thinking activity 3 was the color and size of the image of a piece of cardboard of equivalent fractions. The size of the cardboard cutouts was arranged smaller and smaller from top to bottom. Such arrangement of cardboard drawings resulted in the activation of the empirical-accuracy process. The subject must look at and measure each piece of cardboard to produce an equivalent fraction. Meanwhile, giving a specific color to the cardboard cutout attracted and directed the subject's attention.

CONCLUSION

Thinking activity 1 could condition the occurrence of type 1 default-interventionist interaction. Thinking activity 2 could condition the occurrence of type 2 default-interventionist interaction. Thinking activity 3 could condition the occurrence of type 3 default-interventionist interaction. This research concluded that the default-interventionist interaction occurred because the content and arrangement of the thinking activity conditioned the subjects to pay attention to information gradually and change the subjects' beliefs.

Lecturers were recommended to produce, develop, and research thinking activities on topics other than probability at various levels of education. The default-interventionist interaction was essential to be conditioned when system 1 dominated students' thinking, causing difficulties. Lecturers were recommended to produce, develop, and research thinking activities on topics other than probability at various levels of education. The default-interventionist interaction was essential to be conditioned when system 1 dominated students' thinking, causing difficulties.

Acknowledgments

The authors thank the students of Mathematics Education, Universitas Negeri Malang, Universitas PGRI Madiun, and Universiti Teknologi Malaysia, who have been involved in this research.

Declarations

Author Contribution : S: Conceptualization Framework, Writing - Original Draft, Editing and Visualization.
PD: Writing - Review & Editing, Formal analysis, and Methodology.
WM: Validation and Supervision.
SO: Expert and Concept Consultant.

Funding Statement : This research was funded by the director general of Strengthening Research and Development with the Ministry of Research, Technology and Higher Education of the Republic of Indonesia for supporting and funding this research.

Conflict of Interest : The authors declare no conflict of interest.

Additional Information : Additional information is available for this paper.

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