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Developing the Digital Task Analysis (DTA) framework to enable the assessment and redesign of digital resources in mathematics education

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Abstract

Digital task design is an important issue when integrating technology into mathematics education. However, existing frameworks often are not fine-grained enough for supporting teachers in designing tasks or they only focus on geometric topics. In this paper, we share a case study as the first cycle of our design-based research study that aims to extend and adapt the well-known Dynamic Geometry Task Analysis framework for analyzing further digital materials. The adapted framework is named Digital Task Analysis (DTA) model and can be utilized to analyze, modify, and design digital materials from other mathematical topics. The model focuses on supporting teachers in integrating two essential aspects within digital materials, namely creating cognitively stimulating tasks and exploiting added value of technology. In this paper, we present the first analyses of three cases representing digital materials including visualizations addressing lower secondary mathematics following the DTA model. The results show that the presented DTA model is suitable to analyze such digital materials and has the potential to support teachers in designing, assessing, and modifying digital tasks that support learners in focusing their attention on mathematically relevant aspects of digital resources, and in deepening their awareness of how to formulate targeted tasks for learners.

Keywords: Digital Materials, Lower Secondary Education, Mathematics Education, Task Quality

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During the past decades, the development of technology and technology-based materials started to transform mathematics teaching and learning (Drijvers et al., 2016). Furthermore, the Covid-19 pandemic enhanced digitalization efforts from primary to tertiary school levels (e.g., Borba, 2021). In Austria, for example, from autumn 2021 on, students at lower secondary schools have been or will be equipped with digital devices (laptop or tablet) starting with 5th- and 6th-grade students. However, is this all that is needed for the successful digitalization of a learning environment? Research suggests that is not enough, because one central factor for successful technology integration in mathematics education lies within the role of the teacher (Clark-Wilson et al., 2020; Clark-Wilson & Hoyles, 2017). For example, Drijvers (2015, pp. 147–148) summarizes three factors for such successful integration, all related to teacher activities: the design, the educational context, and the role of the teacher. Among others, the first factor includes the design of tasks, lesson plans, and included student activities (Drijvers, 2015). But how do we prepare pre- and in-service teachers for teaching with technology and integrating digital materials in class to support the learning progress (of teachers and thus of students as well) in a meaningful way?

In this paper, we first outline the issue that led to our research project, and we describe our focus on digital task design. Next, we concentrate on existing frameworks for assessing digital materials and argue the need to adapt them for being applicable to a broader range of mathematical topics and tasks. Afterwards, we present the first cycle of our design-based research study consisting of (i) theoretically developing the DTA model based on the existing DGTA framework and (ii) empirically employing the DTA model on three selected digital materials that represent the cases in our case study approach.

One specific problem in the context of task design and lesson planning is the huge amount of available digital resources; for example, on www.geogebra.org (the website of GeoGebra, a Dynamic Mathematics Software (DMS) widely used in Austrian schools) at the moment more than one million digital materials are publicly accessible (May 2023). Therefore, teachers at least need to be enabled in assessing the quality and suitability of digital resources for their mathematics lessons. Moreover, we consider it necessary to foster this competency already in teacher training programs. That is why in our joint mathematics teacher training program at Johannes Kepler University (JKU) and colleges of education, we currently develop a course for prospective mathematics teachers in our master program that aims at fostering pre-service teachers' skills in designing, creating, and thus assessing digital resources and accompanying tasks at the lower secondary level (Lindenbauer et al., 2022).

First, the concept of resource needs to be defined more precisely. According to Gueudet and Trouche (2009) resources – digital and non-digital – include a broad range of different materials, for example, Internet resources, interactive worksheets, or textbooks. In this paper, digital resources are mainly dynamic, GeoGebra-based materials embedded in online worksheets additionally including further tasks that enable learners to engage interactively with this material. The terms digital resources and digital materials are used synonymously, and tasks are meant as specific prompts or requests to students to engage with a digital resource.

When focusing on the quality and suitability of digital resources, researchers (and/or designers) in mathematics education list various relevant criteria. For internal use in our project, we summarized criteria based on literature resulting in the following, non-comprehensive categories: standards and learning goal alignment, mathematical content, didactical aspects and task design, pedagogical aspects, technology-added value, design and presentation, feedback and assessment, technical aspects, and accessibility (e.g., Donevska-Todorova et al., 2021; Kimeswenger, 2017; Leacock & Nesbit, 2007; Trgalova & Jahn, 2013). Most of these categories encapsulate a range of different aspects (e.g., didactical aspects and task design), and although together they provide a broad overview of relevant criteria, they do not seem to be fine-grained enough to support learners practically when designing digital resources. One central issue in designing (digital) resources with a significant effect on learning is the content and design of tasks (Watson & Ohtani, 2015, p. 3). For our research project about developing a teacher training course, we have thus decided to focus on one specific aspect – digital resources and task design with Dynamic Mathematics Software (DMS). Now, we need a suitable framework for this.

There already exist various theoretical frameworks or models to assess different aspects of digital resources; for example, McCulloch et al. (2021) provide a structured overview of frameworks about learning mathematics with technology organizing them within four related categories based on different perspectives of technology use: (i) students' learning with technology, (ii) teachers' design and evaluation of tools and tasks, (iii) teachers' use of technology for teaching, and (iv) teachers' learning to use technology for teaching mathematics. With this research project, the focus lies on frameworks for designing and evaluating digital tasks. Relevant for this study is the second category, in which McCulloch et al. (2021) summarize the following frameworks: Sinclair's (2003) design principles for pre-constructed dynamic geometry sketches; the Interactive Geometry Software (IGS) framework (Sherman & Cayton, 2015); the Dynamic Geometry Task Analysis framework (DGTA) by Trocki and Hollebrands (2018); evaluating digital tools by their pedagogical, mathematical, and cognitive fidelity (Dick, 2008); and the Digital Instructional Materials (DIM) framework (Thomas & Edson, 2019). In essence, these frameworks and design principles are either too general to support teachers in designing concrete digital tasks or they are narrowing down the focus on the topic of (dynamic) geometry tasks.

Table 1. Overview of frameworks for teachers' design and evaluation of digital tools and tasks summarized and assessed from McCulloch et al. (2021)

Framework	Focus	Characteristics
DGTA Framework (Trocki & Hollebrands, 2018)	2 dimensions: technological actions and mathematical depth;	Focus on geometry, Fine-grained framework suited for task design;
Design principles for pre-constructed dynamics geometry sketches (Sinclair, 2003)	Five design principles for tasks utilizing dynamic geometry sketches;	Fine-grained framework, Focus on geometry, Integrated into DGTA framework;
Fidelity dimensions (Dick, 2008)	Pedagogical, mathematical, and cognitive fidelity	Broad categories, no specific focus on task design;
DIM Framework (Thomas & Edson, 2019)	2 dimensions: effective teaching practices (based on NCTM), and using technology to replace, amplify, and transform teaching (RAT);	Builds on specific NCTM teaching practices with a focus on the US curriculum, Relevant for various technology-based tools, no specific focus on DMS;
IGS Framework (Sherman & Cayton, 2015)	2 dimensions technology used as amplifier or reorganizer, and three goals regarding mathematical actions;	Similar aspects as DGTA, Incorporates RAT framework (see section 2.1.);

From this list of frameworks for designing and evaluating digital tools and tasks (see Table 1), the DGTA framework (Trocki & Hollebrands, 2018) suited as starting point for our research aim although it concentrates on the topic of geometry. Firstly, it already integrates Sinclair's (2003) design principles on geometric tasks. Secondly, fidelity issues as outlined by Dick (2008) are – similar to the quality criteria outlined before – not fine-grained enough for supporting pre-service teachers in learning how to design and write concrete (digital) tasks. For example, pedagogical fidelity would include the aspect of task design but does not provide further information on what to consider when creating a specific mathematics task. Thirdly, the DIM framework builds on specific NCTM teaching practices focused on the US curriculum and further is not specific to DMS as in this study's case. Finally, the IGS framework (Sherman & Cayton, 2015) focuses on geometric tasks and builds on Sinclair's (2003) work, which is already integrated into the DGTA framework, essentially including three mathematical actions: observe, explore, and make and test conjectures. In addition, it considers the role of technology as an amplifier and a reorganizer compared to traditional tools. This latter part will be relevant for the further development of the DGTA framework as outlined in the next section of this paper. Compared with the DIM framework, however, the DGTA model provides a more fine-grained framework especially concerning integrated mathematical actions. Moreover, the DGTA framework is repeatedly cited in recent research about teacher training in mathematics education indicating its relevance in this field of designing digital materials (e.g., Bozkurt & Yiğit Koyunkaya, 2022; Fahlgren et al., 2022).

The DGTA framework from Trocki and Hollebrands (2018) focuses on digital tasks only within the topic of geometry. Due to the development of DMS, which integrates a computer algebra system into a dynamic geometry system, also other mathematical topics such as numbers, algebra, statistics, or calculus move into the focus of digital material designers. However, no similar fine-grained and supportive framework for designing and assessing digital non-geometric tasks has been developed so far. The DGTA framework can be applied to assess the relative quality of digital resources based on students' mathematical activities and thus can enhance digital task quality. Due to the structure and focus of this framework, it supports teachers in (re-)designing tasks that aim at active students' engagement in mathematically meaningful activities (Trocki & Hollebrands, 2018). To exploit this potential, we saw the need to adapt this framework for being applicable to digital tasks in other mathematical topics as well to support (prospective) teachers in designing and assessing mathematically meaningful tasks within digital resources. With this challenge in mind, the study design within the above-mentioned research project addresses the following research issues: How can the DGTA framework of Trocki and Hollebrands (2018) be adapted for being able to assess and modify digital resources in non-geometric topics? How can the adapted framework support prospective teachers in evaluating, modifying, and designing digital materials?

The second issue is part of the overarching research project and will be addressed in upcoming papers. In this article, the focus lies on the first question: we present our first adaptation of the DGTA framework for non-geometric resources and discuss, whether and how the adapted framework is suitable for analyzing and modifying selected digital materials that include visualization aspects by case study research. For the cases presented in this paper, we decided to focus on digital resources including visualizations as in our understanding they are most like geometric tasks, which usually include visual representations of geometric objects. How do we understand the term *visualization*? According to Duval (2006), students can access mathematical objects only through their semiotic representations which he classifies in registers. Here, the non-discursive register contains representations such as iconic figures, diagrams, and graphs which can be interpreted as iconic representations of mathematical objects in the sense of Bruner et al. (1971). Related to such representations, Arcavi (2003, p. 21) defines visualization as "…the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools ...". In this paper, we follow Arcavi's (2003) definition of visualization and apply the term *visualization aspects* to digital materials that contain external, non-discursive or iconic representations such as pictures, images, and diagrams.

The Adaptation of the DGTA Framework – Theoretical Backgrounds

Addressing the problem posed, we present three theoretical frameworks relevant for digital resource and task design on which we refer afterwards, when we present our first version of an adapted model for analyzing digital tasks mainly based on Trocki and Hollebrands' (2018) DGTA framework.

Literature research reveals numerous theoretical frameworks addressing more or less extensive aspects of integrating technology into mathematics education. McCulloch et al. (2021) analyzed and structured several theories into four framing categories (see previous section); with this in mind, our study focuses on the "design and evaluation of mathematics technology tools and tasks" (p. 331) and we start with our central model – the DGTA framework that will form the basic structure of our model.

Trocki and Hollebrands (2018) found that technology-based, dynamic geometry tasks, in which technological actions are coordinated with mathematical tasks, have a higher potential to stimulate students to think mathematically. To this end, the authors developed a theoretical framework that aims at indicating the quality of dynamic geometry tasks and guiding teachers in designing tasks. The DGTA framework relates characteristics of prompts to learners (e.g., questions or instructions) to the way of formulating tasks. It includes two dimensions: What *mathematical depth* does a task allow and what type of *technological action* is involved (Trocki, 2014; Trocki & Hollebrands, 2018). Each dimension consists of seven different levels.

Table 2. Dynamic Geometry Task Analysis (DGTA) framework

Allowance for Mathematical depth	
Levels	*Levels and descriptions*
N/A	Prompt requires a technology task with no focus on mathematics.
0	Prompt refers to a sketch that does not have mathematical fidelity.
1	Prompt requires student to recall a mathematical fact, rule, formula, or definition.
2	Prompt requires student to report information from the sketch. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, procedures, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, procedures, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.
Types of Technological action	
Affordances	*Descriptions*
N/A	Prompt requires no drawing, construction, measurement, or manipulation of the current sketch.
A	Prompt requires drawing within the current sketch.
B	Prompt requires measurement within the current sketch.
C	Prompt requires construction within the current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
F	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise that may be based on testing extreme cases. (Adapted from Sinclair, 2003, p. 312)

Note: From "The development of a framework for assessing dynamic geometry task quality," by A. Trocki and K. Hollebrands, 2018, Digital Experiences in Mathematics Education, 4(2-3), p. 123

The authors define a mathematical prompt as "a written question or direction related to a sketch that requires a verbal or written response" (Trocki, 2015, p.3). Table 2 lists the levels of mathematical depths in this framework. The second dimension involves a categorization of technological actions and consists of technological prompts (i.e., questions or instructions that require a drawing, measurement, construction, or manipulation on a dynamic sketch as outlined in Table 2).

When creating digital resources and corresponding prompts, this model is intended to support teachers in creating high-quality and cognitively stimulating tasks. The following aspects are particularly relevant: At first, prompts must coordinate technological actions with mathematical depth, as this leads to higher quality tasks (Trocki & Hollebrands, 2018, p. 124); secondly, learners are encouraged to go beyond the visualized mathematic content (i.e., to achieve as much mathematical depth as possible); and thirdly, to combine this with the potential of technology (Trocki, 2014, p. 704). In sum, the DGTA framework highlights two aspects: cognitively stimulating tasks (through levels for mathematical depth) and added value of technology use compared to traditional tools (through affordances for technological actions). In particular, these two aspects are relevant guidelines for our overarching design-based study.

When thinking about adapting the DGTA framework to other mathematical topics, the second dimension especially comes into focus as it concentrates on geometric actions and thus needs to be modified for a more general suitability. To generalize these actions for non-geometric topics, we decided to further consider the RAT framework, a theoretical model that covers whether and how technology integration qualitatively changes mathematics teaching and that provides a more general perspective on technological actions. This model consists of three categories depending on whether technology replaces (R), amplifies (A), or transforms (T) teaching practice (Hughes et al., 2006; Thomas & Edson, 2019). Hughes et al. (2006) describe:

- R – Technology as replacement: In this role, technology use does not lead to changes in teaching and learning processes or goals, "technology serves merely as a different means to the same instructional end" (p. 1617).
- A – Technology as amplification: The use of technology makes processes, methods, or goals more efficient; for example, by performing calculations or generating representations quickly and accurately. However, it does not significantly change learning or teaching processes, and students could achieve the same goal without technology.
- T – Technology as transformation: Technology integration has the potential to change students' activities by supporting a reorientation in their thinking that would be difficult or impossible to achieve without technology. The focus of student thinking in technology-enhanced tasks might be on recognizing patterns and making and testing conjectures. The mathematical goal of a task would be difficult to achieve without technology.

When designing digital materials, we want to exploit the potential of technology compared to traditional tools (e.g., paper and pencil). Therefore, it should be used either in an amplifying or transforming way. In the next section we outline, how we have integrated the RAT framework into our new model.

The DGTA framework adapted in this article aims at creating and designing tasks and questions in digital resources. Task design is a central part of mathematics education, not only for technology-based tasks, and thus we additionally consider another framework for cognitively stimulating tasks relevant to all mathematical topics. Smith and Stein (1998) summarize four categories of tasks depending on their cognitive demand to learners: (i) memorization, (ii) procedures without connections to concepts or meaning, (iii) procedures with connections to concepts or meaning, and (iv) doing mathematics. The authors use these categories to classify tasks as 'good', which means "as having the potential to engage students in high-level thinking" (p. 344), depending on the cognitive demand they present to students. Tasks of the first two categories pose lower-level demands than tasks from categories (iii) and (iv), which should lead to higher-level thinking. Especially higher-level demands were also considered when developing the DGTA framework (Trocki & Hollebrands, 2018).

In the next section, we introduce our first, slight adaption of the DGTA framework that integrates the three theoretical models discussed so far and describe our considerations. Henceforth, we call it the *digital task analysis (DTA) model*.

Introducing the Adapted Model – The DTA Model

For the first dimension of *mathematical depth*, Trocki and Hollebrands (2018) drew among others mainly on the work of Smith and Stein (1998) on cognitively stimulating tasks, which can be applied not only to geometry but to all mathematical topics. Therefore, we slightly adapted the DGTA levels of allowance for mathematical depth (Table 2) by reviving and integrating the four categories outlined by Smith and Stein (1998), which results in the following structure for designing digital tasks (Table 3).

Table 3. Adapted model – Allowance for 'Mathematical depth'

Level	Description: Prompt requires …
N	... a technology task with no focus on mathematics
0	... digital material does not have mathematical fidelity
1	... to recall a mathematical fact, rule, formula, or definition. The cognitive demand of the prompt corresponds to *memorization*.
2	... to report information from the applet or to use a procedure without connections to concepts or meaning. The student is not expected to provide an explanation.
3	… to consider the mathematical concepts, procedures, or relationships in the applet.
4	… to explain mathematical concepts, procedures, or relationships in the applet.
5	… to go beyond the current applet and generalize mathematical concepts, processes, or relationships or to make and test assumptions. The cognitive demand of the prompt corresponds to *doing mathematics*.

Based on Trocki (2014), level 1 is related to the category memorization. Next, we supplemented level 2 with the category *procedures without connections to concepts or meaning*. Levels 3 and 4 are related to *procedures with connections to concepts or meaning,* which is already expressed in the original wording. Finally, level 5 has been supplemented by the activity *make and test assumptions*, which requires students to go beyond the content represented within the digital material.

The second DGTA dimension categorizes types of technological actions within a digital material but focuses on geometrical actions (e.g., draw, measure, construct). Therefore, it must be modified to a greater degree than the first dimension to fit also non-geometric mathematical actions. In essence, we adapted the types of technological actions by merging it under the RAT model. Codes A to C (see Table 2) cover actions that could also be performed with paper and pencil. Code D refers to the potential of DGS to modify geometric objects through interactive dynamic activities, and codes E and F highlight the potential of dynamic materials to recognize and investigate invariant relationships (Trocki & Hollebrands, 2018). According to the aforementioned roles of technology (RAT model, see previous section), technology in the DGTA framework tends to act as an amplifier through the activities of codes A to C and in a transforming way for the affordances of codes D and E.

Since prompts for technological actions in geometry cannot or only partially be transferred to other mathematical content, we adapted them by underlying the RAT model as presented in Table 4.

Table 4. Adapted model – Types of 'Technological action'

Code	RAT	Description: Prompt requires ...
N	R	… no action within the applet.
A	A	… a technological action, which result could be also achieved without technology. (*Note: imitation of working with paper & pencil*)
B	A/T	… dragging or using other dynamic aspects of the applet.
C	T	… a manipulation of the applet that enables the recognition of emergent invariant relationship(s) or pattern(s) among or within mathematical objects. (*Note: continuous experimentation, hypothesize: find out 'something' (by chance)*)
D	T	… a manipulation of the applet(s) that may surprise or cause one to refine thinking based on themes within the surprise when exploring the relationships represented. (*Note: continuous exploration, test hypotheses: examine purposefully*)

Code N denotes a prompt that does not require any action with technology and thus can be related to the role of technology as a replacement (R), as technology would be just another means to the same end. For non-geometric tasks, code A encompasses all actions with the digital material where technology acts as amplifier (A), that is, learners could achieve the same goal without technology, albeit more slowly. Related to Trocki and Hollebrands' (2018) framework, code A thus summarizes actions that mimic activities with traditional media. The other codes B, C, and D are essentially based on the DGTA framework and can – except for code B – be assigned to the role of technology as transformer (T) (see Table 4).

METHODS

The aim to adapt Trocki and Hollebrands' (2018) DGTA framework to non-geometric topics is embedded within a larger project about developing a university teacher training course focusing on prospective mathematics teachers' professional knowledge concerning technology integration (Lindenbauer et al., 2022). In essence, we want to further develop a domain-specific theory of mathematics education by interweaving theoretical development and practical application of this theory on developing digital materials in multiple cycles. According to Cobb et al. (2003), this aim can be pursued suitably by employing a design-based research (DBR) methodology. DBR underlies a pragmatic worldview and thus does not propose a specific way of applying research methods but allows all tools necessary to answer the research questions (Cobb et al., 2003; Biesta & Burbules, 2004).

The first cycle of our DBR consists of two phases: (i) theoretically developing the DTA model based on adapting the original DGTA framework, a process we have outlined in the previous section, and (ii) testing our adapted first DTA model by employing it on various digital materials including visualization aspects. Within this pragmatic approach, we have framed the second phase by a case study approach and present these results in this paper. Case study involves the investigation of one or more cases to understand a phenomenon in depth (Yin, 2009). In our study, the phenomenon we want to examine is the suitability of the DTA model for assessing and modifying non-geometric digital resources, and the cases applied for this purpose are selected digital materials for lower secondary mathematics education. On the one hand, the goal is to explore the usability, potential, and flaws of our model for the selected cases, and on the other hand, to examine whether and in what way digital tasks and prompts can be modified according to the DTA model. For both aspects, we want to gain a more general understanding of our DTA model as well as how to proceed for further adaptations and analysis. Therefore, we decided on an exploratory (or instrumental) case study (Stake, 1995; Yin, 2009) in which each analyzed digital resource serves as a case for exploring and thus further developing the DTA model. An exploratory case study is applied to explore an initial and general understanding of a phenomenon or a theory (Yin, 2009; Stake, 1995) – in this study concerning the applicability of the developed DTA model.

Analyzing digital resources from non-geometric topics offers a broad range of possible cases: digital materials for primary, lower, or higher secondary schools; for different content areas; for various teaching phases and technology uses (e.g., exploring concepts or practicing skills) (Büchter & Leuders, 2009; Drijvers et al., 2011); pre- or partial designed digital materials that allow different levels of student activity (Roth, 2017) and more. The original DGTA framework was developed for digital materials that usually include iconic representations of the analyzed geometric objects. As outlined before, for the cases of our first cycle of developing and testing the DTA model, we concentrated on structurally similar tasks and thus chose digital materials including visualizations of mathematical objects (i.e., iconic representations such as circles or strips for fractions, or arrows for numbers). Further decisions were led by the context of our study as it is embedded within the Austrian FLINK project at Johannes Kepler University (see https://www.jku.at/flink-in-mathe/). FLINK is a national digital material development project for lower secondary mathematics education that aims to provide mathematics teachers with pre-designed, DMS-based (in our case GeoGebra) digital resources starting with grade 5. The national curriculum, and thus the FLINK project, divides mathematics content of lower secondary education into four categories: (1) numbers and measurements, (2) variables, (3) geometric figures and solids, and (4) models and statistics (Bundesministerium für Unterricht und Kunst, 2021). As we address non-geometrical tasks and the FLINK project does not offer suitable materials containing visualizations for the content-related categories variables as well as models and statics so far, we focus on digital resources from the topic numbers and measurements. For this paper, we chose three cases represented by pre-designed, GeoGebra-based digital resources including visualizations for grade 5 in which students can explore mathematical concepts. We decided to examine these resources because they include visualizations and are similarly structured as digital materials examined by the original DGTA framework. Digital tasks for exploring mathematical concepts can provide deeper insight into the DTA model because more diverse mathematical actions are required than in drill and practice-oriented tasks. Furthermore, the potential of technology particularly lies in utilizing digital materials to discover, develop, and explore ideas (Ball & Stacey, 2019) and especially such tasks are more difficult to realize with technology (Drijvers, 2018). Therefore, we see the greatest strength of the DTA model in supporting teachers in analyzing and modifying tasks for exploring mathematical ideas.

For analyzing the cases, we proceeded as follows: the first two authors independently examined and assessed each prompt of the chosen digital resources following the guidelines of the DTA model and supported by original literature from Trocki and Hollebrands (2018). To further enhance reliability, we compared, discussed, and justified our codes and suggestions until we reached a well-founded joint result for each prompt that follows the DTA model. In the next section, we will present these results.

RESULTS AND DISCUSSION

The digital resources presented each consist of a GeoGebra-based applet implemented within a digital worksheet that includes additional prompts for students. We now describe and analyze our results for the three chosen cases and further suggest modified prompts following our adapted DTA model as summarized in Tables 3 and 4.

Case 1 – The Concept of Fraction as Part of a Whole

The first resource discussed in this paper addresses grade 5 students who explore the concept of fraction. In particular, the material introduces the terms denominator, numerator, and fraction bar in its numerical representation as well as a linked representation of the presented fraction as part of a whole circle.

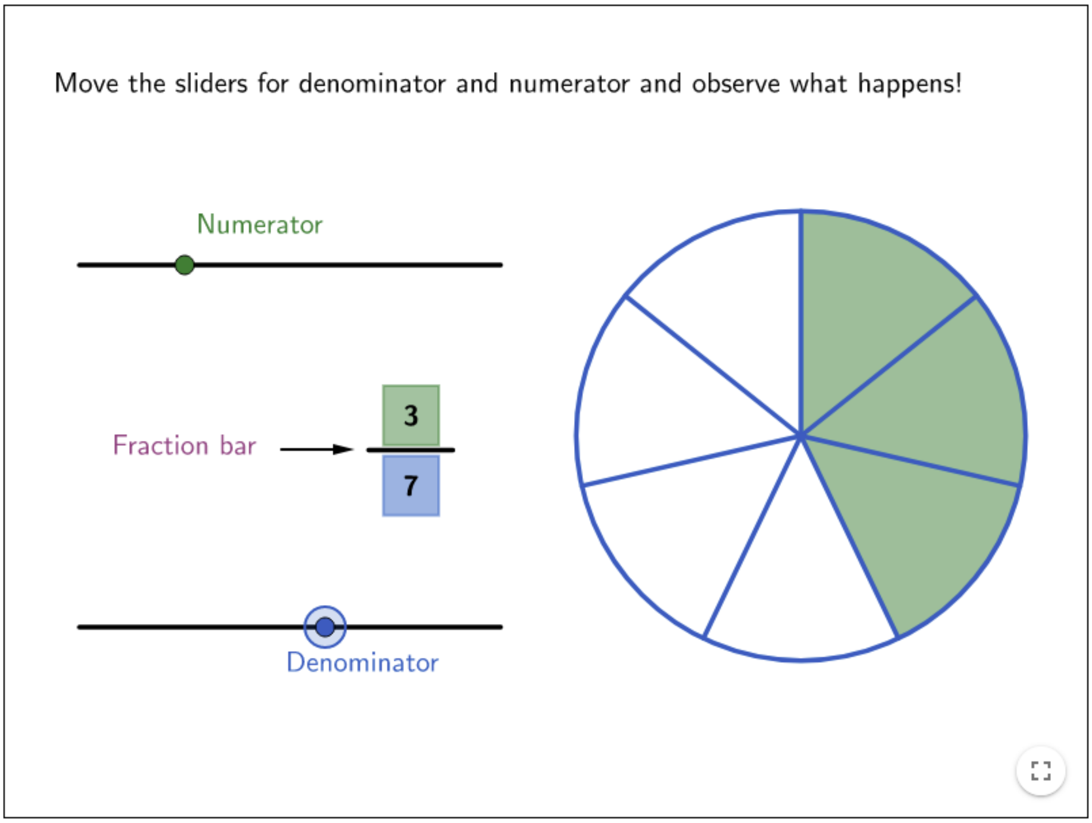

Figure 1. Digital resource 'Denominator, numerator, fraction bar' (https://www.geogebra.org/m/r4ryzr2q)

Students can explore the numerical representation of fractions and its meaning in the visualized representation via changing values of the denominator and/or numerator with sliders and observe how the respective linked representations change.

This digital resource contains the following prompts (translated into English) either within the applet presented in Figure 1 or below as accompanying tasks:

1) Move the sliders for the denominator and numerator and observe what happens.
2) The denominator becomes larger and … (select all correct answers)
    a) … the subdivisions remain the same size.
    b) … the subdivisions become finer.
    c) … the subdivisions become coarser.
3) The denominator becomes smaller and … (select all correct answers)
    a) … the subdivisions remain the same size.
    b) … the subdivisions become finer.
    c) … the subdivisions become coarser.
4) Set the numerator and denominator to the same value! What can you recognize?
5) What happens if you change the denominator, and the numerator stays zero?

Now we analyze these prompts utilizing the presented DTA model as presented in Tables 3 and 4. The first prompt, 'Move the sliders for the denominator and numerator and observe what happens.' consists of both, a mathematical and a technological component, which requires a student to drag sliders and thus fulfills at least technological action (TA) B. At best, this TA supports students to build a connection between the value of the numerator (or denominator respectively) and its meaning concerning the fraction as part of the whole represented as a circle, which would be the number of highlighted equal parts (or the number of equally sized parts of the circle). In such a case, this prompt would allow for recognizing relationships (or patterns) within the digital materials and thus could be categorized with TA C. According to Sherman and Cayton (2015), who also developed a framework for evaluating digital tasks, making mathematically meaningful observations and/or looking for invariant relationships can be one essential goal when working with digital resources. In this digital material's first prompt, for guaranteeing mathematically meaningful observations, we would suggest specifying the wording of the task as specified below, which additionally would categorize prompt 1 with code C.

Regarding the allowance for mathematical depth (MD), the first prompt is ambiguous. As mentioned in the previous paragraph, *making mathematically meaningful observations* is a suitable goal for digital tasks. However, the task '… observe what happens' can be coded differently depending on students' prompted behavior. First, it could be coded with MD N, as the task does not guarantee that students will focus on mathematically relevant information visualized or presented in this digital task. Lindenbauer (2018; 2020), for example, examined grade 7 students working with pre-designed digital materials concerning functions and discovered an intention-reality discrepancy between the mathematical content digital materials are intended to visualize and what students observed and interpreted – especially, when they focus on visual aspects of abstract mathematical concepts within a digital material. Second, the first prompt could be coded with MD 2 if students focus on the visualized representations of the numerator and denominator and would later discuss their observations because in this case, they will have to report information from the digital material. Third, if students not only report information but also consider, for example, the mathematical relation between the numerator and the number of green-colored parts, this task could also be coded with MD 3. Based on this analysis, we conclude that the prompt 'observe' is too general to guide students' actions in a specific way according to the adapted model and thus suggest the following modification for increasing the quality of prompt 1 in the sense of Trocki and Hollebrands (2018):

'Move the slider labeled numerator and observe what happens within the digital material. Can you explain the relationship between the numerator of the fraction and the visualized circle?'

This prompt requires students to manipulate the slider, which allows students to recognize a connection between numerical and visual representation and to consider and explain this mathematical relationship hence resulting in codes (3, 4; C). A similar worded task can be formulated for examining the denominator of a fraction.

The second and third prompts have the same structure and thus can be analyzed together. Both questions are posed in a multiple-choice format and aim at further examining the connection between the denominator or numerator and its visualization via sectors of a circle. The TA 'move the sliders' mentioned within the applet also refers to the questions posed below within the digital material. In both questions, students should report information from the digital resource resulting in code (2; B); that means that students are required to drag (a slider) and report information from the material, which students are required to relate to pre-defined answers.

Now, we consider prompt 4: 'Set the numerator and denominator to the same value! What can you recognize?' Again, the TA can be coded as B as students would have to drag sliders to set the numerator and denominator to the same values. Furthermore, the prompt allows for MD 2 at best – students must report information from the applet but similar to prompt 1, we cannot guarantee that students will focus on mathematical information within the applet. One issue regarding the quality of this prompt is that it does not invite students actively to experiment dynamically within the digital resource. Dragging sliders to one set of equal numbers for both quantities would be enough to complete the prompt but not for recognizing a pattern between a fully colored circle and the fraction with value one. For producing a higher-level task, we suggest rewording the prompt in the following (or a similar) way:

'Set the numerator and denominator to the same value. Change repeatedly the values of both quantities. Write a conjecture about the relationship between equal values of denominator and numerator, the value of the fraction, and the visualization within a circle.'

In this way, the prompt allows students to consider, explain, and hypothesize about the relationship between various representations of a mathematical object via experimenting within the digital materials thus resulting in code (3, 4, 5; C).

Finally, the fifth prompt 'What happens if you change the denominator, and the numerator stays zero?' requires students to experiment with – and thus drag – the slider representing the denominator. As outlined concerning prompt 1, the mathematical allowance cannot be determined due to the ambiguity of the wording 'what happens …'. At most, this prompt allows for MD 3 – considering mathematical relationships together with code B. We suggest improving the wording in the following way, which would result in a code (3, 4; C):

'Set the numerator to zero and drag the slider for the denominator. Can you determine a relation between the value of the numerator and denominator respectively and the visualization within a circle? Explain!'

For the next case, we remain within the topic of fractions.

Case 2 – Strip Diagram Representing Fractions

As in case one, the second case presents a digital resource within the topic of fractions that aims at examining equivalent fractions. We chose to analyze this particular material because it includes a visualization of different fractions in the form of strips and utilizes dynamic aspects but not in the form of sliders (as in case 1) but as a draggable line. This resource enables learners to study equivalent fractions with numerators from one to ten by dragging the orange line. Whenever the line is positioned at the end of a rectangle representing such a fraction, equivalent fractions left of the orange line are highlighted in blue. For example, Figure 2 visualizes that one-third equals two-sixths equals three-ninths $\left(\frac{1}{3}=\frac{2}{6}=\frac{3}{9}\right)$.

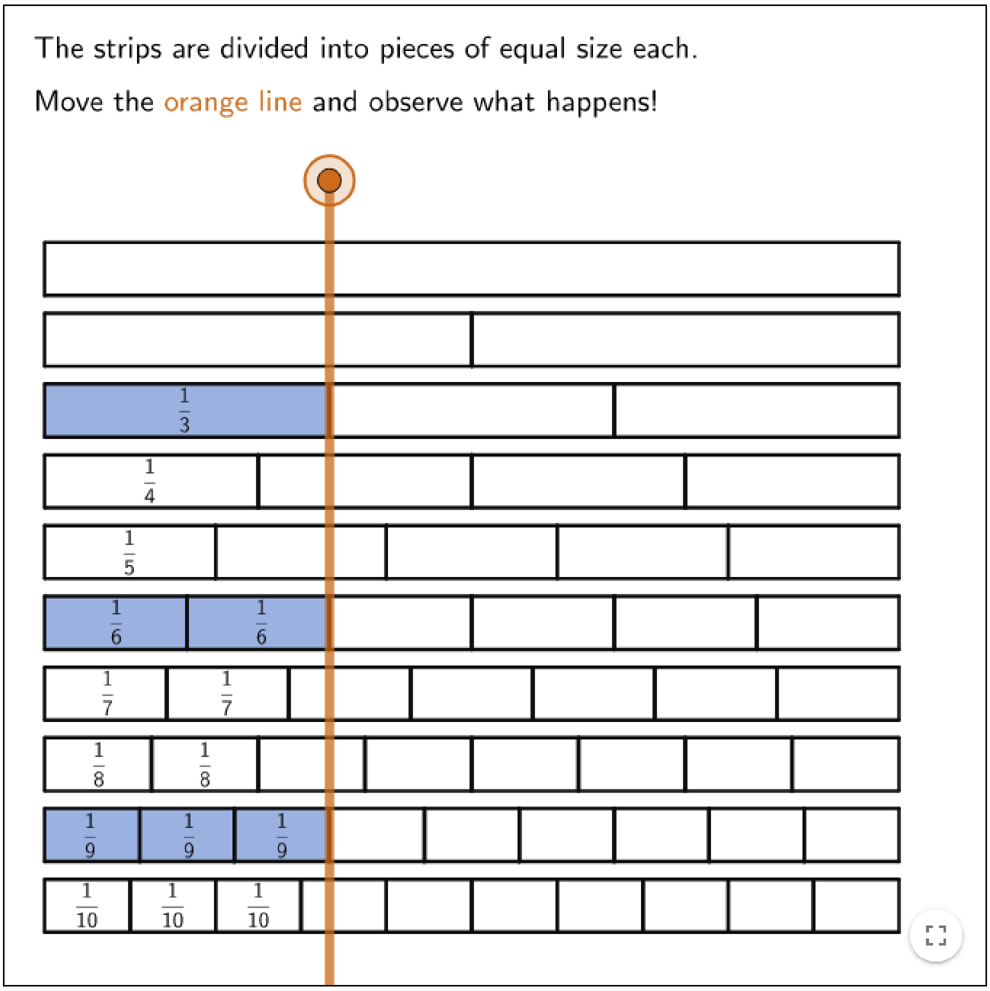

Figure 2. Digital resource 'Strip diagram' (https://www.geogebra.org/m/msxcctam#material/b2qvdutv)

This digital resource contains the following prompts (translated into English), the first one within the applet presented in Figure 2 and the others below as accompanying tasks:

1) The strips are divided into pieces of equal size each. Move the orange line and observe what happens.
2) Move the line to the center. How many quarter pieces fit into a half $\left(\frac{1}{2}\right)$?
3) Move the line to the center. How many tenths fit into a half $\left(\frac{1}{2}\right)$?
4) Move the line so that two-thirds $\left(\frac{2}{3}\right)$ are colored. Which fractions have the same value?

All four prompts consist of a mathematical and a technological component (see Tables 3 and 4). The first prompt 'Move the orange line and observe what happens' can either be analyzed as a standalone prompt or as a prompt preceding all further tasks. As a single prompt, the analysis is very similar to the first prompt in case 1: 'observe' is an ambiguous task for students which could lead to MD N (students do not focus on mathematics), code 2 (students are required to report information from the digital materials), or code 3 or 4 if students either consider or even explain mathematical relevant relations between equivalent fractions. From a technological perspective, this task can at least be coded with TA C as dragging the orange line supports students in relating equivalent fractions from a mathematical point of view or – at best – code D if the fact that fractions color in and out when moving the orange line has a surprising effect and thus stimulates students' thinking. In sum, for specifying the first prompt and thus guiding students' behavior, the first prompt either must be reworded or only be used as an introductory prompt for tasks two to four. The following suggestion for prompt 1 (or an additional one) would lead to the specific code (3, 4; C) and thus to a higher-level task (Trocki & Hollebrands, 2018):

'The strips are divided into pieces of equal size each. Move the orange line until two or more fractions are colored in blue. Can you explain the relationship between two (or more) specific fractions that are colored blue at the same time?'

Prompts 2, 3, and 4 are similarly structured; in both cases, students are required to drag the orange line and report information from the digital resource, but they are not expected to provide any explanation, which results in code (2; B). The technological implementation of this digital material mimics actions with paper-and-pencil but adds value to traditional representations since it automatically colors equivalent fractions. Therefore, this technology integration at least amplifies a mathematical task compared to paper-and-pencil (Hughes et al., 2006). However, the discussed prompts do not encourage students to examine inherent relationships or to explain the meaning of equivalent tasks. Therefore, we suggest following prompts based on our adapted framework as one possible improvement.

'Drag the orange line to the center. How many quarters, sixths, eights, and tenths fit into a half? Can you find a rule for fractions equivalent to $\frac{1}{2}$*? Can you explain this rule?'*

or

'Drag the orange line to the center. How many quarters, sixths, eights, and tenths fit into a half? Can you find a rule for fractions equivalent to $\frac{1}{2}$*? Can you explain why there is no equivalent fraction with denominator 3, 5, 7, or 9?'*

The first modified prompt can be coded (2, 3, 4; C). In the second case, students are required to go beyond the current construction resulting in code (5; C). We could also further enhance this task by encouraging students not to focus on specific fractions but to generalize in the following way:

'Drag the orange line to the center. How many quarters, sixths, eights, and tenths fit into a half? Can you find a rule for fractions equivalent to $\frac{1}{2}$*? Can you formulate and explain a general rule on how to recognize equivalent fractions?'*

This modification would add MD 5 and result in code (2, 3, 4, 5; C). Furthermore, some of these tasks could be decomposed into several subtasks to make them more accessible for younger students. The next case visualizes adding natural numbers.

Case 3 – Adding on the Number Line

The – short – third digital resource we have analyzed for this paper introduces the visualization of adding (three) natural numbers on the number line. It is part of Austria's grade 5 curriculum but could additionally be integrated into primary school. Furthermore, this specific representation of arrows for natural numbers on the number line can be built upon when adding and subtracting integers or vectors.

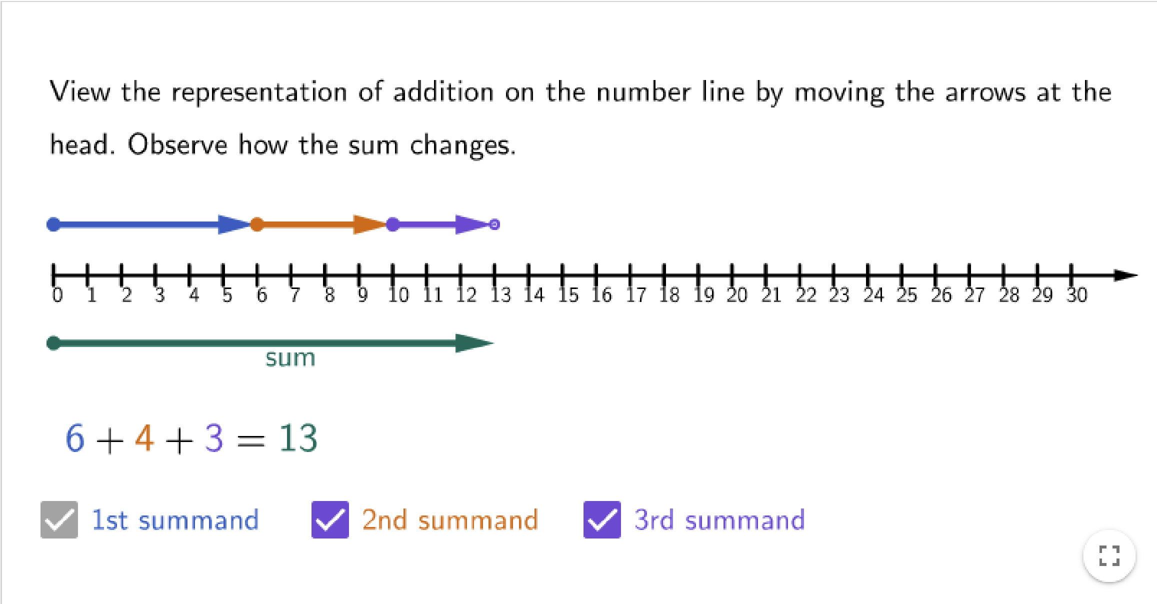

Figure 3. Digital resource 'Adding on the number line' (https://www.geogebra.org/m/kntnvvcr#material/dnxexdjk)

Starting with a first summand (colored blue), students can change the corresponding number below the number line by dragging the blue arrow at the head (see Figure 3). Then, students can display the second summand which results in a sum represented by a green arrow; the corresponding calculation is additionally displayed. At this stage, students can vary both summands from number 1 to a maximum sum. Finally, students can add a third summand that can be manipulated as the other ones. Here is an overview of integrated prompts (translated into English):

1) View the representation of addition on the number line by moving the arrows at the head. Observe how the sum changes.
2) Make the arrow for the second summand as short as possible. What would happen in the figure if the second summand were zero?
3) What would happen in the figure if all summands were 0?

For this task, we interpret the first prompt within the digital applet as an introductory prompt for tasks two and three (otherwise an analysis regarding the adapted model would lead to similar results as in cases 1 and 2), and we present a common analysis of all three prompts.

The prompts require students to move the arrows by the head and thus change the summands and consequently the sum, resulting in code C (see Table 4). Regarding mathematical depth (see Table 3), this task is difficult to assess; basically, students should go beyond the current construction (the smallest value for one summand to set is 1) and hypothesize the consequence of their action on the resulting sum and on the visualization ('what would happen if …'), which would lead to code 5. However, the prompts do not focus on mathematically relevant observations in the figure and thus could be analyzed as MD N. For specifying the prompts according to the framework, we suggest – depending on the specific aim of a lesson – the following task as one possibility (in brackets you can find the associated code):

'Choose one of the summands and move the corresponding arrowhead. How does the sum change if you increase/decrease this summand?' (2; B)

'Make the arrow for the second summand as short as possible. What would happen with the arrow chain – and the sum below the number line – if the second summand were zero? Can you explain?' (4, 5; B)

Due to the specific visualization (e.g., the arrows can only be dragged by the arrowhead) and the small range of mathematical content represented in this digital resource, it is difficult to aim at a higher-level TA. For example, recognizing that the arrows always form an uninterrupted chain could be the result of a code C prompt.

CONCLUSION

The adapted DTA model aims to expand the DGTA framework for the analysis of non-geometric digital resources. In this paper, we present results from the first cycle of our DBR, in particular an analysis of three cases represented by digital materials from non-geometric mathematical topics that include visualizations. For a first expansion of the framework, such materials are a suitable choice as they are structurally close to geometrical tasks, which usually include visual representations of geometric figures but already draw the focus on non-geometric topics such as numbers and measurements. Based on the study of these cases, we conclude that the adapted DTA model helps to analyze digital tasks for lower secondary education including visualizations. For example, applying the model reveals open prompts, such as 'move slider and observe' or 'what would happen if …', that leave room for various student actions. Such prompts cannot be coded unambiguously according to our model leaving room for – at least – two conclusions: either the adapted model is not suitable for visualization tasks, or it reveals actual ambiguity in such prompts. We tend to the second interpretation, as the same would be true when coding geometrical tasks via the original DGTA framework. Furthermore, as we cannot predict students' attention and perception it often seems sensible to adapt such open questions to support learners in focusing their attention on particular, mathematically relevant aspects of the digital resource by formulating prompts regarding mathematical depth more precisely instead of using ambiguous wording such as 'observe what happens'.

We additionally conclude that the DTA model has the potential to support pre- and in-service teachers in modifying tasks as presented in the previous section. Utilizing this model highlights the meaning and goals of prompts and how they can be extended or combined, thus emphasizing the possible learning outcomes achievable through task variation with different prompts. Furthermore, analyzing prompts can support teachers in recognizing connections between prompt and mathematical thinking and thus enables them to deepen their awareness of how to formulate tasks more concisely and targeted; especially in-service teachers can thus integrate their individual students' needs and characteristics. However, the question remains whether our modification would actually result in tasks of higher quality and thus enhance students' cognitive thinking processes as discussed by Trocki and Hollebrands (2018). This question needs to be examined in one of the next research cycles by empirical research in school.

Based on the considerations so far, our DTA model is limited to digital resources including visualizations. The FLINK project, for example, also provides materials for practicing skills that usually do not include dynamic visualizations but provide added value compared with paper-and-pencil tasks by automated feedback. Therefore, in a next step we will examine the applicability and potential limitations of this model to other topics and/or structured tasks. Regarding the allowance for mathematical depth, the usefulness of the adaptation can be expected, mainly because the underlying theoretical considerations, especially the work of Smith and Stein (1998), do not only focus on geometrical tasks. The possible types of technological actions are more difficult to frame; basically, the RAT model appears to be a suitable starting point for framing these actions. However, the technological actions prompted in the outlined cases mainly subsume under codes B and C so far. By combining three geometrical actions of the original framework within one code B, the adapted DTA model provides only four levels of technological actions (including code D which is difficult to achieve – possibly due to the age of learners discussed in this paper), and thus we assume potential finer subdivisions for specific mathematical topics such as algebra, functions, or statistics. Further research will show if the first DTA model could be refined or whether subframes of technological actions are needed depending on the mathematical topic involved.

The second cycle of our DBR will focus on systematic testing of cases based on several characteristics: topic, school level, or structural feature of tasks. Additionally, we have already started to examine the second outlined research question about the support the DTA framework provides for prospective teachers in designing digital resources and tasks for lower secondary education. Due to the abundance of digital materials, prospective teachers need guidance when designing and evaluating digital materials in mathematics education. Also, in-service teachers would profit from specific teacher training about the design and evaluation of digital materials, especially if they are not experienced in integrating technology or technology-based materials in mathematics teaching. Therefore, we will further consider the design of teacher training courses which meet the specific needs of in-service teachers (e.g., time constraints) but already base on their experiences in mathematics teaching with non-digital tools. Hence, applying the DTA model in mathematics teacher training and thus bringing together evidence-based theory and practice seems to be a fruitful way to support (future) teachers in integrating technology successfully in mathematics education and shall be further examined.

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Author Contribution	:	All authors read and approved the final manuscript. EL: Conceptualization, Development of theoretical model, Formal analysis and investigation, Methodology & Data collection, Validation, Writing - original draft preparation, and Writing - review and editing. E-MI: Formal analysis and investigation, Validation, Writing - review and editing. ZL: Writing - review and editing.
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Additional Information : All resources are available on https://www.geogebra.org and on the FLINK project website (https://www.jku.at/flink-in-mathe/).

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