

Non-specialist secondary mathematics teachers learning in study groups by engaging with activities of algebra

Vesife Hatisaru 

School of Education, Edith Cowan University, Perth, Australia
School of Education, University of Tasmania, Hobart, Australia
*Correspondence: v.hatisaru@ecu.edu.au

Received: 3 June 2023 | Revised: 20 October 2023 | Accepted: 1 November 2023 | Published Online: 12 November 2023
© The Author 2024

Abstract

Teaching 'out-of-field' is a present obstacle in mathematics education in many countries, and developing professional learning programs aimed at upskilling non-specialist teachers is urgent. A teacher study group was established wherein two non-specialist teachers of mathematics (Years 7–10; aged 12–16) engaged with algebra to develop a deeper understanding of the subject and its teaching. The study group lasted one school year, during which the teachers actively participated in the learning. Multiple data sources were collected, including reflection forms, open-ended questionnaires, and a storytelling form. Analysis of the teachers' solutions to a sample of algebra problems and self-reports suggests that the study group enabled teachers to acquire new knowledge/skills in mathematics pedagogy, gain a new understanding of how student thinking and understanding develop, adapt new classroom instruction strategies, develop ability/confidence in problem-solving, and develop new knowledge/ability in algebra. The study findings contribute to our understanding of supporting non-specialist mathematics teachers' professional development.

Keywords: Algebra, Professional Development, Study Groups, Secondary Mathematics Teachers, Teacher Learning

How to Cite: Hatisaru, V. (2024). Non-specialist secondary mathematics teachers learning in study groups by engaging with activities of algebra. *Journal on Mathematics Education*, 15(1), 131-150. <http://doi.org/10.22342/jme.v15i1.pp131-150>

In Australia, like in many countries worldwide (Hobbs & Törner, 2019), not all mathematics teachers in schools have mathematics and/or mathematics pedagogy training. These teachers are labeled as 'unqualified' (Australian Mathematical Sciences Institute [AMSI], 2020), 'out-of-field' (Weldon, 2016), or 'non-specialist' (Goos et al., 2020) teachers in mathematics. The related term 'teaching out-of-field' refers to teachers assigned by school principals to teach subjects that do not match their teacher education (Goos et al., 2020). The most recent and comprehensive data collected by the Australian Council for Educational Research [ACER] in 2013 on the qualifications of mathematics teachers in secondary schools has shown that teaching out-of-field is common in Australian schools. This phenomenon happens both in junior (Years 7–10) and senior secondary (Years 11 and 12) years, whilst it is much more frequent in junior secondary years and especially in rural, regional, and remote schools (ACER, 2014). According to this data, 30 per cent of teachers teaching junior secondary students had not studied mathematics at the tertiary level, and 38 per cent of teachers neither had studied mathematics nor mathematics pedagogy (Weldon, 2016). There are calls for supporting non-specialist mathematics teachers to upgrade their

knowledge and skills by supporting their access to ongoing professional development (PL) (du Plessis, 2015; Goos et al., 2020).

Two goals guided me in conducting the research reported in the present paper. First, I wanted to help secondary teachers of mathematics (both specialist and non-specialist) in Tasmania—a rural and regional part of Australia where teaching ‘out-of-field’ is the highest in the country and limited professional development (PD) opportunities for teachers are available (Weldon, 2016)—to enhance their mathematical knowledge needed for its teaching. My goal was to involve mathematics teachers in a PD initiative that would allow them to struggle with mathematics problems, discover their own solutions, think mathematically, and develop mathematical learning (Cuoco, 2013). ‘Learning’ was construed broadly to include any change in teachers’ mathematics knowledge, beliefs, and/or classroom practices (Goldsmith et al., 2014). Second, I wanted to examine the benefits of study groups for accomplishing my first goal. In this paper, I aim to demonstrate how non-specialist teachers’ learning can be supported by the act of engaging in structured mathematical conversations in study groups that last over a period. Three questions are asked:

- 1) What are teachers’ views about the benefits of study groups to teacher learning?
- 2) What do teachers learn in study groups?
- 3) What do teachers change or anticipate changing in their teaching to reflect this learning?

Significance of the Study

The role of teacher knowledge in student mathematics learning has been the subject of considerable research (e.g., Hatisaru & Erbas, 2017; Hill & Chin, 2018; Porsch & Whannell, 2019). It is often suggested that teachers present the mathematical content in a way that supports students to understand mathematics; compute fluently; apply concepts to solve problems; reason logically; and engage with mathematics, seeing it as useful and doable (Kilpatrick et al., 2001). Achieving these expectations can be a daunting task for non-specialist teachers, and sometimes for even specialist teachers (Tatto et al., 2012), because non-specialist teachers may have a paucity of mathematical knowledge needed for teaching, and they themselves may be mathematically less proficient (du Plessis, 2015; Faulkner et al., 2019; Goos et al., 2020). Mathematics teachers – both specialist and non-specialist – need to have continuous in-service training that will enable them to teach all students to gain mathematical knowledge and skills (AMSI, 2020; Darling-Hammond et al., 2017). At the center of this issue is how best to design PD activities for teachers to optimize their mathematics knowledge needed for teaching. The present paper describes how study groups can contribute to effective PL for non-specialist mathematics teachers.

Research studies show that the most effective PD activities focus on topics that may impact the daily instructional practices of teachers (Darling-Hammond et al., 2017; Hill et al., 2020), including topics such as academic content, student thinking, and curriculum materials (Hill et al., 2020). PD activities become more effective when they have active learning aspects (analyzing, discussing) rather than passive activities (mainly listening), have a longer rather than shorter duration, and individual factors are taken into account (e.g., attitudes, motivation) (Thurlings & den Brok, 2018). As a genre of these types of PD opportunities, while study groups have become a popular form of in-service training for teachers, more needs to be learned about such groups. For example, little is known about which design features of such groups support teacher learning (Crespo, 2006; Goldsmith et al., 2014). This study not only extensively documents the formation and progress of a teacher study group but also explicates its affordances in terms of how such study groups can grow into communities that support teacher learning.



Explicating the characteristics and impacts of study groups as an approach to teacher learning will be helpful to mathematics educators, researchers, and policymakers as they design PD activities and/or conduct research (Borko & Potari, 2020; Darling-Hammond et al., 2017). More importantly, teaching 'out-of-field' is a present obstacle in mathematics education in many countries, and the development of PD programs aimed at upskilling non-specialist teachers is a growing field (see Lane & Ní Ríordáin, 2020). The findings presented in this study contribute to our understanding of supporting non-specialist mathematics teachers' knowledge needed to teach the subject and plausibly improving teacher and teaching quality (du Plessis, 2015; Faulkner et al., 2019).

Below, I describe the notion of teacher learning and study groups. There are comprehensive descriptions of the analytic steps of the research to elucidate how the study group implemented in this research had the potential to foster teachers' learning. I end by addressing what study groups can offer to support non-specialist mathematics teachers and possibly specialist mathematics teachers in developing their knowledge in mathematics.

Teacher Learning

To date, substantial research studies have aimed to explain the notion of teacher PD (see Hollingsworth & Clarke, 2017). It is suggested that "if we are to facilitate the professional development of teachers, we must understand the process by which teachers grow professionally," with professional growth defined "as an inevitable and continuing process of learning" (Clarke & Hollingsworth, 2002, p. 947). In this definition, 'change' is perceived as 'growth' or 'learning' – i.e., teachers are considered as learners who experiment in learning and/or teaching contexts, and learning occurs as a result of those experiments (Hollingsworth & Clarke, 2017). In the learning sciences, research is conducted to understand how individuals learn (including teachers), and research is used to design new routines, materials, and curricula to improve learning. In the teacher education and development field, this research concentrates on developing teachers' knowledge, identity, beliefs, and teaching practices in context (Fishman et al., 2014). As Fishman et al. (2014) argue, decades of learning sciences research on teacher learning have demonstrated that learning is first social and distributed. Therefore, effective social support for teacher learning can help teachers develop knowledge, beliefs, identity, and practice. Second, learning is more effective when situated in the learner's context. Situating learning in the teachers' authentic contexts can promote their learning.

The learning sciences perspective on teacher learning is consistent with participatory approaches that view teacher learning in day-to-day teaching practices and in collaborative conversations and discussions in relation to those practices (Kazemi & Franke, 2004; Hill et al., 2020; Putnam & Borko, 2000). Varied forms of participatory approaches to teacher learning, including lesson study (Ponte et al., 2022; Wadjaja et al., 2017), action research (Wongsopawiro et al., 2017), researcher-teacher partnerships (Qi et al., 2021), and professional experimentation (Muir et al., 2021) demonstrate that social support and situativity can enhance teacher learning, including non-specialist teachers' learning (e.g., Lane & Ní Ríordáin, 2020). In this study, I established a teacher study group, operationalised in the participatory approach to teacher learning literature, that was guided by a social and situated view of learning. The current paper describes the design and delivery of the study group and provides evidence of teacher learning.

Teacher Study Groups

A teacher study group comprises small groups of teachers working together on a specific goal (Gersten



et al., 2010). Such groups "... provide teachers with occasions to work together on developing their own mathematical understanding, as well as with opportunities to expand their knowledge of students' mathematical thinking" (Crespo, 2006, p. 29). The groups may be school-based or topic-centered; they function as a discussion group; and they focus on transforming teaching through discussions and reflections (Birchak et al., 1998). They may be independent initiatives that are built and run by teachers to sustain their own learning, or they may be formed and facilitated by researchers, university academics, and various outside professionals. In either situation, teacher study groups offer an alternative to the standard PD workshop model (Stanley, 2011). They include "facilitator-guided discourse and inquiry rather than a "top-down" or "expert-led" study group where teachers play a passive role listening to instruction or watching demonstrations" (Gersten et al., 2010, p. 698). As such, facilitators mostly have double roles, for example, being both an observer or reflector and a participant. The facilitator, as a participant, joins in the conversations, offers their thoughts, and listens to the reactions of the group members to help them articulate their ideas (Birchak et al., 1998). In this study, I straddled the line between the researcher and co-participant. As a researcher, I designed and ran the study group and documented its processes, and as a co-participant, I solved the problems and shared my own thinking with the teachers (Grossman et al., 2001).

Study groups have been employed in numerous studies in the field of mathematics education. Several different aspects of mathematics education have been investigated in these studies, including the value teachers place on participating in study groups (Arbaugh, 2003), the impact of participation in those communities on teachers' knowledge, beliefs, or teaching practices (Kazemi & Franke, 2004); and the forms of talk that occur in those groups as participant teachers work on mathematical problems and analyze their students' work (Crespo, 2006). I drew on this form of teacher PD to gain a window into the nature of teachers' learning. Given the accumulation of research over the past several decades on students' weak learning outcomes in algebra (e.g., Bush & Karp, 2013) and the role of teacher capability in student learning (e.g., Hu et al., 2022), I used activities of school algebra as content domain (Kieran, 2007; Kilpatrick et al., 2001).

Activities of School Algebra

Two aspects of algebra: "(a) algebra as a systematic way of expressing generality and abstraction; and (b) algebra as syntactically guided transformations of symbols" lead to various activities in school algebra (Kilpatrick et al., 2001, p. 256) which can be grouped into three: Generational; Transformational; and Global/meta-level activities (the GTG model) (Kieran, 2007). Kilpatrick et al. (2001) named these activities: Representational; Rule-based; and Generalizing and Justifying activities, respectively. As articulated in Hatisaru et al. (2022), representational activities involve translating verbal statements into symbolic expressions and equations that often (but not necessarily) involve functions. Generally, they include "generating (a) equations that represent quantitative problem situations in which one or more of the quantities are unknown, (b) functions describing geometric patterns or numerical sequences, and (c) expressions of the rules governing numerical relationships" (Kilpatrick et al., 2001, p. 256-257). Rule-based activities include collecting like terms, factoring, expanding, substituting, simplifying expressions, and solving equations. In these activities, the rules for manipulating algebraic symbols are often used to change the form of an expression or equation to an equivalent one (Kilpatrick et al., 2001). Therefore, the main activity in rule-based tasks is often changing the symbolic form of an expression or equation to maintain equivalence (Kieran, 2007). Generalizing and justifying activities include problem-solving, modelling, justifying, proving, and predicting (Kilpatrick et al., 2001). In these types of tasks, the language



and algebra tools are commonly used, though these tools are not exclusive to algebra (Kieran, 2007). Generalizing and justifying activities usually involve examining and interpreting representations that have already been generated or manipulated, and they can give answers to particular questions or conjectures.

I chose these three types of algebra activities as the content focus because they have the potential to support the mathematical knowledge needed for teaching, including the ability to formulate, represent, and solve mathematical problems (Hatisaru et al., 2022; Kilpatrick et al., 2001). The study group was structured around these activities, and the sample of problems shown in Supplementary File 1 (SF 1) was used—the following sections detail how the group was established and implemented before presenting the data analysis.

METHOD

Recruitment of the Participants

In July 2021, member secondary teachers (Years 7 to 10, 12-16 years old) of the Mathematical Association of Tasmania were invited to the study. Amid the COVID-19 pandemic, six teachers expressed interest when most research activities were paused, while two did not attend any of the meetings. Before the actual meetings began, I held a virtual introductory meeting to facilitate introductions between myself and the four volunteer teachers to the study. In this meeting, I oriented the teachers to the study's aims, invited their voluntary participation, and received their feedback regarding the logistic arrangements (e.g., frequency and duration of the meetings). After this meeting, the teachers were emailed an Initial Open-ended Questionnaire (Pre-Q), presented in SF 2, which was designed to learn about their mathematics teaching background and views on mathematics education. Two of the voluntary teachers could not attend study group meetings regularly because of the sudden or unexpected conditions in their schools due to the pandemic. The remaining two teachers—Levi and Mia (pseudonym names)—and I began to meet in September 2021. All study group meetings were held virtually, as COVID-19 transmission and protective measures were in place during the study. The teachers' educational background and teaching context are described in detail in the following section.

Teaching Context of Levi and Mia

The teaching context surrounding the teachers included personal, institutional, and/or broader educational factors that might influence their learning (Clarke & Hollingsworth, 2002). Firstly, like many mathematics teachers in Australia (AMSI, 2020), neither Levi nor Mia are specialist mathematics teachers. Mia holds a Bachelor of Primary Education with some extra units in Years 7 to 10 mathematics, while Levi has a Bachelor of Education degree, wherein he undertook a calculus unit. They taught mathematics across junior secondary years, from Years 7 to 10. Both teachers believed in the need for PL to address the existing gaps in their knowledge and skills. Their words in the Pre-Q, where they reflected on their motivations for participating in the study group, reveal that Mia and Levi hoped to develop their knowledge of mathematics and pedagogy to strengthen their professional capability. Secondly, at the time of the study, Mia had three, and Levi had six years of experience in mathematics teaching. Both teachers placed emphasis on the importance of teacher professional knowledge. They foregrounded the need for teachers to robustly understand the content to support students' success in mathematics. On a different but relevant note, reflected in her words in Pre-Q, Mia also expected students to have a robust understanding of mathematics. For Levi and Mia, teachers could enhance their professional knowledge:

... through PL and through working as a community. This can be within schools (grade and subject teams) or within the broader math teaching community. (Levi, Pre-Q, Excerpt #1)

As early career, non-specialist teachers of mathematics who wanted to enhance their knowledge needed to teach mathematics effectively, both were open to experimenting with new practices, very interested in and engaged with the study group activities, and were pretty attentive to each other and to the researcher's solutions to the problems.

Finally, both Mia and Levi expressed positive views about mathematics, putting value on it. For them, mathematics is: "an enjoyable puzzle to be solved" (Mia, Pre-Q, Excerpt #1) and "learnable. Anyone can learn it. And the more you learn it, the more connections you make, and the more it unfolds and reveals itself" (Levi, Pre-Q, Excerpt #2). The described context in which Levi and Mia taught might contribute to the learnings resulting from the study group.

The Study Group

As a group of three, from September 2021 to March 2022, we met seven times, approximately once a month (see SF 3). Each meeting lasted one hour, and with the teachers' consent, they were recorded. The meetings were usually held after the teachers' school hours. Their attendance was entirely voluntary –neither support for any release time nor a stipend pay for their time was available. I conducted the study group, the author, who is an experienced researcher and lecturer of mathematics education. In this role, I generated all data necessary to investigate the research questions, as well as structuring the meetings and sourcing the problems and relevant readings. During the meetings, I was the facilitator who initiated and facilitated the discussions and a co-participant who joined in the conversations, sharing her thoughts and sometimes her solutions to the problems (Birchak et al., 1998; Grossman et al., 2001).

Usually, one week before each meeting, I emailed the teachers a Reflection Form, presented in SF 4, with the relevant problem recorded in it. The teachers used the Reflection Form to record their solutions, reflect upon the problem (P #) they solved, and make any notes they would want to share with the group. The teachers' solutions to the problems guided the substance and direction of discussions at each meeting. However, the study group was not only intended to solve mathematics problems and share the solutions (Birchak et al., 1998). By discussing some of the key issues and conceptual or theoretical works that underlie the problems, the teachers would find opportunities for enhancing their knowledge of algebra and/or its teaching and learning. Therefore, we also engaged in the theories behind the relevant problems. The activities of school algebra (Kieran, 2007) and types of representation systems (Lesh et al., 1987) were the conceptual frameworks that guided the study group activities. I used the types of representation system framework during the meetings especially to consider making connections among representations.

As summarised in SF 3, meetings usually began with the teachers sharing their solutions to the relevant problem. The teachers could comment on each other's solutions, ask questions, and/or share the specific ways in which their students would solve the problem. I took notes as the solutions were described and sometimes contributed my knowledge of the teaching and learning of mathematics by elaborating on the solutions that the teachers shared. In every meeting, we discussed the algebra principles that underlie the relevant problem. I often asked probing questions or challenged the teachers to reflect on their knowledge of algebra and its teaching. For example, in Meeting #7, to stimulate the teachers' curiosity about the potential implications of the issues in P #7, I noted the concern of Kieran

(1992) raised decades ago and asked: ‘How could we assist students in developing structural conceptions of algebra? For instance, how could these problems (P #7a and P #7b) be taught in the classroom to contribute to that goal? (Hatisaru, 2022). Sometimes, I introduced specific solutions if they did not appear in the teachers’ solutions and/or presented student thinking or strategies available in the existing research (Hatisaru et al., 2022).

In order to support the teachers’ knowledge, we sometimes compared the mathematical sophistication that particular solutions demonstrated. For example, in answering P #6 or P #7, a solution constructed based on numerical examples is less sophisticated mathematically than one using letters (see Section 4.2). The working frameworks – activities of school algebra (Kieran, 2007) and types of representation system (Lesh et al., 1987) – served as sources for continued reflections and elaborations in the meetings as the teachers continued to solve mathematics problems, refresh their algebra knowledge, and learn about student thinking.

At the completion of the study, the teachers shared how and the extent to which their participation in the study group had created value for them and for their classroom practices through using a Storytelling Form (ST Form; see SF 5). They also completed a Final Open-ended Questionnaire (Post-Q; see SF 6) to reflect upon if and the ways in which this experience supported their learning.

Data Analysis

To strengthen research findings through triangulation, multiple sources of data were collected: (a) study group meetings recordings and artifacts, (b) Reflection Forms (seven from each teacher), (c) Pre-Q, (d) Post-Q, and (e) ST Form. Relevant to this paper, three of these data sources were primarily used: Pre-Q, Post-Q, and ST Form. The data from Reflection Forms, and the meetings were only used to find supporting evidence for the findings from the data analysis from these three major sources.

I used inductive content analysis to articulate how the study group experience changed teachers’ knowledge, skills, and practice. Influenced by Hsieh and Shannon (2005), I started by reading all the data repeatedly to gain a sense of the whole. Then, I read the data word by word to identify themes and patterns that appeared to capture key concepts in the research questions (RQs): the teachers’ views about the benefits of the study group (RQ #1); knowledge, skills, and/or abilities gained from the study group (RQ #2); and impact of the study group on teaching (RQ #3). I took notes of my initial impressions and thoughts and labeled them. As this process continued, the labels became major themes, presented in Table 1, directly emerged from the data. Once the emergent themes in the teachers’ responses were identified, I used them to code all data and computed the frequencies (see Table 3).

Table 1. Emergent Themes Grouped into RQs

Key concepts concern RQs	Emergent themes
The teachers’ views about benefits of the study group (RQ #1)	a deeper dive into mathematics teaching and learning interesting problems to think about and solve working with others on mathematics ideas discussions of strategies used by students information and resources (see Figure 1)
Knowledge, skills, and/or abilities gained from the study group (RQ #2)	knowledge/skills in mathematics pedagogy attention to student thinking/understanding/learning ability/confidence in problem solving knowledge/ability in problem solving/algebra

Impact of the study group on teaching (RQ #3)	attention to student thinking/understanding/learning teaching practice
---	--

By way of illustration of the coding process, two Excerpts (E #1) are provided below, with italics added and the coding in brackets:

The range of problems we were given to solve during the study group meetings facilitated conversations [the range of problems], which expanded my ability to solve problems in various ways [ability/confidence in problem-solving]. The importance of developing and demonstrating multiple representations of solutions was the most outstanding learning I gained from the outset of this project [knowledge/skills in mathematics pedagogy]. (Mia, ST Form, E #1)

This really helped me see that having a group share multiple solutions is a really great way of exploring concepts [knowledge/skills in mathematics pedagogy]. I can see myself posing a question to a group and having the class challenge themselves to find as many different solutions as possible – visual, using numbers, etc. [teaching practice] I was often surprised by a novel solution that I hadn't thought of before [knowledge/ability in problem-solving]. (Levi, Post-Q, E #1)

To increase the trustworthiness of the research findings, I took two major quality measures. Firstly, I employed triangulation of the data derived from different persons (Noble & Heale, 2019) and coded the data obtained from Levi and Mia separately. Interestingly, the same themes have emerged in their responses with similar frequencies. Also, both teachers' statements that reflected change are fairly evenly distributed across the key concepts concerning RQs (Table 2), which supports the trustworthiness of data analysis. Secondly, and much more importantly, as wisely recommended by Thorne (2000), I articulated the systematic, rigorous, and auditable analytical steps of the study methods so that these steps are accessible to a critical reader. The relationships between the data and conclusions were made explicit to ensure that the claims made were trusted. As Thorne (2000) says, the reader is now better positioned to judge the trustworthiness of the study findings critically.

Table 2. Emergent Themes in Levi and Mia's Responses Assigned to the RQs

Key concepts concern RQs	Levi	Mia	Total
The teachers' views about benefits of the study group (RQ #1)	8	10	18
Knowledge, skills, and/or abilities gained from the study group (RQ #2)	8	10	18
Impact of the study group on teaching (RQ #3)	8	4	12
Total	24	24	48

RESULTS AND DISCUSSION

Numerous insightful examples of teacher learning resulting from the study group experience are found in the responses of Levi and Mia. Table 3 shows an overview of major themes that emerged from their responses linked to the concepts concerning the RQs. I report the findings for these emergent themes under the relevant RQ.



The Teachers' Views About Benefits of the Study Group (RQ #1)

As a PD activity, the study group provided extensive new information and stimulus for the teachers. The teachers valued these learning opportunities that were created for them. The analysis of their responses revealed that of the 48 teachers' utterances, 18 were dedicated to characterising the study group. [Figure 1](#) shows those characteristics identified in the teachers' responses where various aspects of the study group are reflected well.

Table 3. Emergent Themes in the Teachers' Responses Assigned to the RQs

Key concepts concern RQs	Emergent themes	Total
The teachers' views about benefits of the study group (RQ #1)	characteristics of the study group shown in Figure 1	18
Knowledge, skills, and/or abilities gained from the study group (RQ #2)	knowledge/skills in mathematics pedagogy	11
	ability/confidence in problem solving	5
	knowledge/ability in problem solving/algebra	2
Impact of the study group on teaching (RQ #3)	attention to student thinking	6
	/understanding/learning	
	teaching practice	6
Total		48

The efficacy of the study group was underpinned by its methods. The problems and meetings were organised so that the parts related to each other; group members were presented with practical mathematics pedagogy strategies and the theory behind them; and rich discussions originated in every meeting. It is not surprising that both Levi and Mia explicitly referred to all these features, as well as the others captured in [Figure 1](#). These study group features have contributed to Levi and Mia's learning from the study group. According to Levi, the mathematical focus of the study group; the resources provided; working on specific problems and creating multiple solutions to them; having the group share their solutions; discussions of the solutions; presentation of theory; and presentation of the relevant research literature on student thinking were among the key aspects of the study group that supported his growth. According to Mia, the range of problems and conversations around them; generating multiple solutions to the problems and justifying solutions; discussing students' strategies to the problems; other group members; targeted and deeper PL of a particular mathematics content; resources shared; the readings presenting theory; and being a model of a class situation were the main aspects of the study group that engaged her in learning.

I found that elicited reflections on the study group were touched upon valid points that the study group aimed for:

This is unlike any professional learning I've been involved with. I'd be really keen to participate in similar arrangements – having short, fortnightly, regular opportunities to work with other educators on math ideas is a powerful and valuable idea. (Levi, Post-Q, E #3)

The learning within this study group has enabled a more targeted and deeper dive into one aspect of mathematics teaching and learning, rather than the usual broader professional learning workshop attended for one day. The readings and research provided allowed for

greater insight into the reasons behind the methodology as well as introducing teaching strategies used overseas. (Mia, Post-Q, E #3)



Figure 1. Characteristics of the Algebra Study Group Identified in the Teachers' Responses

Knowledge, Skills, and/or Abilities Gained from the Study Group (RQ #2)

As both teachers explored some of the theoretical concepts in algebra, and some algebraic processes that are related to the teaching and learning of algebra, they acquired new knowledge and skills both in algebra itself and in the teaching and learning of it, which plausibly triggered a shift towards a more conceptual stance in their approaches to mathematical tasks. Three themes have emerged within the subset of 18 mentions of change in the teachers' responses that address the effectiveness of the study group on their knowledge, beliefs, skills and/or confidence in algebra or in problem solving. One is the acquired knowledge/skills in mathematics pedagogy (11 mentions). Structured, research-informed conversations on teaching and learning of algebra led the teachers to begin to attend to the pedagogy of mathematics and develop a broader sense of pedagogy that would support conceptual understanding of mathematics. For Levi, these pedagogies included using suitable problems to elicit algebraic understanding, finding new solutions to a problem, having a group share multiple solutions, and discussing why certain solutions work. The major mathematics pedagogies Mia acquired from the study group were developing and demonstrating multiple solutions to a problem, comparing and contrasting the solutions, linking different solution representations, and making connections within/between representations. Mia provided several reflective and articulate statements such as:

The most important thing I learnt during the study group was that displaying multiple representations of solutions to problems, side-by-side, enables greater understanding and can lead to the use of more efficient strategies. (Mia, Post-Q, E #1)

The discussion of strategies used by students allowed me as a participant to discover ways of connecting between and within student representations that I could not before. (Mia, Post-Q, E #2)

As the teachers explored the types of representation system framework (Lesh et al., 1987) and learned more, they began to use the idea of generating various representations to solve mathematics problems more often. SF 7 summarises their solution strategies to the study group problems as evidenced in their Reflection Forms. Growth in the teachers' ability/confidence in problem-solving or knowledge/ability in algebra (7 mentions in total) was evident in the high value that both teachers attached to the learning opportunities for developing new mathematical skills around formulating, representing, and solving the problems in various ways. For example, Mia shared these reflective words:

On being given the first problem to solve at the beginning of this study group, I was stumped as to think of many different ways to solve the problem. I was still determining my methodology. I took to doing a bit of research to avoid embarrassment. Over the ensuing weeks, my confidence and range of problem-solving strategies significantly increased. (Mia, ST Form, E #2)

In solving P #1, while she did generate numerical and symbolic solutions, unlike Levi, Mia did not undertake a graphical approach. Based on the conversations in that meeting and sharing of the solutions, Mia generated a graphical solution in solving P #2. In her responses to solving routine equations, such as in P #5a, she solved the equations in different ways, such as in the example shown in Figure 2. Whilst (A) and (C) are standard solutions, (B) and (D) are non-standard solutions and have the potential for the development of fluency in students in rule-based activities (Hatisaru, 2021).

P #5a: Solve the equation $2(x + 1) + 3(x + 1) = 10$ for x .

Figure 2. Mia's Solutions to P #5a (Reflection Form #5)

As the study group meetings progressed, the teachers came to value algebraic symbolisation and showed a willingness to understand this notion more by seeking advice from me, the researcher. For example, in solving P #6, in addition to a solution based on the use of numerical examples, Levi generated

the solution given in [Figure 3](#). In this solution, Levi used letters to express the generality, and in completing his solution, he gave a numerical example.

Solution 2 – algebra and substitution – a general solution.
 Let a be larger than b .
 Let $a + b = X$ (the sum)
 Let $a - b = Y$ (the difference)
 We can rearrange the second equation to get $a = b + Y$
 We can substitute that into the first to find $b + b + Y = X$. So, $2b + Y = X$
 It follows that if we are given any Y and any X , we can substitute them into this equation to find one of the numbers, and then find the other one. For example, if they summed to 15 with a difference of 3, then $X = 15$, $Y = 3$, and $2b + 3 = 15$, so $2b = 12$, and then $b = 6$.

Figure 3. Levi's Solution to P #6 (Reflection Form #6)

Understandably, Mia needed clarification as to whether this solution was suitable for the situation in the problem. In expressing her concern about the solution that expresses generality, Mia (and Levi) asked me how the problem could be solved. In response, I shared the solutions in [Table 4](#) that are presented in Hatisaru et al. (2022), developed based on Kieran (1992). Here, to support the teachers' knowledge, we compared the mathematical sophistication that these solutions demonstrate. In the Rhetorical method, a solution is constructed based on numerical examples, and therefore, it is less sophisticated mathematically than one using letters. As the teachers voiced their thinking on the solutions, I introduced terminology from the theoretical works in the teaching and learning of algebra such as the procedural conception of algebra (i.e., using numerical examples like in the Rhetorical method) versus structural conception (i.e., using letters and making generalisations like in the Vietan method) that reveal the development of the use of algebraic symbolism – and plausibly the development of students' algebraic thinking (Kieran, 1992).

Table 4. Solution Methods to P #6

P #6	If you are given the sum and difference of any two numbers, show that you can always find out what the numbers are.
Rhetorical method	For example: Divide the sum by 2, then divide the difference by 2. To get the first number, add the sum divided by 2 to the difference divided by 2. To get the second number, take the difference divided by 2 away from the sum divided by 2.
Diophantine method	Given x is the first number, and y is the second number, assume that $x - y = 2$ and $x + y = 8$. x and y can be found by solving these two equations for x and y , and it is clear that this can be applied for any numbers.
Vietan method	Assume the numbers are x and y . m : the sum of x and y . Then, $m = x + y$ n : difference of x and y . Then, $n = x - y$ Add together: $m + n = 2x$ Find x and substitute back for y . That is, $x = (m + n) / 2$ and $y = (m - n) / 2$

These exchanges within the study group suggest that professional conversations among teachers, structured and facilitated by mathematics educators, may contribute to their mathematical knowledge by means of discussing a lesson (Grandau, 2005; Lachance & Confrey 2003), videotaped lessons of other teachers (Schwartz & Karsenty, 2020), or discussing mathematics problems and their solutions, as in the case of this research. Perhaps this is articulated by Mia best:

One of the greatest resources during this study group was the other group members. The facilitator took on board questions from the group and, between meetings, came back to us with answers to our questions. (Mia, ST Form, E #3)

Impact of the Study Group on Teaching (RQ #3)

According to their self-reports, the study group provided Levi and Mia with several professional experimentation opportunities. These included: (a) having structured and focused discussions around a particular mathematical content and its pedagogy; (b) links between theory and practice and the constant movement between the two; (c) practicing maths problems and struggling with solving them in various different ways; (d) discovering their own, others, and/or students' solutions to (or thinking in) the problems; and (e) creating, using, and connecting mathematical representations during problem-solving. Each of these experimentations played a role in creating change in their practice (Clarke & Hollingsworth, 2002).

In the teachers' responses, two main areas of impact emerged: attention to student thinking/understanding/learning (6 mentions) and teaching practice (6 mentions) (Table 3). The teachers' reflections that reveal the impacts of the study group on their attention to student thinking/understanding/learning are impressive. Perhaps the teachers echoed this impact best when they wrote:

I think as a teacher I know a lot about high school math and the curriculum, but not always a lot about how mathematical thought develops, or how a student may go from a simple understanding of algebra to a more complex one. How do I help students who don't get it to get it, given that 'showing them again, but slower' is clearly not all that helpful? The slides and the readings were helpful and could make the basis of an entire PL session. (Levi, Post-Q, E #2)

My definition of student success prior to doing this project would have been being able to solve the problem using the method of their choice. My definition has evolved to now thinking that being able to discuss and justify solutions and recognising similarities and efficiencies is also a way of measuring success and leads to greater understanding. (Mia, ST Form, E #4)

The development of teachers' understanding of student thinking is intertwined with new teaching practices for supporting their students' learning. As they became more aware of how students' algebraic thinking develops and how students' approaches to algebra problems can be supported during teaching, the teachers began to transform their instructional practice to fit with new pedagogy for creating, using, and connecting mathematical representations. Mia often used the study group resources in her classes

and adapted the study group practices to her instruction in that school year. Levi intended to adapt those practices to his mathematics instruction in the following semesters. Mia, for example, wrote that:

These resources have been invaluable in my practice. My ability and confidence in teaching problem-solving have significantly evolved, particularly when making connections between different types of representations. (Mia, ST Form, E #5)

Levi stated that:

I found this process of finding new solutions valuable, and it's an idea I intend to take forward into my own teaching. I often found my understanding of the problem and the concept was deepened by an unexpected solution, and the discussions of why certain solutions worked were always invaluable. I can see this process benefitting students at all levels of proficiency. (Levi, ST Form, E #1)

Clearly, both Levi and Mia were keen to take an adaptive approach to teaching mathematics, where they intended to use more interpretive pedagogical strategies for effective perspective-taking to support their students in learning mathematics and problem-solving.

CONCLUSION

The potential of study groups to support secondary teachers of mathematics was investigated. A study group was established wherein participating teachers found opportunities to enhance their knowledge and skills that are needed for teaching mathematics in an environment where they were doers of mathematics. The data were generated from multiple sources. The two teachers who joined all study group meetings and completed data collection instruments were non-specialist mathematics teachers. The study group contributed to developing new knowledge for the teachers as their self-reports unfolded. Both teachers (a) acquired new knowledge/skills in mathematics pedagogy; (b) gained a new understanding of how student thinking and understanding develop; and (c) developed ability/confidence in problem-solving or in algebra. These new skills, understandings, and knowledge were linked to the teacher's instruction in the following way. Both teachers expressed increased attention to student thinking, negotiated to create multiple solutions to mathematics problems, and discussed and connected the created solutions in a way that could improve the development of students' understanding. They adapted new classroom instruction strategies, where they utilised, or intended to utilise, more interpretive pedagogical strategies for effective perspective-taking to support their students in learning mathematics and problem-solving.

The effective teaching of mathematics requires teachers to have a robust content understanding and mathematical habits of mind as well as a well-developed pedagogical content knowledge (Cuoco, 2013; Goos, 2013). Neither of the study teachers have relevant specialist training, like many other teachers of mathematics in Australia, and globally (see Hobbs & Törner, 2019). Most issues in the teaching and learning of mathematics were, and are, unfamiliar to them. Their words in the Pre-Q provide some valuable and genuine insights into non-specialist teachers' PL needs, as also elaborated comprehensively by du Plessis (2015). It is revealed that both Levi and Mia are keen to strengthen their mathematical understanding and knowledge of mathematics pedagogy to be able to teach the content



effectively and seek opportunities to learn. This is not surprising because content knowledge and its pedagogy is one area where the needs of non-specialist teachers differ from specialist teachers (Faulkner et al., 2019). Whilst PD opportunities that cover general pedagogy may be relatively widely available for these teachers, alternative PD programmes, like the study group described here, are scarce (see also du Plessis, 2015). The findings in this study show that creating solutions to the given problems, anticipating student reasoning and reviewing sample student solutions, conversations around how the problems can be taught, and interacting with group members played a major role in the teachers' learning. The main implication of this result is that opportunities to experiment with mathematical content within structured trainings can enable non-specialist teachers to fill the gaps in their knowledge, and plausibly, provide opportunities for their students to engage in mathematical activities more conceptually (Muir et al., 2021; Qi et al., 2021).

Research studies show that participatory learning opportunities including preparing and evaluating lesson plans in collaboration with mathematics education researchers (Qi et al., 2021), analysing other teachers' video-recorded lessons and making reflections on them (Schwartz & Karsenty, 2020), experimentation on personalising students' learning (Muir et al., 2021), professional conversations on mathematical tasks and content descriptions of instruction (Grandau, 2005), content explorations (Lachance & Confrey 2003), and self-study action research (Lane & Ní Ríordáin, 2020) enable teachers—both specialist and non-specialist—to extend their knowledge of mathematics and pedagogical practices. As an example of this genre, this study documented a positive link between the provided PD opportunity and teacher learning, as shown in several other studies (also see Borko & Potari, 2020). As both teachers explored some of the theoretical concepts in algebra, and some algebraic processes that are related to the teaching and learning of algebra, they acquired new knowledge and skills both in algebra itself and in the teaching and learning of it, which plausibly triggered a shift towards a more conceptual stance in their approaches to mathematical tasks. This finding suggests that study groups can be fruitfully employed to improve non-specialist teachers' confidence to engage in challenging content knowledge as teachers of mathematics (Hobbs & Törner, 2019) and to enhance both specialist and non-specialist teachers' mathematics and mathematics pedagogy knowledge (Arbaugh, 2003). Further work yet is warranted concerning how these forms of PD opportunities can be expanded to larger cohorts. Specifically, to be able to achieve maximum benefits, researchers and policymakers should partner to shape these efforts and effectively expand them (Darling-Hammond et al., 2017).

The study has the potential to contribute to the works in other contexts as it has extensively documented the formation and progress of a study group that supported teacher learning. This is significant, given that knowing what characteristics of study groups impact teacher development will be helpful to mathematics educators, researchers, and policymakers as they design professional activities and/or conduct research. Finally, the use of Reflection Forms has helped to show the study group's unique features that have created teacher learning. To this end, the study also adds to the accumulating knowledge on various types of learning that can occur from using structured mathematical discussions in participatory PD initiatives. As such initiatives have become more and more prevalent, I see the use of these forms, or potential other tools, as applicable for future PD initiatives, particularly those that seek to promote the cultivation of growth in content and content pedagogy as a central aim.

Limitations of the Study and Future Directions

I acknowledge that the study findings are limited to self-reports and artefacts of two non-specialist teachers of mathematics teaching in Tasmania, Australia. The findings may not apply to non-specialist

teachers of mathematics in different school contexts, or to specialist mathematics teachers. Deeper insights, however, have been gained through this study into the benefits of teacher study groups as a form of PD opportunity. These insights may well inform school leaders, teacher educators, and policy makers about non-specialist mathematics teachers' PL, an issue that requires the teacher development field's close attention as non-specialist teachers usually obtain weak student outcomes (Porsch & Whannell, 2019). Furthermore, the study generates several future research directions.

The research has addressed teachers' learning in a study group as they solve algebra problems and experiment with the theory underlying those problems. Both teachers indicated that PL is most effective when they dive deeper into one aspect of mathematics teaching and learning, embed themselves to solve mathematics problems in various ways, and discuss student solutions. While data from two teachers showed that they set themselves up to create, use and connect solution strategies to mathematics problems in a way that can support the development of student mathematical understanding, further research is needed to determine to what extent this is reflected in enacted mathematics curricula.

One of the challenges in such an approach is to ensure that opportunities for learning, inclusive of content understanding and knowledge of mathematics teaching, are provided. The effective implementation of study groups, and similar forms of PD programmes, requires knowledge and skills on the part of facilitators (teacher educators) as well (e.g., Schwarts & Karsenty, 2020). In this research, the study group meetings were designed very carefully and with great thought, which is what is required from facilitators. This means that facilitators need to have particular professional capabilities. To that end, we need to understand more about what knowledge and skills mathematics teacher educators need to have in order to facilitate these forms of PD programmes effectively, and how mathematics teacher educators' own PL regarding effective facilitation can be enhanced. For relevant investigations see Martin et al. (2022) and Perry and Boylan (2018).

The preponderance of learnings may mirror the unique context of the teachers (Clarke & Hollingsworth, 2002). The teaching context of Levi and Mia—i.e., the need for addressing knowledge and skills gaps, the teachers' career stage, their positive attitudes towards mathematics, and valuing mathematics and its learning—might be contributing factors in their gain from the PD opportunity (Darling-Hammond et al., 2017). While we can assume that PD experiences such as the study group represent effective learning opportunities for teachers who are open to new learnings and to make changes in their teaching practice, we do not yet have an understanding of what this might look like in different contexts where the change environment elements differ. For example, experienced or specialist teachers may respond to learning opportunities differently from early career or non-specialist teachers (Collopy, 2003) and this should be worth investigating in further prospective studies.

Acknowledgements

I thank the University of Tasmania which approved the research, the Mathematical Association of Tasmania for supporting the implementation of, and the teachers who participated in, the study. The thoughts in this paper, however, are solely my responsibilities.

Declarations

Author Contribution : The author takes all responsibilities of the paper.

Funding Statement : This research was unfunded.



Conflict of Interest : The author declares no conflict of interest.

Additional Information : Additional information is available for this paper.

REFERENCES

- Australian Council for Educational Research (ACER). (2014). *Staff in Australia's schools 2013*. Retrieved from https://research.acer.edu.au/tll_misc/20/
- Australian Mathematical Science Institute (AMSI). (2020). *The state of mathematical sciences 2020. 7th discipline profile of mathematics and statistics in Australia*. Retrieved from <https://amsi.org.au/resources/reports/>
- Arbaugh, F. (2003). Study groups as a form of professional development for secondary mathematics teachers. *Journal of Mathematics Teacher Education*, 6(2), 139–163.
- Birchak, B., Connor, C., Crawford, K. M., Kahn, L. H., Kaser, S., Turner, S. & Short, K. G. (1998). *Teacher study groups: Building community through dialogue and reflection*. Urbana, IL: National Council of Teachers of English.
- Bush, S. B., & Karp, K. S. (2013). Prerequisite algebra skills and associated misconceptions of middle grade students: A review. *Journal of Mathematical Behaviour*, 32, 613–632. <http://dx.doi.org/10.1016/j.jmathb.2013.07.002>
- Borko, H., & Potari, D. (Eds.). (2020). *Teachers of mathematics working and learning in collaborative groups. ICMI STUDY 25 Conference Proceeding*. National and Kapodistrian University of Athens. <http://icmistudy25.ie.ulisboa.pt/wp-content/uploads/2020/11/201114-ICMI25Proceedings6.13.2020.pdf>
- Collopy, R. (2003). Curriculum materials as a professional development tool: How a mathematics textbook affected two teachers' learning. *The Elementary School Journal*, 103(3), 287–311. <https://www.jstor.org/stable/1002273>
- Cuoco, A. (2013). *High school teaching: standards, practices, and habits of mind*. Center for Mathematics Education (CME), Education Development Center, Inc. (EDC). <http://cmeproject.edc.org/presentations-publications>
- Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, 18(8), 947–967. [https://doi.org/10.1016/S0742051X\(02\)00053-7](https://doi.org/10.1016/S0742051X(02)00053-7)
- Crespo, S. (2006). Elementary teacher talk in mathematics study groups. *Educational Studies in Mathematics*, 63(1), 29–56. <http://www.jstor.org/stable/25472110>
- Darling-Hammond, L., Hyler, M.E., & Gardner, M. (2017). *Effective teacher professional development*. Learning Policy Institute.
- du Plessis, A. E. (2015). Effective education: conceptualising the meaning of out-of-field teaching practices for teachers, teacher quality and school leaders. *International Journal of Educational Research*, 72, 89-102. <https://doi.org/10.1016/j.ijer.2015.05.005>



- Faulkner, F., Kenny, J., Campbell, C., & Crisan, C. (2019). Teacher learning and continuing professional development. In L. Hobbs & G. Törner (Eds.), *Examining the phenomenon of "teaching out-of-field": International perspectives on teaching as a non-specialist* (pp. 269–308). Springer.
- Fishman, B. L., Davis, E. A., & Chan, C. K. K. (2014). A learning sciences perspective on teacher learning research. In K. Sawyer (Ed.), *The Cambridge Handbook of the Learning Sciences* (pp. 707–725). Cambridge.
- Gersten, R., Dimino, J., Jayanthi, M., Kim, J. S., & Santoro, L. E. (2010). Teacher study group: Impact of the professional development model on reading instruction and student outcomes in first grade classrooms. *American Educational Research Journal*, 47(3), 694-739. <https://doi.org/10.3102/0002831209361208>
- Goldsmith, L. T., Doerr, H. M., & Lewis, C. C. (2014). Mathematics teachers' learning: A conceptual framework and synthesis of research. *Journal of Mathematics Teacher Education*, 17(1), 5–36.
- Goos, M. (2013). Knowledge for teaching secondary school mathematics: What counts? *International Journal of Mathematical Education in Science and Technology*, 44(7), 972–983. <https://doi.org/10.1080/0020739X.2013.826387>
- Goos, M., O'Donoghue, J., Ní Ríordáin, M., Faulkner, F., Hall, T., & O'Meara, N. (2020). Designing a national blended learning program for "out-of-field" mathematics teacher professional development. *ZDM Mathematics Education*, 52, 893–905. <https://doi.org/10.1007/s11858-020-01136-y>
- Grandau, L. (2005). Learning from self-study: Gaining knowledge about how fourth graders move from relational description to algebraic generalization. *Harvard Educational Review*, 75(2), 202–221.
- Grossman, P., Wineburg, S., & Woolworth, S. (2001). Toward a theory of teacher community. *Teachers College Record*, 103(6), 942-1012.
- Hatisaru, V. (2021). A reflection on Star and Seifert's (2006) operationalisation of flexibility in equation solving. *For the Learning of Mathematics*, 41(2), 37-39.
- Hatisaru, V. (2022). How to develop a structural conception of algebra of school students. *LUMAT-B: International Journal on Math, Science and Technology Education*, 7(1), 56–66. <https://journals.helsinki.fi/lumatb/article/view/1834>
- Hatisaru, V. & Erbaş, A. K. (2017). Mathematical knowledge for teaching and student learning outcomes. *International Journal of Science and Mathematics Education*, 15(4), 703-722. <https://doi.org/10.1007/s10763-015-9707-5>
- Hatisaru, V., Oates, G., & Chick, H. (2022). Developing proficiency with teaching algebra in teacher working groups: understanding the needs. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 250–257). Launceston: MERGA.
- Hill, H. C., & Chin, M. (2018). Connections between teachers' knowledge of students, instruction, and achievement outcomes. *American Educational Research Journal*, 55(5), 1076–1112. <https://doi.org/10.3102/0002831218769614>
- Hill, H. C., Lynch, K., Gonzalez, K. E., & Pollard, C. (2020). Professional development that improves STEM outcomes. *Phi Delta Kappan*, 101(5), 50-56. <https://doi.org/10.1177/0031721720903829>



- Hobbs, L., & Törner, G. (Eds.). (2019). *Examining the phenomenon of teaching out-of-field: international perspectives on teaching as a non-specialist*. Springer.
- Hollingsworth, H., & Clarke, D. (2017). Video as a tool for focusing teacher self-reflection: supporting and provoking teacher learning. *Journal of Mathematics Teacher Education*, 20, 457–475. <https://doi.org/10.1007/s10857-017-9380-4>
- Hu, Q., Son, J. W. & Hodge, L. (2022). Algebra teachers' interpretation and responses to student errors in solving quadratic equations. *International Journal of Science and Mathematics Education*, 20, 637–657. <https://doi.org/10.1007/s10763-021-10166-1>
- Hsieh, H.-F., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research*, 15, 1277–1288. <https://doi.org/10.1177/1049732305276687>
- Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: using student work to promote collective inquiry. *Journal of Mathematics Teacher Education* 7, 203–235. <https://doi.org/10.1023/B:JMTE.0000033084.26326.19>
- Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). Macmillan Publishing.
- Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707–762). Information Age Publishing.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. The National Academy Press. <https://doi.org/10.17226/9822>
- Lachance, A., & Confrey, J. (2003). Interconnecting content and community: A qualitative study of secondary mathematics teachers. *Journal of Mathematics Teacher Education*, 6(2), 107–137.
- Lane, C., & Ní Ríordáin, M. (2020). Out-of-field mathematics teachers' beliefs and practices: An examination of change and tensions using zone theory. *International Journal of Science and Mathematics Education*, 18, 337–355. <https://doi.org/10.1007/s10763-019-09964-5>
- Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of Representation in the Teaching and Learning of Mathematics* (pp. 33–40) Hillsdale, NJ: Lawrence Erlbaum Associates.
- Martin, M. M., Goldberg, F., McKean, M., Edward, P., & Turpen, C. (2022). Understanding how facilitators adapt to needs of STEM faculty in online learning communities: a case study. *International Journal of STEM Education*, 9, 56. <https://doi.org/10.1186/s40594-022-00371-x>
- Muir, T., Deed, C., Thomas, D., & Emery, S. (2021). Achieving teacher professional growth through professional experimentation and changes in pedagogical practices. *Australian Journal of Teacher Education*, 46(9), 2. <http://dx.doi.org/10.14221/ajte.2021v46n9.2>
- Noble, H., & Heale, R. (2019). Triangulation in research, with examples. *Evidence Based Nursing*, 22, 67–68. <http://dx.doi.org/10.1136/ebnurs-2019-103145>
- Perry, E., & Boylan, M. (2018) Developing the developers: Supporting and researching the learning of professional development facilitators. *Professional Development in Education*, 44(2), 254–271. <https://doi.org/10.1080/19415257.2017.1287767>

- Ponte, J. P., Quaresma, M., & Mata-Pereira, J. (2022). Teachers' learning in lesson study: insights provided by a modified version of the interconnected model of teacher professional growth. *ZDM Mathematics Education*, 54, 373–386. <https://doi.org/10.1007/s11858-022-01367-1>
- Porsch, R., & Whannell, R. (2019). Out-of-field teaching affecting students and learning: what is known and unknown. In L. Hobbs & G. Törner (Eds.), *Examining the Phenomenon of "Teaching Out-of-Field": International Perspectives on Teaching as a Non-Specialist* (pp. 179–191). Springer. <https://doi.org/10.1007/978-981-13-3366-8>
- Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, 29(1), 4–15. <https://doi.org/10.3102/0013189X029001004>
- Qi, C., Cao, C. & Huang, R. (2021). Teacher learning through collaboration between teachers and researchers: A Case Study in China. *International Journal of Science and Mathematics Education*. <https://doi.org/10.1007/s10763-021-10241-7>
- Schwartz, G., & Karsenty, R. (2020). "Can this happen only in Japan?": mathematics teachers reflect on a videotaped lesson in a cross-cultural context. *Journal of Mathematics Teacher Education*, 23, 527–554. <https://doi.org/10.1007/s10857-019-09438-z>
- Stanley, A. M. (2011). Professional development within collaborative teacher study groups: Pitfalls and promises. *Arts Education Policy Review*, 112(2), 71-78. <https://doi.org/10.1080/10632913.2011.546692>
- Tatto, M. T., Peck, R., Schulle, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)*. International Association for the Evaluation of Educational Achievement.
- Thorne, S. (2000). Data analysis in qualitative research. *Evidence Based Nursing*, 3(3), 68-70. <http://dx.doi.org/10.1136/ebn.3.3.68>
- Thurlings, M., & den Brok, P. (2018) Student teachers' and in-service teachers' peer learning: A realist synthesis. *Educational Research and Evaluation*, 24(1-2), 13-50. <https://doi.org/10.1080/13803611.2018.1509719>
- Wadjaja, W., Vale, C., Groves, S., & Doig, B. (2017). Teachers' professional growth through engagement with lesson study. *Journal of Mathematics Teacher Education*, 20, 357–383. <https://doi.org/10.1007/s10857-015-9341-8>
- Weldon, P. (2016). *Out-of-field teaching in Australian secondary schools*. Policy Insights, Issue 6. Australian Council for Educational Research.
- Wongsopawiro, D. S., Zwart, R.C., & van Driel, J. H. (2017). Identifying pathways of teachers' PCK development. *Teachers and Teaching: theory and practice*, 23(2), 191-210. <http://dx.doi.org/10.1080/13540602.2016.1204286>

