Journal on Mathematics Education
Volume 15, No. 1, 2024, pp. 99-114

Comparative study of means of mathematical communication in Japan, Laos, and Thailand

Duangmanee Ya-amphan[1], Sampan Thinwiangthong[1,*], Phailath Sythong[2]

[1]Mathematics Education Department, Khon Kaen University, Khon Kaen, Thailand
[2]Mathematics Education Department, Savannakhet Teacher Training College, Savannakhet, Laos
*Correspondence: sampan@kku.ac.th

Received: 26 June 2023 | Revised: 7 October 2023 | Accepted: 15 October 2023 | Published Online: 3 November 2023

Abstract

This comparative study analyzed the similarities and differences in the mathematical communication used by teachers and primary school students in Japanese, Lao, and Thai mathematics classrooms. It adopted a qualitative research design, and the targets were teachers and students in Grades 1 - 6 at a selected school in each of the three countries. The data were collected from one classroom at each grade level from the three schools, so eighteen classrooms were involved in the study. The research tools included video recorders, photo cameras, and a field notes form. The data were analyzed by employing the analytical descriptive method based on Pirie's conceptual framework of the means of mathematical communication (1998). The findings revealed that the means of mathematical communication in the three countries were similar in their use of ordinary, mathematical, verbal, and symbolic language—differences in the means of mathematical communication related to how varied the learning materials were. The Japanese classrooms used the most varied means of mathematical communication, demonstrating all six means, while the Thai classrooms used only 5, and the Lao PDR classrooms used only 4. The Japanese classrooms were shown to be focused on the students' diverse self-solution concepts, while some of the Thai and Lao PDR classrooms were based on question-answer interactions between the teachers and students.

Keywords: Comparative Study, Japan, Lao PDR, Means of Mathematical Communication, Thailand

How to Cite: Ya-amphan, D., Thinwiangthong, S., & Sythong, P. (2023). Comparative study of means of mathematical communication in Japan, Laos, and Thailand. *Journal on Mathematics Education, 15*(1), 99-114. http://doi.org/10.22342/jme.v15i1.pp99-114

Results from several assessments by the Trends in International Mathematics and Science Study (TIMSS) have indicated that Japanese students consistently rank in the top five, reflecting the high quality of Japanese education (TIMSS & PIRLS International Study Center, 2019). In addition, the results of the international assessment of the OECD Program for International Student Assessment (PISA) indicate that Japanese students achieve high scores in mathematics (Organization for Economic Co-operation and Development [OECD], 2018). In contrast, Thai students' average mathematics scores have steadily declined (Institute for the Promotion of Teaching Science and Technology [IPTST], 2019). Meanwhile, students in the Lao People's Democratic Republic (Lao PDR) have never participated in the TIMSS and PISA exams due to a lack of management readiness. In terms of educational reform, it was found that Thailand has been undergoing educational reform since 1999 (Chiangkool, 2016), while the Lao PDR has been attempting to reform education continuously (Shadiq & Lukman, 2010; Ministry of Education and Sport, Lao PDR, 2013; Ministry of Education and Sport, Lao PDR, 2015; Ministry of Education and

Sport, Lao PDR, 2020). Several educators (Kaewdaeng, 1999; Phanich, 2014; Inprasitha, 2014; Ansarian, & Mohammadi, 2018) have suggested that shifting from a content-oriented process to a student-centered learning process would make educational reform successful.

Many mathematics educators have emphasized the importance of the mathematical learning process and have suggested that mathematical communication should be developed in mathematics classrooms (Emori, 1993; Emori, 1997; Sierpinska, 1998; Emori, 2005; Cheah, 2007; Sfard, 2008; Smieskova, 2017; Wiriyaudomsatean & Thinwiangthong, 2019; Shanmugam et al., 2020; Uyen et al., 2021; Armiati et al., 2022). Educators and mathematics educators worldwide have recognized the value and benefits of mathematical communication, as clearly demonstrated by the agenda at the APEC Lesson Study 2008 (Isoda & Inprasitha, 2007). Mathematics communication is understood to be an essential part of the learning process that engages students in meaningful learning of mathematics. In addition, research published in the mathematics education community has shown that mathematical communication in the classroom can reflect the quality of educational instruction (Sieprinska, 1998; Emori, 2005). Developing mathematical communication as a skill in learning mathematics can determine the quality of mathematics teaching and learning (Thinwiangthong, Inprasitha, & Loipha, 2012), can help to create meaning, and can allow mathematical concepts to be communicated and exchanged (National Council of Teachers of Mathematics [NCTM], 2000). Mathematical communication is communication about mathematics via having discussions and supplying reasons to support opinions or messages to exchange ideas and reach a mutual understanding. Mathematical representations are expressed in various ways, such as by speaking, writing, mathematical symbols, and feelings or emotions, to convince others to agree with the ideas (Isoda & Inprasitha, 2007). In mathematical communication, the following attributes are to be considered: rigorousness, economy, and freedom of thinking (Emori, 2005). Mathematics educators can consider mathematical communication by considering the means of mathematical communication (Sierpinska, 1998).

The quality of mathematics teaching in the classroom can be determined through mathematical communication, which consists of speaking, writing, listening, and reading related to mathematics (Arani, 2016). Research results on the means of mathematical communication in classrooms have illustrated that teachers and students tend to develop a method for effective mathematical communication by applying visual mediators and specific technical terms, improving the effectiveness of learning mathematics (Ryve, Nilsson, & Pettersson, 2013). In mathematics classrooms, teachers and students can use the six means of mathematical communication: ordinary language, mathematical verbal language, symbolic language, visual representation, unspoken but shared assumptions, and quasi-mathematical language. These methods can help students learn mathematics well (Pirie, 1998). Ordinary language, which is the use of everyday language, is the most basic means of mathematical communication that can be used to learn mathematics. Students, however, must also use mathematical verbal and symbolic language in mathematics classrooms (Pirie, 1998). Visual mediators, as symbolic artifacts (e.g., algebraic expressions and mathematical symbols), are essential tools in mathematical discourse and mathematical communication (Sfard, 2008; David & Tomaz, 2012). Clearly, different frameworks exist for considering the means of mathematical communication. Therefore, researchers, educators, and mathematics educators can choose a framework that best fits the research objectives and the classroom context in which the research area is located. However, suppose we seek to assess the quality of mathematics teaching and learning clearly. In that case, comparative studies must be conducted in order to understand better the similarities and differences in mathematics teaching and learning. Such differences can be reflected in the means of mathematical communication.

Comparative studies are used in all disciplines and are applied to vast research topics, and comparative studies with a focus on international operating conditions or issues are numerous (Hantrais, 1995; Allik et al., 2010; Drobni et al., 2010; Azarian, 2011; Bukhari, 2011). Comparisons of teaching and learning in different cross-cultural classroom contexts, and particularly comparative research on teaching and learning in international classrooms, represent methods that reveal underlying differences and similarities (Stigler & Hiebert, 1999). International comparative studies of mathematics lessons provide international learning opportunities for mathematics educators and opportunities to gain experience in analyzing the best mathematics lessons and analyzing issues based on empirical evidence. In addition, the results of such studies can be applied to improve teaching and learning in the field of mathematics education (Arani, 2016). As mentioned, Thailand and Laos have been attempting to reform education, especially mathematics teaching and learning.

The results from an international comparative study with Japan, a prosperous country in terms of education, can be applied in order to improve the teaching and learning of mathematics in the Thai and Lao contexts. Comparative research on the means of mathematical communication in mathematics classrooms in Japan, Laos, and Thailand is a necessary component of such a comparative study. It is essential to identify similarities and differences in the means of mathematical communication and other related elements used by teachers and students in these countries. Therefore, this comparative research aimed to analyze the similarities and differences in the mathematical communication used by teachers and students in Japan, Laos, and Thailand. The study results affect the quality of mathematics teaching and learning in each country. Mathematics teachers and educators can further use the research results to develop mathematics teaching and learning activities.

METHODS

Research Design

This research was a comparative study (Adiyia & Ashton, 2017) that utilized a qualitative research model, which emphasized protocol analysis (Schoenfeld, 1985). The researcher conducted an analytical narrative analysis based on Pirie's means of mathematical communication analysis framework (1998).

Target Group

The target group were teachers and students in Grades 1 - 6 at a public elementary school affiliated with Naruto University of Education in Japan, the Demonstration School of Savannakhet Teacher Training College in Lao PDR, and Khon Kaen University Demonstration School in Thailand. Data were collected from one classroom at each grade level in each of these three schools, resulting in a total of eighteen classrooms participating in the study. Purposive sampling was used to select the classrooms, and teachers and students in each classroom voluntarily participated in the project.

Research Instruments

Data for this research were collected during the 2019 Coronavirus (COVID-19) outbreak. The researcher was unable to travel abroad to collect the research data. Therefore, the data collection instruments for each country were adapted as detailed below:

1. At the public elementary school affiliated with Naruto University of Education in Japan, video recorders were used in mathematics classrooms (Grades 1 - 6, one lesson each, with a total of six lessons). Classroom videos were authorized to record activities during the 2017-2019 semesters.

2. At the Demonstration School of Savannakhet Teacher Training College Lao PDR, video recorders, photo cameras, and field notes were used in mathematics classrooms, where students were being taught onsite under typical circumstances during the first semester of the Academic Year of 2021 (Grades 1 to 6, one lesson each, with a total of six lessons). The Lao PDR researchers and research assistants recorded videos, took photographs, and recorded field notes on teachers' and students' speech and behaviors that demonstrated the mathematical communication between them.
3. At the Demonstration School of Khon Kaen University in Thailand, video recorders, photo cameras, and field notes were used in the mathematics classrooms (Grades 1 - 6, one lesson each with a total of six lessons). Classes were using a hybrid learning model (a combination of onsite learning and online learning) during the COVID-19 epidemic in the second semester of the Academic Year of 2021. The Thai researcher and research assistants recorded videos, took photos, and recorded field notes on teachers' and students' speech and behaviors that demonstrated the mathematical communication between them.

Data Collection

Data collection based on video recording can be used to learn about learning and teaching structures in detail (Hall, 2000; Lesh & Lehrer, 2000). Data collection as it occurred in the three countries can be detailed as follows:

Data collection in Japan: The researcher could not travel to Japan due to the COVID-19 epidemic. Therefore, data was collected by asking a Professor at the Naruto University of Education for permission to use their recorded open class videos (in Grades 1 to 6, 1 lesson each with a total of six lessons) from the 2017 – 2019 semesters. Written permission to use the videos for research purposes was signed by Prof. Dr. Hiroki ISHIZAKA of the Global Education Course at the Graduate School of the School of Education at Naruto University of Education in Japan. Prof. Dr. Hiroki ISHIZAKA was responsible for open class activities held at the public elementary school affiliated with Naruto University of Education. The classroom videos were recorded for regular educational and research purposes, and the classes were taught as usual under typical circumstances.

Data collection in Lao PDR: A researcher and assistant researchers from Lao PDR managed the collection of video data from mathematics classrooms in Grades 1 – 6, with a total of six classes held at the Demonstration School of Savannakhet Teacher Training College. The classrooms were taught as usual under typical circumstances. The data was collected during the first semester of the Academic Year of 2021 (May - June 2021) in coordination with the third researcher, a lecturer at the Savannakhet Teacher College Lao People's Democratic Republic.

Data collection in Thailand: A Thai researcher and assistant researchers collected video data from mathematics classrooms in Grades 1 – 6, with a total of six classes held at the Demonstration School of Khon Kaen University. During the data collection period, classes were being taught using a hybrid learning model (onsite and online) due to the COVID-19 epidemic. Data collection was conducted during the second semester of the Academic Year of 2021 (December 2021) in coordination with a researcher and assistant researchers, lecturers at the Faculty of Education, Khon Kaen University in Thailand.

Names, surnames, photos, animations, and all information that could have been personally identifiable, were not disclosed. Researchers transcribed the speech and behaviors and outlined protocol with slides of the means of mathematical communication for analysis.

Data Analysis

The first researcher analyzed the Japanese classroom video, the second analyzed the Thai classroom video, and the third analyzed the Lao classroom video. Each researcher studied the video using the analysis protocol and then used an analytical descriptive method to illustrate the details of the means of mathematical communication, as well as other related elements, and selected pictures that demonstrated the teaching and learning activities according to Pirie's conceptual framework of the means of mathematical communication (1998). Then, all three researchers examined the analysis results and verified the validity of the analysis. The six means of mathematical communication were classified as follows:

1. Ordinary language represents the use of language or vocabulary in everyday life, in which students of each age group may use different language due to having different experiences. Students with different domiciles may have different everyday language use or different local languages.
2. Mathematical verbal language represents a method of using mathematical words. The term “mathematical” herein means: “It is related to mathematics or related to a mathematical concept that the student is communicating.”
3. Symbolic language is a method of using mathematical symbols to communicate mathematical concepts. In mathematics classrooms, teachers and students primarily express themselves by writing symbols. The use of symbols is especially important to learning math.
4. Visual representations describe mathematical communication that facilitates each participant's ability to perceive, visualize, and understand clearly. It is a highly effective means of communication.
5. Unspoken but shared assumptions represent a method in which mathematical concepts can be shared among participants without direct or overt communication. This means of mathematical communication usually occurs when the participants, i.e., the sender and the receiver, know each other very well. They are intimate and familiar with each other. Therefore, they are able to share a mutual understanding, so there is no need to talk.
6. Quasi-mathematical language represents a means of mathematical communication most often used by students in mathematics classrooms. Students often use well-known and understood language that has specific mathematical meanings in their classroom contexts. People outside the classroom context may not understand the mathematical meanings of the students’ language. Only the teacher or someone familiar with the students in a particular mathematics classroom context would be able to understand the meaning behind this means of mathematical communication.

Data analysis was carried out using triangular investigation. In order to verify the correctness, accuracy, and reliability of data analysis, after each researcher analyzed the classroom video of one country, all researchers re-analyzed and verified the analysis of all countries’ classroom videos. After that, all of the researchers worked together, discussed, and then summarized their opinions about the results of the analysis.

RESULTS AND DISCUSSION

After completing the descriptive analysis of the eighteen classrooms, (six classrooms each from Japan, Laos, and Thailand), the researchers analyzed the differences and similarities of the means of mathematical communication that had been used in the three countries. The results are shown in Table 1.

Table 1. A summary of the similarities and differences identified in terms of the means of mathematical communication used by the teachers and students in Japanese, Lao, and Thai mathematics classrooms

Similarities	Differences
1) Teachers and students in mathematics classrooms in all three countries had at least three different means of mathematical communication: ordinary language; mathematical verbal language, and symbolic language. These means are important and are a necessary basis for having the students learn mathematics.	1) There were six means of mathematical communication in the Japanese classrooms, five means of mathematical communication in the Thai classrooms, and four means of mathematical communication in the Lao classrooms.
2) The means of mathematical communication used by teachers and students in all of the Japanese mathematics classrooms created opportunities for students to use a variety of learning materials, to analyze their own problem-solving abilities, and to communicate the solution through a variety of means. This was similar to some of the mathematics classrooms in the Lao PDR and in Thailand, in which the teachers had the students analyze their own solutions and had them communicate their solutions through a variety of means.	2) The means of mathematical communication used by teachers and students in all of the Japanese mathematics classrooms were based on and had a focus on a variety of solutions to problems created by students. In this type of mathematics learning, the students played the leading role (student-centered). The means of mathematical communication used by teachers and students in some of the mathematics classrooms in Thailand and the Lao PDR were based on interactions between the teachers and the students. The teachers played the main roles in classroom instruction (teacher-centered).
3) The means of mathematical communication for teachers and students in Japan and Thailand were based on various learning materials designed by the teachers, which highly supported the student's achievements. This was similar to some of the mathematics classrooms in the Lao PDR.	3) The means of mathematical communication used by teachers and students in all of the Japanese mathematics classrooms were based on allowing the students to manipulate the learning materials. Meanwhile, the means of mathematical communication used in the Lao PDR and in Thailand were based on manipulation of the learning materials by the teachers.

The conclusions in Table 1 were derived from generalizations based on the descriptive analysis. Descriptive analysis examples from the research results follow.

1. Example of the means of mathematical communication as used by a Japanese 3rd-grade teacher and students:

At the conclusion stage, the teacher summarized the words that the students had learned together in the class. These were the words for which teachers and students had reached a mutual understanding. They were "separated" and "combined" and the words the teacher had written on the board at the beginning of the class. The teacher asked questions, and students answered in accordance with the following protocols and figures:

Teacher: Students, tell me. What have we learned today? What conclusions did you draw?
Student 1: If the multiplier consists of two identical numbers, we can use the <u>"combined" technique</u>.
Student 2: To solve the problem with the <u>"separated" technique</u>, we must first find the total cost of each type of item. (Find the total price of canned orange juice first and find the total price of oranges first.)

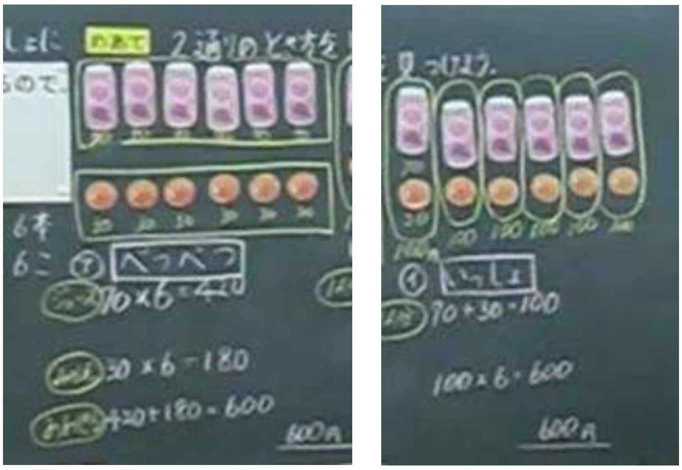

Figure 1. Quasi-mathematical language in the Japanese classroom

From the protocol and Figure 1 above, the teacher and students used the quasi-mathematical language means of mathematical communication. They used the words "separated" and "combined" to describe problems with the use of multiplication properties (distribution properties). "Separated" was obtained from solving a problem using a symbolic sentence $(70 \times 6) + (30 \times 6) = 600$; and "combined" was obtained from solving a problem using a symbolic sentence $(70 + 30) \times 6. = 600$, which meant that the students and teachers in this classroom had come to a mutual understanding of the use of the words "separated" and "combined" in solving problems.

2. Example of the means of mathematical communication as used by a third-grade teacher and students from Lao PDR:

   The teacher discussed the problem's situation by asking students what they noticed on page 79 of the textbook. The students tried to answer the teacher's question according to their understanding, as shown in the following protocol and figure.

Teacher: What number multiplied by 6 equals 30?
Students: 5
Teacher: What other numbers can be multiplied and are equal to 30?
Student 1: 6×5 is also 30.

Teacher: What will the symbolic sentence be?
Student 2: 5×6 or 6×5
Teacher: 5×6 or 6×5. So, how are they related?
Student 1: They are similar.
Student 3: They are reversed.
Student 1: They can replace each other.
Teacher: This symbolic sentence alternates or swaps places.

Figure 2. Ordinary language in the Lao PDR classroom

The above protocol and Figure 2 show a question-and-answer session between the teacher and students. The teacher asked the students, “What number multiplied by 6 equals 30?” and most of the students replied, “5”. The teacher asked the next question, “What other numbers can be multiplied and equal 30?”. Student 1 replied, “6 × 5 is also 30”. The teacher tried to lead students to recognize the commutative property, so she asked, “What will the symbolic sentence be?” Student 2 replied, “5 × 6, 6 × 5”. The teacher did not get the answer she expected, so she kept asking, “5 × 6 or 6 × 5. So, how are they related?” Student 1 replied, “They are similar.” and “They can replace each other.”. At the same time, Student 3 said, “They are reversed.” This question-answer interaction revealed the everyday words the students had used to learn the commutative property of 5 × 6 and 6 × 5, indicating that the teacher and students had used the ordinary language means of mathematical communication.

3. Example of the means of mathematical communication as used by a third-grade teacher and students in Thailand:

The students were trying to figure out the size of the corners of objects using foldable paper. During the presentation and discussion, the students answered the teacher's questions, and the teacher wrote the students’ ideas, as shown in the following protocol and figure.

Teacher: Students, do you know the name of this angle here?
Students: Right angle (an online student’s voice)
Teacher: Does a piece of paper have a right angle?
Student 1: Both are equal angles.
Teacher: Ah, what is the angle?
Student 2: A right angle is an angle that is ninety degrees.
Teacher: (Writes 90° on the board)

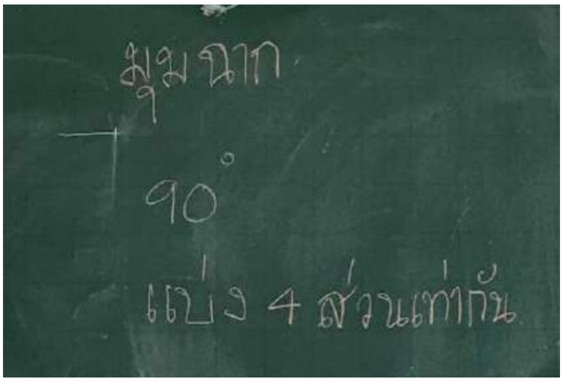

Figure 3. Symbolic language in the Thai classroom

The above protocol and Figure 3 demonstrate a question-answer interaction to reach common ideas during a discussion. The students expressed their ideas by answering questions: "A right angle is an angle that is ninety degrees." Then, the teacher wrote "90°" on the board, indicating that the students and teachers had used symbolic language for mathematical communication.

An interpretative method of analysis was used to conclude the findings, focusing on coming to a deep understanding of the means of mathematical communication and related elements. The findings revealed three similarities in the use of the means of mathematical communication in primary school mathematics classrooms in Japan, the Lao PDR, and Thailand, as follows:

1. The teachers and students in mathematics classrooms in all three countries had at least three different means of mathematical communication: ordinary language, mathematical verbal language, and symbolic language. These three means are essential and provide a necessary basis for the students to learn mathematics. This finding conforms to the Pirie (1998) principle of ordinary language and mathematical symbols in mathematics classrooms, making it familiar to find this type of mathematical communication. In addition, Sfard (2008) and David and Tomaz (2012) note that visual mediators, as symbolic artifacts, are essential tools in mathematical communication.
2. The means of mathematical communication used by teachers and students in all Japanese mathematics classrooms created opportunities for students to use various learning materials, analyze their own problem-solving processes, and communicate solutions through various means. This was similar to some of the mathematics classrooms in the Lao PDR and in Thailand, in which teachers had students analyze their own solutions and communicate them through various means. Due to the learning culture in Japan, learners are allowed to solve problems independently and to learn from their analyses. This correlates with a concept discussed by Stigler and Hiebert (1999), who claimed that teaching is a cultural activity in which teachers and students must have a mutual understanding. The teaching and learning culture in Japanese mathematics classrooms demonstrates the opportunity for students to use an assortment of learning materials and various means of mathematical communication. This is similar to some of the mathematics classrooms in the Lao PDR and in Thailand, in which teachers adjusted their teaching methods and provided the students with more mathematical communication opportunities. Isoda and Inprasitha (2007) suggested that in mathematics classrooms, students should have opportunities to use dialectic features, mathematical ways of explanation to share ideas and understanding, mathematical representation, and competitive and sympathetic attitudes.

3. The means of mathematical communication for teachers and students in Japan and Thailand had been based on the various learning materials designed by teachers, which highly supported each student in achieving the learning target. This was similar to some of the mathematics classrooms in the Lao PDR. Romiszowski (1986) stated that teachers must design and plan the use of learning materials with the aim of developing the students' learning. This correlates with a study conducted by Hamdani (2011), who stated that in order to manage learning to achieve the lesson objectives, teachers need to prepare for the systematic use of media. Using learning materials helps students to communicate their ideas better. Lesh, Post, and Behr (1987) suggested that students can represent concepts by using learning materials. Therefore, teachers need to appropriately design learning materials for learning activities, which would allow students to use a variety of means of mathematical communication.

The findings revealed three differences in the use of the means of mathematical communication in the primary mathematics classrooms of Japan, the Lao PRD, and Thailand. Firstly, it was found that six means of mathematical communication were utilized in the Japanese classrooms, five means of mathematical communication in the Thai classrooms, and four means of mathematical communication utilized in the Loa classrooms. The findings demonstrated that the attribute of "freedom" in using the means of mathematical communication was present in Japanese mathematics classrooms. In the Lao and Thai classrooms, there was also an attempt to create freedom in solving problems and using mathematical communication. Emori (2005) pointed out that freedom in the means of mathematical communication is an essential attribute and that a positive relationship can be shown between this freedom and mastering various mathematical concepts. In classrooms where students are free to think, they can express different concepts through various means of mathematical communication. In the present study, it was clear that there had been more freedom for the means of mathematical communication in the Japanese mathematics classrooms than in the Thai and Lao PDR classrooms.

Moreover, classroom culture also influences the means of mathematical communication, as stated by Stigler and Hiebert (1999), who noted that the teaching and learning culture of Japanese mathematics classrooms demonstrates the opportunity for students to use an assortment of learning materials and to use a variety of means of mathematical communication. In addition, using a wide variety of mathematical concepts resulted from using open-ended questions. This correlates with Shanmugam, Chinnapan, and Leong's study (2020), which revealed that Japanese mathematics classrooms use an investigative approach. The teachers used open-ended questions that could draw upon a wide range of advanced mathematical concepts among students. This gives students the freedom to think mathematically and facilitates the development of students' 21st-century skills, including mathematical communication skills.

Secondly, the means of mathematical communication used by teachers and students in all of the Japanese mathematics classrooms were based on and focused on various problem solutions created by students. Moreover, the students were leading in mathematics learning (learning was student-centered). Takahashi (2006), Isoda (2010), and Shanmugam, Chinnapan, and Leong (2020) all agreed that Japanese mathematics classrooms are problem-solving and investigative-based classrooms in which students can solve problems on their own by using a variety of ideas. The class then discusses the ideas they used to solve the problems. In the present study, students in Japanese classrooms were allowed to use various means of mathematical communication. As for the mathematical communication used by teachers and students in Thailand and Lao PDR, some mathematics classrooms were shown to base instruction on interactions between teachers and students where teachers played the primary role (teacher-centered classroom instruction). This correlates with a study by Inpanya et al. (2015), who found that communication between teachers and students in Lao mathematics classrooms demonstrated the characteristic of rigorousness but not the attributes of economy or freedom, and there needed to be mathematical communication between students. Arani (2016) also discovered that the principal means of mathematical communication used in Iranian mathematics classrooms was speaking rather than writing. The type of interaction seen in the Thai and Lao classrooms is consistent with a teaching approach described by Wood (1998) as IRE (I: Institute a question by the teacher, R: Respond by some of the students, and E: Evaluate for correctness by the teacher), which does not allow for a variety of means of mathematical communication. Moreover, as Black and Wiliam (1998) have pointed out, a short question-and-answer session between the teacher and the students is ineffective, as students do not think critically in such formats concerning their answers.

Thirdly, the mathematical communication used by teachers and students in all Japanese mathematics classrooms was based on having the students manipulate learning materials. Materials manipulation allows students to learn mathematics independently and in a meaningful way. It also allows students to communicate ideas by clearly representing them. This is in line with a study conducted by Nampeng (2020), who showed that students using learning materials can demonstrate a variety of representations, including speaking, writing, drawing, using equipment, and using gestures. Such representations correlate with different means of mathematical communication, including ordinary language, verbal, mathematical, and symbolic language. In addition, when students are given the occasion to use the learning materials together, opportunities for mutual understanding without using spoken language can emerge. Such opportunities correlate with the means of mathematical communication referred to as unspoken but shared assumptions.

Additionally, using learning materials, such as visual mediators (Sfard, 2008), helps students use mathematical communication more effectively. However, it was found that the means of mathematical communication used by teachers and students in the Lao PDR and Thailand were based on manipulating the learning materials. The teacher was the demonstrator and showed the students how to use the learning materials. Therefore, the students did not manipulate the learning materials independently, reducing their chances of producing self-learning and reducing their opportunities to solve problems independently. This led to a reduction in the students' opportunities to use different means of mathematical communication. This finding correlates with findings from studies by Ryve, Nilsson, and Pettersson (2013) and by Goldhaber, Krieg, and Theobald (2020), who revealed that mathematical communication skills are not encouraged in mathematics classrooms because teachers still use traditional teaching methods, which do not facilitate the means of mathematical communication.

CONCLUSION

The analysis of similarities and differences in using various means of mathematical communication in Japanese, Lao, and Thai primary school mathematics classrooms revealed that the teachers and students in all three countries used three means of mathematical communication to create mutual understandings in mathematics (ordinary language; mathematical verbal language; and symbolic language). By providing students with opportunities to use assorted learning materials, various means of mathematical communication can be encouraged, helping students achieve the lesson objectives. Classrooms in which the students play the primary role in learning mathematics (student-centered) appear to offer more diverse approaches to the means of mathematical communication than those in which the teacher plays the primary role (teacher-centered).

The findings from this study suggest that the culture of the mathematics classroom and opportunities that are created for students to use a variety of learning materials are relevant to the means of mathematical communication that teachers and students ultimately use. This research, however, was limited to a profound interpretation of the means of mathematical communication and related elements. The frequency of different means of mathematical communication occurrences in the mathematics classrooms was not measured and could be the focus of further study. Further research should also be conducted to explore how the mathematics classroom culture affects the means of mathematical communication and how learning materials are manipulated to facilitate different means of mathematical communication in the classroom.

Acknowledgments

The authors would like to express their sincere gratitude to the Sumitomo Foundation, which funded the grant for a Japan-Related Research Project (Reg. No. 198717), as well as their gratefulness to Prof. Dr. Hiroki ISHIZAKA, Naruto University of Education, who permitted the use of the Japanese open class videos. The researchers would also like to acknowledge the support of the Faculty of Education, Khon Kaen University, Thailand and Savannakhet Training Collage, Lao PDR.

Declarations

Author Contribution	:	DY: Conceptualization, Writing - Original Draft, Editing and Visualization, Formal Analysis, Methodology, and Validation. ST: Conceptualization, Writing - Original Draft, Review & Editing, Formal Analysis, Methodology and Validation. PS: Conceptualization, Writing - Review & Editing, Formal Analysis, Methodology, and Validation.
Funding Statement	:	This research was supported and funded by the Sumitomo Foundation through a grant for a Japan-Related Research Project (Reg. No. 198717).
Conflict of Interest	:	The authors have no conflict of interest to declare.
Additional Information	:	Additional information is available for this paper.

REFERENCES

Adiyia, M., & Ashton, W. (2017). *Comparative research*. Canada: Rural Development Institute, Brandon University. https://www.brandonu.ca/rdi/files/2017/07/RDI-Comparative-Research.pdf

Allik, J., Realo, A., Mottus, R., Borkenau, P., Kuppens, P., & Hrebickova, M. (2010). How people see others is different from how people see themselves: a replicable pattern across cultures. *Journal of Personality and Social Psychology, 99*(5), 870-882. https://doi.org/10.1037/a0020963

Ansarian, L., & Mohammadi, F. S. (2018). Problem-based learning in action: review of empirical studies. *Pertanika J. Soc. Sci. & Hum, 26*(T), 13-32. http://www.pertanika.upm.edu.my/resources/files/Pertanika%20PAPERS/JSSH%20Vol.%2026%20(T)%20Dec.%202018/02%20JSSH-2672-2017.pdf

Arani., M. R. S. (2016). An examination of oral and literal teaching traditions through a comparative analysis of mathematics lessons in Iran and Japan. *International Journal for Lesson and Learning Studies, 5*(3), 196 – 211. https://doi.org/10.1108/IJLLS-07-2015-0025

Armiati, Fauzan, A., Harisman, Y., & Sya'bani, F. (2022). Local instructional theory of probability topics based on realistic mathematics education for eight-grade students. *Journal on Mathematics Education, 13*(4), 703-722. http://doi.org/10.22342/jme.v13i4.pp703-722

Azarian, R. (2011). Potentials and limitations of comparative method in social science. *International Journal of Humanities and Social Science, 1*(4), 113-125. http://www.ijhssnet.com/journals/Vol._1_No._4;_April_2011/15.pdf

Black, P., & Wiliam, D. (1998). Inside the black box raising standards through classroom assessment. *Phi Delta Kappan, 92*(1), 139-148. https://doi.org/10.1177/0031721710092001

Bukhari, H. A. S. (2011). *What is comparative study*. http://dx.doi.org/10.2139/ssrn.1962328

Cheah, U. H. (2007). Conceptualizing a framework for mathematics communication in Malaysian primary schools. Paper presented at the Third APEC-Tsukuba International Conference Innovation of Classroom Teaching and Learning through Lesson Study III - Focusing on Mathematical Communication, CRICED, University of Tsukuba. https://www.criced.tsukuba.ac.jp/math/apec/apec2008/papers/PDF/10.Cheah_Ui_Hock_Malaysia.pdf

Chiangkool, W. (2016). *The status of Thai education, years 2014-2015: How to reform Thai education to keep pace with the world in the 21st century*. Bangkok: Pimdee Printing. [in Thai]

David, M. M., & Tomaz, V. S. (2012). The role of visual representations for structuring classroom mathematical activity. *Educational Studies in Mathematics, 80*(3), 413–431. https://doi.org/10.1007/s10649-011-9358-6

Drobni, S., Beham, B., & Prag, P. (2010). Good job, good life? working conditions and quality of life in Europe. *Social Indicators Research, 99*(2), 205-225. https://doi.org/10.1007/s11205-010-9586-7

Emori, H. (1993). The mechanism of communication in learning mathematics. In I. Hirabayashi, N. Nohda, K. Shigematsu, & F.-L. Lin (Eds.), *Proceedings of the 17th International Conference for the Psychology of Mathematics Education (PME17)*. (Vol. 2, pp. 230-237), Tsukuba, Japan: University of Tsukuba.

Emori, H. (1997). Mathematics communication. In T. Katsuro (Ed.), *Rethinking Lesson Organization in School Mathematics*. (pp. 44-60). Japan: Japan Society of Mathematics Education.

Emori, H. (2005). *The workshop for young mathematics educations in Thailand 2005 building up the research agenda for the next 10 year, 2006 -2015*. Khon Kean: Khon Kean University.

Goldhaber, D., Krieg, J. M., & Theobald, R. (2020). Exploring the impact of student teaching apprenticeships on student achievement and mentor teachers. *Journal of Research on Educational Effectiveness, 13*(2), 213-234. https://doi.org/10.1080/19345747.2019.1698087

Hall, R. (2000). Videorecording as theory. In A. E. Kelly, & R. A. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education*. (pp. 647-664). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Hamdani. (2011). *Strategi belajar mengajar*. Bandung: CV. Pustaka Setia. http://www.pustakasetia.com/product/strategi-belajar-mengajar/

Hantrais, L. (1995). Comparative research methods. *Social Research Update, 13*(Summer 1995). https://sru.soc.surrey.ac.uk/SRU13.html

Inpanya, T., Thinwiangthong, S., Ban Har, Y., & Inprasitha, M. (2015). Characteristics of mathematical communication in primary classrooms in Lao PDR. Paper presented at the 7th ICMI-East Asia Regional Conference on Mathematics Education (EARCOME 7). Waterfront Hotel, Cebu City, Philippines.

Inprasitha, M. (2014). *Processes of problem solving in school mathematics*. Khon Kaen: Pen Printing. [in Thai]

Institute for the Promotion of Teaching Science and Technology (IPTST). (2019). *PISA 2018 result: summary report*. Bangkok: IPST. [in Thai] https://pisathailand.ipst.ac.th/pisa2018-fullreport/

Isoda, M. (2010). Lesson study: problem solving approaches in mathematics education as a Japanese experience. *Procedia Social and Behavioral Sciences, 8*(2010), 17–27. https://doi.org/10.1016/j.sbspro.2010.12.003

Isoda, M., & Inprasitha, M. (2007). The Third APEC-Tsukuba International Conference Innovation of Classroom Teaching and Learning through Lesson Study III - Focusing on Mathematical Communication (First Announcement), CRICED, University of Tsukuba. https://www.criced.tsukuba.ac.jp/math/apec/apec2008/index_en.php

Kaewdaeng, R. (1999). *Thai education reform*. Bangkok: Matichon. [in Thai]

Lesh, R., & Lehrer, R. (2000). Iterative Refinement Cycles for Videotape Analyses of Conceptual Change. In A. E. Kelly, & R. A. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education*. (pp. 665-708). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of Representations in the Teaching and Learning of Mathematics*. (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Ministry of Education and Sport, Lao PDR. (2013). *Final report of the education sector development plan (ESDP) (2011–2015)*. Vientiane, Lao PDR: Ministry of Education and Sport.

Ministry of Education and Sport, Lao PDR. (2015). *Student learning outcomes in primary education in Lao PDR*. Vientiane, Lao PDR: Ministry of Education and Sport.

Ministry of Education and Sport, Lao PDR. (2020). *Education and sports sector development plan 2021-2025*. Vientiane, Lao PDR: Ministry of Education and Sport.

Nampeng, C. (2020). *Students' representation in using instructional material in mathematics classroom*. Master Thesis in Mathematics Education, Graduate School, Khon Kaen University.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Virginia: NCTM, Inc.

Organisation for Economic Co-operation and Development [OECD]. (2018). *PISA 2018 results (Volume I): what students know and can do*. Paris: OECD Publishing. https://doi.org/10.1787/5f07c754-en

Phanich, W. (2014). *Learning creation to 21st century*. Nakorn Patom: S. Charern Printing. [in Thai]

Pirie, S. (1998). Crossing the gulf between thought and symbol: language as (slippery) stepping-stones. In H. Steinbring, M. Bartolini Bussi and A. Sierpinska (Eds.), *Language and Communication in the Mathematics Classroom*. Reston, VA: NCTM, pp. 7-29.

Romiszowski, A. J. (1986). *Developing auto-instructional materials from programmed texts to CAL and interactive video*. London and New York: Routledge.

Ryve, A., Nilsson, P., & Pettersson, K. (2013). Analyzing effective communication in mathematics group work: the role of visual mediators and technical terms. *Educational Studies in Mathematics, 82*(3), 497-514. https://doi.org/10.1007/S10649-012-9442-6

Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York: Academic Press.

Sfard, A. (2008). *Thinking as communicating: human development, the growth of discourses, and mathematizing*. Cambridge, UK: Cambridge University Press.

Shadiq, F., & Lukman. (2010). *Mathematics curriculum contents analysis: country report Lao PDR*. https://fadjarp3g.files.wordpress.com/2010/12/10-country-report-lao.pdf

Shanmugam, S. K. S., Chinnappan, M., & Leong, K. (2020). Action research in examining the enquiry approach of lesson study in mathematics. *Pertanika J. Soc. Sci. & Hum. 28*(3), 1675 - 1693. http://www.pertanika.upm.edu.my/resources/files/Pertanika%20PAPERS/JSSH%20Vol.%2028%20(3)%20Sep.%202020/05%20JSSH-4813-2019.pdf

Sierpinska, A. (1998). Three epistemologies, three views of classroom communication: constructivism, sociocultural approaches, interactionism. In H. Steinbring, A. Sierpinska, & M. G. Bartolini-Bussi (Eds.), *Language and Communication in the Mathematics Classroom*. (pp. 30-62). Reston, VA.: NCTM.

Smieskova, E. (2017). Communication students' skills as a tool of development creativity and motivation in geometry. *Universal Journal of Educational Research, 5*(1), 31–35. https://doi.org/10.13189/ujer.2017.050104

Stigler, J., & Hiebert, J. (1999). *The teaching gap: best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Takahashi, A. (2006). Characteristics of Japanese mathematics lesson. *Tsukuba Journal of Educational Study in Mathematics, 25*(special issue), 37-44. https://www.criced.tsukuba.ac.jp/math/sympo_2006/takahashi.pdf

Thinwiangthong, S., Inprasitha, M., & Loipha, S. (2012). Adaptation of lesson study and open approach for sustainable development of students' mathematical learning process. *Psychology, 3*(10), 906-911. https://doi.org/10.4236/psych.2012.310136

TIMSS & PIRLS International Study Center. (2019). *TIMSS 2019 international results in mathematics and science*. MA: Boston College.

Uyen, B. P., Tong, D. H., & Tram, N. T. B. (2021). Developing mathematical communication skills for students in Grade 8 in teaching congruent triangle topics. *European Journal of Educational Research, 10*(3), 1287-1302. https://doi.org/10.12973/eu-jer.10.3.1287

Wiriyaudomsatean, T., & Thinwiangthong, S. (2019). Students' words in mathematical communication in classroom using lesson study and open approach. *Journal of Physics: Conf. Series, 1340*, 012090. https://doi.org/10.1088/1742-6596/1340/1/012090

Wood, T. (1998). Alternative patterns of communication in mathematics classroom: funneling or focusing?. In H. Steinbring, A. Sierpinska, & M. G. Bartolini-Bussi (Eds.), *Language and Communication in the Mathematics Classroom*. (pp. 167-178). Reston, VA.: NCTM.