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The didactic phenomenon: Deciphering students' learning obstacles in set theory

Agus Hendriyanto[1,2,*], Didi Suryadi[1,2], Dadang Juandi[1], Jarnawi Afgani Dahlan[1], Riyan Hidayat[3], Yousef Wardat[4], Sani Sahara[1], Lukman Hakim Muhaimin[1]

[1]Department of Mathematics Education, Universitas Pendidikan Indonesia, Bandung, Indonesia
[2]Indonesian Didactical Design Research Development Center (PUSBANGDDRINDO) - PUI-PT, Universitas Pendidikan Indonesia, Bandung, Indonesia
[3]Department of Science and Technical Education, Universiti Putra Malaysia, Selangor, Malaysia
[4]Department of Curriculum and Instruction, Higher Colleges of Technology, Alain, United Emirates Arab
*Correspondence: agushendriyanto@upi.edu

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Abstract

Teachers play a crucial role in disseminating knowledge in educational settings, typically adhering to a credulist-testimonial approach outlined in pedagogical literature. Consequently, students often acquire knowledge through this method, potentially leading to discrepancies between their conceptual understanding and the intended educational objectives. This study investigates the phenomenon of learning obstacles encountered by junior high school students, with a particular emphasis on mathematics education. It is part of a series of Didactical Design Research (DDR) projects aimed at developing effective instructional materials. Employing an interpretive paradigm within the DDR framework, the study adopts a qualitative approach utilizing hermeneutic phenomenology design. Various research tools such as diagnostic assessments, interview guidelines, observation sheets, and audio recordings are employed. Data analysis is conducted using the Constant Comparative Method (CCM). The findings highlight ontogenic, didactic, and epistemological obstacles students face, stemming from factors such as a lack of interest in mathematics, ineffective material presentation, and misconceptions regarding set concepts. These results underscore the importance of educators employing effective teaching strategies to help students overcome these obstacles and succeed in their mathematics lessons.

Keywords: DDR, Didactic Phenomena, Hermeneutic Phenomenology, Learning Obstacle, Set Theory

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Didactical transposition, a prominent concept within the realm of education, refers to the systematic process by which knowledge is adapted from its original form into a format more conducive to educational instruction and comprehension (Banks et al., 2005; Diana et al., 2020; Schneuwly, 2021). In the field of mathematics education, didactic transposition entails the reconstruction of knowledge to render it teachable, meaningful, and applicable (Banks et al., 2005). This theory underscores the challenges inherent in legitimizing teaching content and the disparities between instructed learning and its referential origins (Diana et al., 2020). Such considerations are deeply intertwined with the fundamental aspect of educational practice—the transmission of knowledge across generations (Schneuwly, 2021). Essentially, it involves adapting knowledge to render it more accessible, comprehensible, and utilitarian within a specific educational milieu. Chevallard (1989) posits that didactical transposition entails intricate operations that transform epistemological and cognitive knowledge structures into didactic frameworks suitable for classroom use. These operations may encompass simplification, abstraction, generalization, and creating novel concepts and representations. The overarching objective of didactical transposition is to render knowledge more accessible to students while preserving its core attributes and ensuring its scientific accuracy and validity (Brousseau & Warfield, 2020). Consequently, didactical transposition in education entails a nuanced process of knowledge transformation to enhance its accessibility and applicability for teaching and learning. It involves diverse operations such as simplification, abstraction, and generalization.

In brief, didactical transposition involves translating knowledge from a source domain, like a scientific field, to a target domain, such as a classroom, intending to make it accessible and understandable for learners. However, if this adaptation of knowledge is inappropriate, it can lead to various negative impacts on learning, including learning obstacles (Wilhelmi et al., 2021). The concept of didactical transposition is closely intertwined with the issue of learning obstacles in mathematics education, which refer to the difficulties learners encounter when grasping mathematical concepts, procedures, or structures (Mutambara & Tsakeni, 2022). These obstacles can stem from multiple sources, including the complexity of mathematical content, the cognitive demands of tasks, the language used to convey mathematical ideas or the teaching methodologies employed in the classroom (Duval, 2006). Specifically, Suryadi (2019a) classified learning obstacles into three types based on their root causes: ontogenic, didactical, and epistemological obstacles.

Didactical transposition serves as a means to surmount certain learning obstacles by adjusting mathematical knowledge to enhance its accessibility and comprehensibility for learners (Prabowo et al., 2022). For instance, within didactical transposition, educators may simplify the language used to convey mathematical concepts, furnish concrete examples or visual aids to elucidate abstract ideas or employ diverse teaching strategies to foster comprehension. However, if the adaptation of mathematical knowledge during the process of didactical transposition is inappropriate, it can introduce new learning obstacles or exacerbate existing ones (Brousseau, 2002). For instance, if teachers overly simplify mathematical content, students may fail to develop a profound understanding of the subject matter and encounter difficulties in applying their knowledge in novel contexts. Similarly, if instructors excessively rely on visual aids without allowing students to cultivate their mental representations of mathematical concepts, learners might excessively depend on these aids and confront challenges in independently solving problems (Goldin & Shteingold, 2001).

In essence, the correlation between didactical transposition and learning obstacles in mathematics education is intricate and contingent upon the quality of adaptation of mathematical knowledge. When executed effectively, didactical transposition can aid learners in surmounting certain learning obstacles by rendering mathematical knowledge more accessible and comprehensible (Amade-Escot, 2006). Conversely, inadequate execution of didactical transposition can lead to the emergence of new learning obstacles or exacerbate existing ones (Brousseau, 2002). Suryadi (2019a) contends that overcoming learning obstacles in mathematics necessitates a profound understanding of learners' prior knowledge and their cognitive processes regarding mathematical concepts (Yuen et al., 2003). He suggests that educators should proficiently identify the specific learning obstacles confronted by their students and employ suitable teaching strategies to address them. These strategies may involve presenting diverse representations of mathematical concepts, scaffolding learners' problem-solving efforts, or encouraging them to engage in reflective exercises concerning their cognitive processes.

Numerous studies have shed light on the various learning obstacles encountered by students in the realm of mathematics. Akar and Işıksal-Bostan (2022) uncovered substantial disparities between the knowledge intended to be taught and the content actually delivered by educators, thereby giving rise to learning obstacles. Pratiwi et al. (2019) delineated ontogenic, epistemological, and didactic hurdles within algebraic thinking, while Andini and Suryadi (2017) and Noto et al. (2020) respectively delved into specific barriers related to solving algebraic thinking problems and grasping algebraic forms. Collectively, these studies underscore the importance of understanding and addressing the diverse learning obstacles students encounter in mathematics. In addition to variances in the topics explored, the novelty of this research compared to previous endeavors lies in its foundational approach to analyzing learning impediments. Here, researchers aimed to pinpoint the types of learning obstacles students face in the domain of sets. This endeavor stems from prior findings indicating challenges in the process of translating set knowledge from scholarly domains to pedagogically applicable knowledge, alongside the prevalent uncritical stance of educators in crafting didactic designs. However, challenges arose in scrutinizing the sources utilized by teachers. Consequently, the timely identification of learning obstacles experienced by students is imperative, as it can lead to enhanced learning outcomes, mitigation of misconceptions, tailored support for individual students, and refinement of teacher practices.

Nevertheless, prior research has largely overlooked the specific learning obstacles junior high school students encounter, particularly within the context of design actualization, thereby posing challenges to their academic progress. This underscores a notable gap in the existing literature, as a thorough examination of the learning barriers experienced by students, employing a didactic approach and comprehensive data analysis, could yield valuable insights and deepen our understanding of the subject. Bridging this gap has the potential to refine and advance the design actualization process, consequently impacting the hurdles faced by students in their learning journey. Ultimately, the implications of such research suggest that it could enhance educators' understanding of the learning processes and challenges encountered by junior high school students, facilitating the development of tailored interventions that foster more effective and engaging learning environments.

Our focus on sets stems from several compelling reasons. Firstly, set theory serves as the bedrock of mathematics (Lipschutz, 1986), with all mathematical entities and structures ultimately grounded in set theory (Maddy, 2003). Moreover, set theory finds extensive application across various branches of mathematics, such as algebra, topology, and analysis. Secondly, a pressing need exists for continued examination of sets of material. Linchevski and Vinner (1988) documented teachers' misconceptions regarding sets contextualization, while Fischbein and Baltsan (1999) observed a reluctance among both students and teachers to accept the notion of the empty set due to their perception of sets solely as collections. Recent research by Bingolbali et al. (2021) further reinforced that sets are consistently interpreted as collections containing elements with common properties, thereby discrediting sets with elements lacking a common property as non-sets. Thirdly, several previous studies have highlighted issues with presenting sets of material in textbooks, including deficiencies in content delivery (Muilenburg & Berge, 2005) and knowledge acquisition. For instance, Jamilah et al. (2020) revealed a disparity between students' conceptual images (CI) of sets and the formal concept definition found in scholarly literature, even after accounting for taught knowledge and knowledge intended for instruction. This gap in CI is attributed to ontogenic, didactical, and epistemological obstacles arising from inaccuracies in the material presentation and the didactic scenarios provided.

Navigating the learning process can prove challenging as students encounter various obstacles in pursuing educational objectives. These learning obstacles impede students' progression toward attaining desired learning outcomes (Muilenburg & Berge, 2005). Over time, numerous theories have emerged to elucidate the phenomenon of learning obstacles. One prominent theory is proposed by Brousseau (2002), which delves into how students grapple with obstacles within didactical contexts. Brousseau's theory, also called the didactical situation theory, posits that learning obstacles emerge from the interplay between the learner and the didactical situation (Brousseau, 2002; Brousseau & Warfield, 2020). According to Brousseau (2002), the didactical situation encompasses the teacher, the student, the object of knowledge, and the task or learning scenario, with learning obstacles manifesting when these components fail to interact effectively.

Guy Brousseau's theory of didactical situations offers a framework for understanding the intricate dynamics among educators, learners, and mathematical concepts within educational settings. This theory emphasizes the importance of creating specific instructional environments that empower students to independently engage with mathematical problems, thereby facilitating the construction of new knowledge. It also highlights the conditions necessary for students to participate in activities conducive to developing specific meanings and insights into mathematical concepts (Laborde, 2014; Šipuš et al., 2022). Furthermore, the theory of didactical situations serves as a guiding principle for designing teaching tasks and interventions in mathematics education. It sheds light on how prospective teachers conceptualize task designs, particularly concerning different types of situations, the didactical contract's complexities, and the didactic situation's components (Daher et al., 2022; Vergnaud, 2009).

Moreover, Guy Brousseau's theory of didactical situations holds practical implications, as it has been effectively utilized in devising teaching sequences for primary school settings, showcasing its relevance in educational contexts (Rønning, 2021). Furthermore, it has proven invaluable in exploring the process of knowledge consolidation within mathematics education, offering valuable insights for structuring instructional interventions and dissecting the complexities of knowledge (Pinheiro et al., 2022). The theory has also found application in the didactic engineering of teaching sequences, emphasizing its role in enhancing the learning environment and facilitating the teaching process (Alves et al., 2021; De Sousa & Alves, 2022).

Didi Suryadi, a prominent Indonesian mathematics educator, has applied Guy Brousseau's theory of didactical situations to elucidate students' learning obstacles in mathematics. Suryadi (2019b) contends that learning obstacles in mathematics emerge due to a discrepancy between learners' existing mathematical knowledge and the new knowledge they are expected to acquire. In his research, Suryadi (2019a) identified various learning obstacles experienced by students in mathematics, including ontogenic, didactical, and epistemological obstacles.

Ontogenic obstacles pertain to students' mental preparedness and cognitive maturity in assimilating knowledge. This learning obstacle arises from a disparity between the difficulty level or cognitive demands of didactical situations and students' readiness. Suryadi (2019a) identified three categories of ontogenic obstacles: psychological, instrumental, and conceptual. Psychological ontogenic obstacles involve students' lack of readiness regarding motivation and interest in the subject matter under study. Instrumental ontogenic obstacles involve students' lack of preparation for crucial technical aspects of the learning process, as evidenced by their responses and errors during task completion. Conceptual ontogenic obstacles stem from students' unpreparedness due to previous learning experiences, such as failure to grasp fundamental ideas and prerequisites for supporting material.

Didactical obstacles arise from the didactic system, encompassing factors such as sequencing and curriculum stages, including their presentation in classroom instruction (Fauzi & Suryadi, 2020).

Brousseau (1976) asserts that didactical obstacles must be assessed based on the order of material, both structurally (the interrelationships between concepts) and functionally (the continuity of thought processes), as well as the level of detail in the presentation of the material, whether it is insufficient or excessively detailed. On the other hand, epistemological obstacles arise from students' inadequate understanding and mastery of a concept, problem, or other subject matter that is only relevant in specific situations. This learning obstacle becomes apparent when students can address topics using examples and formats provided by the teacher or textbook but require assistance in responding to questions presented in alternative formats or contexts. Suryadi (2019a) argues that overcoming obstacles in learning mathematics necessitates adopting didactical situations conducive to learning. According to Suryadi (2019b), a conducive didactical situation should involve a teacher who comprehends learners' existing knowledge and can bridge the gap between that knowledge and the new knowledge.

Moreover, the teacher should employ appropriate language and explain clearly to ensure learners grasp the concepts. Suryadi (2019a) also suggests that a conducive didactical situation should incorporate tasks or learning scenarios that are challenging yet manageable. Tasks should be designed to assist learners in building upon their current knowledge while gradually acquiring new knowledge. Additionally, tasks should be contextualized to help learners perceive the relevance of new knowledge to their everyday lives.

In summary, learning obstacles represent a significant hurdle for students in the realm of mathematics. Brousseau's theory of didactic situations provides a valuable framework for understanding the nature of these obstacles and how they arise from the interaction between the learner and the didactic environment. Didi Suryadi's integration of this theory into his research on mathematics learning obstacles has yielded insightful perspectives into the various types of obstacles students face and strategies for overcoming them. By implementing didactical situations conducive to learning, teachers can aid students in overcoming these obstacles and achieving their desired learning outcomes.

METHODS

Research Design

This research is part of the Didactical Design Research (DDR) framework developed by Suryadi (2019a), which implements an interpretive paradigm. Implementing this paradigm allows researchers to construct essential elements as a reference to justify the initial construction of a design (Scotland, 2012). Qualitative research is the type employed in this study, with the design utilizing hermeneutic phenomenology. The application of hermeneutic phenomenology as a research method is warranted for investigating the learning barriers junior high school students face due to its inherent appropriateness for delving into individuals' lived experiences and subjective viewpoints. Hermeneutic phenomenology excels in exploring the intricate complexities of personal experiences, enabling researchers to unveil the underlying meanings and interpretations associated with phenomena like learning obstacles. Through this methodology, researchers can undertake a reflective and interpretative approach that acknowledges the distinct perspectives of participants, thereby offering comprehensive and contextually relevant insights into the challenges encountered within the junior high school learning milieu.

Before embarking on identifying learning obstacles among students, researchers underwent a crucial preliminary stage involving the analysis of the didactic design employed in the learning process. This didactic design encompasses a myriad of materials curated independently by teachers or provided by the government in the form of curriculum, often manifested in textbooks, serving as primary instructional resources within the classroom. This analysis was meticulously conducted, encompassing several vital aspects. Firstly, researchers assessed the appropriateness of the concepts imparted to students with universally accepted scientific knowledge. This necessitated comparing what is presented in the didactic design and the established scientific understanding.

Additionally, researchers analyzed the array of task types based on insights gleaned from praxeology studies. This facilitated an understanding of how learning is structured and presented to students from the perspective of actions taken and techniques employed. Moreover, the research scrutinized whether the task types present in the didactic design were epistemically presented or not. This entails whether the material presented encourages students to comprehend the concepts and contemplate their epistemological underpinnings deeply. This aspect is pivotal to ensure that learning not only revolves around teaching mathematical concepts but also fosters a robust and profound understanding of the scientific principles behind them. After thoroughly analyzing the didactic design, researchers investigated students' learning obstacles.

Participants

The study participants were meticulously selected to ensure a representative sample capable of providing insightful data regarding the phenomenon under investigation. They consisted of students enrolled in two prestigious schools situated in Bandung and Surakarta, respectively. These students were chosen explicitly because they studied sets of material at the junior high school level, which constituted the focal point of the research inquiry. An integral aspect of the participant selection process was ensuring uniformity in the student's primary learning resources. The mathematics textbooks published by the Ministry of Education and Culture of the Republic of Indonesia were utilized in both schools. This deliberate decision aimed to ensure consistency in the didactical transposition phenomenon across the selected schools, thereby facilitating a more accurate comparison and analysis.

The study comprised a total of 183 participants. This larger cohort selected a subset of 24 students for further examination. The selection of these 24 students employed the snowball sampling technique, commonly used in qualitative research, to identify and recruit participants through referrals from initial subjects. This approach enabled the researcher to capture diverse perspectives and ensure that the sample encompassed a broad spectrum of experiences and viewpoints relevant to the research topic (Naderifar et al., 2017). Overall, the characteristics of the participants, including their educational background, level of study, and the consistency of learning resources, were meticulously considered to ensure the validity and reliability of the study findings.

Instrument and Data Collection Technique

The primary instrument utilized in the study was the researcher, who served as the planner and data collector, interpreting the data, analyzing, concluding, and reporting the results. Various tools were employed, including diagnostic assessments of sets of materials, interview guidelines, observation sheets, and audio recording devices. These instruments were developed in accordance with the research objectives and problem construction (Yorulmaz et al., 2021), ensuring language suitability. Prior to data collection, the instruments underwent validation by experts in cognitive mathematical problems (Tutticci et al., 2017). Their feedback included suggestions for improvement, such as enhancing the clarity of mathematical symbol notation, refining problem-solving guidelines, aligning language with research objectives, and streamlining the structure of interview questions for greater efficiency and effectiveness.

In developing instruments for this research, the researchers incorporated insights gleaned from previous studies. Experts and practitioners across various fields, including mathematics teachers, verified the knowledge derived from these studies. This verification process was facilitated through focus group discussions (FGDs), during which these individuals engaged in focused conversations regarding the methodology and research instruments to be employed. Throughout the FGD process, experts and practitioners offered invaluable input and feedback concerning the design and content of the instruments. These discussions were instrumental in ensuring that the developed instruments possess high relevance and accuracy in measuring the investigated variables. Consequently, the instruments are deemed appropriate and reliable for use in this research.

The diagnostic assessment instrument comprised four sub-tasks focusing on understanding the meaning of sets and non-sets, element of set notation, finite and infinite sets, and comparing two sets. The assessment was carried out concurrently with close supervision at each school, involving teachers and other competent individuals to ensure the credibility of the obtained data. Simultaneously, researchers conducted field observations and meticulously documented all observations. The results of students' responses were analyzed and utilized to identify interview subjects. The interview process was conducted sequentially, one participant at a time, using an audio recorder application.

Data Analysis

The researchers employed the constant comparative method (CCM) as a data analysis model, which Glaser and Strauss developed in the 1960s (Glaser & Anselm, 2017). CCM is a qualitative research technique involving comparing data from various sources to discern patterns and themes (Grandgirard et al., 2003). It comprises four fundamental steps: (1) Coding, wherein the researcher reads the data and assigns codes to different segments based on their content; (2) Comparison, where the researcher contrasts codes across different sections of the data to identify similarities and differences; (3) Conceptualization, wherein the researcher begins to group codes into categories and formulate overarching concepts that elucidate the data; and (4) Theoretical saturation, wherein the researcher continues to gather and analyze data until no new codes, categories, or concepts emerge.

Researchers consistently compare and refine their codes, categories, and concepts throughout the CCM process when considering the data. This iterative approach allows researchers to develop a progressively nuanced and sophisticated understanding of the data over time. By continuously comparing and revising their interpretations, researchers generate comprehensive and detailed explanations of the didactical phenomena under investigation.

Validity of Data

Data validation is a critical process in qualitative research aimed at upholding the accuracy and reliability of collected data (Creswell & Guetterman, 2018). This study's data validation methods include triangulation and peer debriefing. Triangulation involves utilizing multiple data sources to corroborate findings, while peer debriefing entails involving other researchers—more than one—to review and offer feedback on the research process and outcomes. This approach aids in identifying potential biases and ensuring the precision of the analysis.

In this research context, data triangulation is paramount to ensure the validity and reliability of findings. It involves comparing information obtained from various data collection techniques and sources. Meanwhile, peer debriefing is conducted through focus group discussion (FGD) activities, where researchers or other experts in relevant fields gather to discuss findings and data interpretations. These discussions in FGDs help test data interpretations, evaluate research methodologies, and provide alternative perspectives on the findings generated.

RESULTS AND DISCUSSION

This research involves a rigorous process comprising various critical steps to ensure the study's quality, particularly regarding data validation. In this research, four categories are employed to test the validity of data and findings, collectively known as trustworthiness: truth value (credibility), applicability (applicability or transferability), consistency (consistency or dependability), and neutrality (neutrality or confirmability). The strategies implemented to ensure credibility in this research include examination by the researcher's team and debriefing with various stakeholders in focus group discussions (FGDs), as shown in Figure 1. FGD activities are also documented on the following page (https://berita.upi.edu/hadirkan-praktisi-hingga-pakar-matematika-mahasiswa-doktoral-pendidikan-matematika-upi-gelar-forum-group-discussion-fgd/).

Figure 1. Documentation of FGD Activities

Transferability can be further enhanced by employing the same data collection methods with different demographic groups or geographical locations. In this instance, the researcher utilized data collection methods in two distinct schools, as illustrated in Figure 2, which documents the approval from the schools involved. This approach allows for the comparison of findings across different educational settings, thereby contributing to the transferability of the study's results.

PEMERINTAH KOTA SURAKARTA
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Nama : AGUS HENDRIYANTO
NIM : 2113215
Prodi : Pendidikan Matematika
Fakultas : Pendidikan Matematika dan Ilmu Pengetahuan Alam
Instansi : Universitas Pendidikan Indonesia

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Demikian surat keterangan ini dibuat, semoga dapat dipergunakan sebagaimana mestinya.

Surakarta, 05 April 2023
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Nama :
NIP :
Jabatan : Kepala

Dengan ini kami menerangkan bahwa :
Nama : Agus Hendriyanto
NPM : 2113215
Program Studi : Matematika
Fakultas : MIPA

Telah melaksanakan Penelitian di SMP Negeri Bandung, dalam rangka memenuhi salah satu tugas mata kuliah, dengan judul :"***Desain Didaktis Materi Himpunan Berbasis Teori Realistics Mathematics Education Bagi Calon Guru Profesional Pendidikan Matematika Melalui Tranposisi Dikdatis***"
Pelaksanaan Penelitian taggal 17 Maret 2023

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Bandung, 30 Maret 2026
Kepala
Pembina Tk.I
NIP. 02

(b)

Figure 2. Certificate of Implementation of Research at (a) Surakarta School and (b) Bandung School

Furthermore, to ensure dependability, the strategy involves engaging all researchers in the analysis process, where each research team member contributes input regarding data analysis, providing additional perspectives. Meanwhile, to achieve confirmability, the researcher adopts a technique involving recording personal feelings, biases, and insights immediately after conducting interviews. Several documentation of the interview activities conducted by the researcher can be observed in Figure 3.

Figure 3. Documentation of Interview Activities

The findings and discussions are organized into four subchapters: learning obstacles in understanding the meaning of sets and non-sets, learning obstacles in understanding set element notation, learning obstacles in understanding finite and infinite sets, and learning obstacles in understanding the similarity of two sets. Each subchapter delves into various subjects tailored to the type of learning obstacle encountered.

Learning Obstacles in Understanding the Meaning of Sets and Not Sets

In this section, we asked five different questions, and subjects were asked to infer and provide their arguments regarding whether the objects presented were sets. The first object is "*The capital city of Indonesia.*" This task has the motivation to identify students' understanding of the meaning of sets. The second and third objects. "$\{2,3,5,7,10\}$ and $\{1,2,3,a,b,c\}$" have the motivation to see how students understand the characteristics of the elements in a set. The fourth and fifth objects. "$\{x|x\ the\ letter\ before\ the\ letter\ a\ in\ the\ Latin\ alphabet\ sequence\}$ and $\{\ \ \}$" are intended to identify students' understanding of the empty set.

The sentence about the capital city of Indonesia that we presented is not a set. It is just a statement, not a property imposed on the object. Two opinions emerged from the students' answers; some said it was a set, and others did not. Arguments against students' views are presented in Table 1.

Table 1. Summary of Students' Answers to the First Object

Argument	Code
Indonesia's capital city is definable and certain. [1]	α_1
Indonesia's capital city can be distinguished from other cities. [1]	α_2
Its elements are Jakarta. [1]	α_3
The name of the city is not mentioned. [2]	α_4
Indonesia's capital city is only one, Jakarta. So, the capital city of Indonesia is not a set because the set consists of several element. [2]	β_1
A set is a unit of several elements, whereas the capital city of Indonesia does not consist of several element (only Jakarta). [2]	β_2

The capital city of Indonesia is not a collection of several cities/contents but only one.	β_3
Indonesia's capital city is only one. It cannot be called a group or set.[2]	β_4
The set is denoted by a capital letter, while its elements are enclosed in curly braces. $\{\quad\}^2$	γ_1

[1]Set

[2]Not a set

Next, about $\{2,3,5,7,10\}$ and $\{1,2,3,a,b,c\}$, both are examples of sets, but some students think they are not sets. Students who claimed both assets also used the same argument as the previous case. Students' opinions on this case are shown in Table 2.

Table 2. Summary of Students' Answers to the Second Object

Argument	Code
$\{2,3,5,7,10\}$ is a set because it is in a sign that is like parentheses indicating they are one unit.[1]	γ_2
$\{1,2,3,a,b,c\}$ is a set. Although there are numbers and letters, they are in the $\{\quad\}$ The sign indicates they are one unit.[1]	γ_3
$\{2,3,5,7,10\}$ is a set because there are curly braces that also have numbers in them that are similar.[1]	γ_4, δ_5
$\{2,3,5,7,10\}$ is a set because its elements can be classified into various sets, for example, such assets that cannot be divided by 4.[1]	ε_1
$\{2,3,5,7,10\}$ set because there is more than one number in a set.[1]	β_5
$\{1,2,3,a,b,c\}$ set because there are some numbers and letters in a set.[1]	β_6
$\{2,3,5,7,10\}$ is not a set because it is not known what the numbers are.[2]	δ_1
$\{1,2,3,a,b,c\}$ is not a set because if you want to include other numbers/alphabets, it is better to create a new set, such as $\{1,2,3\},\{a,b,c\}$.[2]	δ_2
$\{2,3,5,7,10\}$ is not a set because the numbers in the example are random (not prime, even, or odd).[2]	δ_3
$\{1,2,3,a,b,c\}$ is not a set because there are numbers and variables (different types).[2]	δ_4
$\{1,2,3,a,b,c\}$ Although there are curly braces and numbers, it is not a set, but they are not similar. By not similar, we mean there are numbers, and there are also letters.[2]	γ_5, δ_6
$\{1,2,3,a,b,c\}$ is not a set because its elements cannot be known by the origin/name of the set.[2]	ε_2
$\{2,3,5,7,10\}$ It is not a set because the numbers are randomly arranged without a clear reason for grouping them.[2]	δ_7, ε_3
$\{1,2,3,a,b,c\}$ not a set because the group's content does not refer to one specific thing and consists of two different types of content: numbers and letters.[2]	δ_8, ε_4
$\{2,3,5,7,10\}$ is not a set because 2 and 10 are even numbers, and 3,5,7 are odd numbers. The numbers cannot belong to one set.[2]	δ_9
$\{1,2,3,a,b,c\}$ is not a set because 1,2,3 is a set of numbers, and a,b,c is a set of letters. The numbers and letters cannot belong to one set.[2]	δ_{10}

[1]Set

[2]Not a set

Based on Table 1 and Table 2, it can be identified that students' understanding of the set varies, namely α (set only as something with a well-defined element), β (set as a collection), and γ (set must be denoted by a capital letter and expressed in curly braces). At the same time, students understanding of the elements in a set is ε (elements in a set must be subject to a name or classification) and δ (each element in a set must have the same type or characteristic). Lastly, about $\{x|x\ the\ letter\ before\ the\ letter\ a\ in\ the\ Latin\ alphabet\ sequence\}$ and $\{\quad\}$, these are examples of empty sets. Many of the students did not accept this as a set. The reason is all the same: no element in it can be identified. This is a result of students' understanding of sets as β.

Before discussing the type of learning obstacle, it is necessary to emphasize that the set is an undefined part of mathematics, where the set can be understood with the set formation notation. $\{x \in S | P(x)\}$ (Holmes, 1998). Alternatively, a set is formally defined as a collection of objects within a well-defined group. The significance of "well-defined" in this explanation should be emphasized in terms of its intuitive meaning. Every object is subject to properties governed by a "nature" rule, which can be perceived or understood through individual perception or intellectual abilities (Just & Weese, 1996). Students' understanding of sets solely as α, β, or even γ arises from their construction based on perceptual experiences and teacher guidance. Consequently, the knowledge formed represents mere belief without justification. This outcome suggests that students encounter numerous learning obstacles.

It was observed that ontogenic obstacles, encompassing psychological and conceptual barriers, were prevalent in this case. Interview results revealed that many students, including set material, needed more interest in mathematics. The following excerpts from student responses during the interview can serve as evidence:

> *I don't really like material on sets like it needs to be more critical. The material is like elementary school material, for example, mentioning groups of animals, groups of vegetables, etc.*
> *(Interview 1)*
> *Material on sets doesn't relate to other materials, so learning about sets is useless.*
> *(Interview 2)*

Both interview excerpts explicitly indicate that students encounter ontogenic obstacles of a psychological nature. A diminished interest in mathematics can precipitate learning obstacles, reducing motivation and engagement (Skilling et al., 2021). When students are not interested in math, they may pay less attention in class, fail to complete their homework or refrain from asking questions when they need to help understanding something. Consequently, students need to be more engaged in the subject to grasp new concepts and understand the material. This can create a vicious cycle where low interest leads to poor performance and further disengagement.

Additionally, students' low interest in mathematics can foster negative attitudes and beliefs about their abilities in the subject (Gunderson et al., 2012). They may develop a fixed mindset, believing they are "*not good at math*," which can further undermine their motivation and performance. Overall, low interest in mathematics can present a significant obstacle to learning and may lead to various learning obstacles that necessitate effective intervention. Lanigan (2021) highlighted sporadic and reactive engagement among adult learners in mathematics, suggesting that specific situational triggers may impede learning. Sullivan et al. (2006) proposed that a deliberate decision not to engage in mathematics learning may be influenced by classroom culture. Furthermore, Brown (2008) explored epistemological obstacles to the development of mathematical induction, suggesting that students' approaches to mathematical tasks may be influenced by their understanding of the subject.

On the contrary, some students fear learning math in class but not when studying at home with the guidance of a tutor. "*I always sit in the back row because I fear being told to come to the front of the class to do the problems. So, I do not understand some of the material taught by the teacher*," voiced one of the subjects. Furthermore, he elaborated, "*In a week, I take private lessons twice, specifically for math* *subjects. During these lessons, I do my assignments with the tutor.*" This suggests that math can intimidate some students, causing fear and anxiety, impacting their ability to concentrate and comprehend mathematical concepts, and ultimately leading to ontogenic (psychological) learning obstacles. Research indicates that students can experience anxiety and fear when learning math in a classroom setting (Wang et al., 2014). This anxiety may be influenced by genetic factors, such as a predisposition to anxiety and mathematical cognition (Wang, 2014). Classroom culture and the perception of effort influencing achievement can also affect students' engagement with mathematics (Sullivan et al., 2006). Moreover, the presence of math anxiety in student teachers has been observed, with potential implications for their own learning and teaching (Jackson, 2008).

The indication that students experience ontogenic obstacles that are conceptual in nature, in this case, stems from a statement made by one of the subjects, who mentioned, "*I had understood the word set as a collection long before I learned set material, as I know about the set of students*." This student's statement confirms his belief that the set is adequately understood as β. This underscores the notion that students' understanding of sets is not solely derived from their mathematical studies but also from experiences outside of mathematics. Recognizing that students' knowledge of mathematical materials can be shaped by their experiences beyond mathematics is crucial because mathematical concepts and principles extend beyond the confines of the classroom or textbooks (Simamora et al., 2018). Students are continuously exposed to mathematical ideas and thinking in their daily lives, often without realizing it (Furner et al., 2005; Laurens et al., 2017; Marasabessy, 2021). By acknowledging and linking everyday experiences with mathematical concepts, students can deepen their understanding of mathematical material and cultivate a more meaningful and intuitive grasp of the subject. However, evidence suggests that prior knowledge outside of mathematics can also pose learning obstacles. This is because such knowledge may not be fully aligned with the fundamental concepts (Loewenberg Ball et al., 2008).

Furthermore, it is also evident that a conceptual ontogenic obstacle is associated with the conceptual level embedded in the learning design. Students expressed during interviews that they perceived their learning orientation, derived from their conceptual perceptions of α, β, and ε, to be excessively high. Consequently, they encountered obstacles in solving empty-set problems. The cognitive demands that are too high cause students to lose their learning orientation and result in their inability to justify their perceptual understanding related to set knowledge cognitively. Conversely, based on their previous learning experiences, other students feel that their understanding of β is clear, but this leads to the incomplete characteristics of the knowledge they build. Their perception of β indirectly disregards other understandings, as seen in the case of the empty set. The absence of elements in the empty set leads students to assert that it is not a set. However, according to the Schema of Separation, it should be clarified that the empty set is indeed a collection. This condition is referred to by Suryadi (2019a) as a result of conceptual ontogenic obstacles where conceptual challenges are too low, resulting in underachieving students.

The construction of students' knowledge about $\alpha, \beta, \gamma, \delta,$ and ε is a result of the actualization of material in textbooks that are arranged unsystematically. Didactical transposition theory shows that, in general, the knowledge imparted to students by teachers (knowledge imparted) refers to what is called knowledge to be taught (in this case, textbooks) (Chevallard & Bosch, 2020). So the concept of knowledge about the set taught by the teacher is not far from what is in the textbook, including the sequence of material structurally and functionally (de Mello, 2017). When discussing sets, it is crucial to address the concept of the universe of discourse at the outset before delving into the meaning of sets. The universe of discourse, in the context of explaining set material to students, refers to the entire domain or context within which the set is being discussed. The relationship between the meaning of a set and the universe of discourse lies in the fact that the universe of discourse provides the contextual framework within which the set is defined and applied. It assists in determining which objects or elements are pertinent for inclusion in a specific set. The haphazard implementation of set material is the root cause of students encountering didactical obstacles in all instances ($\alpha, \beta, \gamma, \delta,$ and ε).

Mathematics builds on concepts and skills learned in previous grades (Miller & Hudson, 2007). If students have the necessary foundation, they may be able to grasp new concepts. Not only about the unsystematic presentation of material, but the interventions also provided examples of sets and not sets, leading to δ understanding. This is in line with the findings of Bingolbali et al. (2021); students and teachers consider that elements of a set must have the same characteristics, so $\{1,2,3, a, b, c\}$ is considered as not a set. "*There are no examples in the book or examples given by the teacher like this - pointing to the writing* $\{1,2,3, a, b, c\}$" said one subject, and this is in line with the expression of another subject who stated, "*All the examples given have similar elements, all numbers, all animals, all vegetables, or all letters*". This illustrates that students exhibit an over-reliance on examples. Students who overly depend on examples may struggle to apply their knowledge to novel situations or problems. Therefore, in this scenario, students encountered an epistemological obstacle. This finding aligns with research by Sierpińska (1987) and Modestou and Gagatsis (2007), who observed that students' dependence on examples can impede their capacity to transfer knowledge to unfamiliar situations.

Learning Obstacles in Understanding Set Element Notation

In junior high school mathematics curriculum, students engage with the fundamentals of sets and their constituent elements, which constitute a cornerstone of algebraic and geometric principles (Kunen, 1980). Proficiency in comprehending sets and their elements is paramount as it forms the groundwork for grappling with more intricate mathematical concepts such as functions and relations. Notably, mathematical notation plays a pivotal role in facilitating communication within the realm of mathematics, serving as a universal language employed by mathematicians, scientists, and engineers globally to articulate intricate ideas and concepts (Sterenberg, 2008). Consequently, students must grasp and master mathematical notation. However, empirical evidence suggests that students frequently encounter challenges in comprehending mathematical notation due to its inherent complexity, characterized by a plethora of symbols, syntax, and rules (Bardini et al., 2004; Sterenberg, 2008). Nevertheless, adeptness in deciphering mathematical notation empowers students to effectively and precisely communicate mathematical ideas through both written and oral mediums.

The notations of element ($\in$), non-element ($\notin$), and the use of curly braces ($\{ \quad \}$) In sets are important elements that students must understand before learning sets material. In this case, two problems were posed to see if students experienced any learning obstacles in understanding the notation in question. Here are the tasks used to address this issue.

(φ) Suppose $A = \{x|2x = 6, x \in integers\}$ and suppose $B = 3$, does $A = B$?

(ω) Suppose $M = \{r, s, t\}$ Determine whether the following statements are true or false and include your opinion!

(ω_1) $r \in M$

(ω_2) $s \notin M$

(ω_3) $\{r\} \in M$.

The motivation for the φ and ω_3 is the same, which is related to the use of curly braces in sets. The results showed that no students correctly answered φ and ω_3. Many students answered $A = B$ in φ and although in the answer sheet, some stated $A \neq B$ in φ dan *"wrong"* in ω_3, no one provided the correct justification for their statements. Meanwhile, for the cases of $r \in M$ and $s \notin M$, all of them gave correct answers. Table 3 shows students' responses to φ dan ω_3.

Table 3. Summary of Students' Answers on φ and ω_3

Argument	Code
If B is substituted into the equation $2x = 6$, it becomes $2(3) = 6$.[1]	μ_1
Integers start from minus, including zeros and integers. Then $B = 3$ Integers start from minus, including zeros and integers. $2(3) = 6$, then $A = B$.[1]	μ_2
A has a value of 3, and B also has a value of 3, so it can be concluded that A and B have the same value/content.[1]	μ_3
If calculated $A = 2x = 6$ $A = x = \frac{6}{2} = 3$ B is also worth 3.[1]	μ_4
The value $A = 2(3) = 6$, 3 is a whole number that, when multiplied by 2, results in 6. While $B = 3$, so $A \neq B$.[2]	μ_5
r is a value of a part, so it uses parentheses, and since $\in$ indicates it is a part of something.[3]	σ_1
Although r is separate from the set M, because r is also in the set M.[3]	σ_2
r is the set of M, and in that example, the writing is correct, so if read as r is the set of M.3	σ_3
The element of the set M is not just r, but r, s, t.[4]	ϑ_1
$\{r\}$ Is not an element of the set M.[4]	ρ
The writing is wrong, and the statement $\{r\} \in M$ can be written $M = \{r\}$ whereas $M = \{r, s, t\}$.[4]	ϑ_2

[1] $A = B$
[2] $A \neq B$
[3] $\{r\} \in M$ (Correct)
[4] $\{r\} \in M$ (Wrong)

An ontogenic obstacle of conceptual nature was detected in the case of φ and ω, related to the previous response γ. The student who responded γ_1, γ_2, and γ_3 when asked if A in φ and $\{r\}$ in ω_3 sets, the answer was, "*A is a set, but $\{r\}$ is not a set because $\{r\}$ does not have a capital letter, only curly braces."* This answer contradicts his response (μ_5). In the case of φ, he understands that B is not a set and A is a set. However, the answer $A \neq B$ is not because A is a set and B is not a set but because she made a computational error, which resulted in $A = 6$.

Furthermore, she was asked, *"Is it A or x that is worth 6?"* His answer was still "A". This fact shows that the person concerned has not fully understood the set element, even though he understands the concept of writing sets in general. Her response also supports this (ϑ_2) to ω_3. On the other hand, ϑ is a student's response who cannot distinguish between $\in$ and $=$. This phenomenon can be categorized as an instrumental ontogenic obstacle.

Similarly, σ's response is evidence of an instrumental ontogenic obstacle, i.e., they cannot distinguish between sets and set elements. This is consistent with the findings of Lutfi et al. (2021) and

Ferdianto and Hartinah (2020), who both identify instrumental ontogenic obstacles as a significant contributing factor to students' struggles with technical subjects such as triangle and quadrilateral problems, as well as mathematical visualization. These obstacles are characterized by students' lack of preparedness in essential technical aspects. Marquet (2011) further delves into this concept within the context of e-learning and m-learning, emphasizing the challenges arising from integrating disciplinary, pedagogical, and technical components.

Next, regarding the responses $\beta_3, \mu_3,$ and ρ and ρ, which were the responses of a subject (with Waluyo as pseudonym), his understanding of the set was only β and this was consistent when faced with φ and ω. The following is an interview excerpt on the case of φ and ω_3.

Researcher	:	*Why $A = B$ (case φ)*
Waluyo	:	*A and B are the same because they both have a value of 3, and they are not sets.*
Researcher	:	*Is A, not a set? (case φ)*
Waluyo	:	*No, because there is only one value or content of A which is 3*
Researcher	:	*What is the element of M? (case ω_3)*
Waluyo	:	*Elements of M are r, s, t*
Researcher	:	*Then what is this? Pointing to the $\{r\}$ (case ω_3)*
Waluyo	:	*This is r, but it is different from the r in M if this r (pointing to $\{r\}$) stands alone outside of M*
Researcher	:	*What should the correct notation be, then? (case ω_3)*
Waluyo	:	*It should be like this (writing $\notin$)*
Researcher	:	*If so, is $\{r\}$ A set? (case ω_3)*
Waluyo	:	*No, it is not.*

Responses $\beta_3, \mu_3,$ and ρ and ρ consistently rejected sets with one element, but Waluyo accepted that ∅ is a set. When confirmed regarding this, he explained, *"In the book and explained by the teacher, there is such a thing as an empty set."* However, when given a rebuttal, *"The empty set has no elements, why can it be called a group?".* Waluyo was silent and could not explain. This shows that Waluyo constructed knowledge about the empty set based on testimonials only. Concerning learning obstacles, Waluyo's case shows that he experienced epistemological obstacles. Waluyo's reliance on testimonials for constructing knowledge about the empty set may have been hindered by interpretive blindness, a bias that can impede learning from testimony (Asher & Hunter, 2021). This is particularly relevant in the context of non-reductionism in the epistemology of testimony, which argues for the positive epistemic status of some testimonial beliefs (Perrine, 2014). However, the recovery problem in the epistemology of testimony, which posits that audiences may not always reliably recover asserted contents, could have further complicated the learning process (Peet, 2016). These obstacles underscore the necessity for a more nuanced understanding of the role of testimonials in knowledge construction.

Another form of epistemological obstacle that occurs in the case of φ is the response μ. All agreed that the tasks that had been given about identifying elements of a set were only like the cases of $r \in M(\omega_1)$ and $s \notin M(\omega_2)$. The case of φ is very different from the examples or exercises done in the set learning process. Students who do not clearly understand the underlying concepts and principles may struggle to apply them to new and unfamiliar problems (Carpenter et al., 2015). Students need to understand the concept behind the example, not just the specific steps used to solve it. This is often due to a reliance on memorization of steps rather than understanding the subgoals and subtasks involved (Catrambone, 1994). To address this, it is important to help students learn the relations between principles and examples through generating explanations and making analogies (Nokes & Ross, 2007). Providing a conceptual framework that visually illustrates the relationships between concepts can also be beneficial (Ellis & Turner, 2003). Lastly, understanding the meaning of concepts and relationships is crucial, and a meticulous analysis of problem types is necessary to identify sources of difficulty (Quintero, 1983).

Learning Obstacles in Understanding Finite and Infinite Sets

Students' understanding of finite and infinite sets must be discussed because many explanations are incomplete or wrong. This is based on the analysis of textbooks and interviews with teachers. Put, finite sets are only explained as sets whose elements can be counted in number, while infinite sets have elements that cannot be counted or are infinite (As'ari et al., 2017). This kind of explanation makes it undoubtedly challenging to achieve an understanding of the consequences of the definition of finite and infinite sets: "*Suppose A is a finite set, then there is an integer k such that $n(A) = k$, and if there is no k then A is an infinite set."* (Enderton, 1977). It is more ironic that it is explained in the textbook and understood by teachers that infinite sets have no cardinality. This is a fatal mistake because an infinite set is known by the term Aleph $(\aleph)$ to express its cardinality (Enderton, 1977). However, in terms of cognitive development, it is not the time junior high school students recognize the term, Aleph. At least the knowledge conveyed is not out of the basic theory. Next is about the dual meaning in interpreting the notation of the colon "...". While it is interpreted as a notation to express infinite sets, on the other hand, it states *"and so on following the pattern"* for the case of finite sets whose elements are difficult to count.

Indeed, the textbook and the teacher serve as pivotal sources of knowledge for students, significantly shaping their understanding of mathematical concepts (Sievert et al., 2019). When teachers' understanding of mathematical materials requires correction, it can have several adverse effects on students:

1. Misunderstanding: A teacher needs clarity on mathematical concepts to avoid inadvertently conveying misconceptions to students (Alwan, 2011). Consequently, students may develop erroneous beliefs about the topic, which can be challenging to rectify later.
2. Confusion: Inadequate clarity from the teacher regarding mathematical concepts can confuse students and impede their comprehension (Bray, 2011). Confusion may lead to frustration and diminish students' motivation to engage with the subject matter.
3. Inaccuracy: If a teacher's understanding of mathematical material is flawed, it may disseminate incorrect information to students (Sitorus & Masrayati, 2016). Inaccuracies can lead to errors on assignments and exams, adversely affecting students' academic performance.
4. Limited Instruction: A teacher's limited grasp of mathematical material may restrict their ability to provide students with a comprehensive understanding of the topic (Ganal & Guiab, 2014). Consequently, students' capacity to apply knowledge to real-world scenarios may be curtailed.

Addressing these issues necessitates ongoing professional development for teachers to enhance their understanding of mathematical concepts and pedagogical strategies, ensuring effective instruction and fostering optimal student learning outcomes.

This phenomenon has a vast potential to shape false knowledge, so something known as a didactical obstacle emerges. All students agreed that $\{2,4,6,8,\dots\}(\tau_1)$ was an infinite set and $\{2,5,3,7,\dots,23\}(\tau_2)$ It was a finite set. However, when asked, *"What is the difference between "..." in τ_1 and τ_2"* no student answered. Even though τ_1 can be written as $\{2,\dots,4,8,6\}$ or "..." in τ_2 can be filled with any number because the set does not pay attention to the order of writing its elements, for example $\{1,2,3\} = \{3,1,2\}$. The concept of didactical obstacles, which can lead to the formation of false knowledge, is explored in various contexts. Serradó et al. (2005) and Prediger (2008) discuss the influence of these obstacles on learning probabilistic knowledge and the multiplication of fractions, respectively. Bingolbali et al. (2021) further examine the didactic phenomenon in the context of knowledge of sets. These studies highlight the need to consider these obstacles in the teaching and learning process. Buford and Cloos (2018) add a philosophical perspective, presenting a dilemma for the *"knowledge despite falsehood"* strategy, which is relevant to forming false knowledge.

The next task, $\{x|x \text{ is a word that can be formed from the letters } a, i, s, w\}$ (τ_3) and $\left\{1,2,\{3\},\left\{\{1\},\{2,3\},1\right\}\right\}(\tau_4)$ became the object of ontogenic and epistemological obstacle indications on finite and infinite sets. Many say that τ_3 is a finite set because the words that can be formed are limited, while others state that τ_3 is an infinite set because its elements are very many. When confronted with the question, *"Is it an infinite set when there are so many elements?",* then an example was shown *"Suppose S is the set of rivers on earth. Is S an infinite set?"* The student who answered τ_3 as an infinite set stated that S is a finite set, how many are there? *"It is hard to count,"* he replied. Furthermore, interpreting the words *"very many elements"* as an infinite set result from constructing their prior knowledge outside of mathematics. This is evidence that an actual conceptual ontogenic obstacle occurs in the case of τ_3. Other research has shown that students can experience ontogenic learning obstacle when they construct mathematical understanding through their experiences outside of mathematics, leading to errors in translation among mathematical representations (Afriyani et al., 2019).

Students' inability to state τ_3 as an infinite set because *"there is no need to pay attention to the repetition of letters in forming words from the letters a, i, s, w"* and incorrectly mentioning the cardinality of τ_4 is evidence that students experience instrumental ontogenic obstacles. This relates to the student's promise to pay attention to important technical matters. According to other research, this obstacle may stem from a lack of understanding of the underlying mathematical principles, as seen in other studies on epistemological obstacles in mathematics learning (Moru, 2009). Concerning epistemological obstacles, both teachers and students stated that τ_3 and τ_4 were types of problems that they had never encountered. *"An example of a problem that I often encounter, for example, E is the set of letters in the word "matahari","* said one of the subjects. Similarly, with τ_4, no examples or problems wrote the set in the set. The diversity of types of exercises students carry in learning mathematics can result in epistemological barriers. This also includes students' habits of only solving routine questions with the same difficulty level.

A variety of research studies have highlighted the influence of different forms of physical activity on the cognitive processes involved in mathematical learning among students. Supandi et al. (2021) observed that students may encounter epistemological challenges when they struggle to transfer their mathematical knowledge to novel or intricate problems, a phenomenon particularly notable in tasks involving integer operations and word problems. Brown (2008) and Bolden and Newton (2008) further investigated these challenges, with Brown focusing on the complexities of mathematical induction and Bolden examining the barriers to implementing inquiry-based teaching methods in elementary mathematics. Taken together, these investigations underscore the importance of adopting a more diverse and innovative approach to mathematics education, one that enhances students' capacity to apply mathematical concepts across a range of problem-solving scenarios.

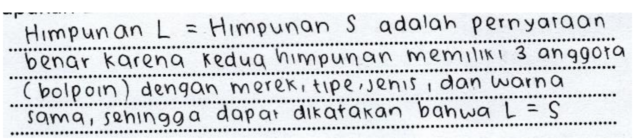
Learning Obstacles in Understanding the Similarity of Two Sets

The fundamental skills expected from students in the Indonesian school mathematics curriculum following the study of sets include the ability to solve contextual problems (Indonesia, 2018). Consequently, the material presented in the textbook is heavily imbued with contextual nuances (As'ari et al., 2017). This emphasis on contextualization in mathematics teaching allows educators to integrate mathematics beyond its abstract realm (Ramos-Rodríguez et al., 2022). However, chaos may arise when mathematics is divorced from its real-world context (Hendriyanto et al., 2023). Therefore, the task format in this section is designed employing contextualization, as referred in Figure 4.

Lukman and Sani each have three ballpoint pens with identical brand, type, and color (identical). Suppose each of Lukman and Sani's ballpoint pens becomes a set L and a set S determine and explain whether $L = S$?

Figure 4. Form of Contextual Problem about the Similarity of Two Sets

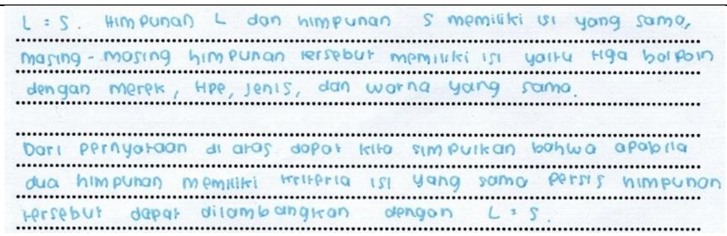
In practice, the explanation of the similarity of two sets is unified with the explanation of two equivalent sets. *"two sets A and B are said to be equal if and only if $A \subset B$ and $B \subset A$, notated by $A = B$. If $n(A) = n(B)$, then set A is equivalent to set B"* (As'ari et al., 2017). However, students may have misconceptions about interpreting two sets as equal because they do not fully understand the concept of sets and their properties. Figure 5 and Figure 6 are the real evidence found.


Himpunan L = Himpunan S adalah pernyataan benar karena kedua himpunan memiliki 3 anggota (bolpoin) dengan merek, tipe, jenis, dan warna sama, sehingga dapat dikatakan bahwa L = S

English version:
Set L = set S is a true statement because both sets have 3 elements (ballpoint pens) with the same brand, type, type, and color, so it can be said that $L = S$

Figure 5. Siti's (Pseudonym) Answer on the Topic of Two Equal Sets

Figure 5 shows that the suspicion of confusion between similarity and equivalence was answered during the interview. In the interview, Siti explained, *"I understand that the pens owned by Lukman and Sani are different, but the number is the same."* This expression shows that Siti confuses the idea of two sets that are the same (i.e., have the same elements) with the idea of two sets with the same number of elements. Furthermore, her hesitation and inability were demonstrated when asked to make her mathematical modeling. It adds to the belief that what is understood about two sets being equal is that all three have the same number of elements. Shindi (pseudonym), on the other hand, understood the concept of the similarity of two sets (Figure 6) correctly.


L = S. Himpunan L dan himpunan S memiliki isi yang sama, masing-masing himpunan tersebut memiliki isi yaitu tiga bolpoin dengan merek, tipe, jenis, dan warna yang sama.

Dari pernyataan di atas dapat kita simpulkan bahwa apabila dua himpunan memiliki kriteria isi yang sama persis himpunan tersebut dapat dilambangkan dengan L = S.

English version:
$L = S$ set L and set S have the same content. Each set has the content of three ballpoint pens with the same brand, type, type, and color.
From the above statement, we can conclude that if two sets have the exact content criteria, then the set can be denoted by $L = S$

Figure 6. Shindi's Answer on the Topic of Two Sets being Equal

The important thing distinguishing the answers in Figure 5 and Figure 6 is that in Shindi's answer, it is written, *"If two sets have the same content criteria, then they are the same set."* This shows that Shindi understands the concept of two sets being the same ($A = B \Leftrightarrow A \subset B \wedge B \subset A$). However, Shindi's answer to the problem (Figure 4) was incorrect because $L \neq S$ should have been (for example, if a pen is symbolized by p, then $L = \{p_1, p_2, p_3\}$ and $S = \{p_4, p_5, p_6\}$). Shindi's inability to identify the different elements in the contextual problem resulted from the presentation of material that focused too much on surface-level applications without providing students with a deep understanding of the underlying mathematical concepts. For example, when explaining the example of E being the set of letters in the word *"matahari".* Everyone understands that $E = \{m, a, t, h, r, i\}$, this is the translation of the explanation: *"In a set, the elements must be different from each other or in writing the elements of a set there must be no repetition* $(\propto)$." In the context of set E, the explanation $\propto$ is acceptable. However, the lack of further explanation for recurring elements in contextual problems can give rise to didactical obstacles, as illustrated in Shindi's case. Meanwhile, Siti's case exemplifies a form of conceptual ontogenic obstacle. The absence of problems akin to Figure 4, resulting in all students being unable to provide the correct answer, manifests the epistemological obstacle.

Shindi's struggle with grasping mathematical concepts due to superficial teaching methods is a prevalent issue, as evidenced in Maarif et al. (2021) study on sequence and series problems. This challenge is further compounded by the contextual nature of knowledge, as explored by Vassallo (2001), Guazzini (2018), and Giunchiglia (1993). Vassallo's (2001) analysis of epistemological challenges underscores the necessity of a contextualized approach to understanding knowledge, while Guazzini's (2018) proposal for resolving the Symbol Grounding Problem highlights the significance of context in establishing objective reference points. Giunchiglia's (1993) theory of reasoning with contexts emphasizes the importance of considering the environment in which cognitive processes unfold. Contextual learning in mathematics involves teaching mathematical concepts and skills that emphasize their practical application in real-world scenarios and their interconnectedness with other disciplines (Smith & Morgan, 2016). While this approach effectively helps students recognize the relevance of mathematics in their lives, it is essential to recognize that it can potentially compromise the integrity of fundamental mathematical principles and concepts. In addition to the examples mentioned earlier, contextualized learning may dilute mathematics by oversimplifying or distorting mathematical concepts to make them more "accessible" to students (Wood, 2013). This can result in students possessing a superficial understanding of concepts (Nicol, 2010) but lacking the ability to apply them in complex situations or relate them to other mathematical ideas.

In conclusion, while contextual learning can be a valuable tool for engaging students and demonstrating the applicability of mathematics to real-life scenarios, educators must ensure that it upholds fundamental mathematical concepts and principles. Teachers should strive to teach students a deep understanding of mathematical concepts and provide them with the skills and strategies necessary for applying them across various contexts. Furthermore, once teachers have successfully contextualized mathematics and fostered students' acceptance of the subject, they should guide students in recontextualizing the mathematics they have learned and encourage them to explore its applications further.

CONCLUSION

Upon analyzing the data and deliberating on the findings, this study concludes that students encounter various learning obstacles about the material on sets. These obstacles encompass ontogenic, didactic, and epistemological factors. An ontogenic obstacle emerges from students' diminished interest in mathematics learning, coupled with their inadequate understanding of the significance of material on sets within mathematics, thereby leading to psychological ontogenic impediments that can detrimentally affect their overall learning journey and academic performance. Additionally, students' lack of readiness concerning essential aspects of sets of material, such as understanding set concepts, contributes to instrumental ontogenic barriers when tackling subsequent topics, including set elements, finite and infinite sets, and comparing two sets. Moreover, conceptual ontogenic hurdles manifest when students need help to fully grasp the underlying essence of the set concept, as formal definitions may appear overly intricate or overly simplified.

Didactical obstacles arise when the presentation of the material needs to facilitate effective learning about sets. For instance, the unstructured arrangement of the material may lead students to reject sets with a single element and empty sets, while an overemphasis on examples of similar problems and tasks can result in students dismissing elements with differing characteristics (e.g., {1, 2, 3, a, b, c} as a set). Moreover, an excessively contextualized delivery method is the primary cause of didactical hurdles. Additionally, teachers' incomplete understanding of specific topics is crucial to address, as this can pave the way for the perpetuation of didactical obstacles across generations. These obstacles persist and become entrenched over time.

Epistemological obstacles to the set concept emerge when students require assistance in comprehending the overarching meaning of sets. A robust foundation is essential for students to grasp the essence of sets, as misconceptions may arise while constructing more advanced knowledge, including understanding element notation, finite and infinite sets, and comparing two sets. Another form of epistemological obstacle within the function concept involves students' inability to tackle non-routine problems that demand a higher cognitive level than typical tasks.

Didactic design encompasses a teaching approach that highlights the teacher's role in imparting knowledge to students and how the realization of this knowledge, often referred to as the curriculum, is executed. The emergence of learning obstacles among students is not inherently attributed to the curriculum itself but rather to inaccuracies in implementing didactic design, both in terms of material design within textbooks and the delivery by teachers during instruction. Mathematics poses challenges for many students, and presenting the material in an appropriate, clear, engaging, and accessible manner is crucial to mitigate barriers to learning. To alleviate such obstacles, educators should endeavor to offer engaging mathematics materials that are appropriately leveled and provide ample scaffolding to support student learning. This study suggests that students require more opportunities to practice applying set theory concepts to problem-solving scenarios and to receive feedback on their efforts. Overall, when the didactic design fails to captivate students, constrains their autonomy, emphasizes rote memorization, and neglects individual differences in learning styles and abilities, it can lead to learning impediments. Effective teaching demands a nuanced approach that considers individual students' unique needs and interests. Consequently, this research on the analysis of learning obstacles in set materials could hold significant implications for developing more suitable materials, more effective teaching strategies, educational policies, and interventions to enhance student learning and achievement in mathematics.

However, this research still possesses several limitations. Firstly, it solely scrutinizes set theory, with the scientific knowledge referenced being confined to Cantor's version of set theory. Additionally, this study exclusively concentrates on the phenomena encountered by students, overlooking the process of knowledge inheritance from its original source through various institutions. It is also crucial but still needs to be studied. Recommendations for future research encompass analyzing other topics or exploring set theory with more contemporary scientific knowledge. Understanding how curriculum developers and teachers conceptualize a subject is also imperative. Ultimately, developing an effective didactic design to alleviate learning obstacles is paramount.

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