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# Construction of reflective thinking: A field independent student in numerical problems

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## Abstract

In educational settings, reflective thinking is often overlooked, with an excessive emphasis on final answers, resulting in students needing more ability to evaluate and reconstruct their problem-solving processes. The ability for reflective thinking is required by students in solving problems, including numerical problems. This study uses a qualitative approach to focus on field-independent students' numerical problem-solving processes. The data collection technique begins by administering the Group Embedded Figures Test (GEFT), a valid and reliable numeracy problem instrument, and conducting in-depth interviews. Two students with similar initial mathematical abilities and field-independent cognitive styles were selected as research subjects. Findings reveal that these students face challenges such as lengthy problem descriptions and a lack of confidence but gradually develop strategies, emphasizing repeated problem analysis, concept interconnections, and error awareness. Researcher-provided scaffolding facilitates critical reflection, enabling the construction of new ideas. These results have practical implications for teachers, suggesting the need to design lessons that cater to diverse cognitive styles, providing more complex problems to field-independent students to enhance their problem-solving skills.

**Keywords**: Field Independent Student, Numeracy Problems, Reflective Thinking Process, Scaffolding

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In general, various student reactions occur when given mathematical questions. Some students can solve them smoothly, some want to answer the questions but don't know how to solve them, and some even let the questions pass by. Not all mathematical questions automatically become a problem for students (Hamidah & Suherman, 2016). When students know the steps to solve them, it is no longer a problem.

When solving problems, students think in their minds to arrive at the answers (Sriwongchai et al., 2015). This aligns with Kim and Hannafin (2011), stating that the thinking process occurs when students try to solve mathematical questions through problem-solving steps. Therefore, teachers must provide unstructured problems with multiple approaches to solving them related to real-life situations and involving various contexts or current issues (Hodnik Čadež & Manfreda Kolar, 2015).

Several previous studies documented that one way to enhance students' thinking processes is by providing numerical problems (Ariyana & Suardipa, 2023; Gittens, 2015; Xiao et al., 2019). Numerical problems require applying mathematical knowledge (Goos et al., 2013; Liljedahl, 2015), problem-solving strategies, and reasonable estimations (Zevenbergen, 2004). Studies in several developed countries have shown that an individual's numeracy skills negatively impact their well-being (Bruine de Bruin & Slovic, 2021). Someone with low numeracy skills may need help in getting a job, have low self-confidence, and earn a lower income. In contrast, the opposite is true for individuals with higher numeracy skills (Indefenso & Yazon, 2020).

Schleicher (2013) explains that good numeracy skills are the best protection against unemployment, low income, and poor health. Numeracy skills are needed in all aspects of life, both at home, in the workplace, and in society (Jain & Rogers, 2019). Globally, numeracy is also one of the skills assessed in the Program for International Student Assessment (PISA).

Based on the PISA results in 2018, in the aspect of numeracy, 71% of Indonesian students scored below the minimum competence level (Ahmad & Latif, 2021). The Ministry of Education responded to the low numeracy literacy by implementing Minimum Competency Assessments (MCA) as a national assessment in accordance with the Ministry of Education and Culture Regulation No. 43 of 2019. This is because MCA is designed to measure literacy and numeracy skills based on the characteristics of PISA problems. In Indonesia, numeracy is included as one of the evaluation criteria in the new government policy since October 2019, replacing the National Examination and implemented in 2021 (Megawati & Sutarto, 2021). The implementation of character surveys and MCA, which includes numeracy and literacy as a replacement for the National Examination, is expected to encourage improvements in the quality of education in Indonesia.

A focus on numeracy, especially in mathematics education, will provide contextual learning experiences by providing stimuli that immerse students in the problem (Ariyana & Suardipa, 2023). The context given should ideally be built from situations, conditions, and facts close to students' daily lives (Geiger et al., 2015). This is in line with Tout and Gal (2015) stating that numeracy can be observed when students solve problems in real-life contexts involving information about mathematical ideas represented in various ways. For instance, consider the numeracy problem where a student must decide the most effective and efficient solution. Andi wants to buy "serabi kinca" for his sibling at home. A vendor sells "serabi kinca" in packs of 3 for Rp. 2,000 and packs of 4 for Rp. 2,500. If Andi has Rp. 8,000, what is the maximum number of "serabi kinca" he can buy? Is it possible for Andi to get some change? The expected answer is that Andi buys 3 packs of "serabi kinca" with 4 pieces each, so 3 x 4 = 12 pieces, and he spends 3 x Rp. 2,500 = Rp. 7,500, getting Rp. 500 as change. However, it is possible that a student might immediately answer that Andi buys 4 packs of "serabi kinca" with 3 pieces each, spending 4 x Rp. 2,000, so Andi wouldn't get any change despite still getting 12 "serabi kinca" packs. This answer is given spontaneously without further thinking, in this case, without evaluating/considering various available options (Thanheiser, 2010). In a simplified manner, the "serabi kinca" problem is often encountered in real life, emphasizing the importance of precision and careful consideration in decision-making. An individual with numeracy skills not only knows and uses efficient methods but also evaluates whether the obtained results are reasonable (Baker et al., 2020) and is aware of using appropriate and inappropriate mathematical reasoning to analyze situations and draw conclusions (Goos et al., 2014). The ability to consider ideas, arguments, or specific situations involves reflective thinking critically and objectively.

Many experts have emphasized the importance of reflective thinking in the learning process (Gürol, 2011; Lee, 2005; Pagano & Roselle, 2009; Sezer, 2008). For students, reflective thinking is a directed cognitive process towards problem-solving, demanding them to analyze, evaluate, motivate, and attain deep meaning (Gürol, 2011). Mathematical reflective thinking provides an opportunity to learn how to think about the best strategies in achieving learning goals and helps integrate their thinking abilities through assessment (Pagano & Roselle, 2009; Sezer, 2008). For teachers, accommodating students' reflective thinking processes means paying attention to how problems are solved and why students answer in a certain way so that math problems are not just about the end result (Lee, 2005). Therefore, teachers should be capable of designing an appropriate learning environment, observing the students' learning processes, and providing appropriate scaffolding while considering the differences in students' cognitive levels.

Hong and Choi (2011) stated that reflective thinking is the ability to view a problem or situation from various perspectives, consider possible implications and consequences, and find the best solutions based on deep understanding. Reflective thinking can help individuals apply numeracy effectively (Anghileri, 2006). In other words, numeracy problems based on real-life situations, where the solutions cannot be obtained through routine procedures (Megawati & Sutarto, 2021), require mathematical reflective thinking processes. Reflective thinking allows individuals to consider the numerical implications of a situation, evaluate the accuracy of statements based on available data, and identify and avoid biases or numerical reasoning errors. Furthermore, reflective thinking can help apply relevant mathematical concepts in problem-solving or decision-making involving numbers (Steen, 2001a). Several preliminary studies related to mathematical reflective thinking have been conducted, as shown in Table 1.

**Table 1**. Reflective Thinking Process

| **Dewey** (Kitchener, 1984) | **Surbeck** (Surbeck, 1991) | **Mezirow, Kember** (Kember et al., 2008) | **Lee** (Lee, 2005) | **Zehavi & Mann** (Zehavi & Mann, 2005) |
|---|---|---|---|---|
| Pre-Reflective | Reacting | Habitual Actions | Problem Context | selection of techniques |
| Reflective | Elaborating | Understanding | Problem Definition | monitoring of the solution process |
| Post-Reflective | Contemplating | Reflection | Seeking possible solution | insight or ingenuity |
| | | Critical Reflection | Experimentation | conceptualization |
| | | | Evaluation | |
| | | | Acceptance/ rejection | |

This research's constructed reflective thinking process, with reference to previous studies, presented in Table 2. Numerous studies related to the process of reflective thinking have been conducted. Based on the research findings, reflective thinking is highly essential in the teaching practices of prospective teachers and pre-service teachers (Gürol, 2011) and as a form of professional development for educators (Bell et al., 2011; Kim & Silver, 2016; Mirzaei et al., 2014). Several studies indicate that reflective thinking can stimulate critical thinking in teachers (Choy & San, 2012) and students through ideas, insights, questions, and actions (Chen et al., 2019; Richard, 2010; Yuek Ming & Abd Manaf, 2014) and university students (Ghanizadeh, 2017). A scale for assessing reflective thinking in reflection activities was previously developed by Basol and Evin Gencel (2013). Furthermore, the analysis of the reflective thinking abilities of junior high school students based on their initial mathematical abilities was examined (Salido & Dasari, 2019; Noer, 2020) and analyzed reflective thinking abilities in the context of two-variable linear equations systems, relations, and function problems (Hidayat et al., 2021), as well as in the context of fractions based on gender (Rasyid et al., 2018).

**Table 2**. Descriptor Reflective Thinking Process

| Reflective Thinking Process | Descriptor of Reflective Thinking Process |
|---|---|
| Reacting | a. Experiencing the difficulty of a problem. In this step, students recognize the existence of a problem and identify it.<br>b. Stating/writing what is known in the problem.<br>c. Stating/writing what is asked in the problem.<br>d. Finding the relationship between what is known and what is asked in the problem. |
| Seeking possible solution; | Proposing several possible answers to a problem. (In this step, students develop various possibilities and solutions to solve the problem.) |
| Elaboration | a. Developing ideas to solve the problem by gathering the required data.<br>b. Utilizing previous knowledge and experiences and associating them.<br>c. Connecting relevant concepts to problem-solving. |
| Critical Reflection | a. Reviewing the obtained answers.<br>b. Correcting and explaining if any errors are found while solving the problem.<br>c. Drawing conclusions from the solved problem. |

Dwiyanti et al. (2022) explore their research and show that the reflective thinking process of students varies depending on their different cognitive styles. Therefore, the way someone assesses and thinks will also differ. These individual differences in processing information are known as cognitive styles. Furthermore, cognitive styles are classified into field-independent (FI) and field-dependent (FD) (Witkin et al., 1977). With these different cognitive styles, students will likely approach problem-solving differently according to their reflective thinking process and perception of the given problem (Hamer & Collinson, 2014). To determine if this is indeed the case, further investigation is needed. From several previous studies, no research has examined the construction of reflective thinking processes in solving numerical problems from a field-independent cognitive style perspective. Hence, the researchers believe that it is necessary to examine the reflective thinking process of students in solving numeracy problems, specifically those related to number concepts while considering the cognitive style, particularly the field-independent cognitive style.

## METHODS

This research design is qualitative, with a grounded theory approach aimed at constructing a new theory, accompanied by explanations that support this theory. This theory is 'grounded' in data that has been collected and analyzed during the research, which consists of processes, actions, or interactions discovered within it (Creswell & Clark, 2017). This research will develop a new theory from the data collected and analyzed concerning the phenomena that occur among seventh-grade junior high school students when solving numeracy problems related to number concepts using the stages of reflective thinking. The data source for this research is the 8th-grade students in the Academic Year 2022/2023, who serve as the research subjects. The data in this research consists of the results of written tests of students' work and data from interviews during clarifications of written answers. In the initial stage, 32 students completed the mathematics proficiency test and the Group Embedded Figures Test **(**GEFT) Test, which was developed to determine their field-dependent or field-independent category (Kahtz & Kling, 1999; Mamonto et al., 2018; Mefoh et al., 2017). Subsequently, the researcher provided numeracy problems and interviewed subjects categorized as field-independent. The numeracy problems given to the students were modified questions developed by Putra et al. (2016), which the researcher used as an instrument to explore students' mathematical reflective thinking processes. These questions have been tested for validity, practicality, and potential effects.

Subject selection is repeated and continuously until data saturation is achieved, which means having the same or consistent patterns from several research subjects. Based on this data saturation, we will now discuss two subjects with Field Independent cognitive styles, namely S-01 and S-02. The justification for selecting these two research subjects is based on data saturation, students' capability, considering the smoothness of oral communication, seeking input from mathematics teachers, and equivalent initial mathematical abilities. The process of selecting research subjects can be seen in Figure 1.

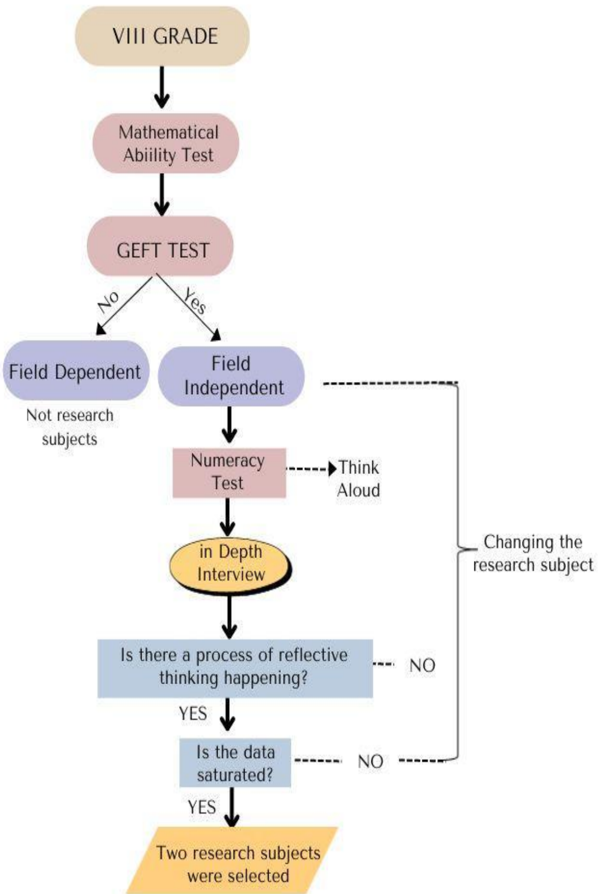



**Figure 1**. Selecting Research Subjects

After the first data collection (Mathematics proficiency test), a second data collection was conducted to obtain valid data, which involved completing the reflective thinking test. The third data was gathered during an in-depth interview between the students and the researcher to understand their reflective thinking process. The first, second, and third data were collected at different times. By comparing these three data sets, the reflective thinking process of students with the field-independent cognitive style was identified as valid data. The data collection techniques used in this research were interviews and tests. The data on students' mathematical reflective thinking processes were obtained using the main and auxiliary instruments. The main instrument was the researcher themselves, who directly interacted with the research subjects. Researchers conducted interviews to delve deeper into the reflective thinking processes of junior high school students in solving numeracy problems, considering cognitive styles that cannot be represented in others. The auxiliary instrument consisted of mathematical reflective thinking questions, a Mathematics proficiency test, documentation, and interview guidelines. To ensure that the data remains unbiased, triangulation is employed. This study uses time triangulation, which involves cross-checking through written tests and interviews at different times or in different situations. If the data obtained shows consistency (many similarities), then the data from reflective thinking ability tests and interviews are considered valid. The interview guidelines for each stage of the reflective thinking process are referred to in Table 3.

**Table 3**. Interview Guidelines

| Reflective Thinking Process | Interview Guidelines |
|---|---|
| Reacting | a. What do you think when you read this problem?<br>b. What comes to your mind when you read this question?<br>c. What information do you know?<br>d. What additional information do you know that might help in solving it?<br>e. Is the information provided in the problem sufficient? Please provide your reasons! |
| Seeking possible solution; | a. What problems are present in the problem?<br>b. What topics are included in the problem?<br>c. What strategies did you use when solving the problem?<br>d. Are there any other strategies you can use to solve the problem? |
| Elaboration | How do you solve the problem presented in the question? |
| Critical Reflection | a. What can you conclude from the problem?<br>b. Are you confident that the method applied to answer the problem is correct? |

Data credibility is established through time and source triangulation. Time triangulation in this research involves cross-checking through tests of mathematical proficiency, numeracy tests, and interviews conducted at different times or in different situations. Source triangulation performed by the researcher entails comparing written data from research subjects obtained from numeracy problem-solving tests with verbal data from research subjects gathered through in-depth interviews. It means the data is considered valid if there is consistency between the results obtained from the second and third data collection, which depicts the students' mathematical reflective thinking process. The data analysis technique is performed based on the stages proposed by Miles and Huberman (1994), as follows: (1) data reduction, which involves verifying the students' work and excluding data that does not support the research; (2) data display, which involves clarifying and identifying organized and categorized data, allowing for drawing conclusions; (3) conclusion drawing/verification, which entails drawing conclusions or verifying findings. The activities carried out in the data reduction phase include sharpening, selecting, focusing, abstracting, and transforming raw data obtained in the field into meaningful data. In this research, the raw data obtained from field research is reduced to obtain the data that is genuinely needed to describe the reflective thinking process of students with a field-independent cognitive style in numerical concepts. The collection of data, after reduction, is organized and categorized. Subsequently, the data is presented in a more straightforward narrative format, allowing for conclusions to be drawn from the information. Drawing conclusions involves summarizing the data and verifying the accuracy of the data collected regarding how reflective thinking with a field-independent cognitive style is applied in solving numerical problems.

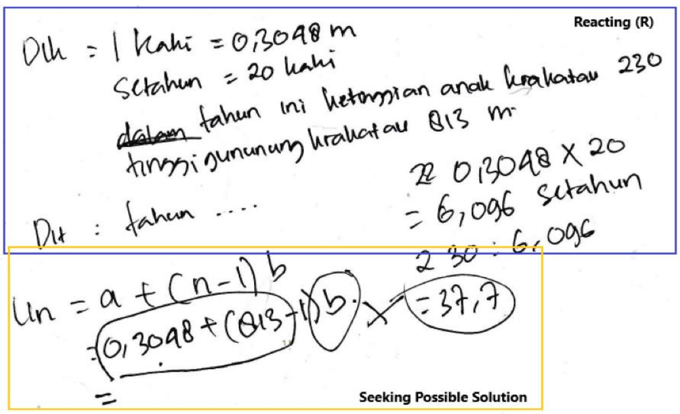
## RESULTS AND DISCUSSION

The following describes the subject selection based on the cognitive style of 8th-grade students, categorized according to their scores obtained from the GEFT Test as the instrument for cognitive style and Mathematics proficiency test. Additionally, the selected subjects clearly communicated their reflective thinking process in solving numeracy problems. The selected subjects are summarized in Table 4.

**Table 4**. Description Cognitive Style

| No | Kode | Mathematics proficiency test | GEFT Score | Cognitive Style |
|---|---|---|---|---|
| 1 | S-01 | Moderate | 18 | Field Independent |
| 2 | S-02 | Moderate | 15 | Field Independent |

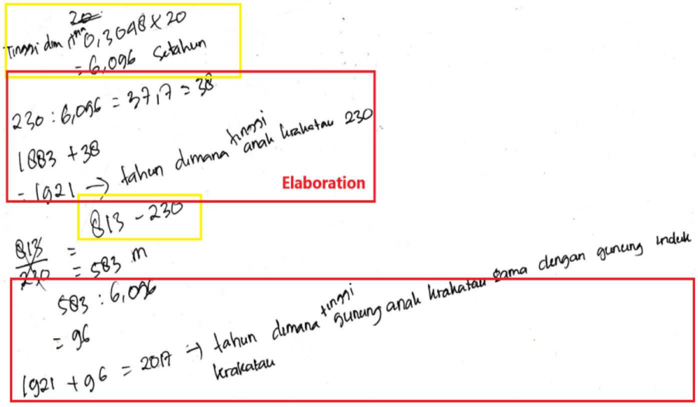
The instrument for the test of mathematical reflective thinking with numeracy characteristics given to the students is a modified version of the questions developed by Putra et al. (2016). These questions have been tested for validity, practicality, and potential effects. The answers of student S-01 in solving these questions can be seen in Figure 2.




Known : 1 feet = 0,3048 m
1 year = 20 kaki
This year, the height of Anak Krakatau is 230 meters
The height of Mount Krakatau is 813 m
Questioned: Year…?
Un = a + (n-1) b
= 0,3048 + (813 – 1) b = …
0,3048 × 20 = 6,096 (one year)
230 : 6,096

**Figure 2**. S-01 Answer (1)

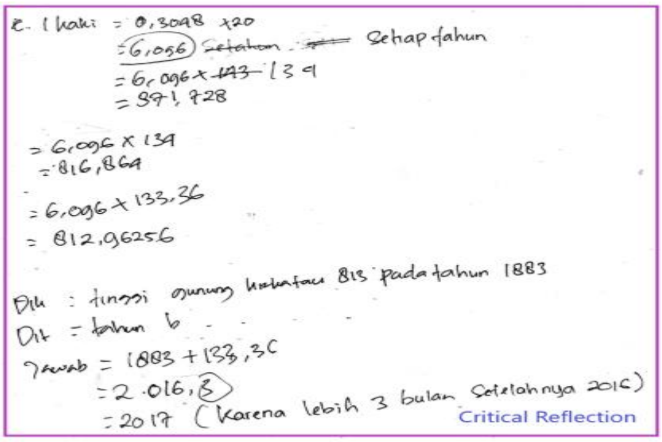
Figure 2 shows that S-01 went through the reacting process where they could write down the information given in the problem (what is known and what is asked). Then, S-01 looked for several possible solutions, initially attempting to solve the problem using an arithmetic sequence and writing the formula completely as Un = a + (n-1) b. Realizing the mistake when substituting an as the height of the mountain and b as the years, S-01 eventually tried another strategy, as seen in Figure 3.




Yearly height increase = 0,3048 × 20
= 6,096
230 : 6,096 = 37,7 = 38
1883 + 38 = 1921 (The year when the height of Anak Krakatau is 230 meters)
813 – 230 = 583 m
583 : 6,096 = 96 year
1921 + 96 = 2017 (The year when the height of Anak Krakatau is the same as Mount Krakatau)

**Figure 3**. S-01 Answer (2)

S-01 reread and understood the problem, identifying unclear information, namely the year when the height of Anak Krakatau reached 230 meters. In the Elaboration stage, S-01 first calculated the increase in height each year by converting the unit from feet to meters, multiplying the annual increment (20 feet) by 0.3048 (1 foot = 0.3048 meters). Then, S-01 searched for the current year when the height of Anak Krakatau reached 230 meters. To answer when the height of Anak Krakatau would be the same as the parent volcano, S-01 obtained the result by adding the current year to the difference in the following years. In the critical reflection stage, S-01 reviewed the steps of the answer and hesitated whether rounding off was allowed. Furthermore, S-01 asked themselves if they could directly find the year when the height of Anak Krakatau would be the same as the parent volcano by using the annual increment of the parent volcano. Finally, S-01 explored other possible answers, attempting to find a number close to 813 meters and performed the initial addition operation with the year of the parent volcano eruption, which was in the year 1883 (see Figure 4).




1 feet = 0,3048 × 20 = 6,096 (1 year)
= 6,096 × 143 134 = 371,728
= 6,096 × 134 = 816,864
= 6,096 × 133, 36
= 812, 96256
The year when the height of Anak Krakatau is the same as Mount Krakatau
= 1883 + 133,36 = 2016,3
= 2017
(Because there are decimal numbers, so the next year

**Figure 4**. S-01 Answer (3)

To support these findings, the researcher also conducted an interview with S-01. The following is an excerpt from the interview with subject S-01 aimed to explore their reflective thinking process when solving numeracy problems.

Researcher : After reading the problem, what comes to your mind?

S-01 : Oh, it's a long problem... I need some clarification about how to solve it.

Researcher : If you still need clarification, try to understand it again. First, find the critical information in the problem.

S-01 : It's known that 1 foot is 0.3048 meters. It increases by 20 feet each year. The height of Krakatau Volcano is 813 meters, and Anak Krakatau Mountain is 230 meters. The question is... can I answer c first?

Researcher : Sure, go ahead.

S-01 : So, c asks about the year when Anak Krakatau's height equals Krakatoa's height.

Researcher : Good, it means you understand the critical information in the problem. Now, what did you search for first to find the year when Anak Krakatau's height was the same as Krakatoa's?

S-01 : Anak Krakatau's height keeps increasing every year. Can I use the nth-term formula?

Researcher : Try it first; no need to cross it out if it needs to be corrected.

S-01 : (after trying, looks confused) It doesn't seem easy; a represents the height while b represents the difference in years before and after, so they're different... This formula will not work.

Researcher : Different how? S-01: The variables are different, so I can't use the nth-term formula. Let me try again... It's known that 1 foot is 0.3048 meters. It increases by 20 feet each year. The height of Krakatau Mount is 813 meters, and Anak Krakatau Mountain is 230 meters. Let's see... 20 feet = 0.3048 x 20 = 6.096. This is the height in one year. Now, how many years for 230 meters... the result is 37.7 years. I'm looking for the year when the height is 230 meters, right? But can I round off 37.7 years to 38 years?

Researcher : Yes, you can, because it's more than 37... Okay, next, try to find the current height first while checking if your steps are correct.

S-01 : Alright, so the height when Anak Krakatau is 230 meters = 1883 + 38 years = 1921. Then the difference is the height of Krakatau Mountain minus the Anak Krakatau Mountain, which is 583 meters. Next, to find the year when the height is 583 meters = 583 : 6.096 = 96 years. So, the height when Anak Krakatau is the same as the parent is 1921 + 96 years = 2017.

Researcher : From the various steps you took, what can you conclude?

S-01 : Well, initially, I didn't pay much attention to the phrase "at this time"... I realized I needed to know the current year first...

Researcher : Do we have to find the current year?

S-01 : Hold on... the expected final height is 813 meters, and it increases 6.096 meters every year... so how many years to reach 813 meters? Let me try... I multiply hundreds, which is closest to 813 meters. If I multiply by 143, the result is 871.728. That's too much, right? Let's try 134, the result is 816.864, 3 too many. If I try 133, the result is 810, 3 too few. So, it's between 134 and 133. *pauses and looks confused*.. Can I use decimals?

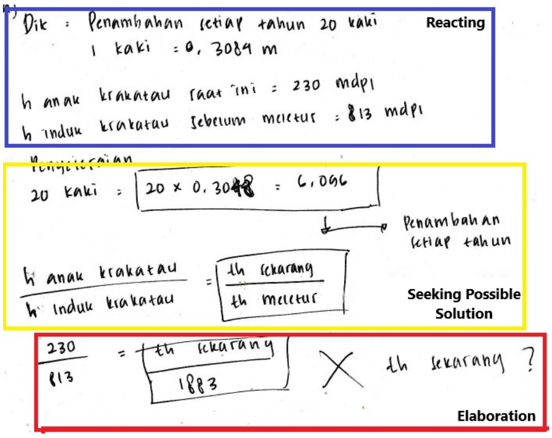
Researcher : Go ahead and try, find a number between those.

S-01 : Alright, with 133.36, the result is 812.96. Now, it's closer... so the height of Anak Krakatau is the same as the parent in 1883 + 133.36 = 2016.36, which is still the same as before... 2017.

Researcher : What can you conclude from this step? S-01: It's simpler and faster, it turns out...

Based on the interview excerpt, S-01 experienced confusion at the beginning of solving the problem because numeracy problems are preceded by stimuli that require careful reading first. With this confusion, S-01 tried to find several vital pieces of information in the problem and attempted several solution steps. During critical reflection, S-01 could criticize those steps and construct a new understanding through trial and error. After confirming, S-01 found that their last step was more straightforward and faster.

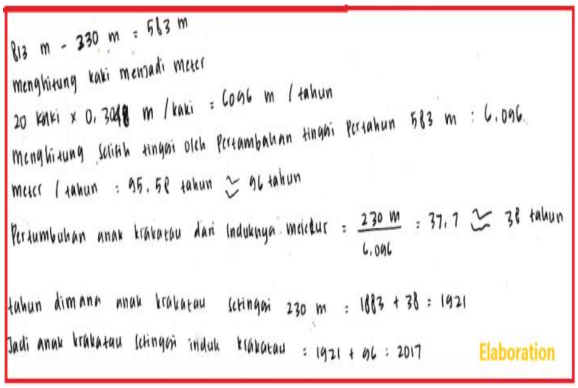
To further strengthen the data on how field-independent students solve numeracy problems through reflective thinking, the researcher conducted another interview with the second research subject, coded as S-02. The answers to S-02 when solving the problem can be seen in Figure 5.




Known : An increase of 20 feet every year.
1 feet = 0.3048 meters.
h Anak Krakatau Mount is 230 mdpl
h Krakatau Mount before the eruption was 813 mdpl
Answered :
20 feet = 20 × 0,3048 = 6,096

**Figure 5**. S-02 Answer (1)

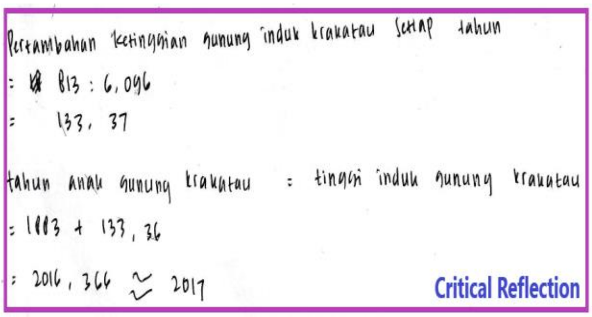
Based on the answers from S-02 in Figure 5, S-02 went through the reacting process where they could write down the information present in the problem, what is known, and what is asked. S-02 correctly interpreted the problem, such as using 'h' to symbolize height and 'mdpl' to symbolize meters above sea level. Next, in the Seeking Possible Solution stage, S-02 wrote the ratio formula as (height of Anak Krakatau)/(height of Krakatau Volcano) = (Current Year)/(Eruption Year) and correctly substituted the relevant numbers into the ratio formula. Then, in the solving stage, S-02 calculated (230 × 1883) : 813, resulting in 532.7. In the Critical Reflection stage, S-02 was still confused because the year obtained was 532, and they questioned why the year's result went backward. S-02 repeated the process, as seen in Figure 6.




813 m – 230 m = 583 m
Converting feet to meters
20 feet × 0,3048 m/feet = 6,096 m (every year)
Calculating the difference in height by the annual height increment = 583 m : 6,096 m/year = 95,58 year 96 year
The growth of Anak Krakatau from the eruption of Krakatau =
The year when the height of Anak Krakatau reached 230 meters = 1883 + 38 = 1921
So the year when Anak Krakatau is the same height as Krakatau = 1921 + 96 = 2017
Elaboration

**Figure 6**. S-02 Answer(2)

In the Elaboration stage, S-02 calculated the height difference between the Krakatoa Volcano and its offspring, converted the units from feet to meters, and calculated the current year and the year when the height difference with the Krakatoa Volcano occurred. In the Critical Reflection stage, S-02 was confident that the answer was correct this time. Next, S-02 tried to directly divide the height of Krakatoa Volcano at the eruption time by its annual growth in height. Then, S-02 added it to the year when Krakatoa Volcano erupted, as shown in Figure 7.




The increase in the height of Mount Krakatau every year = 813 : 6,096 = 133.37
The year when Anak Krakatau Mountain is the same height as Krakatau Mountain = 1883 + 133,36
= 2016,366  2017

Critical Reflection

**Figure 7**. S-02 Answer (3)

To support these findings, researchers also conducted an interview with S-02. The following is an excerpt from the interview with subject S-02 to explore their reflective thinking process when solving problems with numerical characteristics.

Researcher : After reading the problem, what comes to your mind? S-02: This is the first time I've done a problem like this, ma'am. It isn't very clear, and I'm unsure if I can do it. Researcher: It's almost similar to story problems... Just take your time to understand it first. Write down the necessary information given in the problem...

S-02 : Yes, ma'am. It's given that the height increases by 20 feet each year, where 1 foot is equal to 0.3084 meters. The current height of Anak Krakatau is 230 meters above sea level, and the height of the parent Krakatau before the eruption was 813 meters above sea level. Now, what's the next step?

Researcher : In which year was Mount Krakatau's height not recorded, Belva? It's mentioned in the problem, right?

S-02 : Oh, yes, ma'am. It erupted on August 27, 1883. So, I need to find the year when Anak Krakatau's height equals its parent's.

Researcher : Do you understand what is known and what is being asked in the problem? What mathematical concept are you going to use? S-02: I should use ratios.

Researcher : Just give it a try, Belva.

S-02 : Okay, ma'am. The height of Anak Krakatau divided by the height of the parent Krakatau is equal to the current year divided by the eruption year. So, 230 meters divided by 813 meters equals the current year divided by 1883. The result is 532.76, rounded to 533, right, ma'am? But it can't be the year 533, can it? (Smiling to herself). My approach seems to be wrong, ma'am...

Researcher : You can try a different strategy, Belva, but don't erase what you've written above... Just write the new approach below.

S-02 : Yes, ma'am. Let me try something else first... For instance, I can calculate the height difference. So, 813 meters minus 230 meters equals 583 meters. It increases by 20 feet each year, which is 6.096 meters per year. Then I can calculate the difference in height per year, which is 583 meters divided by 6.096 meters, equal to 95.58 years, rounded to 96 years. So, when the height is 230 meters, it means 38 years after the eruption of Mount Krakatau in 1883. That gives us the year 1921. The height of Anak Krakatau, being the same as the parent Krakatau, equals the year 1921 plus 96, which is the year 2017. Is it correct, ma'am?

Researcher : Are you sure about the method you used, Belva?

S-02 : I'm still not entirely sure, ma'am, as I've never done this before... But what if I directly divide 813 meters by the height increase per year? (S-02 tries to solve the problem using this new approach)

S-02 : Turns out the result is the same, ma'am... 2017. Now I'm confident, ma'am. Oh, I got confused with all the lengthy information earlier...

Based on the excerpt from the interview, S-02 was not confident in solving the problem given by the researcher because they had never encountered a similar problem before. With the researcher's guidance to understand the problem, S-01 tried to find some critical information about the problem. In the reacting phase, S-02 must be completed by writing down what was known and asked in the problem. However, during the interview, S-02 demonstrated a good understanding and ability to interpret the problem well. This can be seen from how S-02 wrote mathematical symbols. Then, S-02 attempted to use the ratio formula to solve the problem. However, when S-02 obtained the final answer, they seemed uncertain. S-02 then used a different method, and even after getting the same result as the initial approach, they remained still determined. During the critical reflection, S-02 attempted another approach to validate their answer by dividing the height of the parent Krakatau by the yearly increase (in meters) and then adding the eruption year. After getting the same result as the initial approach, S-02 became confident and concluded that only some of the numbers given in the problem were used in the solution.

Based on the problem-solving process and in-depth interviews conducted by the researcher with two students with a field-independent cognitive style, the following outlines their reflective mathematical thinking process in solving numerical problems.

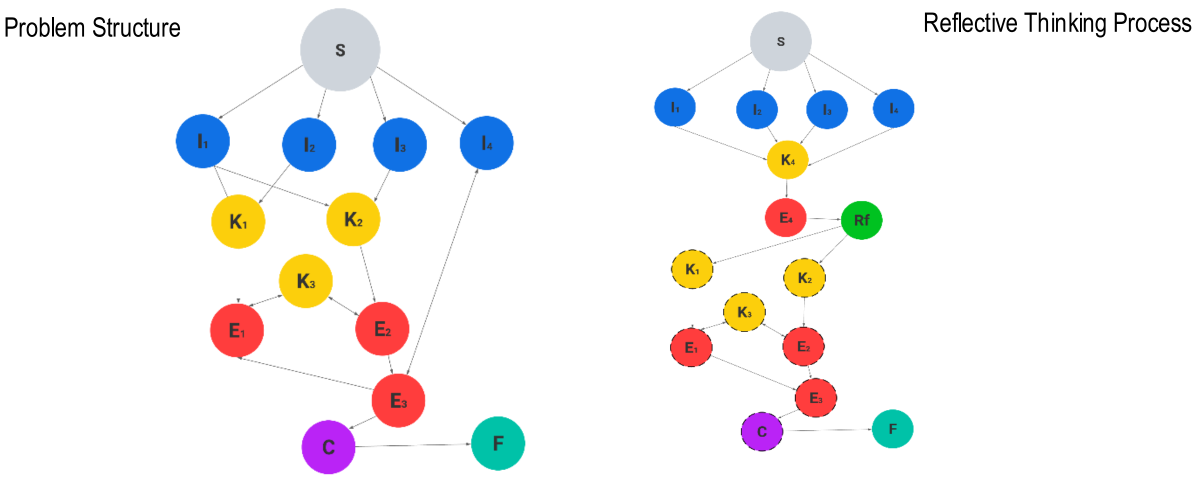



**Figure 8**. Reflective Thinking Process Field Independent's students

Figure 8 shows that the thinking structure of students with a field-independent cognitive style differs from the problem structure. An explanation of the coding process of students' reflective mathematical thinking in solving numerical problems with the characteristics of numeracy in Figure 8 is provided in Table 5.

**Table 5**. Construction of the Reflective Thinking Process of Field-Independent Cognitive Style Students

| Code | Remarks |
|---|---|
| S | Identification of the height of Anak Krakatau |
| $I_1$ | Identification of the length unit, where 1 feet = 0.3048 meters |
| $I_2$ | Identification of the height of Krakatau Mountain |
| $I_3$ | Identification of the year when Krakatau Mountain erupted |
| $I_4$ | Connecting the concept of length units with operations using rational numbers |
| $k_1$ | Linking the concept of the difference between integers with operations using rational numbers |
| $k_2$ | Using the concept of number rounding |
| $k_3$ | Connecting the concepts of ratios, nth term formula, time units, and operations with integers |
| $k_4$ | Calculating the time when the height of Anak Krakatau is 230 meters |
| $E_1$ | Calculating the time when the height of Anak Krakatau is 583 meters |
| $E_2$ | Calculating the time when the height of Anak Krakatau is 230 meters |
| $E_3$ | Calculating the year when Anak Krakatau's height is the same as its parent's height using the nth term/ratio formula |
| $E_4$ | Drawing conclusions based on the answers obtained within the context of the problem. |
| $C$ | Identification of the height of Anak Krakatau |
| $F$ | Finished |
| . | Reflective Thinking Process |

The data obtained from the interviews and responses of S-01 and S-02 is further analyzed and triangulated to obtain valid data. Then, this valid data is used to understand the reflective thinking process of students with a field-independent cognitive style in solving numerical problems presented in the following Table 6.

**Table 6**. Triangulation of Data on Reflective Thinking Process of Field-Independent Students.

| Reflective Thinking Process | Field Independent |
|---|---|
| Reacting | • Reading the given numerical problem repeatedly and marking important keywords.<br>• Absorbing information effectively from the given numerical problem.<br>• Able to identify the information presented in the numerical problem.<br>• Providing interpretations of the problem using mathematical symbols. For example, representing height as 'h' and meters above sea level as 'mdpl.'<br>• Repeatedly checking the relationships between variables mentioned in the problem. For instance, words like "currently," 1 foot = 0.3048 meters, increase in mountain height per year, etc. |
| Seeking possible solution; | • Trying various possible answers. |

| | |
|---|---|
| | ● Rethinking the questions in the problem and connecting them to similar problems previously solved. For example, using the concept of ratios or the nth term formula.<br>● Reconsidering the relationship between the unit "foot" and the length unit "meter."<br>● Providing explanations for the mistakes made. |
| Elaboration | ● Actively engaging in specific considerations while solving problems.<br>● Able to connect multiple mathematical concepts. For example, calculating the increase in mountain height per year, solving operations involving fractions and integers. |
| Critical Reflection | ● Believing in the correctness of the obtained answer.<br>● Actively constructing other ideas or strategies that can be used to solve the problem.<br>● Able to draw conclusions from the final answer.<br>● Connecting the final answer obtained with everyday life or their own experiences in problem-solving. |

Based on the analysis of responses and transcripts of in-depth interviews (S-01 and S-02), it is evident that both students experienced confusion (perplexity), lack of confidence, and uncertainty when solving numerical problems. The situation where students encounter confusion is considered a positive initial stage in the reflective thinking process for solving numerical problems (Pagano & Roselle, 2009). S-01 and S-02 read the numerical problems repeatedly to trace what is known asked and establish connections between them. This occurs because numerical problems have characteristics such as being stimulus-driven, related to real-life situations (Steen, 2001b), difficult to solve (Scott, 2016), and having multiple solutions that cannot be solved procedurally (routine problems), providing opportunities to view the problem holistically by considering cause and effect and its relation to each question (Muhaimin et al., 2023)

Both students attempted different problem-solving approaches, with S-01 using the nth term and S-02 employing the ratio formula. According to Piaget's theory, problem-solving involves an adaptive process that includes assimilation and accommodation. The accommodation process in reflective thinking occurs when students experience a discrepancy between the problem's structure and their thinking structure caused by perplexity. Subsequently, adjustments are made to align the thinking structure with the problem's structure through reflection. By exploring various possible answers, s-01 and S-02 adjusted their thinking structures to the problem's structure. S-01 tried to find the increment of Anak Krakatau's height each year, while S-02 attempted to calculate the difference between Anak Krakatau's and the parent Krakatau's height. To build a more complex thinking structure, students can receive assistance from teachers and peers or use learning media. Scaffolding is provided according to the student's needs to facilitate their own knowledge construction (Darling-Hammond, 2019; Hmelo-Silver, 2004).

In this study, the researcher provided scaffolding to S-01 and S-02 during the critical reflection phase by prompting questions that require more complex thinking, for example: "Are you confident with your problem-solving steps? Here, you are looking for the height difference between Krakatau Mountain and Anak Krakatau. Then, you add the year when the height reaches 583 meters to the year when Anak Krakatau's height reaches 230 meters. So, from these two steps, what year are you actually looking for the height of?" With the scaffolding provided, students with a field-independent cognitive style can construct new ideas in problem-solving. This aligns with (Van de Pol et al., 2010) statement that when teachers provide scaffolding (such as hints, questions, corrections, etc.), it can enhance students' thinking process at specific cognitive levels. Nevertheless, the final decision remains with the students on how they respond to the scaffolding provided by teachers or peers in redesigning their work.

Based on the findings of this study, the researcher added the concept of constructive thinking, which occurs when students encounter perplexity and overcome it by constructing new knowledge. Students with characteristics of constructive reflective thinking processes can be given more complex and higher-level difficulty questions. In order to foster constructive reflective thinking, knowledge must be actively constructed by students (Rokaya, 2021) and it is important to identify prior knowledge before acquiring new knowledge (Dubinsky, 1991), create a learning environment that allows students to learn from each other (Thayer-Bacon, 2000), prepare assignments and student learning activities, and anticipate any responses and questions that students may pose (Watson & Mason, 2006). This reflective thinking process is built based on Bloom's cognitive level theory (Revised), which can be seen in Figure 9.

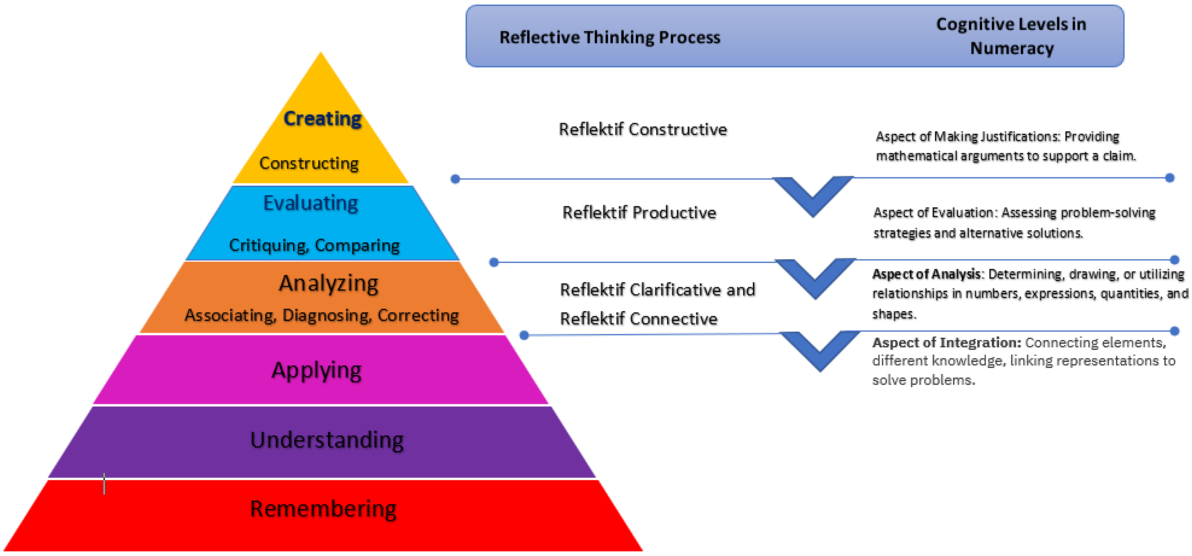



**Figure 9**. Categories Of Reflective Thinking Process

In this study, constructive reflection was developed after the researcher provided students with scaffolding. With the scaffolding, students became confident in their reflective thinking process, enabling them to identify and correct their mistakes in problem-solving (Kusmaryono et al., 2020). As a result, during the interviews, they could provide explanations with appropriate reasons. This finding aligns with the view that teachers can give scaffolding to help students develop their cognitive structures and become independent learners (Puntambekar, 2022). In addition to scaffolding, this process of constructive reflective thinking it involves the full awareness of students, which can be seen when they take action, explore various possible answers, elaborate on ideas, and engage in critical reflection. This is consistent with the statement of Gagatsis and Patronis (1990) that when solving a problem, full awareness is required, beginning with perplexity (which may be due to the subject's very limited knowledge), followed by a change in the situation with the discovery of examples that prove that the subject's intuitions are not in line with the actual facts. In this circumstance, the subject is compelled to re-examine the problem situation (resulting in a more meaningful and systematic reorganization process, and partially reconstruct their notions via a series of long computations or short deductive steps). Finally, the person is able to "see" and trust in the solution through mature mental representations.

Based on the findings of this study, an additional categorization/classification of reflective thinking processes was made, building upon previous research by Suharna et al. (2022), where three categories of reflective thinking processes were identified: clarifying reflection, connecting reflection, and productive reflection. (Kholid et al., 2020) categorized reflective thinking processes into three groups: assumptive, virtual, and connective, while Rahmi and Zubainur (2020) divided them into three categories: reflective thinking for action, reflective thinking for evaluation, and reflective thinking for critical inquiry. Lastly, Samuels and Betts (2007) divided reflective thinking into four mechanisms: revisiting reflections, using structure, taking responsibility, and metacognition.

## CONCLUSION

Students with a field-independent cognitive style respond well to numerical problems even if they haven't encountered them before. When solving problems, students with a field-independent cognitive style can break down the components of the problem, respond with various strategies, and connect the concepts of operations with integers and rational numbers to other mathematical concepts. Scaffolding is considered effective in assisting students with a field-independent cognitive style in constructing alternative ideas, leading to the discovery of more straightforward ways to solve numerical problems.

The limitations of this research include presenting only one numerical problem, involving only two subjects with a field-independent cognitive style, and the absence of a quantitative study that establishes the correlation between numeracy and reflective thinking processes. Based on the research findings, the researcher provides recommendations for further development. First, the reflective thinking process of students with a field-dependent cognitive style in solving numerical problems must be investigated. Second, the appropriate scaffolding to aid the reflective thinking process of students with both field-independent and field-dependent cognitive styles should be further examined. Third, learning materials should be developed that accommodate reflective thinking processes in solving numerical problems.

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| | | |
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