Journal on Mathematics Education
Volume 15, No. 2, 2024, pp. 451-472

How do Indonesian students learn function concepts? A praxeological analysis of textbook

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Received: 10 August 2023 | Revised: 15 February 2024 | Accepted: 23 February 2024 | Published Online: 6 March 2024

Abstract

The conception of functions, defined as the relationship between magnitudes or sets of ordered pairs, varies among students depending on the contextualization of the concept within the curriculum, notably in school textbooks. This investigation endeavors to scrutinize the approach taken by Indonesian textbooks in introducing the function concept at the lower secondary school level. An eighth-grade mathematics textbook was scrutinized utilizing praxeology, the fundamental construct of the Anthropological Theory of the Didactic. The analytical process unfolded in three main phases: examination of the praxis block, analysis of the logos block, and evaluation of the textbook's praxeological structure in collaboration with experts and educators. The examination revealed that the Indonesian textbook organizes functions into three distinct local praxeological frameworks: functions as sets, bijective functions, and functions as relationships between magnitudes. The praxis primarily emphasizes tasks and techniques for functions formulated by sets, shaping the landscape of function learning in Indonesia. Consequently, a notable epistemological gap within logos stems from the disparity between two conceptions of functions: functions as sets and analytical expressions. These findings underscore the necessity for an alternative praxeological arrangement of functions, mainly to bridge the divide between the set-theoretical definition and the analytical expression of a function.

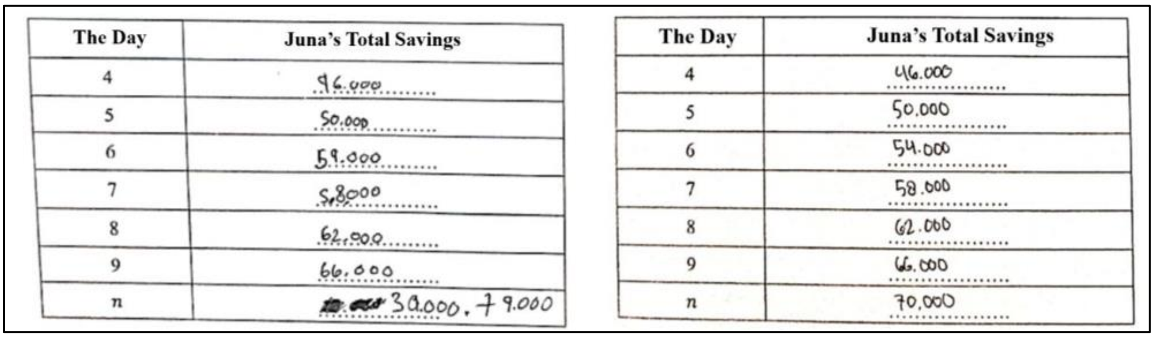
Keywords: Anthropological Theory of the Didactic, Functions, Praxeology, Textbook Analysis

How to Cite: Utami, N. S., Prabawanto, S., & Suryadi, D. (2024). How do Indonesian students learn function concepts? A praxeological analysis of textbook. *Journal on Mathematics Education, 15*(2), 451-472. http://doi.org/10.22342/jme.v15i2.pp451-472

The historical evolution of the concept of a function encompasses two primary phases. Initially, it was approached through curves and analytical expressions—a perspective often referred to as the old concept (Kleiner, 1989; Ponte, 1992). This viewpoint focused on studying the regularities in relationships between changing quantities, where one quantity depended on another (Biehler, 2005). The notation of function, $f(x)$, was subsequently initiated by Euler (Kleiner, 1989). As mathematical understanding progressed, the concept of a function underwent a significant transformation. The newer concept, rooted in set theory, extended the notion to a correspondence between two non-empty sets, provided they met a specific criterion: each element in the first set was paired with exactly one element in the second set (Vinner & Dreyfus, 1989). This concept is often called as the Dirichlet-Bourbaki's definition.

The dual nature of functions raises a crucial question: what concept of a function should be introduced to beginning students? The introduction of function commonly relies on the Dirichlet-

Bourbaki's definition, where some studies reported that the secondary school curricula and textbooks typically present this definition (Bardini et al., 2014; Markovits et al., 1986; Vinner & Dreyfus, 1989). However, Kleiner (1993) argued that it would be more meaningful to introduce functions as formulas or rules rather than as a set of ordered pairs. Notably, students, though informed about a function in terms of the set of ordered pairs (x, y), their subsequent exposure to function learning often involves formulas such as polynomials and trigonometry (Tall, 1996). These tendencies have proved challenging for students, particularly in overcoming their limited understanding of 'function' to specific representations (Denbel, 2015; Muzaffer, 2013; Ponte, 1992; Wilkie & Ayalon, 2018). For Indonesian students, a common misconception emerges where a function is perceived merely as a correspondence between elements of two sets, overlooking the underlying 'rule' that generates this correspondence (Jannah et al., 2019; Septyawan et al., 2019). One example of students' work in solving functional thinking problems, conducted by Utami et al. (2023), indicated that Indonesian students still have problems with formulating a function (Figure 1). Therefore, critically examining the current constraints in presenting functions in the school curriculum becomes crucial to addressing these issues.



The Day	Juna's Total Savings
4	46.000
5	50.000
6	54.000
7	58000
8	62.000
9	66.000
n	30.000. + 4.000

The Day	Juna's Total Savings
4	46.000
5	50.000
6	54.000
7	58.000
8	62.000
9	66.000
n	70.000

Figure 1. Students' Work in Making Formula of a Function (Utami et al., 2023, p. 919)

An essential part that explicitly reflects the learning goals in a curriculum is the textbook used by teachers and students (Fan et al., 2013; Valverde et al., 2002). Textbooks are designed to deliver the learning materials in the curriculum into an organized sequence of materials that teachers and students can use (Sosniak & Perlman, 1990). According to TIMSS researchers, over 70% of teachers rely on textbooks as their primary teaching resources (Mullis et al., 2012). In Indonesia, an overwhelming 93% of teachers favor using textbooks (Human Development Department: East Asia and Pacific Region, 2010). The intricacy and content of tasks presented in textbooks constitute the primary components of students' mathematical engagement, significantly influencing their ability to solve specific types of problems (Macintyre & Hamilton, 2010; Törnroos, 2005; Xin, 2007).

Regarding studies on textbook analysis, some researchers primarily focused on the broader mathematical content featured in textbooks. For instance, Stylianides (2008) conducted a study focused on examining the presence of reasoning-and-proving tasks in mathematics textbooks. Another noteworthy investigation by Wijaya et al. (2015) highlighted that a mere 10% of tasks in Indonesian textbooks were context-based. Comparative studies of textbooks have also undertaken content-specific analyses; for example, in geometry content, a study by Abdullah and Shin (2019) explored how the activities in learning-related concepts exhibit variations between Korean and Malaysian textbooks. A distinction emerges as Indonesian textbooks feature more routine problems than their Australian counterparts (Hidayah & Forgasz, 2020). For the context of functions, Mesa (2004) conducted a well-known textbook analysis, classifying the typical problems used in functions along with their available techniques. While these content-specific investigations provide valuable insights, they are constrained to the types of tasks featured in textbooks and do not address the specific mathematical knowledge each task imparts.

Addressing a gap in prior studies, our approach draws upon the Anthropological Theory of the Didactic (ATD) (Chevallard, 2006; Chevallard & Bosch, 2020). According to ATD, mathematical knowledge emerges from human activities related to the acquisition of such knowledge, which can be modeled within the framework of the praxeology (Bosch & Gascón, 2014; Chevallard, 2006). Two closely interconnected elements lie at the basis of praxeology: *praxis* and *logos*. *Praxis* is defined as a human activity generated by fulfilling a specific task (T) performed by a particular technique (τ). Since human actions inherently involve reasoning, the part of the *logos* completes the *praxis*, comprised of a technology (θ) as the discourse explaining these techniques (the term derived from *techno* as technique and *logos* as knowledge; the knowledge of the technique) and a theory (Θ) as a more general justification for the technology (θ) (Chevallard, 2007; Chevallard et al., 2015). These quadruplets $[T/\tau/\theta/\Theta]$, shown in Table 1, form a praxeological organization.

Table 1. Four Elements in Praxeology

***Praxis* block**		***Logos* block**	
Type of task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
The type of problem given	A way of solving the problem	A way of explaining and justifying the technique	A way of justifying the technology that will lead to applying more abstract knowledge

Source: Adapted from Chevallard and Sensevy (2014, p. 40)

Furthermore, studies employing praxeology for textbook analysis often confine their examination to the *praxis* components. Three illustrative instances include evaluations of Indonesian textbooks concerning proportions (Wijayanti & Winsløw, 2017) and rational numbers (Putra, 2020), along with an investigation by Aoki and Winsløw (2022) focused on Japanese elementary school arithmetic. Notably, the theoretical dimension of praxeology tends to remain implicit within textbooks. This aligns with findings from González-Martin et al. (2013), whose study revealed a predominant emphasis on the *praxis* block in the praxeology found within the sampled textbooks. Explorations into the theoretical aspect of praxeology are comparatively scant but can be found in studies such as Takeuchi and Shinno's (2020) comparative study between Japan and England concerning symmetry and transformations. In the case of functions, a study by Wijayanti (2018) assessed the common learning of functions in Indonesia, provided the *logos* block that functions are presented by the naïve set-theory, and identified the potential gap of introducing functions from proportionality. Therefore, this study elaborated Wijayanti's study, that is, by conducting a praxeological analysis of how two conceptions of a function, i.e., sets or ordered pairs and relationships between magnitudes, are delivered in the Indonesian textbook (Wijayanti, 2018).

In doing the praxeological analysis of the textbook, this study also adapted different types of praxeology, which is helpful in identifying the praxeological organization of functions within the textbook. The types are classified into *point praxeology* (i.e., involve a singular type of task-technique-technology-theory), *local praxeology* (i.e., a collection of types of task-techniques organized around a shared technology and a theory), and *regional praxeology* (i.e., all types of task-techniques-technology unified by a common theory) (Bosch & Gascón, 2014). This nuanced categorization illuminates the diverse ways in which praxeological organization manifests in educational contexts, offering a comprehensive framework for understanding and enhancing the teaching and learning of mathematics.

The various constraints associated with praxeological organizations, namely, point, local, and regional, are elucidated through another construct within the Anthropological Theory of the Didactic (ATD), known as the levels of didactic co-determinacy (Artigue & Winsløw, 2010; Bosch & Gascón, 2014). This perspective underscores that the process of knowledge transposition traverses nine institutional levels, each effectively conditioning and constraining the others, as illustrated in Figure 2.

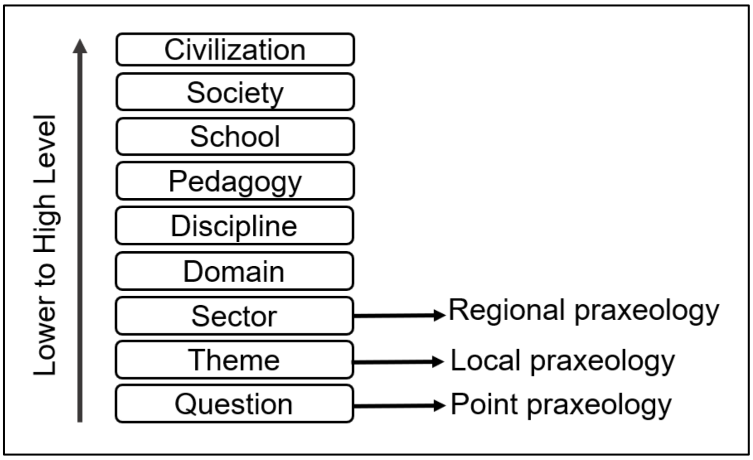


Figure 2. Levels of Didactic Co-determinacy (adapted from Artigue & Winsløw, 2010, p. 6)

This study is focused on investigating the praxeological organization across various levels as they progress from *question* to *sector*. Drawing upon insights from Artigue and Winsløw (2010), a conceptual hierarchy is established: a *question* corresponds to a point praxeology, a *theme* embodies a local praxeology, and a *sector* can engender a regional praxeology. In the specific context of this recent investigation, functions are conceptualized as a sector. For instance, functions represented as sets of ordered pairs assume the role of a *theme*, while the *question* is encapsulated by a particular problem (and associated technique) articulated within the textbook. Given that only one sector, namely functions, is scrutinized in this study, the praxeological organization will primarily focus on local praxeology.

Building upon the previous elucidation, this study's objective is to thoroughly examine the *praxis* and *logos* dimensions of praxeology in introducing functions within Indonesian secondary school textbooks. Acknowledging the constraints inherent in textbooks for adequately articulating the *logos* component within the task-oriented techniques provided, this research suggests augmenting the *logos* (theoretical discourse) through a collaborative small-group discourse involving mathematics experts and educators. This discourse envisages meticulously evaluating the suitability of tasks related to mathematical knowledge, mathematics pedagogy, and the pragmatic facets of mathematics education in schools, thereby generating an alternative praxeological framework for comprehending the function concept.

METHODS

In the process of constructing and applying praxeological analysis for this investigation, empirical data were extracted from a textbook utilized in an Indonesian lower secondary school. The choice of an Indonesian textbook serves as an exemplary instance, facilitating the formulation and examination of fundamental inquiries concerning the organization of practical knowledge intended for student engagement. This inquiry anticipates yielding two significant contributions: firstly, it furnishes a comprehensive understanding of how the concept of function is introduced in schools, particularly within the context of Indonesian education; secondly, it presents a lucid and reproducible demonstration of praxeology's application in the analysis of textbooks. This deliberate incorporation of praxeology in scrutinizing textbooks is crafted to be a valuable asset for fellow researchers aspiring to undertake analogous analyses grounded in this theoretical framework.

Education System Background

Indonesia's educational framework spans twelve years of mandatory general education divided into three tiers: six years of elementary school (for students aged 7-12), three years of lower secondary school (for students aged 13-15), and three years of upper secondary school (for students aged 16-18). Presently, the official curriculum adopted in Indonesia is the *Merdeka* Curriculum. However, amid the COVID-19 pandemic (2020-2022), the Indonesian Ministry of Education granted autonomy to Indonesian schools to choose (or integrate) from three official curricula: The Emergency Curriculum, the 2013 Curriculum, and the *Merdeka* Curriculum. In the context of this paper, textbook investigation constitutes a vital component of a longitudinal study initiated in 2022 aimed at crafting lesson plans related to the topic of functions. Consequently, the textbook analyses in this research align with the textbooks employed by our school participants. At the time of this study, the school adhered to the 2013 Curriculum.

Data Collection

In this investigation, initial data were gathered from a mathematics textbook. This textbook was the primary source for identifying the four components of praxeology: task, technique, technology, and theory. As discussed earlier, this study does not solely focus on textbook analysis; however, the chosen textbook aligns with the 2013 Curriculum. Despite not being part of the more recent *Merdeka* Curriculum, it is essential to note that teaching functions still necessitate a revision from the previous curriculum.

Moreover, the textbook used was *Matematika untuk SMP/MTs Kelas VIII Semester 1 Kurikulum 2013 Revisi* (Adinawan, 2016) (translate: *Mathematics for Grade VII Semester 1 Lower Secondary School 2013 Revised Curriculum*). It was released by a private Indonesian publisher, which has been approved by the Ministry of Education and legally distributed to be used as a learning resource in schools. Although Indonesian publishers publish multiple versions of textbooks, all adhere to the same learning objectives outlined in the official Indonesian curriculum. Thus, the selected textbook, which our school participants also use, can serve as one of the representative choices for this study. Additionally, permitting research using the textbook, the author stated a disclaimer in the introduction (in English translation): "*Constructive criticism and suggestions from users of this book are very much expected for the improvement of future publications…*" (Adinawan, 2016, p. vi).

Following the examination of data derived from the textbook, this study also engaged in a focused group discussion involving mathematics experts and educators, lasting approximately 2 hours and 30 minutes. The objective of this discussion was to address the logos aspect of the textbook, thereby mitigating potential bias in interpretation from the researchers' perspectives. The discussion was guided by a set of questions designed to elicit insights from the participants. These questions included inquiries such as: How do experts and teachers perceive the suitability of each task concerning its content, context, and the interconnection between tasks? What are the observations regarding the techniques presented in the textbook? How is the technological discourse, whether explicit or implicit, depicted in the textbook compared to the conceptualization of the function concept by mathematicians? Furthermore, in what ways does the textbook facilitate the integration of task-technique-technology to assist students in developing a comprehensive understanding of functions?

Data Analysis

The data analysis process unfolded in two distinct phases. In the initial phase, the focus was directed towards scrutinizing the practical elements of the textbook. This involved a thorough examination of the praxis components embedded within the text. Following this, the analysis transitioned into the second phase, which involved a deeper exploration of the theoretical or *logos* aspect of the textbook. Additional elaboration on the specifics of these analytical phases will be presented in the subsequent sections.

In the first phase of analysis, the focus was on identifying the practical elements outlined in the textbook. Specifically, the analysis aimed to categorize the various task types related to functions presented within the learning materials. In accordance with the textbook, eight distinct materials were identified in the introductory phase of learning functions, namely, the definition of a function, representation of non-formula-based functions, determining the number of functions generated from two finite sets, understanding one-to-one correspondence, notation and depiction of formula-based functions, exploration of functions derived from generalized patterns, graphical representation of formula-based functions, and application of functions. These materials encompass a comprehensive range of topics essential for understanding functions, providing a structured framework for learning within the textbook.

While the textbook delineates eight distinct materials for learning functions, this recent analysis has explicitly focused on the first five task types. This selective emphasis is informed by insights gleaned from a previous interview conducted with a mathematics teacher. During the interview, a participant disclosed that students were primarily exposed to the first five task types, predominantly due to time constraints. It is worth noting that data collection transpired amidst the challenging circumstances of the COVID-19 pandemic, which notably impacted available learning time. Given that this study constitutes an initial step towards designing a function learning approach tailored to the participants, the textbook analysis herein aligns with students' prior learning experiences regarding functions. However, it is acknowledged that a more comprehensive analysis encompassing all task types is slated for future investigation and elaboration.

Furthermore, based on their intended objectives, each task type comprises two types of problems: examples and exercises. Example problems are designed to illustrate concepts, and solutions are provided to guide students through the problem-solving process. Conversely, exercise problems are intended to provide practice opportunities for students and typically mirror the structure and difficulty level of the example problems. Students are encouraged to employ the strategies demonstrated in solving example problems to tackle exercise problems effectively. Given this framework, the focus of the textbook analysis in this study was primarily directed toward examining the example problems, considering them as indicative of the practical tasks students are expected to navigate.

After scrutinizing the *praxis* block, the analysis shifted towards examining the *logos* block, which entailed providing explanations, justifications, and rationales for the existing task techniques. A comparative analysis examined the textbook's definitions, examples, and properties related to functions. Similarly, the analysis aimed to uncover the theoretical discourse employed to facilitate students' comprehension of functions, assessing whether the emphasis leaned towards the traditional concept, the new concept, or a balanced integration of both. As a result, the recent analysis emphasized evaluating the coherence in organizing these theoretical elements within the textbook.

Along with these analyses, both components of praxeology investigated from the textbook will be further elaborated with the preliminary analysis of functions concepts (which have been described in the theoretical framework). This elaboration is expected to suggest improving the quality of praxeological organization in the textbook according to the didactical perspective of functions (how students should construct functions as mathematical knowledge). All stages in the research process performed in this study are depicted in Figure 3.

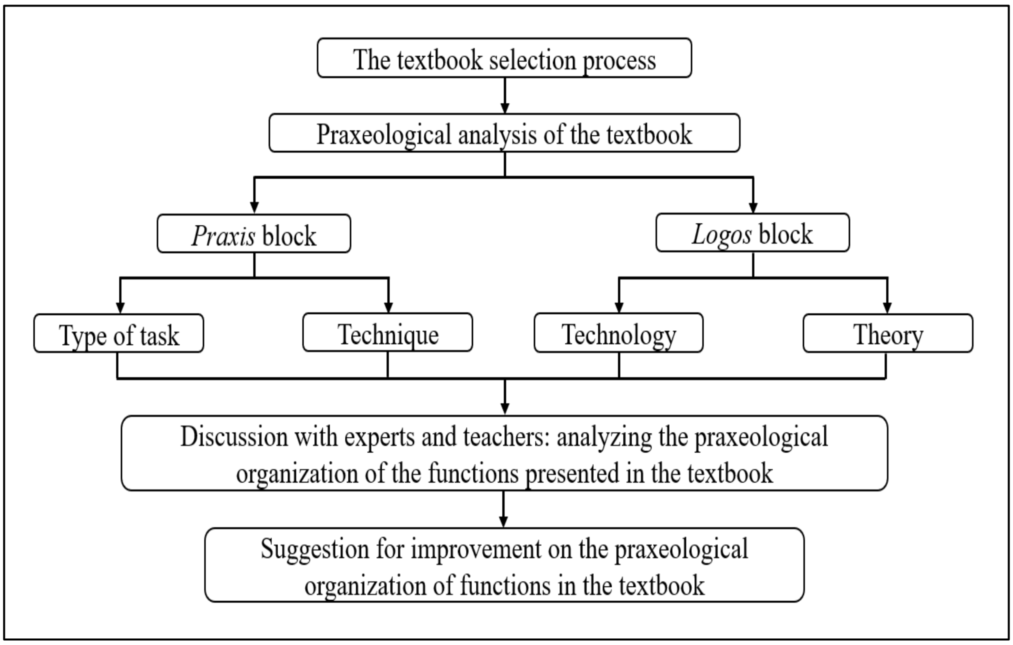


Figure 3. The overall research processes

RESULTS AND DISCUSSION

Praxis Block

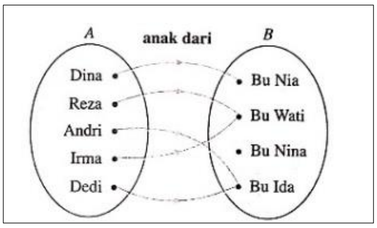
In this study, the initial examination focused on the first five task types, denoted as T_1 to T_5, within the selected textbook. Analyzing these tasks (presented as example problems in the textbook) essential in portraying students' mathematical activity, particularly as they represent the introductory tasks encountered when studying functions. Table 2 presents the five types of tasks and their respective tasks in the chosen textbook. For instance, task type 1 (T_1) comprises of two tasks ($t_{1,1}$ and $t_{1,2}$). Moreover, the sequential arrangement of tasks holds significance in this research, as the interconnectedness between tasks plays a pivotal role in facilitating students' knowledge of functions. To illustrate, the final work on each task posed in T_1 should be helpful for students in working on the subsequent tasks in T_2. By this means, the mathematical knowledge constructed by students while solving tasks in T_1 should be adequately equipped for application in solving tasks in T_2.

As outlined in Table 2, students are tasked with addressing nine primary objectives, categorized into five distinct types, to construct their understanding of the concept of a function. The task sequence commences with T_1, focusing on guiding students to comprehend and employ the function definition to classify examples and non-examples of functions. Moreover, T_1 introduces problems starting with everyday examples, which is recommended to facilitate students' comprehension of the function concept (Dauidenko, 1997). Nevertheless, relying on a single example, as depicted in task $t_{1,1}$, is less effective than exposing learners to diverse examples. It corresponds to the definition of a theory (in this study is regarded as a concept), proposed by Hamilton and Ghatala (1994), as specific attributes from an object, which distinguishes that object with others. Therefore, engaging students with a task to classify characteristics of functions and non-functions at their first encounter could deepen their understanding of the nature of functions.

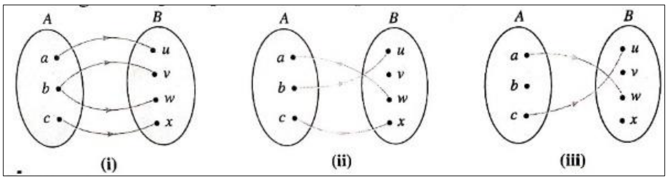
Table 2. Types of Tasks and Tasks for Introducing Functions

<table>
<tr><th>Type of Tasks (T)</th><th>Task (t)</th></tr>
<tr><td>T_1: to identify a relation as a function</td><td>

$\mathrm{t}_{1,1}$:
The picture below shows a Venn diagram of a relation “the child of” from set A (the children’s name) to set B (the mother’s name).



Describe the characteristics of both sets’ relation from the diagram.
$\mathrm{t}_{1,2}$:
Classify whether the given relation below is a function or not function.



</td></tr>
<tr><td>T_2: to represent non-formula-based functions</td><td>

t_2:
Given the set $K = \{a, b, c, d\}$ and $L = \{1, 2, 3\}$.

a. Draw a Venn diagram showing the function f, if $a \to 1, b \to 3, c \to 1$ and $d \to 3$.
b. Represent f in Cartesian diagram!
c. Represent f in sets of ordered pair!

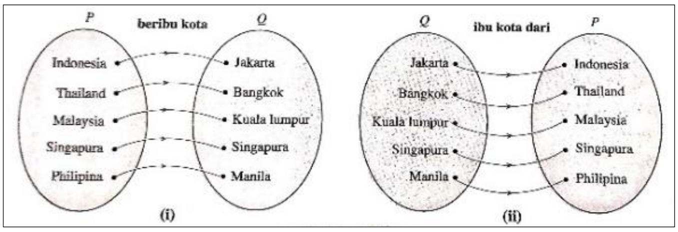
</td></tr>
<tr><td>T_3: to find a conjecture of the number of functions generated by 2 finite sets</td><td>

$\mathrm{t}_{3,1}$:
Find the formula of the number of functions that can be made from the set A to the set B by completing the following table:

<table>
<tr><th>No.</th><th>Banyak anggota A, $n(A)$</th><th>Banyak anggota B, $n(B)$</th><th>Banyak fungsi dari A ke B</th><th>Banyak fungsi dari B ke A</th></tr>
<tr><td>1.</td><td>2</td><td>1</td><td>$1 = 1^2 = n(B)^{n(A)}$</td><td>$2 = 2^1 = n(A)^{n(B)}$</td></tr>
<tr><td>2.</td><td>1</td><td>2</td><td>$2 = 2^1 = n(B)^{n(A)}$</td><td>$1 = 1^2 = n(A)^{n(B)}$</td></tr>
<tr><td>3.</td><td>3</td><td>2</td><td>$8 = 2^{-} = n(B)^{n(\_)}$</td><td>$9 = \_^{-} = n(\_)^{n(\_)}$</td></tr>
<tr><td>4.</td><td>2</td><td>3</td><td>$9 = 3^{-} = n(B)^{n(\_)}$</td><td>$8 = \_^{-} = n(\_)^{n(\_)}$</td></tr>
</table>

(translation from left to right: the number of elements in A, the number of elements in B, the number of functions from A to B, the number of functions from B to A)

</td></tr>
</table>

Type of Tasks (T)	Task (t)
	If the number of elements in the set A and B is n(A) and n(B), respectively, then: • The number of functions from A to B is $n(\_)^{n(\_)}$ • The number of functions from B to A is $n(\_)^{n(\_)}$ $\mathrm{t}_{3,2}$: Given the set $A = \{1, 2, 3, 4, 5\}$ and $B = \{x \mid 5 < x \leq 10 \in odd\ number\}$. Determine the number of functions that can be made from: a. A function from A to B b. A function from B to A
T_4: to identify a function as a one-to-one correspondence	$\mathrm{t}_{4,1}$: The picture below shows two Venn diagrams representing two relations. P is the set of country and Q is the set of capital city. The relation from P to Q is "the capital city is" (i), while the relation from Q to P is "the capital city of" (ii).  Describe the characteristics of both sets' relation from each diagram. $\mathrm{t}_{4,2}$: Find how many one-to-one correspondences that can be made if: 1) $n(P) = n(Q) = 2$, $n(P) = n(Q) = 3$, $n(P) = n(Q) = 4$. Then, based on your observation, how many one-to-one correspondences that can be made if $n(P) = n(Q) = n$?
T_5: to represent a function with an algebraic equation	$\mathrm{t}_{5,1}$: Given a function $f: x \longrightarrow 3x - 1$. Determine: a. The function formula, b. The function's value for $x = 4$, c. The image of f under $x = 5$. $\mathrm{t}_{5,2}$: A function is defined with a formula $h(x) = -2x + 5$. Determine: a. $h(n + 1)$, b. The value of a if $h(a) = -17$.

Notably, T_1 utilises non-formula-based functions as examples, denoting functions that cannot be expressed by algebraic formulas (e.g., $f(x) = y$) and solely rely on pairing elements from two sets. Consequently, T_2 tasks students with representing a non-formula-based function through different forms, starting with a Venn diagram, followed by a graphical representation and sets of ordered pairs. Moreover, the textbook, with T_3, guides students to formulate a conjecture of how many functions can be generated from two given sets, prompting them to employ this conjecture in addressing subsequent questions.

Notably, while facilitating the development of the conjecture, the textbook provides illustrative examples of functions using Venn diagrams. For instance, if set A contains one element and set B comprises two elements, the textbook depicts two distinct functions from A to B through two pairs of Venn diagrams. Despite these exemplifications, the assigned task for students is to complete a table summarizing the showcased examples in the textbook (Table 2). Once students derive their conjecture, they are expected to use it to solve the subsequent task.

Task type T_4 focuses on one-to-one correspondence or bijective functions. Initially, the context of the task introduces two sets of Venn diagrams, each portraying a function either from set P to set Q or vice versa. Employing these examples, the textbook prompts students to discern shared characteristics in both diagrams. The objective is for students to recognize that a function with distinctive attributes—wherein all elements in both sets have partners, and each element has one partner—exists, identified as one-to-one correspondence.

Furthermore, the knowledge acquired through solving task type T_1 is directly applicable to solving task type T_2, demonstrating a clear interconnection between the two tasks. Similarly, the insights gained from task type T_2 are instrumental in addressing task type T_3, highlighting a seamless flow of knowledge between these tasks. However, T_3 tasks seem solely focused on determining the number of functions generated from two finite sets and lack contribution to the subsequent tasks, specifically task type T_4, lacking a clear *raison d'être* or purpose as emphasized by Chevallard (2006).

The task types spanning from T_1 to T_4 exclusively showcase non-formula-based functions, which underscore the operational conception of a function, defined as the assignment of a quantity to another quantity (Sfard, 1991) or as an input-output assignment (Doorman et al., 2012). Within task types $\mathrm{T}_1/\mathrm{T}_2/\mathrm{T}_3/\mathrm{T}_4$, the operational conception is presented in a simplified manner, focusing on assigning values to both sets without delving into the associated processes, algorithms, or actions for transitioning from one set to another.

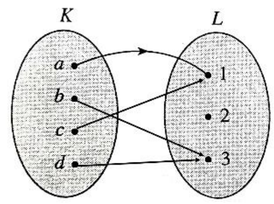
In contrast, T_5 introduces the function formula, denoted as $f(x) = y$, extending its focus to encompass both the operational and structural conception of a function (Sfard, 1991). In this context, the examples of functions in T_5 depart from the set-theoretical definition and the formation of sets with explicitly listed elements. Instead, they are expressed through variables x (representing the domain set) and y (representing the range set).

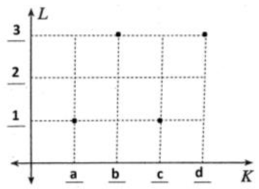
To identify the technique used in each task types, this study initiated an examination of the strategies presented in the textbook for solving each task. The objective was to systematically categorize and describe these strategies. As a general overview, it became evident that the textbook explicitly provides techniques for each task type, given that the tasks analyzed are illustrative examples tailored to aid students in constructing their understanding of function-related concepts, particularly the definition and associated properties. Subsequently, this study extended its analysis to evaluate the effectiveness of these techniques when executed by students. The findings, including a detailed description of the techniques associated with each task type, are described in Table 3.

As summarized in Table 3, the techniques employed across various tasks ranges from visuospatial and formal to numerical and symbolic approaches (Tall, 1996). The visuospatial technique hinges on the students' visual judgement. For instance, $\tau_{1,1}$ prompts students to scrutinise the Venn diagram, illustrating the relationship between two sets and subsequently identifying the characteristics of that relationship based on their visual observations. Moreover, the formal technique emphasizes functions as a correspondence between two sets, exemplified in $\tau_{1,1}$, $\tau_{1,2}$, τ_2 part c, and $\tau_{4,2}$. The numerical technique, as manifested in $\tau_{3,2}$, engages students to do numerical calculations.

Table 3. Techniques for Tasks Types Relating to Function Introduction

<table>
<tr><th>Type of Task (T)</th><th>Technique (τ)</th><th>Description of technique</th></tr>
<tr><td>T_1</td><td>$\tau_{1,1}, \tau_{1,2}$: Visuospatial and formal</td><td>$\tau_{1,1}$:
The characteristic of relation between the set A and B is described below.
• Every child has a mother, there is no child who does not have a mother. Thus, all members in the set A assign to the member in set B.
• Every child has only a mother, there is no child has more than a mother. Thus, each member in the set A assigns to only one member in the set B.

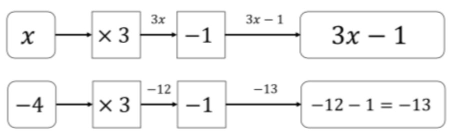
$\tau_{1,2}$:
• Picture (i) is not a function because there exists b, a member of A, which has more than one pair member in B.
• Picture (ii) is a function because each member of A has one pair member in B.
• Picture (iii) is not a function because there exists b, a member of A, which does not have pair member in B.</td></tr>
<tr><td>T_2</td><td>τ_2: Visuospatial, graphic, and formal</td><td>τ_2:
a. Venn diagram


b. Cartesian diagram


c. Sets of ordered pair
$\{(a, 1), (b, \mathbf{3}), (\boldsymbol{c}, \mathbf{1}), (\boldsymbol{d}, \mathbf{3})\}$</td></tr>
<tr><td>T_3</td><td>$\tau_{3,1}, \tau_{3,2}$: Visuospatial, numerical</td><td>$\tau_{3,1}$:
<table>
<tr><th>No.</th><th>Banyak anggota A, $n(A)$</th><th>Banyak anggota B, $n(B)$</th><th>Banyak fungsi dari A ke B</th><th>Banyak fungsi dari B ke A</th></tr>
<tr><td>1.</td><td>2</td><td>1</td><td>$1 = 1^2 = n(B)^{n(A)}$</td><td>$2 = 2^1 = n(A)^{n(B)}$</td></tr>
<tr><td>2.</td><td>1</td><td>2</td><td>$2 = 2^1 = n(B)^{n(A)}$</td><td>$1 = 1^2 = n(A)^{n(B)}$</td></tr>
<tr><td>3.</td><td>3</td><td>2</td><td>$8 = 2^3 = n(B)^{n(A)}$</td><td>$9 = 3^2 = n(A)^{n(B)}$</td></tr>
<tr><td>4.</td><td>2</td><td>3</td><td>$9 = 3^2 = n(B)^{n(A)}$</td><td>$8 = 2^3 = n(A)^{n(B)}$</td></tr>
</table>
</td></tr>
</table>

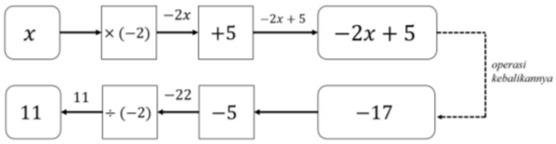
<table>
<tr><th>Type of Task (T)</th><th>Technique (τ)</th><th>Description of technique</th></tr>
<tr><td></td><td></td><td>Based on the table, the number of functions that can be made from the set A to the set B is $n(B)^{n(A)}$ and the number of functions that can be made from the set B to the set A is $n(A)^{n(B)}$.

$\tau_{3,2}$:
$A = \{1, 2, 3, 4, 5\}$, then $n(A) = 5$.
$B = \{x|5 < x \leq 10, x \in odd\ number\}$.
$B = \{7, 9\}$, then $n(B) = 2$.
a. $n(B)^{n(A)} = 2^5 = 32$ (the number of functions from A to B)
b. $n(A)^{n(B)} = 5^2 = 25$ (the number of functions from B to A)</td></tr>
<tr><td>T_4</td><td>$\tau_{4,1}, \tau_{4,2}$:
Visuospatial and formal</td><td>$\tau_{4,1}$:
In picture (1), each country is assigned to only one capital city, and each capital city is assigned to only one country (picture ii).

$\tau_{4,2}$:
<table>
<tr><th>Banyak anggota P</th><th>Banyak anggota Q</th><th>Banyak korespondensi satu-satu dari P ke Q</th></tr>
<tr><td>2</td><td>2</td><td>$2 = 2 \times 1$</td></tr>
<tr><td>3</td><td>3</td><td>$6 = 3 \times 2 \times 1$</td></tr>
<tr><td>4</td><td>4</td><td>$4 \times 3 \times 2 \times 1$</td></tr>
<tr><td>n</td><td>n</td><td>$n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$</td></tr>
</table>
(translation from left to right: the number of elements in P, the number of elements in Q, the number of one-to-one correspondences from P to Q)
Thus, if $n(P) = n(Q) = n$, then the number of one-to-one correspondences that can be made from the set P to Q is: $n \times (n-1) \times (n-2) \times \ldots . \times 2 \times 1$</td></tr>
<tr><td>T_5</td><td>$\tau_{5,1}, \tau_{5,2}$:
Symbolic</td><td>$\tau_{5,1}$:
a. The function formula of f is $f(x) = 3x - 1$
b. Two ways of determining the function value when $x = -4$
First way:
$f(-4) = 3(-4) - 1$
$= -12 - 1$
$= -13$
Second way:


c. The image of f under $x = 5$ is $f(5) = 3(5) - 1 = 14$

$\tau_{5,2}$:
a. $h(x) = -2x + 5$
$h(n+1) = -2(n+1) + 5$ (change x with $n + 1$)
$= -2n - 2 + 5$
$= -2n + 3$</td></tr>
</table>

Type of Task (T)	Technique (τ)	Description of technique
		b. Two ways of determining the value of a if $h(a) = -17$ First way: $h(x) = -2x + 5$ $h(a) = -2a + 5 = -17$ (change x with a) $-2a = -17 - 5$ $-2a = -22$ $a = -22 \div -2$ $a = 11$ Second way: 

Finally, symbolic techniques come into play when tasks involve functions expressed through algebraic formulas $f(x) = y$. Thus, a comprehensive analysis on the technique component indicates that the textbook predominantly leans towards visuospatial and formal techniques in teaching functions, aligning with the set-theoretical definition of a function.

Nevertheless, this study reveals that nearly all techniques for each task type are exclusively provided within the textbook. This observation aligns with Wijayanti's (2018) examination of the praxeological organization of linear functions in Indonesian textbooks, indicating a prevailing trend where various techniques are predominantly illustrated through examples and subsequently reappear implicitly in exercises that draw upon those techniques. This limitation guides students towards imitation rather than fostering active engagement in developing problem-solving strategies. Notably, the implementation of "technique" in the textbook does not fully correspond with Chevallard's (2019) conceptualization; a technique should involve an action, implying active participation by the individual assigned the task. Consequently, the textbook's approach risks encouraging students to memorize problem-solving procedures without cultivating a deep understanding of the underlying content, making new knowledge vulnerable to being forgotten (Ivie, 1998).

While a more comprehensive investigation of the practical block could have provided insights into the distinctive characteristics of each task and technique, this study opts for an alternative approach by focusing on the analysis of the two elements comprising the *logos* block: technology (the discourse of techniques) and theory that justifies the technology. Assessing this block in textbooks presents challenges due to its implicit appearance. To address this, the results of the *logos* block analysis in this study will be supplemented by discussions with mathematics experts and teachers. The subsequent section will delineate a detailed analysis of the *logos* block.

Logos Block

The analysis of the *logos* block serves as a discourse for the *praxis* block (Bosch & Gascón, 2014; Chevallard, 2007). Within the textbook, this study identified three technological discourses pertaining to functions, subsequently organizing the *praxis* block into three local praxeologies. The first three task types (T_1, T_2, and T_3) and their respective techniques (τ_1, τ_2, and τ_3) align cohesively within a shared technological discourse (θ_1), which is organized into the first local praxeology. The technological discourse of the first local praxeology exhibits the Dirichlet-Bourbaki's definition of a function (Vinner & Dreyfus, 1989). In their perspective, a function is conceived as a correspondence between two sets, a subset of the Cartesian product $A \times B$.

Furthermore, the rationale behind task type T_3 and its associated techniques (τ_4) finds justification in a distinct technological discourse pertaining to bijective functions (θ_2), labelled as the second local praxeology. However, the second local praxeology does not contribute to enhancing students' comprehension of functions at this level.

Lastly, the technological discourse revolving around functions as analytical expressions (θ_3) appears to underpin the rationale for justifying the last task type, T_5, and the set of its techniques, τ_5, made up the last local praxeology. In this study, 'intended' technological discourse is considered since the technology component of praxeology based on the researchers' perspective and would be further validated through discussions with experts and teachers.

Initiating the *logos* block analysis, this study undertook a comprehensive comparison of each technology (θ) outlined in the textbook with the scholarly definition, herein referred to as the formal definition, derived from the preliminary analysis through literatures. Each technology (θ) from both the textbook and scholarly sources is described in Table 4. This comparative framework serves to highlight the nuances and divergences between the textbook's portrayal of these technologies and the established formal definitions discerned from scholarly literature.

Table 4. Comparison between Textbook and Formal Definition of Each Technology (θ)

Technology (θ)	Textbook's definition	Formal definition
θ_1: Set-theoretical definition of a function	A function from the set A to B is a relation pairing each element of A to exactly one element in B	Let A and B be two sets, which may or may not be distinct. A relation between a variable element x of A and a variable element y of B is called a functional relation in y if, for all, there exists a unique which is in the given relation with x. The function refers to the operation which in this way associates with every element $x \in A$ the element $y \in B$ which is in the given relation with x.
θ_2: Bijective functions	A set A is said to be a one-to-one correspondence with a set B if each element of A is paired to exactly one element of B, and each element of B is paired to exactly one element of A. Thus, the number of members of sets A and B must be equal.	• A function $f: A \rightarrow B$ is said to be bijective if and only if f is injective $\wedge$ f is surjective. • A function $f: A \rightarrow B$ is said to be injective if and only if $\forall a, b \in A : a \neq b \Longrightarrow f(a) \neq f(b)$, • A function $f: A \rightarrow B$ is said to be surjective if and only if $\forall b \in B, \exists a \in A : b = f(a)$.
θ_3: The analytical expression of a function	If a function f assigns each x an element of the set A to y an element of the set B, then f can be written as $f: x \rightarrow y$. y is the image of x under f which can be denoted by $f(x)$,	A function of a variable quantity is an analytical expression composed in any manner from that variable quantity and numbers or constant quantities, which then denotes by $y = f(x)$. In the symbol $f(x), x$ is a quantity that can be varied arbitrarily over the domain of f. As x

Technology (θ)	Textbook's definition	Formal definition
	therefore obtaining a relation $f(x) = y$. In $f(x) = y,\ x$ is the independent variable, while y is the dependent variable.	varies, so does the number $f(x)$, but with one difference; where the value of x can be varied arbitrarily within the domain f, the value of $f(x)$ is determined once the value of x is specified. For this reason, x sometimes called the independent variable and $f(x)$ the dependent variable.

Discussions with Experts and Teachers regarding the Textbook's Praxeological Organization

Through a praxeological analysis, this study has identified that for students to comprehend functions, the textbook exhibits that functions constitute a combination of three local praxeologies. These praxeologies are designated as "locals" due to their task types and techniques sharing discourses from three distinct technology (θ) (Bosch & Gascón, 2014). These three technologies are substantiated by a single theory (Θ), specifically the theory of functions.

The praxeological organization analyzed from the textbook underwent thorough examination by mathematics experts and teachers, leading to an extensive discussion. This dialogue centered on the comparative analysis of the textbook's praxeological organization and the preliminary study of the function concept. The primary focus was on understanding how this organization could influence students' comprehension of functions, resulting in valuable insights that, in turn, contributed to recommendations for new praxeological organizations for introducing functions. The results of the discussion with experts and teachers are described as follows.

Initially, the study observed that the textbook initiates the local praxeological organization of functions by illustrating examples as pairings between sets, $P_1 = \{(T_1, T_2, T_3)/(\tau_1, \tau_2, \tau_3)/\theta_1/\Theta\}$. Subsequent discussions with experts and teachers highlighted the need for modifications to enhance this organizational approach. As described in Table 2, it is evident that task type T_1, comprising task $t_{1,1}$ and $t_{1,2}$, aims to guide students in defining the concept of a function. However, insights from experts and teachers suggest that task $t_{1,2}$ could serve as a more effective introduction since exposing new students to examples and non-examples can help them identify critical characteristics of a function. This approach contrasts with task $t_{1,1}$, which explicitly provides a single example. Moreover, there are no issues found for T_2. Nevertheless, feedback from experts and teachers indicated that task type T_3 needs to be more abstract for eighth-grade students. This task involves making conjectures that require using polynomial equations (e.g., τ_3 in Table 3), a topic beyond the student's current knowledge. In addition, the teachers argued that this type of problem is highly time-consuming. Therefore, it is recommended that this task be omitted from the praxeological organization, as it needs more meaningful learning opportunities for the students.

In contrast to the formal definition of θ_1 outlined in Table 4, the textbook's definition of a function is notably simpler. This simplification is reasonable, given that the scholarly definition of a function tends to be more abstract and potentially challenging for students to grasp. However, insights gained from discussions with experts underscored that the fundamental essence of a function lies not merely in the pairing of two sets, as presented in the textbook, but rather in "the operation or the rule" that links both sets (Kleiner, 1989). In this context, the study found that in the initial stage of introducing functions, the textbook reduces the meaning of "the operation".

Furthermore, this study analyzed the fourth task type-technique, explicitly grounded in the properties of a bijective function (θ_2), leading to the identification of the second local praxeology: $P_2 = \{T_4/\tau_4/\theta_2/\Theta\}$. A comparison with Table 4 reveals that the technological discourse (θ_2) in the textbook align relatively closely with the formal definition. However, insights from experts in this study emphasised the crucial sequence in learning bijective functions, asserting that it should ideally commence or be followed by the understanding of surjective and injective functions—two properties noticeably absent in the textbook. Teachers also proposed that introducing bijective functions might not be necessary at this stage, as these properties are typically employed in the study of inverse functions, a topic not addressed in the current learning. Consequently, a recommendation is made to exclude this task from the existing praxeological organization.

Finally, the analysis addresses the fifth task type-technique, elucidated through the interpretation of a function as an analytical expression, resulting in the identification of the third local praxeology: $P_3 = \{T_5/\tau_5/\theta_3/\Theta\}$. While the textbook presents a similar definition with the function formula $f(x) = y$, experts, and teachers contend that grasping functions in this context necessitates viewing them as a mathematical relationship between two sets, where the rule can be formulated using an expression. They argue that the absence of exposure to formula-based functions in the examples provided from T_1 to T_4 may leave students with insufficient knowledge to cope with T_5.

To address this, it is suggested that additional tasks be integrated before T_5., aiming to familiarise students with both non-formula-based and formula-based functions. Consequently, in T_5, students are guided to determine the functional relationships between quantities by making tables of values, determining the rate of change, and generating the expression of relationships using the formula $f(x) = y$. This suggestion aligns with the aspects of function proposed by Doorman et al. (2012) that at the first stage, functions are explored as an input-output assignment, which involves the execution of calculation processes. Progressing from there, the material could advance to functions as a dynamic covariation process, elucidating how changes in one variable's value can result in changes in another variable's value. Finally, the functions can be viewed as a mathematical object. Therefore, these preparatory tasks can be positioned before $t_{5,1}$ and $t_{5,2}$ (as outlined in Table 2) since $t_{5,1}$ and $t_{5,2}$ emphasize using function formulas.

The collaborative discussion with experts and teachers has resulted in formulating an alternative reference praxeological model for functions in the textbook, visually represented in Figure 4. The uncolored diagram denotes the praxeological components already present in the existing textbook, while the red-dashed diagram indicates task types that could be omitted from the model. Additionally, the grey diagram signifies components that could be introduced to enhance the completeness of the model. In this alternative representation, only two local praxeologies are identified: the task types-techniques sharing elements with functions derived from the set-theoretical definition (θ_1) and those justified by the analytical expression of a function (θ_2).

In summary, applying praxeology in textbook analysis, explicitly employing the discussion method to interpret the theoretical block, elucidates specific patterns in how the textbook structures its content to introduce the function concept. The findings of this study, aligning with those of González-Martin et al. (2013) and Wijayanti (2018), suggest that definitions, properties, and examples are predominantly presented as memorization lists, lacking an emphasis on cultivating the technological-theoretical foundations that validate the techniques employed for solving designated tasks. Moreover, there appears to be a deficiency in establishing connections between the three local praxeologies organized in the textbook.

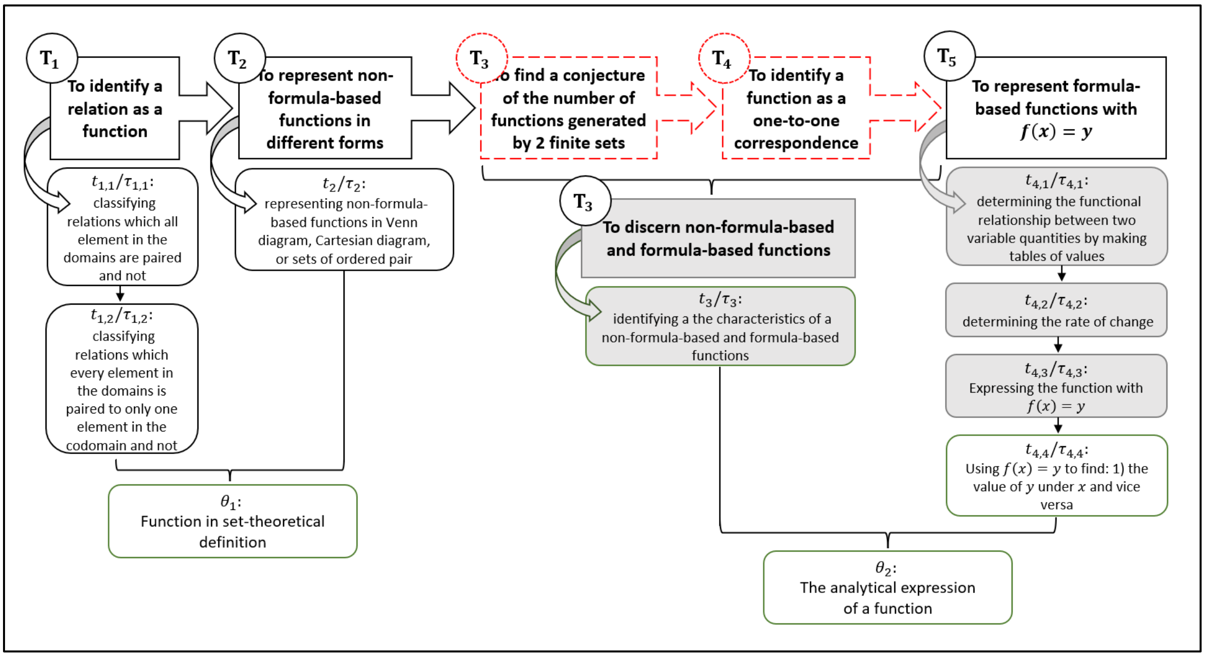


Figure 4. Alternative Praxeological Organization of Functions

CONCLUSION

This study analyzed the concept of function in the Indonesian lower secondary textbook, employing the notion of praxeology. Focusing on the *praxis* block, the analysis revealed that T_1/τ addressing the definition and examples of a function, T_2/τ focusing on function representations, T_3/τ making conjectures of the number of functions made from two sets, T_4/τ pertaining to the definition and examples of one-to-one correspondence, and T_5/τ representing functions in terms of formula. Almost all techniques for each task type are presented exclusively by the textbook, and each task is usually solved with one solution method. Subsequently, the *logos* block leads to three significant results. Regarding the first technology (θ_1), the definition of functions relies on naïve set theory as pairings between sets. The second technology (θ_2) discusses bijective functions. The last technology (θ_3) addresses functions as analytical expressions denoted by $f(x) = y$. All technological discourses are justified by a general theory, namely functions (Θ).

Various studies focus solely on analyzing textbooks at the *praxis* level of praxeology, while some have delved into the *logos* block. This paper presents an alternative approach by involving experts and mathematics teachers in discussions to achieve analysis at the level of the logos. Furthermore, the findings of this study carry significant implications for the teaching and learning of functions. An epistemological leap occurs during the transition between technologies (θ_1, θ_2, and θ_3). Hence, it is crucial to establish connections between different technologies when designing learning activities centered around the concept of functions.

Further research may be warranted to augment the findings of this study in two key aspects. Firstly, it's essential to acknowledge that this study exclusively examines the analysis of local praxeologies within a textbook. The results could be enriched by expanding the scope to include a broader level of praxeology, such as regional or global. Secondly, it is essential to extend this study to conduct a comparative analysis of how the concept of functions is taught in the classroom setting. Such an analysis could shed light on potential similarities and differences between the knowledge organized in textbooks and the actual lessons delivered in classrooms.

Acknowledgments

The authors would like to thank two mathematics education lecturers and two mathematics teachers for participating in this study.

Declarations

Author Contribution	:	NSU: Conceptualization, Writing - Original Draft, Editing and Visualization. SP: Writing - Review & Editing, Formal analysis, and Methodology. DS: Validation and Supervision.
Funding Statement	:	This research was supported by the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia under the Magister-Doctoral Fast Track Scholarships Program (PMDSU).
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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