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Toward proficiency: Developing a multiplication mathematical content knowledge test for pre-service mathematics teachers in Indonesia and Türkiye

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Abstract

This study aimed to develop a specific Mathematical Content Knowledge (MCK) test focused on multiplication for pre-service mathematics teachers in Indonesia and Türkiye. No assessment tool exists to measure MCK related to multiplication among pre-service mathematics teachers. We used convenience sampling to obtain data from third- and fourth-year pre-service teachers in Indonesian and Turkish mathematics teacher education programs. The test items were administered to 423 Indonesian pre-service mathematics teachers and 413 Turkish pre-service mathematics teachers, and their responses were analyzed using factor analysis. The MCK test was found to be reliable, with 18 items that were grouped into four components: procedural knowledge of multiplication (C1), understanding of multiple representations (C2), conceptual knowledge of multiplication (C3), and anticipating students' thinking (C4). The study results suggest that a valid MCK test helps evaluate multiplication in these four components. Additionally, the study findings indicate that Indonesian pre-service teachers score higher in the C1 and C4 categories, while Turkish pre-service teachers score higher in the C2 and C3 categories.

Keywords: Comparative Studies, Indonesia, Mathematical Content Knowledge Test, Multiplication, Pre-Service Mathematics Teachers, Türkiye

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There is increasing interest in exploring mathematics teaching knowledge, such as in teachers' courses (Max & Amstutz, 2019). Moreover, there is a growing interest in distinguishing between mathematical content knowledge and knowledge for teaching (Charalambous et al., 2020) and developing prospective teachers' mathematics teaching knowledge through studying the history of mathematics (Jankvist et al., 2020). In addition, researchers have developed tests to assess mathematical content knowledge (MCK) utilizing diverse approaches, such as multiple reasoning assessments (Kosko & Singh, 2018), and evaluate the teaching of mathematics through mathematical processes (Alsina et al., 2021).

Developing research on mathematical knowledge, as mentioned above, is closely related to Shulman's (1986) idea that teachers must master content knowledge and pedagogic content knowledge for effective teaching. In the case of mathematics teaching, Hill et al. (2004) refer to this as mathematical knowledge for teaching (MKT), which includes both subject matter knowledge (SMK) or MCK and PCK. Teachers with poor content knowledge tend to teach more rigidly and struggle to connect mathematical concepts and students, while those with strong MCK can support PCK development (Krauss et al., 2008; Wu, 2012). Furthermore, Ball, Bass et al. (2008) define MCK as "… teachers' need to know the material they teach; they must recognize when their students give incorrect answers or when the textbook gives an inaccurate definition" (p. 399). Therefore, pre-service teachers must master MCK for effective teaching. The literature suggests that mathematics teaching preparation impacts student success. (Burroughs et al., 2019).

Previous literature indicates that pre-service teachers often need help with understanding the concept of number and counting operations (Ma, 2010; Southwell & Penglase, 2005). Many studies have shown that students learn and use counting procedures repetitively (Tall & Razali, 1993). This phenomenon may be linked to pre-service teachers' MCK, often grounded in memorization instead of comprehension in teacher education. It is widely accepted that mathematics should be taught with understanding (Jankvist et al., 2020). MCK plays a critical role in pre-service teachers' mastery of multiplication.

Nonetheless, the fact is that procedural symbolism is frequently introduced in class without being connected to students' conceptual understanding (Walter & Gerson, 2007). The concept of multiplication is frequently presented as repeated addition, representing multiplication as multiple clusters of the same size (Larsson, 2016). Ma and Kessel (2018) define whole number multiplication as the product of two numbers, which yields a third number that includes as many units as one number taken or as many times as the units in the other. However, an appropriate definition of multiplication and scaling is required for the multiplication of fractions from a measurement standpoint. This may apply to all rational numbers and overcome the limitations associated with the model area and repeated addition (Sidney et al., 2019).

Some scholars use the term multiplication in the context of mathematical thinking skills and beliefs. For instance, Kosko and Singh (2018) use multiple reasoning and develop the Multiple Reasoning Assessment (MRA) to evaluate students' performance based on completing mathematical operations. Other studies employ phrases such as multiplicative thinking (e.g., Siemon et al., 2006), number sense multiplication (e.g., Ghazali et al., 2004) in real-world scenarios, and beliefs in multiplication (Novikasari & Dede, 2021). Larsson (2016) associates mastery of multiplication content with proficiency in three components: a model for multiplication, counting, and arithmetical properties. Hickendorff et al. (2019) demonstrate students' weaknesses in multiplication, including difficulties in understanding the concepts, procedural fluency, and flexible strategy selection. Therefore, it is crucial to enhance pre-service teachers' conceptual knowledge of mathematics and assist them in justifying why procedures work for fraction multiplication (Whitehead & Walkowiak, 2017). Multiplication is a fundamental mathematics concept taught to primary and middle school students. In Türkiye, the multiplication of natural numbers is introduced in Grade 2 of primary school, while in middle school, multiplication is applied to fractions, decimals, and rational numbers (MEB, 2018). Likewise, in Indonesia, multiplication is initially introduced in Grade 2 of primary school, and in Grade 6, students are taught decimal and fraction multiplication. At the middle school level, fraction, decimal, and rational-number multiplication are reintroduced (Kemendikbud, 2013). Therefore, both countries expect their middle school students to master the multiplication of real numbers. Research in mathematics education that involves cross-national contexts is vital because it can provide insight into the unique cultural and pedagogical practices in mathematics education (Cai & Ni, 2011).

Curriculum reforms in Türkiye began in 2004 and involved a shift from a subject-centered to a student-centered model of education (Babadoğan & Olkun, 2006). The new mathematics teacher education program curriculum, developed by MEB and YÖK, included courses in mathematical content, pedagogical knowledge, and general culture (Dede, 2012). In Indonesia, the curriculum changed from competence to student-centred in 2006. The changes followed the Minister of Higher Education Number 44 of 2015 regulation concerning National Higher Education Standards, which established minimum standards for graduate learning outcomes, including attitudes, knowledge, general skills, and special skills. The teacher education programme was subsequently created per established national standards, focusing on the scientific core, technological development, and higher education characteristics (Kemendikbud, 2013). After the curriculum reforms, both countries applied a student-centred constructivist approach emphasizing students' active construction of knowledge (Beswick, 2019; Zajda, 2021). The focus is on creating meaningful experiences, which is the central tenet of constructivism (Cox, 2005). Although both countries use the same constructivist approach to teach multiplication, differences in practice can be observed due to each country's social and cultural context (Vistro-Yu, 2013).

As a result, this study aims to develop a test to assess the knowledge of pre-service mathematics teachers in Indonesia and Türkiye regarding multiplication. The selection of these countries is based on several factors: i) They share the same educational philosophy of constructivism. ii) Pre-service mathematics teachers in both countries are trained to teach at the middle school level. iii) Indonesia is known for embracing Eastern cultures, while Türkiye has effectively combined both Eastern and Western cultures, creating a well-balanced blend (Dede, 2012). Kawanaka et al. (2012) suggest that cultural differences can impact teaching the same mathematical content using diverse approaches. iv) This comparison may provide valuable insights into the literature and discussion on the mastery of mathematical content knowledge in multiplication. v) No studies have examined the development of the mathematical content knowledge (MCK) test in multiplication by conducting cross-national studies in Indonesia and Türkiye in the literature on MCK multiplication.

The study by Copur-Gencturk et al. (2019) suggests that many teachers lack conceptual knowledge of the mathematics they are expected to teach, and multiplication is a problematic mathematical content area (Mack, 2000). To improve students' success, it is essential to provide thorough training to pre-service mathematics teachers (Morris & Hiebert, 2017). Therefore, further investigation is necessary to assess the mastery of mathematical content knowledge (MCK) among pre-service mathematics teachers by developing an appropriate test (Mohammadpour & Maroofi, 2023). This study aims to develop a test focused on evaluating MCK in multiplication among pre-service mathematics teachers in Indonesia and Türkiye, with the goal of designing a test to evaluate MCK in pre-service mathematics teachers regarding multiplication and examining the profiles of Indonesian and Turkish pre-service mathematics teachers on this topic.

METHODS

Sample

The sample for this study was selected through convenience sampling, which allowed for a more efficient selection of respondents in the vast regions of Indonesia and Türkiye (Jager et al., 2017). A total of 423 Indonesian pre-service mathematics teachers from eight universities in Indonesia and 413 Turkish pre-service mathematics teachers from eighteen universities in Türkiye were included in the study during the 2019/2020 academic year. Indonesian and Turkish pre-service teachers were enrolled in primary and secondary school mathematics teacher education programs. Both groups of pre-service mathematics teachers have taken similar courses in mathematics and mathematics education, such as Calculus, Introduction to Algebra, Geometry, Analysis, Mathematics Teaching Methods, Evaluation, Learning

Psychology, and Learning Technology and Material Design.

Research Design and Process

The study utilized a research design consisting of two phases– 1) determining a framework from multiplication items and 2) developing and validating the test. In the first phase, a framework was developed to evaluate pre-service teachers' knowledge of multiplication by selecting applicable multiplication items in classroom situations. According to Shulman (1986), teachers should understand a concept and its reasons. Additionally, Ball, Thames et al. (2008) suggest that pre-service teachers should be able to break down a concept to make it clear and meaningful to their students as part of their teaching assignments.

Phase 1: The framework for evaluating pre-service teachers' knowledge of multiplication was determined by considering various theories related to multiplication as content elaboration. Specifically, MCK assessment frameworks previously developed by Ball, Bass et al. (2008) and Tatto et al. (2008) were explicitly used for the multiplication topic, and the researcher formulated it as a test indicator. To examine pre-service teachers' mathematical teaching knowledge, the class situation context was presented, as suggested by Koponen et al. (2019). This context of knowledge is necessary for future teaching assignments. According to Llinares (2018), providing students' thinking can improve pre-service teachers' mathematical content knowledge.

The concept of real number multiplication, which includes both conceptual and procedural knowledge, as well as the representation issue and the connection between multiplication and number characteristics, was presented in a classroom context. Whitehead and Walkowiak (2017) state that pre-service teachers need to master essential conceptual and procedural knowledge, such as understanding multiplication concepts in various types of numbers and counting the multiplication of integers, decimals, and fractions. The real number multiplication representation issue was included in some items. as per Ball, Bass et al. (2008), mastery of the various concept models is essential in teaching mathematics. Furthermore, to examine the effect of multiplication, the connection between multiplication and numbers is crucial (Christou & Vamvakoussi, 2023), as multiplication does not necessarily result in a higher value. Initially, 22 MCK items were developed, considering the knowing, applying, and reasoning aspects. Phase 2: The test was developed and validated through multiple-stage revision with a small sample size. To begin, three mathematics education experts from Indonesia and Türkiye initiated the development of the test items' indicators, content, and context. All test items were presented in a multiple-choice format, as Haladyna (2004) suggested, for testing conceptual knowledge, procedural knowledge, and other cognitive skills. Initially, 22 MCK items were developed, and translation guides were employed in the translation process to ensure language validity, as recommended by Beaton et al. (2000).

In the first revision stage, the test was administered to two Indonesian and two Turkish experts in mathematics education and education who were proficient in Indonesian, Turkish, and English. Each expert independently translated the test from English to their respective language. In the second phase, the synthesis stage, an English language expert retranslated the test from the Indonesian and Turkish languages to English. In the final stage, the researcher analyzed the changes made during the translation process.

Factor Analysis and Item Analysis

The current study used Principal Component Analysis (PCA) to analyze the factor. The Factor Analysis program was used to determine the significance of the PCA outcome. In addition, the analysis program utilized Parallel Analysis (PA) to determine the significant components, variable loadings, and analytic statistics of response data obtained from the multiplication test. Franklin et al. (1995)suggest that the PA method accurately determines these outcomes.

During the initial analysis of the 22 items in the survey, Kaiser Mayer Olkin (KMO) and Bartlett's Test of Sphericity (BTS) were employed. The KMO score obtained was 0.592, and the BTS was 1084.9; 231, p<0.001. Kaiser (1974) suggests that a KMO score of at least 0.5 is required. However, it was decided to remove items with a factor loading basis of less than 0.30 during item analysis (Field, 2013). The factor analysis was then repeated to obtain qualifying items. After applying the Parallel Analysis method, four components were obtained for the 18 remaining items in the MCK.

RESULTS AND DISCUSSION

Factor Analysis

The item analysis method used in the study resulted in the disposal of four items with a factor loading of less than 0.30, leaving 18 items from 836 pre-service teacher respondents. The Cronbach's alpha MCK was calculated to be 0.799, which meets Hair et al. 's (1998) recommendation of a minimum acceptance limit of 0.60 to 0.70 for the internal consistency reliability coefficient.

Regarding the subscales framework of MCK, the study employed the PA and Promin rotation methods (Lorenzo-Seva, 1999) to analyze the item responses. The rotated structure matrix table (refer to Table 1) shows the procedure used to identify components in MCK items with an eigenvalue greater than 1. The final KMO value for MBQ was 0.603, and the BTS was B=974.8; 153; p < 0.001. Kaiser (1974) suggests a factor analysis with a KMO value higher than 0.50 and BTS can be performed.

Table 1. Rotated structure matrix of MCK Items

	C1	C2	C3	C4	α
M3 Rule in decimal multiplication	0.396				
M5 Routine fraction multiplication problem	0.539				
M6 Routine decimal multiplication problem	0.621				
M12 Decimal multiplication algorithm	0.389				
M16 Rule in the multiplication of negative integer and positive integer rule	0.666				0.804
M19 Rule in integer multiplication	0.606				
M22 Rule in fraction multiplication	0.500				
M4 Representation of multiplication strategy		0.773			
M7 Representation of integer multiplication		0.706			0.828
M8 Representation of fraction multiplication		0.668			
M13 Multiplication of two numbers always results in a more significant number			0.470		
M15 Correlation between decimal multiplication and fraction multiplication			0.426		0.837
M20 The result of fraction multiplication compared to its factor			0.786		
M21 The result of decimal multiplication compared to its factor			0.810		
M1 Analysis of multiplication misconception				0.599	
M10 Analysis of student's mistake in counting				0.336	
M14 Analysis of difficulty in fraction multiplication				0.586	

M18 Analysis of difficulty in decimal multiplication	0.563	0.727

Table 1 shows that the 18 items in MCK are categorized into four components. Component 1 (C1) consists of seven questions (M3, M5, M6, M12, M16, M19, M22), which are the basis of multiplication and procedural knowledge. Star and Stylianides (2013) distinguish between two types of knowledge: conceptual and procedural. Conceptual knowledge pertains to principles, definitions, and concepts, while procedural knowledge concerns recognizing symbols, rules, and counting procedures (Levin, 2018). The questions in this study for the conceptual knowledge test were designed to require pre-service teachers to select the best answer about the concept and meaning of multiplication with real numbers.

On the other hand, the procedural knowledge questions aimed to analyze the algorithms used by students, with pre-service teachers asked to calculate real number multiplications. Star (2005) suggests that students have mastered the procedure for solving these problems if they can use them automatically. Questions that fall under procedural knowledge include routine problems using standard procedures (Kablan & Uğur, 2021). The questions in C1 contain questions about routine problems and rules related to multiplication problems. For example, item M19 asks.

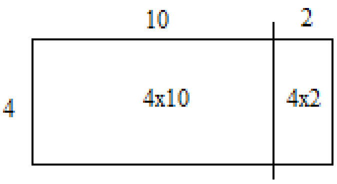
> M19: "Multiplication of 3 × 2 means 3 + 3," with answer choices AGREE, DISAGREE, or SOMETIMES TRUE.

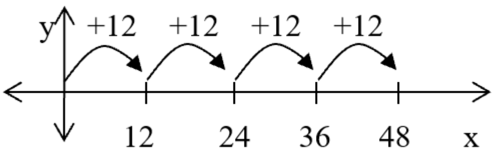
The following three items (M4, M7, M8) constitute component 2 (C2), which focuses on pre-service teachers' general understanding of the model and form of representation of real number multiplication. This component is referred to as the understanding of multiple representations by the researcher. C2 reveals that understanding the model may differ from the concept of multiplication (Larsson, 2016). It links the use of multiplication content with the mastery of the subsequent three components: models for multiplication, counting, and arithmetical properties. Hino and Kato (2019) suggests that introducing representations in multiplication can establish a preliminary understanding of proportional relationships that can be progressively applied across grade levels. For instance, an example item in C2 is M7.

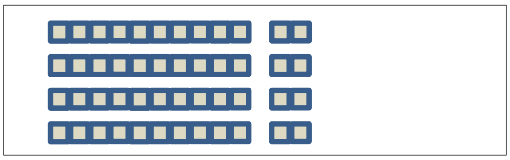
> M7: "Mr. Erdem thinks it is crucial to use various representations when teaching multiplication, such as using various images. Which model below cannot be used to demonstrate that 4 × 12 = 48?"

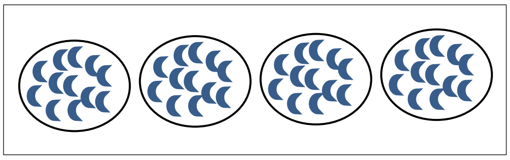
Component 3 (C3) consists of four items: M13, M15, M20, and M21. These items contain conceptual knowledge related to number multiplication. Conceptual knowledge refers to an explicit or implicit understanding of the principles governing a domain and the interrelations between knowledge pieces within that field (Rittle-Johnson & Schneider, 2014). As Star (2005) suggests, it is essential to understand principles and interrelations in this domain. An example of an item in C3 is M20.

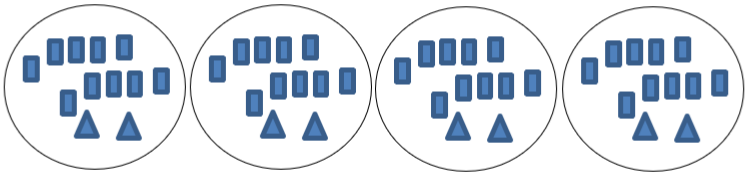
> M20: "The result of the fraction multiplication is smaller than the factor," with answer choices AGREE, DISAGREE, or SOMETIMES TRUE.

a. 

b. 

c. 

d. 

e. 

Component 4 (C4) consists of these four items: M1, M10, M14, and M18. It measures pre-service teachers' ability to anticipate student thinking, particularly regarding error analysis and anticipating students' misconceptions about multiplication content in classroom situations. As noted by Ball, Thames et al. (2008), before teachers can effectively teach material, they must be able to identify when students make mistakes. This requires both content knowledge and an understanding of how students learn multiplication. Grossman (1989) also suggests that teachers need a theoretical understanding of how students learn specific material in terms of their knowledge of the material itself. An example of an item in C4 is M18.

> M18: "Ahmet, your friend is a teacher who teaches in class 6. He teaches that 3 × 1 ≠ 30 and 0.3 × 1 = 0.30. Ali is a student who cannot understand the reason for that calculation. Choose the best suggestion for your friend."

This is about general rules of the position of the decimal sign.

a. Decimal as a base-ten fraction. I think my friend should explain that 3×1 ≠ 30 and 0,3×1 ≠ 0,30. Because 0,3 × 10 = 0,30
b. Decimal as a base-ten fraction. Ahmet should explain to the student that 3×1 ≠ 30 and 0,3×1 $=\frac{3}{10} \times 1= \frac{3}{10} = \frac{3,0}{10}$ = 0,30

c. Ahmet should explain the concept of place value in decimal numbers.
d. Ahmet should explain that place value and base-ten fraction are different concepts so the students can understand the concept of multiplication.

Indonesian and Turkish Pre-service Teachers' Multiplication Performance

Based on the factor analysis outcomes, pre-service teachers from Indonesia and Türkiye have four components that make up their multiplication content knowledge: procedural knowledge of multiplication (C1), understanding multiple representations (C2), conceptual knowledge of multiplication (C3), and anticipating students' thinking (C4). Additionally, Figure 1 displays the mean score chart for each country.

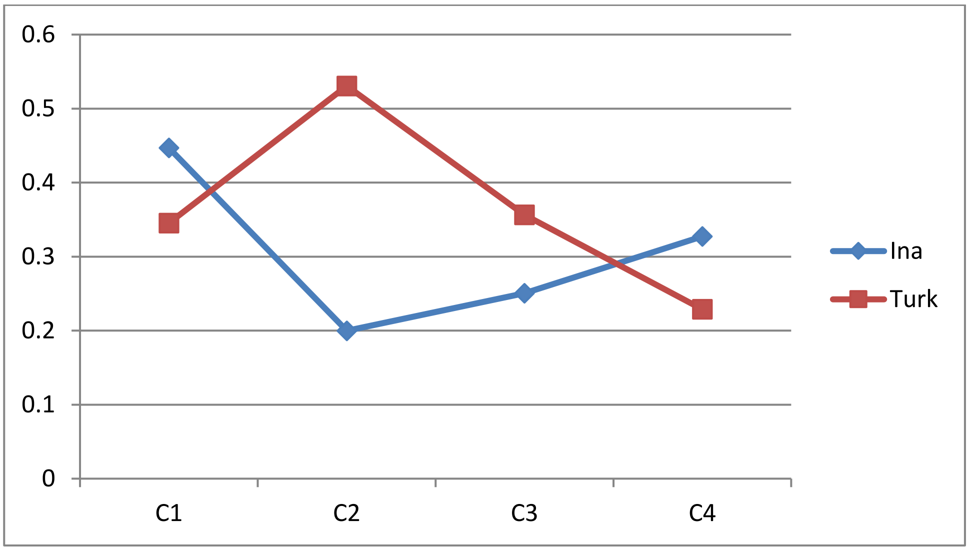


Figure 1. The mean score of items in the four components in Indonesia and Türkiye

It is clear from Figure 1 that the mean scores for C1 and C4 are higher for Indonesian pre-service teachers than for Turkish pre-service teachers. Conversely, Indonesian pre-service teachers have lower mean scores in C2 and C3 compared to their Turkish counterparts. The independent samples t-test results, conducted using a statistical package program, indicate a p-value below 0.05, indicating a significant difference between the mean scores of Indonesian and Turkish pre-service teachers in all four components: C1, C2, C3, and C4. Little attention has been given to special assessments for measuring pre-service teachers' multiplication content knowledge. Therefore, this study aims to identify the components of an assessment framework that can accurately measure mathematical content knowledge in multiplication. The study identifies four components: procedural knowledge of multiplication (C1), understanding of multiple representations (C2), conceptual knowledge of multiplication (C3), and anticipating students' thinking (C4).

The identification of C1, as Ball, Thames et al. (2008) suggested, is a general knowledge area that all pre-service teachers must master before teaching content. For example, one type of required knowledge is defining a concept. The mastery of the concept of multiplication in real numbers can be tested using an item that assesses the concept of multiplication of integers and fractions. According to Hickendorff et al. (2019), most students struggle with this question as they cannot explain the concept of number multiplication and are weak in procedural fluency. Another item in C1 assesses pre-service teachers' procedural counting capability by asking them to count fractions and decimal numbers and explain the decimal multiplication algorithm. Shulman (1986) and Max and Amstutz (2019) suggest that the teacher's ability to calculate and explain the algorithm is essential for teaching mathematics. Simon et al. (2018) found that while most students can calculate fraction multiplication, only a few can explain multiplication settlement algorithms.

The next component of the assessment framework is understanding multiple representations (C2), which requires pre-service teachers to create representations for integer and fraction multiplication. Ball et al. (2005) emphasize the importance of multiple representations in helping students understand a concept. However, research by Huang and Kulm (2012) suggests that pre-service teachers need help to create representations for algebraic expressions. Ding (2016) also shows that pre-service teachers often provide abstract number sentences without reasoning for word problem context, indicating inadequate representation and explanation indicators. However, representation is related to C1 and C2, as it concretizes the abstract nature of C1 (e.g., Venkat, 2015).

The third component is conceptual knowledge of multiplication (C3), which requires pre-service teachers to understand the validity of the reason for the relation in number multiplication, whether it generates a smaller or bigger number. Ball, Thames et al. (2008) suggest that teachers must understand mathematical topics' principles and relations. In this case, pre-service teachers connect integer multiplication with fractions and decimals. However, research by Christou and Vamvakoussi (2023) shows that students often have the misconception that multiplication always produces bigger numbers. Additionally, C3 is related to C1 (e.g., Zhang et al., 2019), as understanding calculation operations contributes to understanding number operations in the learning process.

The last component is anticipating students' thinking (C4). Ball et al. (2005) argue that teaching multiplication cannot be limited to using an algorithm for correct calculation; instead, teachers should explain, listen, check student work, analyze errors, and have reasonable hypotheses about their causes. Valanides (2000) states that teachers' inadequate content knowledge can cause misconceptions in students. C4 items present problems related to misconceptions of multiplication, analysis of the type of students' multiplication errors, and choosing a valid reason to solve difficulties in multiplication that students commonly encounter. Busi and Jacobbe (2018) provide evidence that analyzing students' work and anticipating their thinking are vital in building pre-service teachers' knowledge for effective mathematics teaching.

Based on the study, it is found that the mean scores for all components are significantly different between Indonesian and Turkish pre-service teachers. Indonesian pre-service teachers have a higher mean score in C1 and C4, indicating better basic knowledge of multiplication and error analysis in multiplication. However, they need help with representation, which is essential for understanding multiplication concepts. This finding aligns with previous research identifying a tendency in Indonesia to introduce formulas without involving the concept, leading to weak representation skills among pre-service teachers.

On the other hand, Turkish pre-service teachers have a higher mean score in C2 and C3, indicating better skills in representation and conceptual understanding of number multiplication. This finding aligns with literature suggesting that making representations and possessing conceptual knowledge are crucial teacher knowledge components that connect foundation knowledge and operational knowledge. Regarding the use of textbooks, the research finds that Indonesian textbooks emphasize more on procedural knowledge and memorization. In contrast, Turkish textbooks develop students' procedural knowledge and conceptual understanding in a balanced way. This difference in textbook design may explain the variation in mean scores between the two groups.

Finally, it is worth noting that Indonesian pre-service teachers have a higher mean score in anticipating students' thinking (C4). This skill is essential as it enables teachers to identify the source of students' errors and analyze their work effectively, leading to more effective teaching strategies, especially regarding procedural knowledge (C1).

CONCLUSION

This study aims to develop a test to measure MCK in multiplication for pre-service teachers in Indonesia and Türkiye. The test has four main components, namely C1, C2, C3, and C4. The study results show a significant difference in MCK levels between the two groups, with pre-service teachers in Indonesia demonstrating higher proficiency in C1 and C4. In comparison, their counterparts in Türkiye showed higher proficiency in C2 and C3. Although there are similarities in the curriculum reform in the two countries, emphasizing student-based learning and mathematics pedagogy, the differences in this study suggest that different practices and cultures could cause these findings.

However, studying has several limitations. First, data were obtained online and offline, which may impact the findings. Thus, further research is needed to have the same sampling method, whether entirely online or offline, to ensure the accuracy of the results. Moreover, identifying pre-service teachers' mathematic learning experiences, including conducting in-depth interviews with them and observing the teaching methods implemented in their schools, can provide a deeper understanding of what factors influence their MCK levels.

In conclusion, cross-cultural research can provide additional insight into how culture and values in a society influence pre-service teachers' mathematical understanding and abilities. By making these efforts, we can better understand MCK differences between prospective teachers in Indonesia and Türkiye and explore teaching strategies that can improve their multiplication abilities.

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Declarations

Author Contribution	:	IF: Conceptualization, Writing – Original Draft, Methodology, Data Collection YD: Validation, Supervision, Data Collection, Review and Editing, Data Collection
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