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Students' mathematics communication behavior: Assessment tools and their application

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Abstract

Mathematics communication ability is an essential component of mathematics that students should have. However, the mathematics communication ability of students, especially in Indonesia, still needs to improve. This study offers a new and different view of mathematics communication to improve it. This study aims to develop an assessment tool for students' mathematics communication to identify the problems so teachers can focus on improving those areas. Not only the cognitive domain of the students, but this study also includes assessments of the affective and psychomotor domains as well. The reason is that cognitive, affective, and psychomotor aspects are interconnected in mathematics communication. The study of these three domains is called behavior. The assessment tools consist of the mathematics communication behavior analytical rubric and appropriate mathematics test problems. This study is developmental research with three phases: the development of the analytical rubric, the development of mathematics tests, and the application. The participants in this study are two mathematics education experts and 240 students in the 8th grade from seven schools, each located in a different city. The findings of this research show that the developed assessment tools can be used to assess students' mathematics communication behavior.

Keywords: Behavior, Developmental Research, Differentiated Learning, Mathematics Communication, Measurement Tools

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Mathematics communication ability is an essential component of mathematics that students should have to share and gain knowledge from literature, teachers, and peers. This is essential because mathematics communication is a learning process in expressing and implementing mathematics ideas orally, visually, and in writing, using mathematics components (NCTM, 2000; Sari & Yuberta, 2022; Tate, 2020). From a mathematics literacy perspective, communication is one of the crucial components/indicators of mathematics literacy (OECD, 2013; 2017; 2019; 2023). Furthermore, mathematics communication is also necessary to improve students' learning outcomes and logical thinking (Syamsuddin et al., 2020). Stein et al. (1996) also said communication will affect students' reasoning ability. Moreover, Widada's (2019)

research found that mathematics communication contributes to problem-solving skills.

However, the mathematics communication ability of the students, especially in Indonesia, is still low and needs improvements. This is in line with Rusyda et al. (2020) research that found students in the calculus course have low mathematics communication ability; students needed help understanding the problem or the concept and could not apply the concept. Fatihah et al. (2022) found that students' written communication in all aspects(average) is included in the poor category, especially in using graphs, tables, and diagrams, explaining events or experimental results, and concluding. Based on this study, the reasons are that students do not know the existing concepts, do not want to try more when learning mathematics, and lack practice in presenting their answers. Rustam and Ramlan (2017) found that students' mathematics communication skills are categorized as low because students are less accustomed to providing non-routine problems related to mathematics communication indicators.

Much research has been done to improve students' mathematics communication ability. Armiati et al. (2022) developed a Local Instructional Theory (LIT) on probability topics using realistic mathematics education and found that the LIT effectively improves students' mathematics communication ability. Furthermore, Zananti et al. (2023) researched developing mathematics problems using the Bangka Belitung traditional house context to train mathematics communication skills and found that the problems/questions are valid and practical to train the students. On the other hand, Febriyanti et al. (2023) found that Padlet-based RADEC model online learning can improve students' mathematics communication ability. Problem-based learning assisted by Baamboozle (Kajori & Hendriana, 2023), flipped classroom model using digital media (Putri et al., 2023), APOS-Mathematics worksheet (Yerizon & Musdi, 2018), geometry instructional device based on Van Hiele's theory (Musdi & Andila, 2020), and react model based learning devices (Sastri et al., 2018) also enhance mathematics communication ability.

Unlike previous research, this study offered a different view of mathematics communication to improve it. This study provided assessment tools for the students' mathematics communication to know where the problem the students is, so the teacher can focus on improving that part. This study includes not only the cognitive domain of the students but also an assessment of the affective and psychomotor domains. The reason is students' affective and psychomotor domains is as important as the cognitive domain. Moreover, mathematics communication also connects cognitive, affective, and psychomotor skills. This is in line with Disasmitowati and Utami (2017) research that found the students' mathematics communication skills are related to psychomotor aspects. Furthermore, Kaur and Prendergast (2022) research also found that students' mathematics communication connected to their enjoyment and self-confidence (affective).

A study of three domains, cognitive, affective, and psychomotor, is called behavior (Rohati et al., 2022). Behavior is all physical and mental activities that lead to students to maintain a condition or change a particular state to another, accompanied by the drive to succeed (Rohati et al., 2023). Warner (2008) said that behavior is one of the components in the students' learning process that will help students grow their mathematical ideas. The students' behavior is also different; for example, based on Muir et al. (2008), there are three categories of students in solving a problem: naive, routine, and sophisticated.

Studies about the behavior of students have already been conducted for several students' mathematics skills, and there is no study about mathematics communication behavior yet. Rohati et al. (2023) explored the mathematics reasoning behavior in junior high schools using a grounded theory and found that there are four levels of students in mathematics reasoning behavior, which are imitative, algorithmic, semi-creative, and creative reasoning behavior. Gunawan et al. (2019) researched the behavior of understanding mathematics concepts of junior high schools' students and found that there

are three categories of students' behavior in understanding mathematics concepts which are instrumentalist, semi-relationalist, and relationalist. Next, Muir et al. (2008) researched students problem-solving behaviors and found three categories of students to describe mathematics problem-solving behavior, which are naive, routine, and sophisticated. On the other hand. Harisman et al. (2019) investigated students' behavior in problem solving and found the extension of Muir et al. (2008) categories in problem solving behavior, which are naive, routine, semi-sophisticated, and sophisticated. For developing assessment tools, there is research done by Rohati et al. (2022) that develop analytical rubrics to assess students' mathematical reasoning behavior and the result shows that the analytical rubric is ready to use.

In the end, as an effort to improve students' mathematics communication, this study aims to develop assessment tools for mathematics communication behavior. The assessment tools are included analytical rubrics and mathematics test. In this paper, how to use the assessment tools also showed representatively.

METHODS

This study is developmental research (Miller, 2023). It's divided into 3 phase, namely development of analytical rubric of mathematics communication behavior, development of mathematics test, and application. The details of the process can be seen in Figure 1.

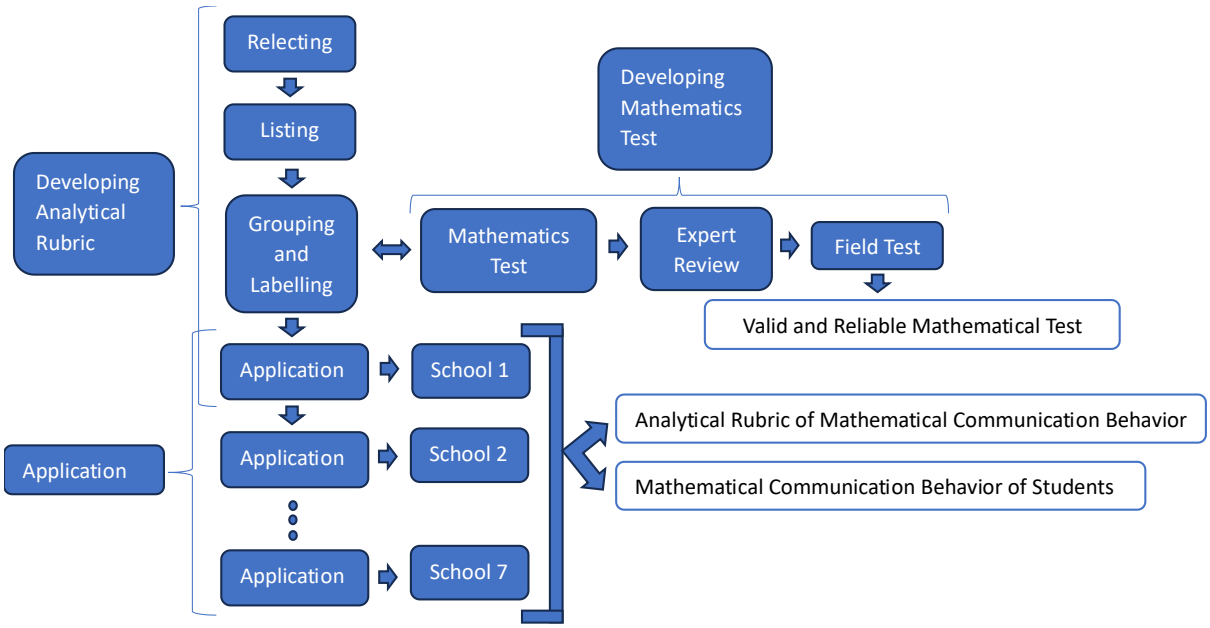


Figure 1. Method and Research Process

Research Model

The development of the analytical rubric of mathematics communication behavior will be based on Stevens et al. (2023) method, which is divided into 4 stages, which are: Reflecting (stage 1), Listing (stage 2), Grouping and Labeling (stage 3), and Application (stage 4) as shown in Figure 1. In reflecting, all of preparation is conducted, for example: the reason of development and what will be developed based on the opinion of the researcher, observation, and discussion with experts (Isaacson & Stacy, 2009).

Furthermore, the component of the mathematics communication behavior rubric needed is listed. In grouping and labeling, based on the list stage, the list is grouped and represented in the table. Lastly, the rubric will be tested to determine whether the assessment tools can define students' mathematics communication behavior in the application phase.

In developing the mathematics test, the questions are expected to cover all the indicators/elements of the analytical rubric and be able to make students show their behavior based on the indicators. The method used was a validity test and a reliability test (Sürücü & Maslakçi, 2020). The validity test method used was a bivariate test, and the reliability used was reliability analysis with the help of the SPSS application.

The application phase was conducted in all cities in West Sumatra: Padang, Pariaman, Padang Panjang, Bukittinggi, Payakumbuh, Solok, and Sawahlunto. The objectives of using several cities are to make the students varied, and the result of this study will be more representative (can be used widely). In the application, the method used was a semi-structured interview (Adams, 2015).

Participants

The participants of this study are two mathematics education experts and 240 students in 8th grade. The experts will review the analytical rubric and mathematics test developed. Experts will also suggest while the study is conducted. The students are divided into two groups: 29 for field test of the mathematics test and 211 for the application. The 29 students are from one school and one class in Padang Panjang. The 211 students are 30 from Padang Panjang, 27 from Bukittinggi, 34 from Payakumbuh, 30 from Solok, 28 from Sawahlunto, 31 from Padang, and 31 from Pariaman. In each city, the students are from one school and one class selected using purposive random sampling (Palinkas et al., 2019) to shorten the duration of the research and avoid changing the student's condition (Students expected to have the same last learning topics).

Data Collections

In developing analytical rubrics, data were collected by literature review, especially in deciding the dimensions and indicators. The dimension and indicators should be beneficial to the learning process and assessment because they should be able to explicitly describe each criterion in work (Brookhart, 2013). After the rubric was developed, two experts reviewed it before it was used to develop mathematics tests and applications, as shown in Figure 1. Furthermore, stage 4 (application) is conducted in seven schools in West Sumatra to make this rubric actual.

In developing mathematics tests, the data is the review result of the two experts and the test result of 29 8th-grade students. These students will answer the mathematics test question, and then the test result will be used to test the mathematics test's validity and reliability.

The application data were collected in several steps. First, the students will be given the mathematics test. Three students in each city were chosen based on their answers, which were different from each other, so they had different ideas for the discussion situation in the interview section. The three students selected were interviewed simultaneously with semi-structured interviews to examine the students' mathematics communication behavior. The students will be given a stimulus to make each mathematics communication behavior rubric indicator appear.

Data Analysis

In developing analytical rubrics, the rubric's evaluation used expert review to improve the rubric (Gezie

et al., 2012). The suggestions from 2 experts are analyzed. Furthermore, in application, the actual condition of students will be observed. The observation was conducted on students to see whether the indicators/components of the rubric appeared or not. The observation was conducted in the interview session.

Expert review was also conducted in developing mathematics tests, and the experts' suggestions were analyzed. Next, the field test results of 29 students were also analyzed. The student's answers will be marked on a scale of 0-4, and the result will be used to test the mathematics test's validity and reliability. Students' answer marks will be based on Table 1.

Table 1. Students' Answer Mark Guidance

Score	Students' Answer
0	No answer
1	There is an answer but completely wrong
2	Students have the idea to answer the question but not executed well
3	Students have the idea and can answer, but there is non-fatal mistake
4	Students completely correct

RESULTS AND DISCUSSION

Developing Analytical Rubric of mathematics Communication Behavior

Stage 1 Reflecting

The result of reflecting on the performance that will be assessed is mathematics communication behavior, which is the topic of this study. Furthermore, experts suggest developing a new rubric of mathematics communication behavior along with the proper mathematical test. There is no rubric about mathematics communication behavior yet, which will be an innovation in mathematics education, especially in mathematics communication. The snippet of the discussion is (E=Expert, I= Interviewer),

I :So.. about the issue of mathematics communication, we have an idea that we will try to develop assessment tools to make teachers easier in developing students' mathematics communication. Also, we add some novelty that not only assesses the cognitive aspect that some previous research did, but we also try to assess three domains of the students, which will be called the behavior of the students. What do you think about that?

E1: There are several things that we should remember. First of all, when people hear about communication, they tend to think that this is practical or psychomotor when people share their ideas or knowledge orally. Different from this, when it comes to mathematics communication, teachers tend to look at the students' answers, how they understand the problem, and the question, answer, and conclusion in their answers. These two, of course, are true, but it is better to see it not only in answering or the practical, but both. You said that you will also assess the affective of the students; this is also important; for example, the confidence of the students will affect their answers, orally or verbally. So, in conclusion, I agree with your solution.

E2: As E1 said, communication is in three domains, or it can be like that. I also agree with your solution, but of course, if you want to see the students' behavior, you should develop the rubric of the behavior because there is no rubric about this yet and try to find it(the indicator) at the actual condition of the students. Furthermore, you also need to develop

the test questions that will be able to make the indicators of behavior appear. This will also be the novelty of your research.

Stage 2 Listing

At this stage, the component/indicator of mathematics communication behavior is listed. The list is based on literature and researcher opinion, accompanied by students in 8th grade condition. Moreover, this list is based on what we hope the students will show in the assessment phase/application. The list includes domains in mathematics communication behavior and indicators of each domain. The rubric will also define the behavior into three categories of students: low, moderate, and high, which will be called self-contained (low), informative (moderate), and communicative (high) to make it easier to call and represent their mathematics communication behavior level.

Stage 3 Grouping and Labelling

The list was grouped into domain indicator and mathematics communication behavior categories. The purpose of the table is to make the rubric of mathematics communication behavior easier to use. The rubric of mathematics communication behavior can be seen in Table 2.

Table 2. Mathematics Communication Behavior Rubric

Domain	Indicator	Mathematics Communication Behavior Categories		
		Self-Contained	Informative	Communicative
Cognitive	1.Organize and consolidate their mathematics thinking through communication	Not gaining more insight/understanding during presentations or asking questions	Gain more insight/understanding but cannot convey back the understanding gained	Gain more insight/understanding and can convey the understanding gained
	2.Communicate their mathematics thinking coherently and clearly to peers, teachers, and others	Unable to communicate their mathematics thinking coherently and clearly to peers, teachers, and other people	Able to communicate some of their mathematics thinking coherently and clearly to peers, teachers, and other people	Able to communicate their mathematics thinking coherently and clearly to peers, teachers, and other people
	3.Analyze and evaluate the mathematics thinking and strategies of others	Inability to analyze and evaluate other people's mathematics thinking and strategies	Students are able to understand other people's mathematics thinking and strategies, but are unable to convey it back	Students are able to understand the mathematics thinking and strategies of other people, and are able to convey them back
	4.Use the language of mathematics to express mathematics ideas precisely	Not able to use appropriate mathematics language to convey mathematics ideas	Able to use appropriate mathematics language to convey mathematics ideas but only partially	Uses appropriate mathematics language to convey overall mathematics ideas

Domain	Indicator	Mathematics Communication Behavior Categories		
		Self-Contained	Informative	Communicative
Psychomotor	6.Using mathematics tools.	Able to use simple mathematics tools to communicate mathematics thinking (Example: Calculator).	Able to use mathematics tools on a smartphone to communicate mathematics thinking (Example: PhotoMath).	Able to use mathematics tools on a computer/laptop to communicate mathematics thinking (Example: Geogebra).
Affective	7.Self-confidence	Conveying mathematics thinking is done with nerves	Conveying mathematics thoughts is done with confidence but there is still a trembling tone	Conveying mathematics thinking is done well and confidently

The cognitive domain contains four indicators based on the National Council of Mathematics. The indicators use the National Council of Teachers of Mathematics (NCTM, 2000) for the cognitive domain. The reason for using NCTM is because the indicators can be used to assess students' communication not only based on their communication (present) but also their understanding (receiving) and the quality of arguments. For the psychomotor, students are expected to use mathematics to present their answers/ideas; for affective, the students are expected to have confidence in presenting.

After developing the analytical rubric, an expert review was conducted. Based on the expert review, the analytical rubric is good, but some revisions should be made. The experts' suggestions can be seen in Table 3.

Table 3. Expert Suggestion on Analytical Rubric

No	Suggestion
1	The presentation of some indicators is not clear
2	There is also one domain that can be included in the rubric, which is meta-cognitive

Based on the suggestions, the rubric was revised, and meta-cognitive is included, but this rubric will be called meta-communication. The metacognitive aspect was also included because it has been confirmed to be a primary factor in problem-solving (Temur et al., 2019). Meta-communication is a performance in realizing error. The meta-communication will be added to the rubric as the 5th indicator "control" below the cognitive domain, as seen in Table 4.

Table 4. Meta-Communication

Domain	Indicator	Mathematics Communication Behavior Categories		
		Self-Contained	Informative	Communicative
Meta-Communication	5.Control	Unable to realize delivery errors even though stimulus has been given	Able to realize delivery errors after being given a stimulus	Able to realize delivery errors without being given a stimulus

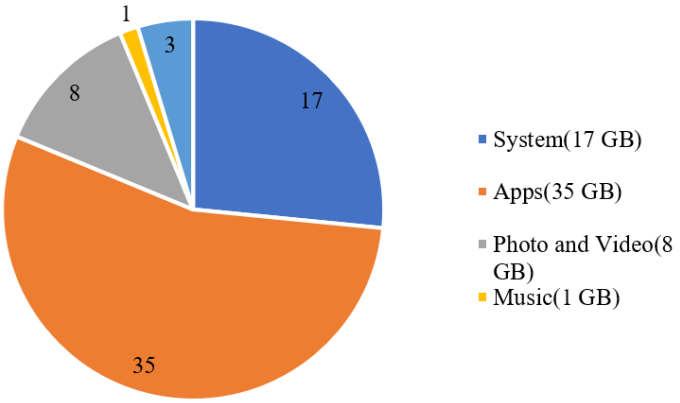
Developing Mathematics Test

The mathematics test will be used to find the variation in students' answers so the discussion in the interview can happen. The test was developed based on the rubric and expected that the test could

explore the student's mathematics communication behavior. The mathematics test questions can be seen in Table 5.

Table 5. Mathematics Communication Behavior Questions

Case	Problems
1	Pay close attention to pie chart to answer questions number 1 and 2!



To make a documentary video assignment, Andi needs a 60-minute video with an estimated size of 14 GB. Because Andi's smartphone storage was almost full, Andi had to delete several files/applications.

1. If the files or applications that can be deleted are indicated in the table below

File/App	Size
Mobile Legends	6,6 GB
PUBG Mobile	11,2 GB
Lazada	0,35 GB
Photo Album 1	0,7 GB
Photo Album 2	1,5 GB
Indonesian Learning Videos	3 GB
mathematics Learning Videos	1,85 GB
Music	1 GB

   What files/applications did you delete and explain your reasons? Show your calculations mathematically!
2. Present the contents/storage status of Andi's SmartPhone after inserting the documentary video assignment file in the form of a table and pie chart!

2	Mr Arya has several chickens and several coops. When Mr. Arya placed 1 chicken in each cage, it turned out that 1 chicken was extra and he didn't get a cage. When Mr. Arya placed 2 chickens in each cage, it turned out that he had 2 extra cages. How many chickens and cages does Mr. Arya have? Explain your answer!
3	Budi has a rectangular garden measuring 538 m x 114 m and will be planted with corn. If Budi's corn must be 0.6 m away from the next corn, illustrate Budi's garden after being planted with corn and count how many corns trees Budi planted in the garden!

After developing this test, experts reviewed the problems qualitatively and said that the mathematics test can be used in the field test for the valid and reliability test. A field test to 29 students in 8th grade has been done. The field test result was used to check the validity and reliability of the test. The result of validity test and reliability test can be seen in Table 6.

Table 6. Mathematics Test Validity and Reliability Test Result

Question Number	Score
Validity	
1	0.764
2	0.497
3	0.721
4	0.813
Reliability	
0.665	

Table 6 shows that the mathematics test is valid and reliable. This is because the score result of the test is bigger than the $t_{table,}$ which is 0.367 for 5% significance and 0.470 for 1% significance. Based on the expert review and field test, the mathematics test is good.

Application—Stage 4

Three students for each city in West Sumatra were chosen based on the variation of the answers to the mathematics test. Based on the observation and interview, decide for each student, in each indicator, what the categorization of the students is. The interviewee and the result can be seen in Table 7. (S = Self-contained, I = Informative, C = Communicative)

Table 7. Interviewee and Interview Result

City	Interviewee	Cognitive				Meta-Communication	Psychomotor	Affective
		1	2	3	4	5	6	7
Padang	Dayna	C	C	C	C	I	S	C
	Fajar	C	C	C	I	C	S	I
	Irsyad	S	S	I	S	S	S	S
Pariaman	Abdul	I	I	S	I	S	S	I
	Dhila	C	C	C	C	C	I	C
	Sherin	C	C	C	I	I	I	C
Padang Panjang	Diah	C	C	S	C	I	S	C
	Fatur	C	I	C	C	I	I	C
	Prosa	C	C	I	C	I	S	C
Bukittinggi	Janet	I	C	S	I	I	I	I
	Cika	I	I	S	C	S	I	I
	Zakiya	I	C	I	C	I	I	I
Payakumbuh	Amel	C	I	C	C	C	I	C
	Sarah	C	C	C	C	C	I	C
	Nana	C	C	C	C	C	I	C
Solok	Aisyah	C	C	C	C	C	C	I
	Revi	S	S	I	I	C	C	I
	Rifqah	C	I	C	I	C	C	I
Sawahlunto	Sania	C	I	I	I	I	S	C
	Taqiyah	C	C	C	C	C	S	C
	Viona	C	C	I	C	I	S	C

Table 7 shows all the rubric components. This also means that the assessment tools can assess or define students' mathematics communication behavior. The representatives for each indicator are:

1. Organize and consolidate their mathematics thinking through communication

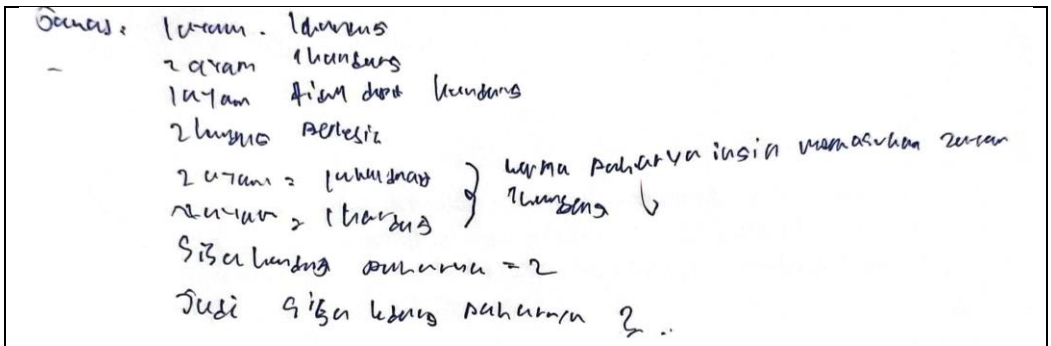
In this indicator, students will hear the presentation of other students that have different answer and try

to understand the presentation. The purpose of this activity is to measure how their understanding after listening to different/ correct answers. The result shows that in this indicator, two students are categorized as self-contained, four as informative, and fifteen as communicative.

For the self-contained category, students did not gain more understanding of the presentation, as Irsyad's behavior shows. The interviewer chose question case 2,

> "Mr. Arya has several chickens and several coops. When Mr. Arya placed one chicken in each cage, it turned out that one was extra, and he didn't get a cage. When Mr. Arya placed two chickens in each cage, it turned out that he had two extra cages. How many chickens and cages does Mr. Arya have? Explain your answer!"

and he will hear the explanation of Fajar because Fajar's answer is correct. Irsyad and Fajar's answer for case 2 can be seen in Figures 2 and 3.



Translation:
Answer: 1 chicken 1 coop
2 chickens 1 coop
1 chicken does not have coop
2 coops left
2 chickens = 1 coop] Because Mr. Arya want to put 2 chickens
1 chicken = 1 coop] 1 coop
Mr. Arya's coop left =2
Thus, Mr. Arya's coop left is 2

Figure 2. Irsyad's Answer for Case 2

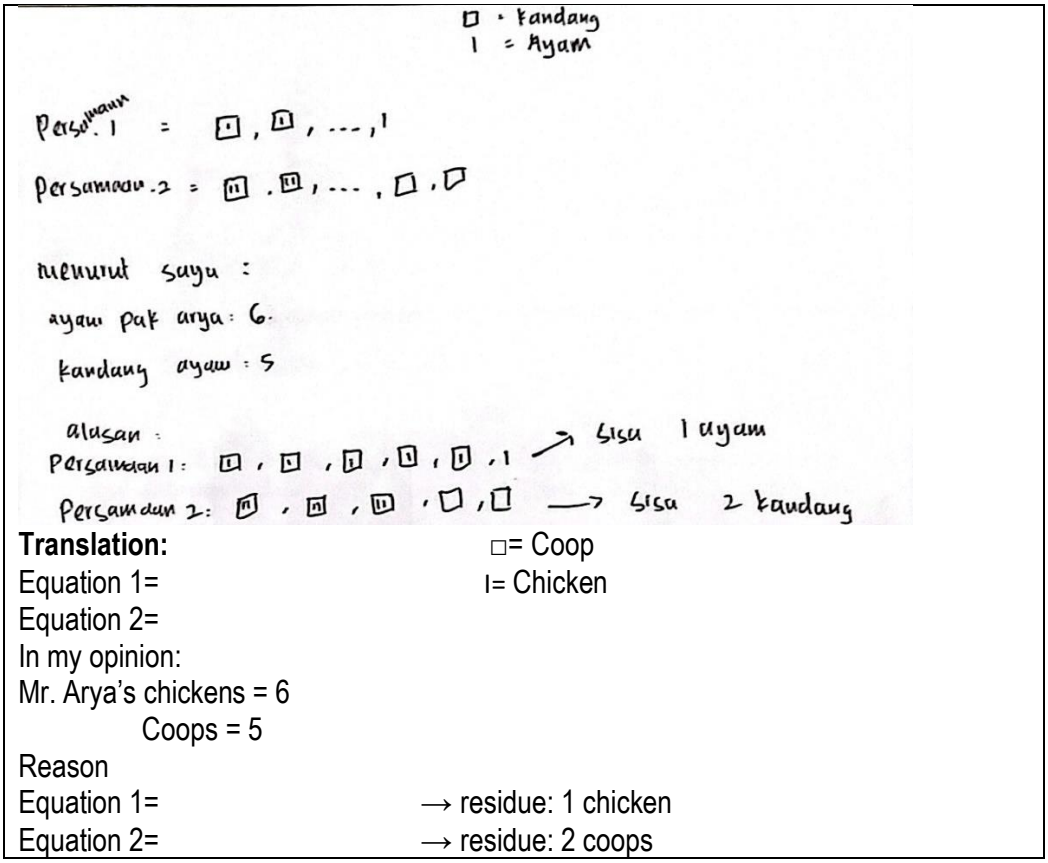
Figure 3 shows that Fajar's answer is correct although the perfect answer should be on algebraic form. The answer that expected is

> "If x is the number of chicken and y is the number of coops, then based on the given, the model is $x - y = 1$ and $y - \frac{x}{2} = 2$,
> $x - y = 1 \rightarrow y = x - 1$, then substituted it to $y - \frac{x}{2} = 2 \rightarrow x - 1 - \frac{x}{2} = 2 \rightarrow \frac{1}{2}x = 3$,
> Thus, $x = 6$, substitute to $x - y = 1$, hence $y = 5$."

Fajar chose to use trial and error with several trial and found that the chicken is 6 and the coops are 5. Before having the trial and error, Fajar said that based on the given, chicken should be even number, and coops should be odd number.

After Fajar explained his answer for case 2 to Irsyad, then the discussion happened. The snippet of the discussion is. (I=Interviewer, IR: Irsyad)

I: "What do you think about Fajar's answer, try to compare it with yours?"
IR: "I did not understand the Fajar's explanation as well as the question for case 2. I don't know how to solve it I was just answering randomly."



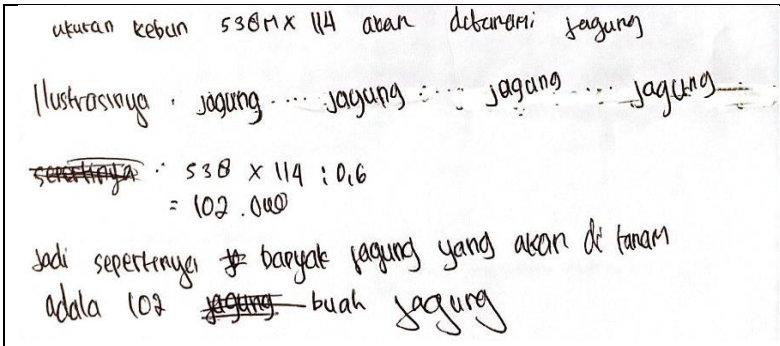
Translation: □= Coop
Equation 1= I= Chicken
Equation 2=
In my opinion:
Mr. Arya's chickens = 6
Coops = 5
Reason
Equation 1= → residue: 1 chicken
Equation 2= → residue: 2 coops

Figure 3. Fajar's Answer for Case 2

For informative category, students tend to understand other students' explanation but cannot convey the understanding gained. In this indicator Janet show this behavior. In the discussion, case 3 was chosen,

"Budi has a rectangular garden measuring 538 m x 114 m and will be planted with corn. If Budi's corn must be 0.6 m away from the next corn, illustrate Budi's garden after being planted with corn and count how many corns trees Budi planted in the garden!"

because for other case Janet, Zakiya, and Cika have relatively same answers, but unfortunately there is no correct answer between them. Thus, interviewer gave explanation for case 3. Janet's answer for case 3 can be seen in Figure 4.



Translation:
The garden dimension is 538 M x 114, which will be planted with corn.
Illustration: Corn…. Corn… Corn…. Corn
538 x 114 : 0.6
=102.000
So, the number of corns that can be planted are 102 corns

Figure 4. Janet's Answer for Case 3

Janet needed to improve in illustration, modeling, execution process of multiplication, and also the conclusion. Nevertheless, after the interviewer explained the correct answer, Janet understood. The snippet of the dialog is (I=Interviewer, J=Janet)

I: "To answer this question, the garden illustration can help us. The illustration is,

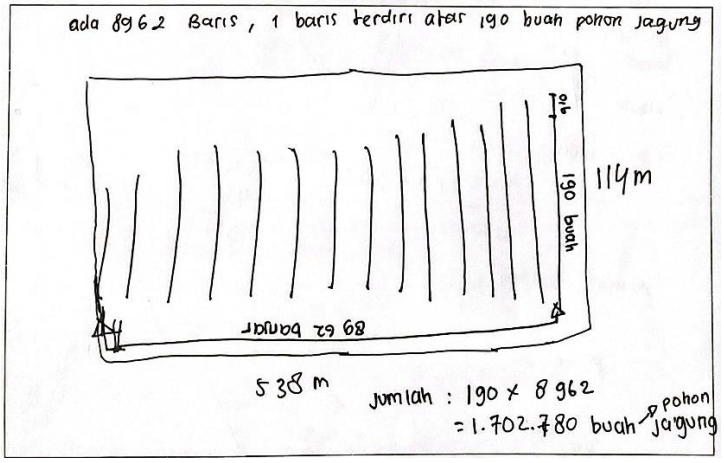
$$\begin{bmatrix} v & \cdots & v \\ \vdots & \ddots & \vdots \\ v & \cdots & v \end{bmatrix}$$ 114 m

538 m

Because the corn should be 0.6 m away from the next corn, then the number of corns should be $\frac{538}{0.6} \times \frac{114}{0.6} = 170366.667$ corn, because the corn cannot be on decimal, then the number of corns is only 170366. Do you understand?"
J: "I understand"
I: "Explain it again to me, Cika, and Zakiya!"
Janet explained it again to her friends, but, during her explanation, Janet got confused and asked interviewer.

For the communicative behavior, in this indicator Aisyah show it. In the discussion, case 3 was chosen because for other cases Aisyah, Rifqah, and Revi have relatively same answers. Aisyah's answer for case 3 can be seen in Figure 5.

For this case, Aisyah made mistake at the "8962", it should be 896.667. Then, interviewer asked Aisyah to present her understanding to her friends. Aisyah showed that she is completely understand to the problem and her solution and able to re-explain it to her friends.



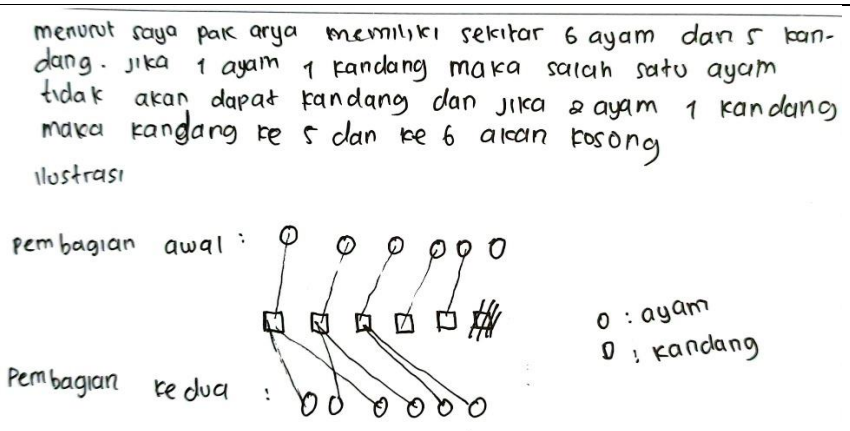
Translation:
There are 8962 lines, 1 line contained 190 corns
Total= 190 x 8962
= 1702780 corns

Figure 5. Aisyah's Answer for Case 3

2. Communicate their mathematics thinking coherently and clearly to peers, teachers, and others
In this indicator students will present the answer. Based on the explanation, interviewer will assess how is their idea presenting. Based on the study, there is 2 students that self-contained, 6 informative, and 13 communicative.

The self-contained students in this category chosen was Irsyad. Irsyad was unable to communicate his answer, based on him he said that he just answered randomly. Furthermore, Irsyad said that he also did not understand the cases.

For the informative category, Aisyah is chosen. The representative of this indicator is case 2. Aisyah's answer for case 2 can be seen in Figure 6.



Translation:
Based on my opinion, Mr. Arya has 6 chickens and 5 coops. If 1 chicken for 1 coop, then one of the chickens does not have coops, and if 2 chickens for 1 coop, then coops 5th and 6th will empty.
First distribution:
o: chicken
□: coop
Second distribution:

Figure 6. Aisyah's Answer for Case 2

In this case, Aisyah can explain her answer but only read her answer. Aisyah cannot explain why her answer ended with choosing five coops and six chickens. She just said that it comes to her mind.

For the communicative category, Fajar was chosen, and Fajar's answer that chose can be seen in Figure 2. Fajar can explain his answer entirely and systematically. First, he illustrated the coop and the chickens using □ to describe the coop and I to illustrate the chicken. Then, he illustrated/made the modeling/ equation based on the given. Because he has □ and I, he explained that he started to trial error to get the answer. Finally, he found the answer: Mr. Arya has six chickens and five coops.

3. Analyze and evaluate the mathematics thinking and strategies of others

In this indicator, students will read and understand other student's answers. After they read, they will be asked to present their understanding of their friend's ideas and strategies. Based on the study, four students were self-contained, six were informative, and 11 were communicative.

In the self-contained category, Diah showed this behavior where she could understand Fatur's answer for case 3. The informative category for this indicator is Prosa, where she understood Fatur's answer. Still, when the interviewer asked her to explain it to her friends, Prosa stopped in the middle and got confused. For communication, Fatur was understand completely Prosa's answer and can describe Prosa's idea.

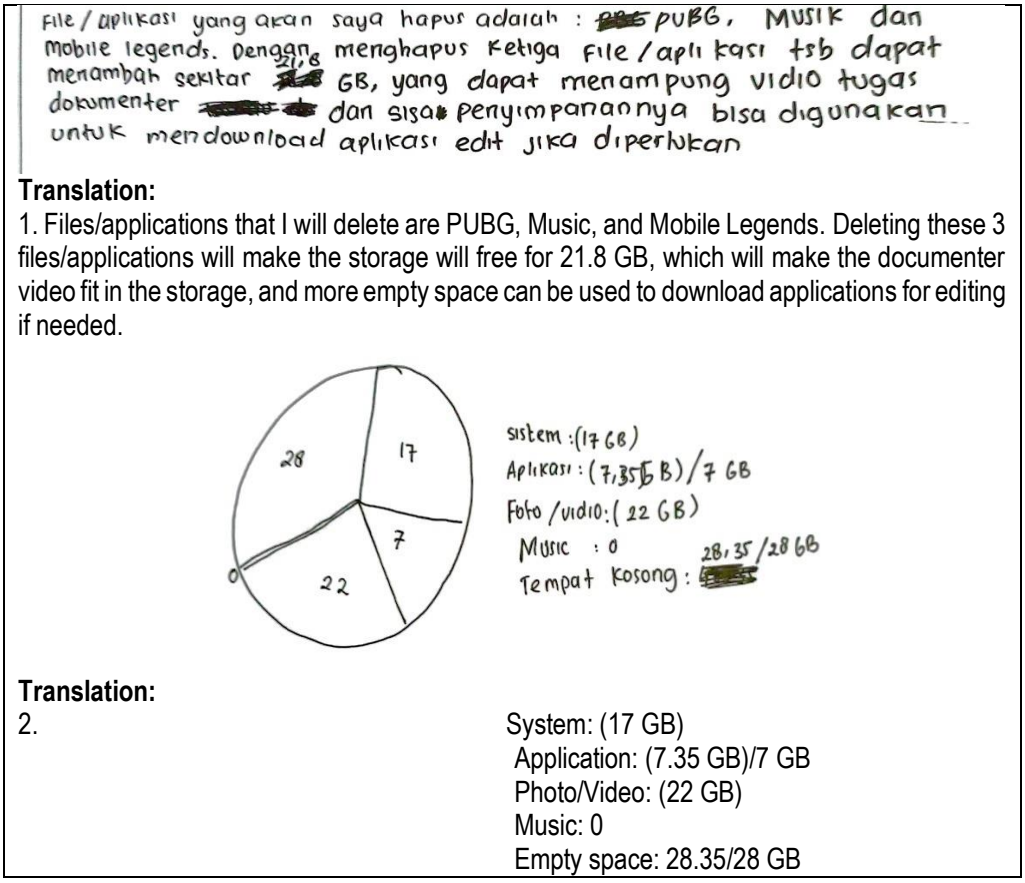
4. Use the language of mathematics to express mathematics ideas precisely

For this indicator, the interviewer uses students' language orally and verbally. The interviewer analyzes students' answers and how they present their ideas, systematical symbols, and arguments. The result shows that one student is self-contained, seven are informative, and 13 are communicative.

For self-contained, Irsyad was chosen. He said he did not understand the question and just answered randomly, as shown in Figure 1. This indicates that not only is his answer random and there is no systematical, but he also needs help understanding the question.

Furthermore, Aisyah is chosen for informative students. Aisyah cannot precisely present her answer, although she is correct. The representative of this can be seen in Figure 7. It shows that Aisyah was correct about his answer to delete three files and free the storage spaces. But for the question, too, Aisyah cannot present the pie chart. Her first mistake was that there needed to be a clear indicator of the system or other files. Second, she also needed to remember that she should have the new storage status in the table.

For the communicative category, the representative chosen is Fajar. His answer can be seen in Figure 2. From a verbal perspective, his answer is reasonable. However, there is a sentence of his method (trial and error), but orally, Fajar presented his answer well and did it systematically. His answer indicates that he is a communicative student.



Translation:

1. Files/applications that I will delete are PUBG, Music, and Mobile Legends. Deleting these 3 files/applications will make the storage will free for 21.8 GB, which will make the documenter video fit in the storage, and more empty space can be used to download applications for editing if needed.

Translation:

2. System: (17 GB)
Application: (7.35 GB)/7 GB
Photo/Video: (22 GB)
Music: 0
Empty space: 28.35/28 GB

Figure 7. Aisyah's Answer for Case 1

5. Control

In this indicator, first, the interviewer asks the students what their answer is before the discussion. If a student is wrong but there is no realization of him, then the interviewer will give him a stimulus. Irshad showed this behavior for the self-contained because all of his answers were wrong. The interview gave him a stimulus, but there was no response or better understanding of him. For information, Zakiya showed it when her answer for case 1 question 2 was wrong where she did not write down the table, and after stimuli of the interviewer, "What is a table?" she realized. Amel showed this behavior for communication when, right before the discussion, he said, "Can I change the answer of case 1? I made a mistake".

6. Using mathematics tools

In this indicator, students were asked to present their answers (case 1 question 2) using tools, smartphones, laptops, or other tools. An interesting finding is that only Solok students can use laptop tools. When the interviewer asked them "How?" they said they had already learned some tools on their laptop, including GeoGebra. In other cities, students only can use smartphones or even cannot use smartphone apps like Photo Math.

7. Self-confidence

In this indicator, the interviewer assesses through their expression and intonation. There are a lot of students who can present clearly, students who are a little nervous, and some students are nervous. The result is one self-contained, eight informative, and 12 communicative.

The result of this study shows that the rubric can be used as an assessment tool along with the

mathematics test. The rubric uses four domains: cognitive, meta-communication, affective, and psychomotor, different from previous studies with only 2-3 domains. Rohati et al. (2022) developed an analytical rubric to analyze three domains, which are cognitive, meta-reasoning, and affective. Gunawan et al. (2019), Harisman et al. (2021), and Muir et al. (2008) also used three domains: cognitive, meta-cognitive, and affective. The difference is in the psychomotor of the students. In this study, psychomotor is also included because it is essential in the learning process and communicates their idea. This is in line with Disasmitowati and Utami (2017) research that found the students' mathematics communication skills are related to psychomotor aspects. Muir et al. (2008) study also differs in the presentation of the indicators; they do not use the indicators, and the rubric directly shows the category of the student's behavior.

The result also shows that the mathematics test developed is valid and reliable. The context used for the mathematics test problem was only a contextual problem. This is because the focus on the mathematical test is how the students can show the indicator of the rubric. For this problem, the context can be varied based on the needs of the learning process. The context of the problem can be sports (Nizar et al., 2018; Yansen et al., 2019), the pandemic covid-19 (Nusantara et al., 2021), the sailing context (Efriani et al., 2019), or even the city context (Dasaprawira et al., 2019).

The teachers and researchers can use the assessment tools to assess students' behavior in mathematics communication ability. This assessment will benefit as learning, for learning, and of learning (Schellekens et al., 2021). The student assessment results can be based on differentiated learning purposes to improve students' mathematics communication ability and learning achievements (Bal, 2016). For Indonesia especially, this is in line with the need for a "Merdeka" curriculum, the learning process for the students based on the students' condition/characteristics (Hasanah et al., 2022). Finally, based on the assessment, the teachers can use tested learning strategies, like reciprocal teaching, to improve students' mathematics communication abilities (Qohar & Sumarmo, 2013).

CONCLUSION

The assessment tools (analytical rubric and the mathematics test) of mathematics communication behavior are ready. The rubric contains four domains: cognitive, meta-communication, psychomotor, and affective. The students' mathematics communication behavior category can also be defined as self-contained, informative, and communicative. The mathematics test developed is also already valid and reliable.

For further research, it is expected that the study on this topic can have more samples and cover all of the students. Furthermore, studying learning tools based on the students' mathematics communication behavior is necessary to improve students' mathematics communication. The learning tools developed are expected to have varied learning strategies, including assessing and enhancing students' mathematics communication in all domains in the analytical rubric. Lastly, the mathematics communication behavior can also be a variable of a study, and these assessment tools and how to apply them can be the guide of the researchers.

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Declarations

Author Contribution	:	EM: Conceptualization, Writing - Original Draft, Editing and Visualization. HS: Writing - Review & Editing, and Methodology. A: Formal Analysis, Visualization, and Project Administration. YH: Writing – Review & Editing, Validation, and Supervision.
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