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Designing learning trajectory to support preservice mathematics teachers' skills in creating and implementing realistic mathematics tasks

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Abstract

In mathematics teaching and learning, mathematics tasks embedded in realistic contexts are crucial for developing mathematical concepts, procedures, and the application of mathematical knowledge. Despite this, mathematics teachers often encounter challenges in designing and implementing such realistic mathematics tasks. Therefore, this study aims to construct a learning trajectory to enhance preservice mathematics teachers' abilities to create and implement Realistic Mathematics Tasks (RMTs). Employing a design research methodology, the study comprises three phases: preliminary design, teaching experiment, and retrospective analysis. The data presented in this article are from the first cycle, encompassing all three phases. The research involved four fourth-year preservice mathematics teachers from Sanata Dharma University in Yogyakarta, Indonesia, participating in a preliminary experiment conducted over three months outside their regular coursework. Data were collected through pre-course written tests, observations, interviews, journal reflections, and analyses of the preservice teachers' work. The findings indicate that the learning trajectory effectively raised awareness among preservice teachers about key characteristics of realistic mathematics tasks. This heightened awareness was evident in their reflections and work. Additionally, observations of their teaching practices revealed that preservice teachers valued student reasoning in solving RMTs. However, they faced difficulties in classroom management and guiding students to construct new knowledge actively through their experiences in solving RMTs.

Keywords: Design Research, Didactical Strategies, Preservice Mathematics Teachers, Realistic Mathematics Tasks

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Education aims to bridge the gap between theory and practice, facilitating students' ability to meet the demands of contemporary life (Nuthall, 2004). It is widely accepted that mathematics instruction should prioritize enhancing students' capability to apply mathematical concepts to solve real-world problems (NCTM, 2000). In the Programme for International Student Assessment (PISA), the Organisation for Economic Co-operation and Development (OECD) defines "mathematical literacy" as the capacity to utilize mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena across various contexts. Realistic Mathematics Education (RME) is a domain-specific instructional theory that emphasizes the use of tasks grounded in realistic situations to develop students' mathematical

concepts and procedures, as well as their ability to apply mathematical knowledge (van den Heuvel-Panhuizen & Drijvers, 2020).

Despite the significance of real-world contexts in mathematics education, Indonesian students continue to exhibit low performance in solving mathematics problems situated in real-world contexts. According to PISA results, the mathematical literacy skills of Indonesian students remain significantly below the OECD average (OECD, 2016; 2019; 2023). The PISA results released by the OECD at the end of 2022 reveal that almost no Indonesian students achieved level 5 or 6 mathematical proficiency (OECD, 2023). This indicates that these students struggle to model complex situations mathematically and to select, compare, and evaluate appropriate problem-solving strategies for such situations. In 2021, the Indonesian government addressed this issue by replacing the National Examination with a minimum competency assessment and character survey. Inspired by PISA, the minimum competency assessment measures students' reading and mathematical literacy in grades four, eight, and eleven (Pusat Asesmen dan Pembelajaran, 2020). To support this transformation, mathematics teachers in Indonesia need to facilitate learning that provides students with opportunities to develop their mathematical literacy. Thus, experiences in solving real-world tasks in mathematics classrooms are crucial for developing students' ability to formulate, employ, and interpret mathematics in various contexts (Hwang & Ham, 2021).

Various terms describe mathematical tasks situated in real-world contexts, such as "word problems" (Depaepe et al., 2010; Verschaffel et al., 2020), "story problems" (Große, 2014; Neef et al., 2003), "modeling tasks" (Blum, 2011; Borromeo Ferri, 2018; Chan, 2013; Lee, 2013), "authentic tasks" (Kramarski et al., 2002), and "context-based tasks" (Wijaya, 2015). Word problems are often seen as mathematical problems presented in words that require only simple interpretation (Niss et al., 2017), similar to story problems (Neef et al., 2003). However, other scholars (Depaepe et al., 2010; Große, 2014; Verschaffel et al., 2020) view word or story problems as opportunities for mathematical modeling, facilitating a deeper understanding of mathematical concepts. This perspective aligns with the notions of "modeling tasks" and "authentic tasks," where modeling tasks involve translating between reality and mathematics (Blum, 2011; Borromeo Ferri, 2018; Chan, 2013; Lee, 2013). Similarly, "context-based tasks" are situated in real-world settings, requiring mathematical modeling (Wijaya, 2015).

The last three terms are closely related to a concept known as realistic mathematics tasks (RMT), as conceptualized within Realistic Mathematics Education (RME) theory. In RME, the term "realistic" pertains not only to the connection with the real world but also to problem situations that are real in students' minds (Gravemeijer & Doorman, 1999; van den Heuvel-Panhuizen, 2005; van den Heuvel-Panhuizen & Drijvers, 2020). Realistic situations can refer to tasks situated in daily life, the fantasy world, or the formal world of mathematics (van den Heuvel-Panhuizen, 2005; van den Heuvel-Panhuizen & Drijvers, 2020). Realistic mathematics tasks (RMTs) contain rich contexts that require students to mathematize reality. In RME, mathematizing is the core mathematical activity, serving as a way to reinvent mathematics (Gravemeijer & Doorman, 1999). Therefore, RMTs enable students to bridge the gap between informal and formal mathematics (van den Heuvel-Panhuizen, 2005; Widjaja & Dolk, 2010). In this study, the term "realistic mathematics tasks" refers to mathematics tasks that are meaningful to students and support their mathematization process.

Mathematics teachers play a central role in adapting existing tasks by selecting and modifying them, or in designing new tasks, as well as sequencing, implementing, and evaluating these tasks (Sullivan et al., 2015). However, several studies have indicated that preservice teachers often experience difficulties in creating mathematics tasks situated in real-world contexts (Isik & Kar, 2012; Kohar et al., 2019; Şengül & Katranci, 2015). In Indonesia, where teachers cannot rely on mathematics textbooks to

provide students with realistic tasks (Wijaya et al., 2015a), teachers are expected to create their own realistic tasks. Furthermore, Wijaya et al. (2015b) found that a majority of Indonesian mathematics teachers in their study used a directive teaching approach to facilitate students in solving context-based tasks. Such teaching practices may not provide students with the opportunity to explore the didactic potential of real-world mathematics tasks. Therefore, it is essential to support mathematics teachers, particularly inexperienced or preservice teachers, in creating mathematics tasks that align with the intended learning objectives and implementing them to facilitate the process of mathematical reinvention.

To address these challenges, several studies have proposed efforts to support mathematics teachers or preservice mathematics teachers in creating real-world problems (Sevinc & Lesh, 2018, 2021; Siswono et al., 2018). Siswono et al. (2018) focused on preparing teachers to develop their problem-solving knowledge for teaching by posing problems. However, this study did not emphasize supporting teachers in gaining a deeper understanding of the characteristics of real-world mathematics tasks or enhancing their skills in implementing such tasks. Sevinc and Lesh (2018) concentrated on improving preservice teachers' understanding and skills in writing realistic mathematics problems, but their approach did not specifically aim to help preservice teachers create a set of realistic mathematics problems that promote students' understanding of a particular mathematics topic. Additionally, while Sevinc and Lesh (2021) involved preservice teachers in implementing realistic mathematics problems in a classroom setting, they did not focus on preparing preservice teachers to effectively enact these problems with students.

To address this issue, this study aimed to develop a local instruction theory that offers a framework and exemplary instructional activities for preservice teachers to create and implement realistic mathematics tasks. Local instruction theory, developed through design research, provides teachers (or lecturers) with a reference framework and instructional activities that can be adapted to the actual learning of their students for a specific topic or concept (Gravemeijer, 2004; Gravemeijer & Cobb, 2006).

In this study, teachers' skills in implementing realistic mathematics tasks (RMTs) were captured through the didactical strategies proposed for implementing RMTs. To analyze the didactical strategies that preservice teachers propose for helping students grasp formal mathematics by exploring RMTs, the researcher employed the Theory of Didactical Situations (TDS). TDS is a robust theory for describing classroom interactions in mathematics, providing unique tools for analyzing didactical phenomena that emerge during instruction (Nyman, 2017). The didactical strategies of the preservice teachers were identified within the design of didactical situations and the negotiation of the didactical contract between the teacher and the students.

METHODS

Research Methods

This research aimed to contribute to an empirically grounded instructional theory for preservice mathematics teachers in creating and implementing Realistic Mathematics Tasks (RMTs). To achieve this, the study employed a design research methodology. Design research is a research methodology focused on developing theories about the learning process and the means to support that learning (Gravemeijer & Cobb, 2006).

Research Subjects and Setting

This article focuses on the preliminary experiment of the study. Data collection was carried out at a private university in Yogyakarta, Indonesia. The primary participants were preservice secondary mathematics teachers. The preliminary experiment, conducted during the first semester of the 2022-2023 academic year, took place in laboratory settings with four preservice teachers. These teachers worked in small groups of two. This phase served as the pilot experiment, bridging the preliminary design and the teaching experiment (Jupri, 2015; Prediger & Krägeloh, 2015; Wijaya, 2008).

Phases in the First Cycle

This study followed the phases of design research proposed by Gravemeijer (2004), namely the design phase, the teaching experiment phase, and the retrospective analysis phase.

1. Design phase

In this phase, the researcher determined the learning goals, created a set of teaching activities, and formulated conjectures regarding preservice teachers' thinking and strategy. To lay the groundwork for determining learning objectives and activities, the researcher analyzed mathematics textbooks and previous studies on task design and implementation. The goals for the instructional activities were aimed at developing preservice mathematics teachers' skills in creating and implementing Realistic Mathematics Tasks (RMTs).

Following this, the researcher developed a sequence of learning activities and conjectures about the preservice teachers' responses. As preservice teachers engaged in creating and implementing RMTs, which are cognitively demanding tasks, the learning process was enriched by encouraging active social interactions. This approach allowed them to collaborate and continuously reflect on their experiences. Consequently, the instructional activities were structured around the principles of collaborative and reflective teaching practices. These practices are known to be effective in empowering preservice teachers to actively enhance their professional skills in creating and implementing RMTs.

Additionally, the study employed the problem-posing approach proposed by Stoyanova and Ellerton (1996), which consisted of structured, semi-structured, and free problem-posing scenarios as instructional tools for preservice teachers. In structured scenarios, preservice teachers were asked to develop a realistic mathematics task based on a given real-world context. In semi-structured scenarios, they were tasked with reformulating a word problem with a camouflage context into a realistic mathematics task. In free scenarios, preservice teachers were encouraged to create a realistic mathematics task using a context of their choice. The incorporation of these diverse problem-posing scenarios aimed to enhance preservice teachers' experiences in creating RMTs across various contexts.

Furthermore, guided by the principles of Realistic Mathematics Education (RME) and the characteristics of RMTs, the researcher selected several real-life contexts for problem posing. The characteristics of RMTs also served as guidance in formulating preservice teachers' reflection forms to analyze the quality of their posed problems. Additionally, the RME principles guided the researcher in designing learning activities to help preservice teachers formulate a lesson plan for implementing RMTs.

Three main ideas were translated into eight activities, as shown in Figure 1. These activities were expected to be completed within ten meetings, each lasting approximately 2.5 hours. Prior to the teaching experiment, the researcher conducted a pre-course written test and pre-course interview. The pre-course written test aimed to identify preservice teachers' initial conceptions of realistic mathematics tasks. Meanwhile, the pre-course interviews aimed to identify preservice teachers' prior learning experiences that might shape their conceptions of RMTs, their prior experiences in creating RMTs, and their initial

conceptions of RMTs. Moreover, after the teaching experiment, the researcher conducted post-course interviews to gain insight into how the learning activities developed preservice teachers' competencies in creating and implementing RMTs.

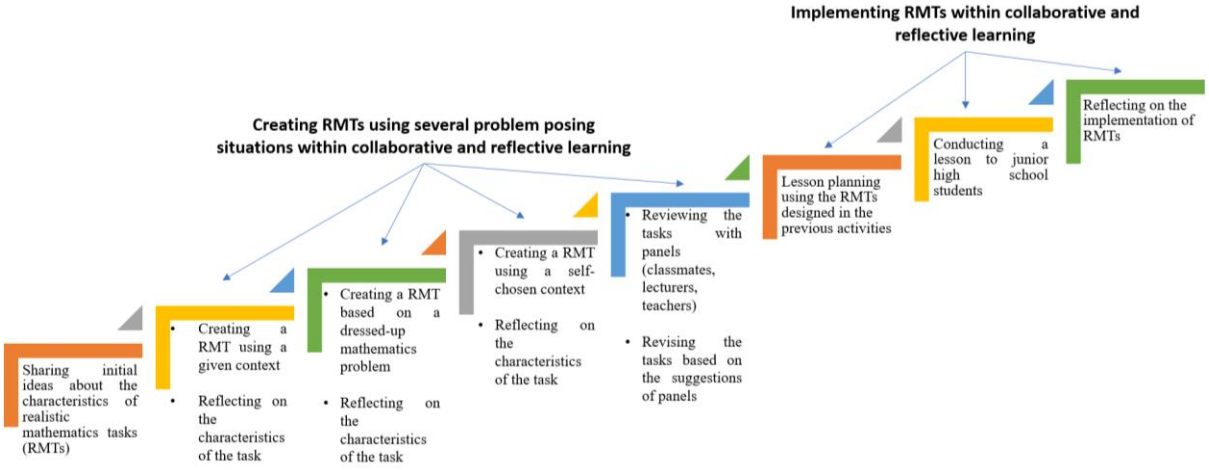


Figure 1. Hypothetical Learning Trajectory

2. Teaching experiment phase

The teaching experiment phase in design research involves all activities in which the researcher directly interacts with the participants. During this phase, the set of instructional activities was implemented, revised, and redesigned (Bakker & van Eerde, 2015; Gravemeijer & Cobb, 2006). Consequently, the Hypothetical Learning Trajectory (HLT) was subject to change throughout this phase. The HLT served as a guide for the researcher and the teacher, indicating what to focus on in teaching, observing, and interviewing (Bakker & van Eerde, 2015).

3. Retrospective analysis phase

The primary goal of retrospective analysis is to contribute to the development of a local instruction theory (Gravemeijer & Cobb, 2006). Therefore, during this phase, the researcher analyzed the entire data set gathered during the teaching experiment.

Methods of Data Collection

To capture the implementation of the instructional activities and the development of teachers' skills in creating and implementing RMTs, the researcher employed multiple instruments. Data were collected through pre-course written tests, observations of preservice teachers' teaching practices, interviews, journal reflections, and analysis of preservice teachers' work.

Analysis of Data

The researchers analyzed the data from the different sources qualitatively. Specifically, in analyzing pre-course written tests and preservice teachers' works, the researchers employed several criteria to assess the quality of tasks created by preservice teachers, which were considered as the characteristics of RMTs. The characteristics of RMTs are outlined as follows.

1. The authenticity of the contexts

Mathematics tasks situated in real-world contexts should possess authenticity (Stacey, 2015). Regarding the authenticity of the real-world context, Wijaya et al. (2015a) differentiate between two types of contexts, namely "camouflage context" and "relevant and essential context".

2. Supporting the process of mathematization

Solving realistic mathematics tasks requires students to transform the given situation into a mathematical expression through the process of mathematization. Therefore, the tasks should clearly incorporate mathematization activities to facilitate students' construction (van den Heuvel-Panhuizen, 2005). The opportunity for mathematization can also be realized by formulating a task into an open-ended task (van den Heuvel-Panhuizen, 2005).

3. Plausibility and sufficiency of the information

Well-designed tasks should be plausible. Plausible problems refer to those that do not contain conflicting information or invalid assumptions (Bonotto, 2013). By ensuring plausibility, it becomes possible to determine whether the problem provides sufficient information or not.

4. Reading demand

Mathematics tasks situated in real-world contexts require reading comprehension (Leiss et al., 2019). Teachers can use a visual representation of the context to assist students in handling tasks that involve extensive reading (van den Heuvel-Panhuizen, 2005). Visual representations serve as vehicles for contextual understanding, helping students visualize the situation and fostering their engagement with the task (van den Heuvel-Panhuizen, 2005).

5. Cognitive demand

Realistic mathematics tasks also require students to demonstrate their ability to tackle a diverse range of cognitive demands. According to OECD (2019), the cognitive demands of mathematics tasks can be categorized into three clusters: reproduction, connection, and reflection. In mathematics instruction, tasks falling under the connection and reflection cluster should be included to support students in developing competence beyond procedural and conceptual knowledge.

Furthermore, the researchers evaluated the quality of the solutions provided by preservice teachers for their designed tasks. To analyze these solutions, the researchers differentiated three solution steps, as employed in the study of Hartmann et al. (2021): mathematical model, mathematical result, and interpretation.

Additionally, analyzing the preservice teachers' teaching practices provided insights into the didactical strategies they proposed for facilitating students' learning. To this end, the researchers employed the Theory of Didactical Situations (TDS). TDS is a powerful theory used to capture the dynamics of mathematics classrooms because it offers unique analytical tools to study didactical phenomena that arise during instruction (Nyman, 2017). The didactical strategies of the preservice teachers were identified by investigating the design of didactical situations and the negotiation of the didactical contract between the teachers and the students.

RESULTS AND DISCUSSION

The result of this study indicated that the learning trajectory played an important role in supporting preservice teachers' skills in creating and implementing RMTs as they progressed through different learning activities. There were eight learning activities completed within the teaching experiment phase that were carried out in 10 meetings, each lasting approximately 2.5 hours. First, the researcher explored preservice teachers' initial ideas about the characteristics of RMTs by asking them to solve and analyze several mathematics tasks with different types of contexts, information provided, and also cognitive demands. This first activity aimed to help preservice teachers to conceptualize the characteristics of RMTs. Second, the preservice teachers collaboratively created RMTs using different problem-posing situations. During the process, they reflected on their experiences in creating RMTs. Third, the RMTs were reviewed by the researcher as the facilitator, classmates, and a junior high school teacher. After the review process, each group revised their RMTs based on the comments and suggestions given. Fourth, the preservice teachers were guided to formulate a lesson plan accommodating RMTs as a source as well as a starting point for teaching mathematics. To prepare for this, the researcher engaged the preservice teachers to discuss classroom norms, formulate conjectures regarding potential students' answers, and discuss several didactical strategies that can be employed when students are engaged in solving RMTs. Next, the preservice teachers conducted the lesson with junior high school students. Lastly, they reflected on their experiences in implementing RMTs. The findings gathered in the teaching experiment and retrospective analysis phases are presented in detail below.

Findings from the Teaching Experiment

Prior to the teaching experiment, the researcher administered a written test to evaluate the preservice teachers' initial conceptions of realistic mathematics tasks, as reflected in the tasks they created. Each preservice teacher was required to develop three mathematics tasks, each targeting different problem-posing skills: structured, semi-structured, and free problem-posing. In terms of context authenticity, 91.7% of the tasks developed by the preservice teachers were categorized as having camouflage contexts. Although these tasks were situated in real-world scenarios, they were considered to have camouflage contexts because the contexts could be disregarded when solving the problems. Essentially, these tasks were bare mathematics problems "dressed up" in a real-world guise, not requiring students to mathematize the situations since the mathematical operations needed to solve the tasks were evident. Only one task (8.3%) fell into the category of tasks with a relevant and essential context. This result may indicate the preservice teachers' initial conceptions of RMT, where they perceived every mathematics task presented in a word problem as realistic.

The findings from the pre-course interviews supported this result, revealing that the preservice teachers had limited learning experiences in solving realistic mathematics tasks. Most of the problems they encountered in school were bare mathematics problems or word problems with camouflage contexts. Regarding the cognitive demand of the tasks, reproduction and connection tasks were equally prevalent, each accounting for 50% of the tasks, with reflection tasks entirely absent. In terms of the type of information provided in the tasks, the majority were tasks with missing information (83.3%), while the remaining tasks had matched information. These findings underscored the preservice teachers' initial understanding and approaches to creating realistic mathematics tasks, highlighting the need for instructional activities to address these gaps.

1. Preservice Teachers' Skills in Creating RMTs

To prepare preservice teachers to create RMTs, the lecturer guided them to share their initial ideas about the characteristics of RMTs by analyzing several mathematics tasks. Four tasks, each representing

different types of contexts, information provided, and cognitive demands, were chosen for this activity. By exploring these tasks and discussing their characteristics, the preservice teachers began to understand that realistic mathematics tasks require reasoning about the context or daily life experiences. They also realized that realistic mathematics tasks do not contain explicit mathematical operations needed to solve the problem, nor do they include missing or superfluous information.

After analyzing and discussing the characteristics of various mathematics tasks, the preservice teachers participated in problem-solving activities focusing on three different problem-posing skills: semi-structured, structured, and free problem-posing. In semi-structured problem posing, the researcher provided a specific context to each group and asked them to generate a realistic mathematics task from that context. In structured problem posing, the researcher gave two typical word problems commonly found in school mathematics textbooks. An example of a common mathematical problem is illustrated in Figure 2.

> It is known that the price of 2 pens and 3 books is Rp 7,100. The price of each pen is Rp 1,300. How much does 1 book cost?

Figure 2. Common word mathematical problems given in structured problem-posing

In this activity, the researcher instructed the preservice teachers to reformulate the given problems and generate new mathematical tasks that significantly differ from the original ones, adhering to specific criteria such as increased complexity, realism, and engagement for junior high school students. Additionally, during the free problem-posing phase, each group was required to select a realistic context and develop a corresponding mathematics task. The preservice teachers were explicitly reminded that their tasks should be original and not replicated from other sources.

The preservice teachers were given the opportunity to present their initial tasks, receiving comments and suggestions from other groups. The researcher also provided feedback during these discussions. Additionally, each group was asked to complete a reflection journal, guided by several prompts, to explore their experiences in creating Realistic Mathematics Tasks (RMTs). Furthermore, the researcher invited a junior high school mathematics teacher to review and offer comments or suggestions on two tasks created by each group of preservice teachers.

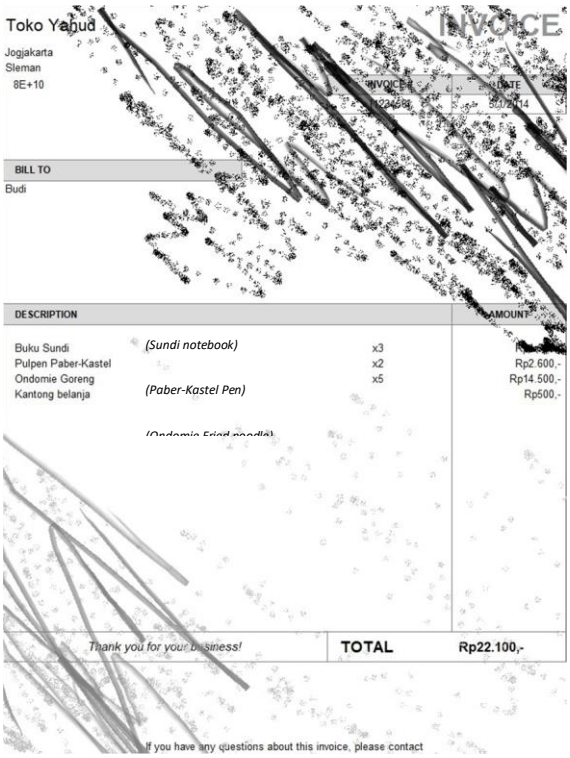
The analysis of the preservice teachers' work indicated that the tasks created by both groups adhered to the characteristics of RMTs. An example of one such RMT designed by the preservice teachers is illustrated in Figure 3.

The example of the RMT presented above was compatible with mathematical principles, plausible, provided sufficient information, and contained comprehensible text. Additionally, the task met the specific characteristics of RMTs, including an essential and relevant context, higher cognitive demand, and variability in the amount of information provided, which could be either more or less than required to solve the task. The solution steps provided by the problem posers demonstrated the ability to develop an accurate mathematical model, achieve correct mathematical results, and offer a clear interpretation.

These findings were consistent with the responses documented in the preservice teachers' reflective journals. One group emphasized that the primary consideration in creating RMTs was ensuring that the contexts were grounded in students' personal experiences or in situations commonly encountered in daily life. Similarly, the other group highlighted several key considerations for creating RMTs: the ability of students to visualize the given context, the coherence and meaningfulness of the context, and the presence of a problem that instills a sense of urgency, thereby motivating students to find a solution. The

reflections from both groups demonstrate their profound understanding of the essential characteristics of RMTs.

Ani's father bought Ani 2 Peber-Kastel brand pens and 3 Sundi brand notebooks at the prices stated on the receipt.



Since the receipt is dirty, some parts of the writing cannot be read clearly. Determine the total amount of money Ani needs to prepare if she wants to buy 3 Peber-Kastel brand pens and 5 Sundi brand notebooks for herself!

Figure 3. The example of RMT created by Group 2

2. Preservice Teachers' Skills in Implementing RMTs

After creating the RMTs, the lecturer facilitated a discussion on classroom norms that foster a conducive learning environment among the preservice teachers. Additionally, the lecturer encouraged the preservice teachers to anticipate students' strategies for solving the RMTs and to explore various didactical strategies to support students during problem-solving activities. Subsequently, the lecturer guided the preservice teachers in formulating an 80-minute lesson plan aimed at enhancing students' mathematical learning through the exploration of RMTs. The principles of Realistic Mathematics Education (RME) served as the key reference in developing the lesson plan. The teaching practices were conducted at a private junior high school in Yogyakarta, with one group implementing the lesson plan in 7th grade and the other group in 8th grade.

Based on the observation during the implementation of RMTs in the mathematics classroom, the researcher identified several didactical strategies that facilitate students learning mathematics by exploring RMTs. These didactical strategies are outlined as follows:

a. The design of didactical situations
   - Reviewing students' prior knowledge
     Both groups encouraged students to revisit previous mathematical concepts that were relevant to the upcoming lessons. This approach was intended to help students establish connections between their prior knowledge and the new topic, as well as to identify and address any misconceptions they might have.
   - Using the context that can be imagined by students
     As outlined in the lesson plan, the preservice teachers initiated the learning process by presenting RMTs to their students. These tasks were framed in contexts that were relatable for junior high school students. For instance, Group 1 used the scenario of restocking goods in an electronic store. To introduce this scenario, the preservice teachers posed a series of questions to ensure that all students understood the context of restocking goods and could envision how this process would be carried out in a store.
   - Presenting RMTs as a way of introducing new mathematical knowledge
     Each group used RMTs as the initial focus of their instruction. For example, Group 2 utilized a scenario involving shopping receipts and weighing goods with market scales to enhance students' understanding of linear equations with one variable and their problem-solving abilities. The preservice teachers in Group 2 motivated students to explore these scenarios, encouraging them to develop their own informal strategies and insights to address the RMTs.
   - Offering manipulative materials
     To support students in solving the tasks, the preservice teachers provided manipulatives to facilitate their exploration of solutions. In the initial cycle, only Group 2 supplied manipulatives to their students. This group provided market scales and encouraged students to investigate the weighing process to determine the weight of specific items. Through this hands-on exploration, students developed informal strategies for solving linear equations with one variable.
   - Using specific questions to guide students in solving RMTs
     The preservice teachers employed targeted questions to assist students in solving RMTs by drawing on their existing knowledge and personal experiences. For example, they posed questions such as, "What information can you extract from the given problem?", "What approach might be effective for solving this problem?", and "Can you explain the meaning of your results?". These questions were designed to encourage critical thinking and motivate students to articulate their reasoning.

b. The negotiation of didactical contracts
   - Establishing classroom norms
     Preservice teachers across various groups worked to establish classroom norms designed to foster an engaging learning environment and promote student involvement. These norms, which were implemented during teaching practice, included practices such as raising hands to ask or answer questions and maintaining silence and attentiveness when others were speaking. However, there were instances of inconsistency among the preservice teachers in reinforcing these norms. Consequently, the classroom atmosphere, especially during group discussions, was not always conducive to effective learning.

- Fostering active students' participation
  To promote active student participation, the preservice teachers organized students into groups to facilitate peer-to-peer exchanges of ideas. Additionally, they encouraged students to confidently share their thoughts during both group and whole-class discussions.
- Focusing on student reasoning
  To foster active student participation, the preservice teachers emphasized the importance of student reasoning rather than merely focusing on correct or incorrect answers. They frequently posed thought-provoking questions to scaffold students' critical thinking about tasks and solutions. Examples of such questions included, "What conclusions can be drawn from the results?" and "Does your solution make sense?". This approach encouraged students to recognize that both incorrect and correct solutions are valuable for discussion and learning. It also assisted teachers in identifying misconceptions and gaps in understanding.
- Valuing students' informal strategy
  The preservice teachers guided students in solving RMTs by encouraging them to use any logical and meaningful strategy they preferred. During the lessons conducted by Groups 1 and 2, several students sought a formula to solve the problems. However, the teachers emphasized the freedom to employ any strategy that made sense to them. This focus on the meaningfulness of the approach underscored the value of informal strategies, motivating students to engage actively in problem-solving and to approach problems in their own unique ways.

Although the preservice teachers demonstrated a range of didactical strategies, they encountered several challenges in their teaching practices. Reflections from both groups indicated difficulties with classroom management, time management, guiding student comprehension, and formulating effective scaffolding questions. In their reflection journals, both groups proposed ideas for enhancing future teaching practices. They planned to encourage students to articulate their opinions more freely, establish consistent classroom norms to foster a conducive learning environment, use tasks with diverse contexts to deepen understanding of RMTs, and provide more stimulating questions to better scaffold students' thinking during RMT problem-solving.

Findings from the Retrospective Analysis

Overall, there was a positive outcome in the preservice teachers' skills in creating and implementing RMTs in mathematics classrooms. Findings from the pre-course written test identified major difficulties, such as creating tasks with essential and relevant contexts, tasks with higher cognitive demand, and tasks that required students to analyze the information needed to solve the problems. Comparing these results with those from the pre-course test, it is evident that the preservice teachers addressed these identified difficulties during the learning process. The quality of the mathematics tasks posed by the preservice teachers during the learning process demonstrated that all tasks met the characteristics of RMTs.

Post-course interview data revealed that engaging in various problem-posing activities enriched the preservice teachers' experience in creating RMTs. This finding aligns with previous research, which has shown that problem-posing activities positively impact preservice teachers' ability to develop meaningful mathematics tasks (Bonotto, 2013; Şengül & Katranci, 2015). Additionally, the preservice teachers' performance in implementing RMTs in the classroom indicated their effective use of various didactical strategies to support meaningful mathematics learning through RMT exploration.

Consequently, the positive results observed in the preservice teachers' skills in creating and implementing RMTs suggest that similar outcomes are expected in the design of the next cycle.

However, it is important to acknowledge that the preservice teachers faced challenges in classroom management and in guiding students to actively construct new knowledge through solving RMTs. Analysis of post-course interview data indicated that these difficulties might have stemmed from the preservice teachers' limited opportunities to assess students' characteristics prior to the teaching practice. This observation is supported by Santrock (2021), who emphasized the importance of understanding students' characteristics as a key factor influencing effective teaching practice. Therefore, incorporating carefully planned activities designed to familiarize preservice teachers with their students' characteristics could enhance their performance in implementing RMTs.

Additionally, several modifications were introduced to the problems discussed during the initial meetings. These adjustments focused primarily on analyzing the characteristics of various mathematics tasks. Moreover, changes were made to the mathematical topics addressed by each group of preservice teachers to better align with the topics scheduled for teaching in schools during the implementation of Cycle 2.

CONCLUSION

This study developed a learning trajectory to enhance preservice mathematics teachers' skills in creating and implementing RMTs. The learning trajectory consisted of eight activities conducted over ten meetings. These activities included collaborative and reflective practices focused on both creating and implementing RMTs.

In the process of creating RMTs, the preservice teachers developed problems through various problem-posing situations, including structured, semi-structured, and free problem-posing scenarios. To evaluate and improve the quality of the RMTs, the tasks were reviewed by the researcher, peers, and a teacher. Additionally, the preservice teachers formulated and implemented a lesson plan for junior high school students, applying their developed RMTs in practice.

The researcher concluded that the learning trajectory significantly enhanced preservice teachers' skills in creating and implementing RMTs, based on the results and retrospective analysis of the preliminary experiment. This conclusion is substantiated by the high quality of the mathematics tasks developed by the preservice teachers, all of which conformed to the characteristics of RMTs. Furthermore, the preservice teachers effectively employed various didactical strategies in their implementation of RMTs in mathematics classrooms, which facilitated meaningful learning experiences for students.

However, challenges persisted, particularly in classroom management and guiding students to actively construct new knowledge through RMTs. This suggests that teacher education programs should focus on providing targeted support and guidance in these areas to improve preservice teachers' ability to effectively incorporate RMTs into their teaching practices.

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Declarations

Author Contribution	:	VFR: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Visualization, and Writing-original draft. AFHG: Conceptualization, Formal analysis, Funding acquisition, Methodology, Supervision, Validation, and Writing-review & editing.
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Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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