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The effects of gradual release of assistance instruction on students' heuristics, confidence, and attitude toward independent problem-solving

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Abstract

The 'Gradual Release of Assistance (GRA) instruction' was implemented among pre-service teachers in a problem-solving course designed to reduce students' resistance to independent problem-solving. This approach aimed to enhance students' heuristic skills, confidence, and attitudes toward problem-solving. The GRA instruction encompassed activities with progressively decreasing levels of instructional support across three stages: the maximum assistance stage, the medium assistance stage, and the independent problem-solving stage. Utilizing an embedded multiple-case design and a simple time-series analysis of six individual cases, this research explored how pre-service teachers applied heuristics and their confidence and attitudes toward problem-solving. The study revealed improvements in heuristic application among cases with high and medium mathematical abilities, as well as increased confidence and positive attitudes toward problem-solving within this cohort. However, there were no notable improvements in using different heuristics among cases with low mathematical ability. The findings discussed the observed changes and consistencies, offering plausible explanations that underscore the significance of GRA instruction in alleviating students' reluctance to engage in independent problem-solving.

Keywords: Attitude, Confidence, Gradual Release of Assistance, Heuristics, Problem-solving

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A fundamental objective of mathematics education is to develop students' independent problem-solving skills (Son et al., 2020). While students are traditionally taught computational techniques, manipulation of mathematical relationships, translation of concepts, and content knowledge, these instructional components are ultimately aimed at enhancing problem-solving abilities (Caviedes et al., 2023). Given the curriculum's strong focus on problem-solving, it is essential to provide effective instruction that supports the development of these skills. As a result, pre-service teachers must engage in diverse problem-solving tasks and learn various methodologies and strategies to teach problem-solving effectively.

Problem-solving is an essential competency for future educators within the framework of the Philippine K to 12 Mathematics curriculum. This curriculum places a strong emphasis on both problem-solving and critical thinking. As part of this educational mandate, students are expected to develop a range of foundational problem-solving skills, including understanding and knowledge acquisition, estimation, computation, visualization and modeling, representation and communication, conjecturing, reasoning, proving, decision-making, and the application and integration of concepts. According to Surya et al. (2017), fostering these foundational skills is essential, as they significantly influence students' learning processes and their confidence in tackling problems. These initial skills are critical at various stages of problem-solving, ultimately empowering students to solve problems independently.

In independent problem-solving, students tackle challenges solely using their own knowledge and abilities, without assistance from peers, adults, or external resources. When presented with a "practice problem," students must independently navigate each step of the solution (Reisslein et al., 2007). Successful resolution of these practice problems signifies the attainment of independent problem-solving skills. For pre-service teachers, mastering this stage is essential, as it prepares them for high-stakes assessments such as licensure exams, enhances their ability to teach problem-solving effectively (Kurniati et al., 2019), and strengthens their analytical skills for addressing diverse situations (Sari et al., 2019). Given the persistent challenges in solving mathematical problems, it is imperative for the community of mathematics educators, students, and researchers to invest time in developing innovative strategies and exploring related factors that can enhance students' problem-solving capabilities (Erbilgin & Macur, 2022). Consequently, this study examines various aspects that may contribute to students' resistance to independent problem-solving.

Firstly, the perceived challenges embedded in problem-solving tasks can discourage students from engaging with these problems, as the sudden demand on their cognitive resources can overwhelm their ability to focus on understanding the task (Schwonke et al., 2009). While some students find problem-solving to be an exciting and stimulating way to connect mathematics to real-world scenarios, others exhibit mixed reactions and attitudes toward mathematical problem-solving (Russo & Minas, 2020). In particular, some students experience heightened stress or anxiety when faced with difficult problems (Abdollahi et al., 2018). This sense of being overwhelmed can lead to panic, impairing their capacity for clear thinking. Others may respond by procrastinating or avoiding the problem altogether, which can result in feelings of hopelessness and frustration.

Independent problem-solving is defined as a culminating stage in problem-solving practice, where a student solves practice problems using only their own knowledge and skills, without any assistance from teachers, peers, or other resources (ter Vrugte et al., 2017). To mitigate the cognitive demands of problem-solving, instructional assistance or scaffolding is recommended (Salden et al., 2009). The progression from full assistance to partial assistance, and eventually to independent problem-solving, occurs as students demonstrate increased proficiency and understanding (Schwonke et al., 2009). Examples of instructional assistance in problem-solving include worked examples, tutored problem-solving, faded-worked examples, metacognitive prompts, erroneous examples, and both tutored and untutored problem-solving (Adeniji & Baker, 2022; 2023; Avdyli et al., 2017; McLaren et al., 2016; Salden et al., 2009, 2010; ter Vrugte et al., 2017).

It is crucial to understand how pre-service teachers approach and demonstrate solutions to problem-solving tasks (Setiyani et al., 2023). Research shows that students can employ various strategies and present different solutions at each stage of problem-solving (Jagals, 2013). Additionally, before engaging students in independent problem-solving activities, it is important to assess their attitudes toward the process (Harun et al., 2021) and their confidence in problem-solving (Jagals, 2013). Since certain factors can negatively impact students when faced with non-routine problems, it is essential to explore how affective domains, such as confidence and attitudes, influence their problem-solving abilities (Mutohir et al., 2018). Factors like insufficient proficiency in independently solving non-routine problems, anxiety due to a lack of knowledge and skills, and a general aversion to problem-solving can all discourage students from attempting to tackle problems.

As Anwar et al. (2017) observed, scaffolding plays a crucial role in supporting students' learning, particularly for those who struggle to complete problem-solving tasks independently. Based on this, the study hypothesized that restructuring the instructional approach for non-routine problems through the provision of scaffolds and instructional support could reduce, if not eliminate, students' reluctance to engage with non-routine problems, potentially enabling them to reach a level of independence in problem-solving. Therefore, the study aimed to examine the impact of the Gradual Release of Assistance (GRA) and the introduction of heuristics on students' problem-solving processes, as well as their confidence and attitudes toward problem-solving.

This study operated under the premise that pre-service teachers could cultivate independent problem-solving skills by engaging with a diverse range of closed non-routine problems. Through scaffolded learning facilitated by the GRA instructional approach, they would become familiar with various heuristics. To explore these assumptions, the study aims to conduct a multiple-case analysis focusing on pre-service teachers' application of heuristics during problem-solving processes and affective learning outcomes, specifically their confidence and attitudes, as they progress through the stages of GRA instruction, from maximum support to independent problem-solving. The study seeks to answer the following questions:

1. How do students' problem-solving processes and use of heuristics improve at each level of instructional assistance?
2. How does students' confidence in problem-solving evolve from the stage of maximum assistance to independent problem-solving?
3. How do students' attitudes toward problem-solving become more favorable as they transition from maximum assistance to independent problem-solving?

As indicated in the literature, this study employed heuristic strategies to tackle non-routine problems, which included mathematical problems both with and without real-world elements integrated into the problem statement (Ünlü, 2018). This approach was designed to enhance the teacher's ability to scaffold students' learning by enabling them to formulate and assess hypotheses, thereby encouraging the exploration of multiple ideas (Son et al., 2020). The intervention used in the study, the Gradual Release of Assistance (GRA) instruction, as depicted in Figure 1, was rooted in the principles of scaffolding (Vygotsky & Cole, 1978) and the Gradual Release of Responsibility (Pearson & Gallagher, 1983). In this instructional model for teaching heuristics, students were provided with six types of instructional support, organized in a top-to-bottom sequence of decreasing assistance embedded within the activities.

Gradual Release of Assistance Instruction model

Cognitive support involves assistance related to understanding a task, developing specific techniques, and breaking tasks into smaller, more manageable components (Leerkes et al., 2011). The instructional cognitive support in this study was gradually reduced, as illustrated by the funnel-shaped model in Figure 1, and was categorized into three stages: maximum assistance (including worked examples and tutored problem-solving), medium assistance (comprising faded worked examples and metacognitive prompts), and independent problem-solving (featuring erroneous examples and untutored problem-solving).

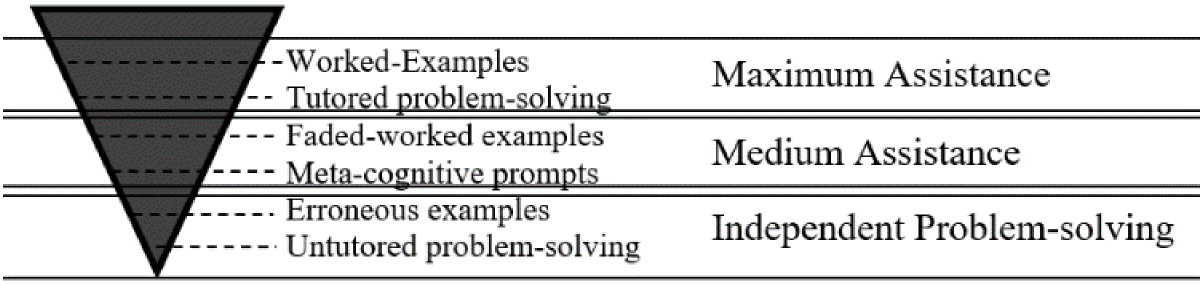


Figure 1. The Gradual Release of Assistance Instruction

A worked example includes the problem presentation, solution process outline, and the completed solution (Adeniji & Baker, 2022; 2023). In tutored problem-solving, students actively engage with the problem and are expected to develop a problem-solving schema, with teacher feedback and hints provided during points of difficulty to support schema development (McLaren et al., 2016). Faded worked examples present partial solutions, allowing students to complete the missing steps. Additionally, metacognitive questions or prompts are used to help students understand the problem's language, identify relationships between variables, and convert textual information into mathematical symbols (Avdyli et al., 2017). Students then work with erroneous examples, where they are tasked with identifying, correcting, and justifying the errors (McLaren et al., 2016). Finally, McLaren et al. (2016) also explain that untutored problem-solving requires students to solve problems without teacher or external assistance.

In this study, confidence is defined as the degree to which pre-service teachers are willing to tackle non-routine problems, both when supported by learning scaffolds from teachers and peers, and eventually independently (Gok, 2012). Confidence is assessed within the intellectual domain (Gunawan et al., 2022; Hendriana et al., 2018; Jagals, 2013). Additionally, attitude refers to the pre-service teachers' reactions to problem-solving scenarios, including their responses and behaviors that reflect their knowledge and feelings about problem-solving (Arslan et al., 2014).

METHODS

Research Design

The research employed an embedded multiple-case study design to explore various aspects of pre-service teachers' processes in learning independent problem-solving. This design allowed for the examination of cognitive and affective factors specific to each individual case. In contrast to a single-case study, this approach is comparative. The design was guided by several components: research questions, propositions, cases, linking logic, and interpretation criteria.

Defining the Research Questions and the Study Propositions

The research questions outlined earlier are aligned with the following propositions considered in this case study. Specifically, the study hypothesizes that if pre-service teachers engage with the Gradual Release of Assistance (GRA) instruction, then:

P1. the pre-service teachers' solution processes in solving problems from the maximum assistance stage to independent problem-solving will improve in terms of the use of different heuristic strategies;

P2. the pre-service teachers' confidence towards solving problems will improve over time up to the independent problem-solving stage; and

P3. the pre-service teachers' attitudes towards problem-solving will become positive over time up to the independent problem-solving stage.

The Case in the Study

In this study, the unit of analysis was the individual pre-service teacher, with six cases corresponding to the six participants. Each case was bounded by both the constructs under study and the time constraints, as participants progressed through hierarchical stages in learning to solve mathematical problems. Consequently, data collected from an individual case at one stage could not be used to describe the learning processes in other stages.

Linking Logic

For linking analysis, the study utilized the simple time-series analysis technique as recommended by Yin (2018). This method was chosen to track changes over time from the initial stage (maximum assistance) to the final stage (independent problem-solving), in alignment with the research questions and propositions. The study began on Day 1 of the problem-solving class and concluded on Day 12, spanning a total of six weeks. Each week, the class met for 90 minutes per session, with two sessions per week, amounting to a total of 18 hours of instruction over the course of the study. Prior to these sessions, the pre-service teachers had not received formal training in problem-solving. The researcher also documented the time intervals for each stage of the instruction.

Interpretation Criteria

Initially, the raw data collected from individual cases using various instruments were transcribed into text. The time-series analysis employed chronological sequencing, so data occurrences were organized in chronological order. Significant statements regarding participants' experiences during the independent problem-solving learning process were identified through coding. These codes were then transferred to a spreadsheet, where they were filtered, arranged, and sorted according to the research objectives. To enhance the credibility of the research, coding was performed three times to ensure thorough reading, re-reading, coding, and re-coding of the transcribed texts. Additionally, the transcribed evidence was reviewed four times during the stages of coding, editing, generating meaning units, and condensing these units into themes. Memos were written throughout the processes of transcribing, coding, and thematic analysis. These memos also included follow-up questions for participants as part of the cross-checking procedure. The transcripts and descriptions were returned to the participants for member-checking, which helped reduce researcher bias and address the confirmability of the study.

Table 1 displays the third round of coding for the output of Student 2 on Problem No. 1. For instance, each significant statement was assigned a code, such as "DP" for the "Developing a Plan" phase, accompanied by a brief memo like "The first strategy the student considers." This memo was interpreted within Meaning Units and categorized under the subtheme "Trial and Error/Heuristics." These subthemes were subsequently consolidated into a broader theme, "Heuristics Strategies."

Table 1. Sample Coding of Students' Responses

Coding No. 3						
Problem No.	**Transcript**	**Codes**	**Memos**	**Meaning Units**	**Subthemes**	**Themes**
1	First, know what is asked. Then state all the given. Set it all aside and then back again to what is asked. Know all possible solutions that you could give. And then, see where you are going to start, from the bottom or from the beginning.	DP: Setting up order of solution	Somehow, recalling the important actions to consider when solving.	Initially, the students when confronted with a problem, the student has to recall the procedures a problem should be solved.	Listing the steps to be taken	Specific solution's steps
1	I am planning to use Trial and error.	DP: Planned heuristic-strategy Trial and error	The first strategy the student has in mind.	When the student was asked to solve the problem, trial and error was the first strategy the student thought he would use.	Trial and error heuristics	Heuristics – strategies
1	I will use Trial and error.	Trial and error	The students seem to be more familiar with the phrase 'trial and error' referring to the 'Guess and check' heuristics.	This implies they have used trial and error in previous mathematics classes – in factoring general trinomials.	Trial and error heuristics	Heuristics - Strategies

Analysis of Case Study Evidence

The researcher linked evidence to the propositions using simple time-series analysis, strengthened by replication logic. Initially, individual cases within the group were identified, and a case study was conducted to develop the evidence for each case. This evidence was then used to replicate other cases with similar outcomes or minimal variation until a sufficient number of cases were established (Yin, 2018). If the individual cases consistently supported the propositions through literal replication, this would provide strong and convincing support for the propositions, leading to a conclusive case report.

Conversely, if the individual case studies yielded contradictory results, the original hypotheses were modified or retested with a different group of cases. Any evidence that contradicted the initially proposed propositions was considered a rival explanation. The researcher evaluated these rival explanations by collecting additional data and investigating alternative influences, focusing on supporting the validity of other factors rather than merely rejecting the contrasting evidence. After addressing these rival explanations, the researcher revisited and revised the initial propositions and conclusions.

Instruments

An activity book was utilized to document participants' daily problem-solving solutions before, during, and after the implementation of the gradual release of assistance instruction. The activity book comprised two sections: the aptitude test and scaffolded activity sheets. The aptitude test featured ten closed, non-routine problems, each designed to apply a specific heuristic strategy. The scaffolded activity sheets supported the gradual release of assistance instruction, incorporating six types of instructional support that progressed from maximum assistance to independent problem-solving. The ten test items, representing ten distinct strategies, are detailed in Table 2. These heuristics were randomly represented across problem tasks, which were consistent throughout all six types of instructional materials, each containing ten problems. The representation of heuristics via problem tasks was maintained across the three stages of the Gradual Release of Assistance (GRA).

Table 2. Order of Heuristics as used in the Activity Book

Aptitude test/ Six types of instructional assistance	Heuristic-strategies
1	Working backward
2	Finding a pattern
3	Adopting a different point of view
4	Solving a simpler and analogous problem
5	Extreme case reasoning
6	Drawing a diagram
7	Intelligent guessing and testing
8	Accounting for all possibilities
9	Organizing the data
10	Logical reasoning

For example, scaffolded *Activity 1.5.b.* illustrates the heuristic strategy of 'working backward' during the maximum assistance stage (worked examples).

Activity 1.5.b. "The sum of two numbers is 12, and the product of the same two numbers is 4. Find the sum of the reciprocals of the two numbers."

It may not be apparent in this problem that working backward is the best heuristic to use. Nonetheless, the solution starts from what is being asked in the problem, the sum of the reciprocals of the two unknown numbers which is represented by $\frac{1}{x} + \frac{1}{y}$; performing the operation on fractions and substituting the given values yield the answer required of the problem: $\frac{x+y}{xy} = \frac{12}{4} = 3.$

Another data source was semi-structured qualitative interviews, which were employed to capture participants' personal views on confidence and attitudes. Although a prepared interview protocol was used, the questions were flexibly worded, allowing for rephrasing and clarification as needed during the interviews. Unlike highly structured interviews, this approach permitted the interviewer to adapt questions in real-time, fostering a two-way interaction. Interviewees could query the questions, and the interviewer could pose follow-up and probing questions, facilitating more productive data collection. The interviewer, who was also the researcher, used open-ended questions to avoid leading or yes/no responses. The open-ended questions focused on two main areas: confidence and attitudes.

Confidence questions aimed to gather data on how participants' willingness to tackle non-routine problems was influenced by the gradually reduced instructional assistance. These questions explored participants' abilities to accept challenges when faced with problematic situations. Attitude questions, on the other hand, elicited participants' reactions and responses to various problem-solving scenarios. During the interviews, participants were presented with different problem-solving situations and asked about their reactions. Responses from the initial interview served as baseline data, which were compared with data from subsequent interview sessions. These follow-up interviews assessed changes in attitudes and required participants to provide further insights into how such changes occurred. These questions were crucial in the absence of direct observations of participants' behavior by the researcher. To ensure reliability, the researcher corroborated participants' views by asking about them in multiple ways and at different times. Sample questions for assessing confidence and attitudes included:

"What was it like when you solve problems with open resources around you compared to working with problem-solving without instructional support?"
"What if you were given activity sheets right now with only one instruction, that is "Show your solution to the problem," embedded on it, how would you react?"

In addition to the interview, a 30-item questionnaire for the attitude toward problem-solving was administered (Buzon, 2008; Tapia & Marsh II, 2004). Each item was rated by the participants using a scale of 1 – 4 where four (4) was the highest (*4 – Not at all true; 3 – Slightly true; 2 – Generally true; and 1 – Very true*). This Mathematical Problem-solving Attitude Inventory was originally used by Buzon (2008) to determine the attitude of the nine preservice math teachers through the implementation of the problem-solving course. Hence, the tool was found to be the most appropriate instrument in the study. The following are sample statements in the questionnaire:

"Mathematical problems make me feel as though I am lost in a jungle, and I cannot find my way out."
"I enjoy going beyond the assigned problems and I try solving more than what is expected

of me."

Research Locale and Participants

The target participants for this study were third-year pre-service teachers specializing in mathematics, who were enrolled in a Problem-Solving course during the first semester of the academic year. These students were preparing to teach mathematics to secondary-level students in grades 7 through 12. Prior to this study, the participants had completed seven mathematics courses: History of Mathematics, Advanced Algebra, Trigonometry, Geometry, Logic and Set Theory, Calculus with Analytic Geometry, and Elementary Statistics and Probability.

Informed consent was obtained from all participants as part of the research requirements. Ethical considerations included conducting a briefing session to explain the scope and duration of the study, the potential risks and benefits of participation, securing informed consent, and ensuring confidentiality by using pseudonyms to protect participants' identities.

Out of the original ten participants, six were selected for case categorization using the two-phased approach outlined by Yin (2016). The first phase involved collecting quantitative data on the group, which included obtaining participants' grade point averages (GPAs) from their previous major subjects and administering a questionnaire on attitudes toward problem-solving. Participants were categorized based on their mathematical ability: those with low and high mathematical abilities were identified as having the lowest and highest GPAs, respectively, within the same affective category. Additionally, participants with medium mathematical abilities were selected based on GPAs that fell between the highest and lowest and were purposively chosen according to their attitudes to provide a comprehensive range of data for the study, presented in Table 3.

Table 3. Categorization of Cases in terms of Mathematical Ability and Attitude towards Problem-solving

Students according to Mathematical ability	Students according to attitudes toward problem-solving	
	Positive	Negative
High	*Alfredo*	*Berto*
Medium	*Maria*	*Nena*
Low	*Xavier*	*Yolly*

The second phase involved establishing criteria for stratifying or reducing the candidate cases. Participants were categorized based on their mathematical ability, as indicated by their GPAs (low, medium, and high) (Sanjaya et al., 2018), and their affective scores (negative and positive) (Buzon, 2008). Participants with a mean score of 2.0 or below on the attitude inventory were classified as having a 'negative' attitude, while those with a mean score above 2.0 were classified as having a 'positive' attitude. Table 3 presents the participants, identified by aliases, classified into six categories. Additionally, statistical tools used included the arithmetic mean to summarize the results of the Mathematics Problem-Solving Attitude Inventory, and the weighted mean to determine the participants' GPAs from their previous mathematics courses.

Data Gathering Procedures

The problem-solving class was conducted over a period of no less than six weeks, incorporating scaffolding and a gradual release of assistance instruction. Initially, each student, including the participants, completed an aptitude test and a questionnaire assessing their attitudes toward problem-solving. Subsequently, different heuristic strategies were clearly defined and demonstrated through “worked-examples”. Each heuristic was introduced and discussed following the completion of each “worked-examples”, with the expectation that students would become familiar with all ten heuristics. These strategies were then applied in subsequent activities following the “worked-examples”.

In the tutored problem-solving phase, students were assigned a new set of tasks. During this stage, they were encouraged to ask questions and received hints from the teacher as needed. In addition to providing hints, the teacher suggested which heuristics to apply for each task.

Following the tutored problem-solving phase, students progressed to the medium assistance stage, where they engaged with faded-worked examples. In this phase, students worked on activities that included partially completed solutions and were provided with suggested heuristics (see Figure 2(a)). Additionally, metacognitive prompts and heuristic suggestions were incorporated into the activity sheets to further support students (see Figure 2(b)).

In the independent problem-solving stage, students were presented with erroneous examples and tasked with independently identifying, explaining, and correcting the mistakes in the activity sheets. Finally, in the untutored problem-solving stage, students were given tasks to solve independently, without any hints or suggestions. This sequence of activity-based scaffolds was designed to facilitate the gradual release of assistance in mathematics problem-solving through the use of an activity book.

Activity 3.2.a
Understanding the problem

Name:		Date:	

***Find the units digit of 8^{19}.**

Some concepts of understanding in this problem was already provided; however, it is not enough to understand the whole context. Continue the answer in the question and write it in the space provided below.

Solution:
You begin this problem by examining the increasing powers of 8 and see if there is a pattern that you can use.

$8^1 = 8$
$8^2 = 64$
$8^3 = 512$
$8^4 = 4{,}096$

The solution to this problem was already started; however, the process needs your help to finish it. Continue the solution and write it in the space provided below.

(a)

Activity 4.3.b
The problem-solving process

Name:		Date:	

Given the sequence of integers 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, …, where each positive integer *n* occurs in a grouping of *n* consecutive terms, how many terms are needed so that the sum of the reciprocals is 500?*

You can work on the solution to this problem by considering the prompts given below. Write your solution in the space provided. (If you have a solution other than the suggested prompts, you are allowed to do so.)

What heuristic strategy do you plan to use?

We can solve the problem by examining an organized listing of the terms.

Show your solution:

- ✓ *How can I list all the possible terms in the sequence?*
- ✓ *How about grouping all the terms with the same denominator?*
- ✓ *To check my answer, what other solution can I suggest other than organizing the data?*

(b)

Figure 2. Sample Activities in the Activity Book

Qualitative interviews were conducted at the beginning of the study and at the end of each stage of the Gradual Release of Assistance (GRA) instruction, with four sessions per stage, each lasting five to ten minutes. The researcher conducted all interviews, which focused primarily on gathering data regarding participants' confidence and attitudes.

Transcription Type: *Individual Interview 1*
Duration: *6:49*

Researcher: What can you say about the process of reflection?
Student 3: Para sa akin… reflection ay parang giving your point of view on something. It is more on stating how o paano at ano ang mga natutunan.
Researcher: How was your experience, if you have, when reflecting after engaging in an activity?
Student 3: Mahirap po magsulat ng reflection since hindi naman ako fun of writing ng katulad niyon, pero I admit nakakatulong ito ma-improve yung writing skills o ability ko.
Researcher: How was your experience, if you have, when reflecting after engaging in a problem-solving activity?
Student 3: Wala pa po eh kung after problem solving activity? Wala pa po.

Figure 3. Sample Interview Transcription

Daily activity sheets were completed and submitted at the end of each session. Interviews were audio-recorded and meticulously transcribed, with a portion of the transcriptions shown in Figure 3. Additionally, the results of the analysis were individually communicated to the participants as part of their assessment and feedback in the course.

RESULTS AND DISCUSSION

Cognitive Domain

In the study by Kurniati et al. (2019), pre-service teachers often approached problems using a 'trial and error' method, which was interpreted as a lack of thorough investigation into the problem, despite 'trial and error' being a recognized heuristic by George Polya. Consequently, these teachers were unable to generate useful evidence for constructing a correct argument or solution. Similarly, this study observed that during the aptitude test, students frequently resorted to 'guessing and testing' or 'trial and error' strategies when unsure of how to proceed (see Figure 4).

Barbara took a 20-question multiple-choice test. The test was scored +5 for each correct answer, -2 for each wrong answer, and 0 if the question was omitted. Barbara scored a 44, even though she omitted some questions. How many questions did Barbara omit? *	trial and Error. Substitute the numbers from 0 to 9 as a value of d. since no definite solution is appropriate to use, trial and error process is the most convenient for me to use.

Figure 4. Sample 1 Problem and Students' Solution-statements during the Aptitude Test

This approach resulted in a lack of clear direction and effective problem-solving before the initial stage. Additionally, one student successfully solved a problem using the 'working backward' heuristic but mistakenly categorized it as 'trial and error' (see Figure 5).

Nancy breeds New Zealand rabbits for a hobby. During April, the number of rabbits increased by 10%. In May, 10 new rabbits were born, and at the end of May, Nancy sold one-third of her flock. During June, 20 new rabbits were born, and at the end of June, Nancy sold one half of her total flock. So far in July, 5 rabbits have been born, and Nancy now has 55 rabbits. How many rabbits did Nancy start with on April 1st? *	I am planning to use Trial And Error for this. First, I get the total numbers of the rabbit left from April to July w/c is 55 and then less 5 and Assume that half of it plus 5 is the number of rabbit until june. And then less 20 and Assume that the 2/3 of rabbit left is 80 + 1/3 = 120 less 10 and less 10% = 100 rabbits.

Figure 5. Sample 2 Problem and Students' Solution-statements during the Aptitude Test

Despite being a representative of students with high mathematical ability, Berto initially scored low on the aptitude test. This lower performance can be attributed to the test's closed-response and non-routine problem tasks, which required skills not primarily covered in their previous mathematics courses. However, Berto’s performance improved significantly during the maximum assistance stage. For instance, as shown in Figure 6, Berto's approach of listing all candidate answers enabled him to identify the smallest integer required by the problem.

Evelyn, Henry, and Al play a certain game. The player who loses each round must give each of the other players as much money as the player has at that time. In round 1, Evelyn loses and gives Henry and Al as much money as they each have. In round 2, Henry loses and gives Evelyn and Al as much money as they each then have. Al loses in round 3 and gives Evelyn and Henry as much money as they each have. They decide to quit at this point and discover that they each have 24 Pesos. How much money did they each start with? *

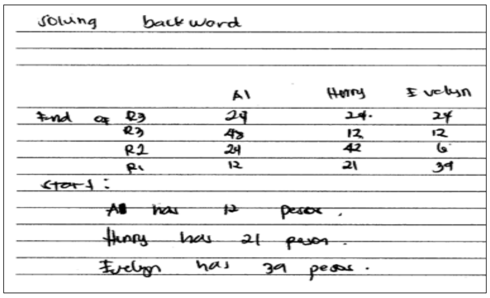


Figure 6. Berto’s Solution-statements during the Maximum Assistance Stage

Similarly, Figure 7 illustrates Alfredo's effective use of the ‘working backward’ strategy. Notably, the other three participants—Maria, Alfredo, and Nena—employed similar strategies and heuristics as Berto in their problem-solving approaches.

***Find the smallest integral value for x for which $\frac{12}{x+1}$ yields an integer.**

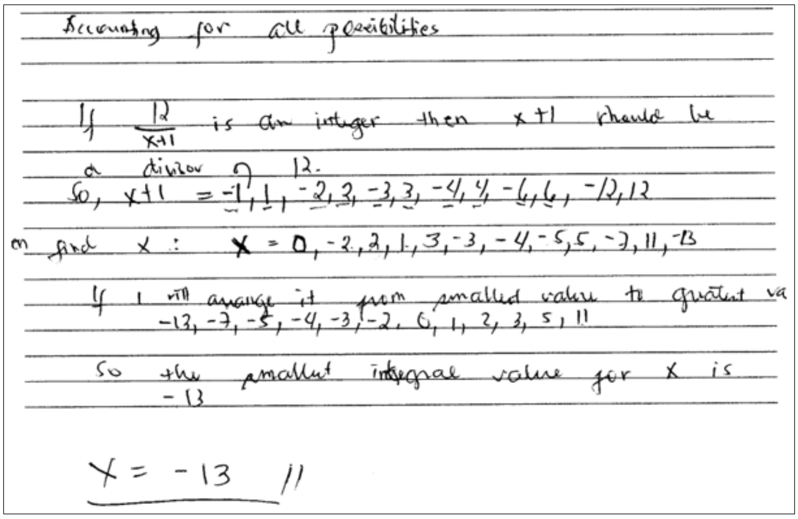


Figure 7. Alfredo’s Solution-statements during the Maximum Assistance Stage

In contrast, during the independent problem-solving stage, participants demonstrated that they did not rely solely on ‘trial and error’ or ‘guessing and testing.’ As shown in Table 4, Berto effectively applied various heuristics during this stage. Similar results were observed for Maria, Alfredo, and Nena, who also utilized different heuristics effectively.

The spontaneous use of various heuristics to find correct answers indicates that the participants became more proficient in solving non-routine problems. This suggests that the Gradual Release of Assistance (GRA) instruction effectively enabled participants to independently tackle non-routine problems by leveraging the heuristics they learned throughout the stages of instruction. Consistent with Setiyani et al. (2023), scaffolding proves beneficial for students with a field-independent cognitive style, aiding them in understanding concepts and devising simpler solutions to numerical problems. The observed improvement in participants' problem-solving abilities—evidenced by their shift away from solely relying on the 'guessing-checking' method—is particularly notable among those with high and medium mathematical abilities. The acquisition of problem-solving heuristics has thus better prepared these pre-service teachers for future challenges involving non-routine problems. Consequently, this finding supports the use of GRA instruction as a foundational approach for teaching heuristics in non-routine problem-solving, especially for pre-service teachers with high and medium mathematical abilities.

Table 4. Berto's Heuristics for the Problems in the Independent Problem-solving Stage

Heuristics used	**Number of activities in the Independent Problem-solving Stage**
Finding a pattern	2
Simple and analogous problem	2
Drawing a diagram	2
Intelligent guessing and testing	3
Accounting for all the possibilities	2
Logical reasoning	3

The introduction of the ten heuristics through 'worked examples' effectively reduced cognitive load during this stage (Adeniji & Baker, 2023). By minimizing unnecessary cognitive demands, participants were better able to absorb and apply these heuristics, as evidenced during the independent problem-solving stage. Consequently, participants with high and medium mathematical abilities demonstrated a strong performance by solving problems independently, indicating significant improvement and effective learning.

However, in the cases of Xavier and Yolly, who both exhibited lower mathematical abilities, there was a noticeable decline in performance during the independent problem-solving stage despite their high performance with different strategies during the maximum and medium assistance stages. Table 5 illustrates that, although Xavier and Yolly were familiar with various heuristics, they applied them correctly to only three out of ten and two out of ten problem tasks, respectively, during the independent problem-solving stage.

Table 5. Xavier's and Yolly's Implementation Results in the Independent Problem-solving Activities

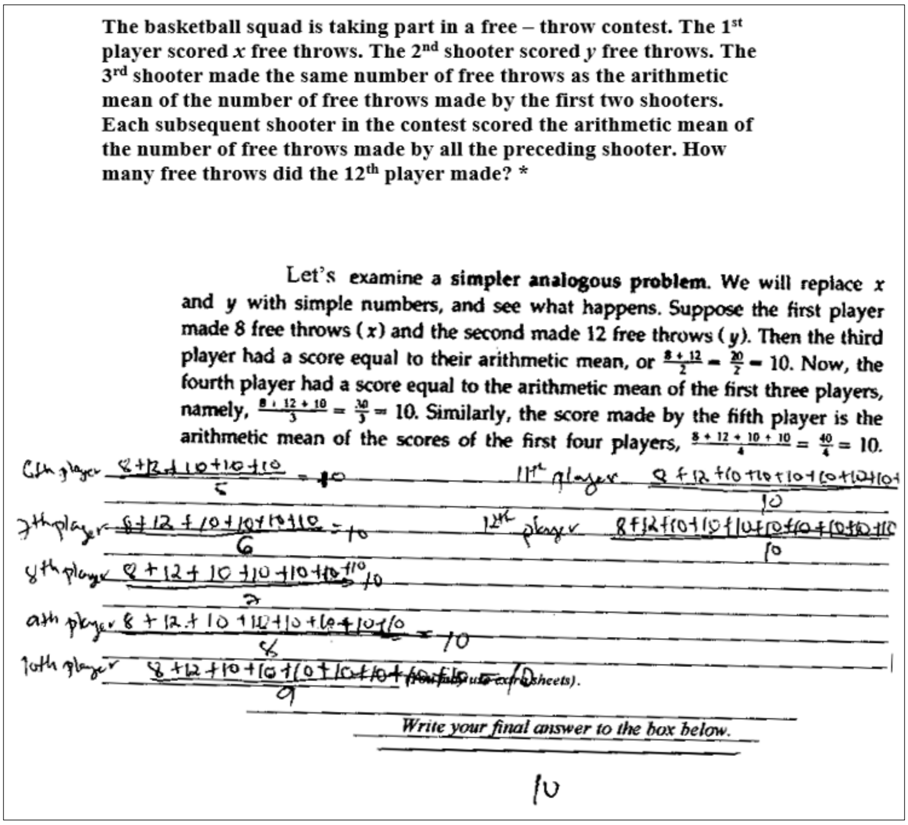
Implementation result	**Number of activities in *Xavier's* Independent Problem-solving stage**	**Number of activities in *Yolly's* Independent Problem-solving stage**
Correct Implementation	*3*	*2*
Wrong Implementation	*6*	*8*
Incomplete solution	*1*	*0*
Different strategies applied	*4*	*2*

Table 6 indicates that, during the medium assistance stage, Berto did not correctly apply the suggested strategies of "solving a simple and analogous problem" and "intelligent guessing and testing" in tasks 3.4.b and 3.7.b, respectively. Similarly, in the maximum assistance stage, he failed to correctly apply these strategies in tasks 6.4.b and 6.8.b.

Table 6. Berto's implementation results in the activities with medium assistance

Implementation of heuristics results	Medium Assistance Stage	Independent PS Stage
Correct Implementation	*3.1.b, 3.2.b, 3.3.b, 3.5.b, 3.6.b, 3.8.b, 3.9.b, 3.10.b*	*6.1.b, 6.2.b, 6.3.b, 6.5.b, 6.6.b, 6.7.b, 6.9.b, 6.10.b*
Wrong Implementation	*3.4.b, 3.7.b*	*6.4.b, 6.8.b,*

In applying the heuristic "solving a simple and analogous problem," as illustrated in Figure 8, Berto failed to properly interpret the hint "Suppose the first player made 8 free throws." This phrase was meant to simplify and make the problem analogous to the original one, rather than provide a definitive answer. The number "8" was not part of the actual problem's information and thus was not required for solving the original problem. Consequently, Berto misunderstood that the heuristic "solving a simple and analogous problem" was intended to illustrate and help formulate an expression for the number of free throws made by the 12th player, given an arbitrary value x for the first player's free throws, rather than to directly solve the original problem.


The basketball squad is taking part in a free – throw contest. The 1st player scored x free throws. The 2nd shooter scored y free throws. The 3rd shooter made the same number of free throws as the arithmetic mean of the number of free throws made by the first two shooters. Each subsequent shooter in the contest scored the arithmetic mean of the number of free throws made by all the preceding shooter. How many free throws did the 12th player made? *

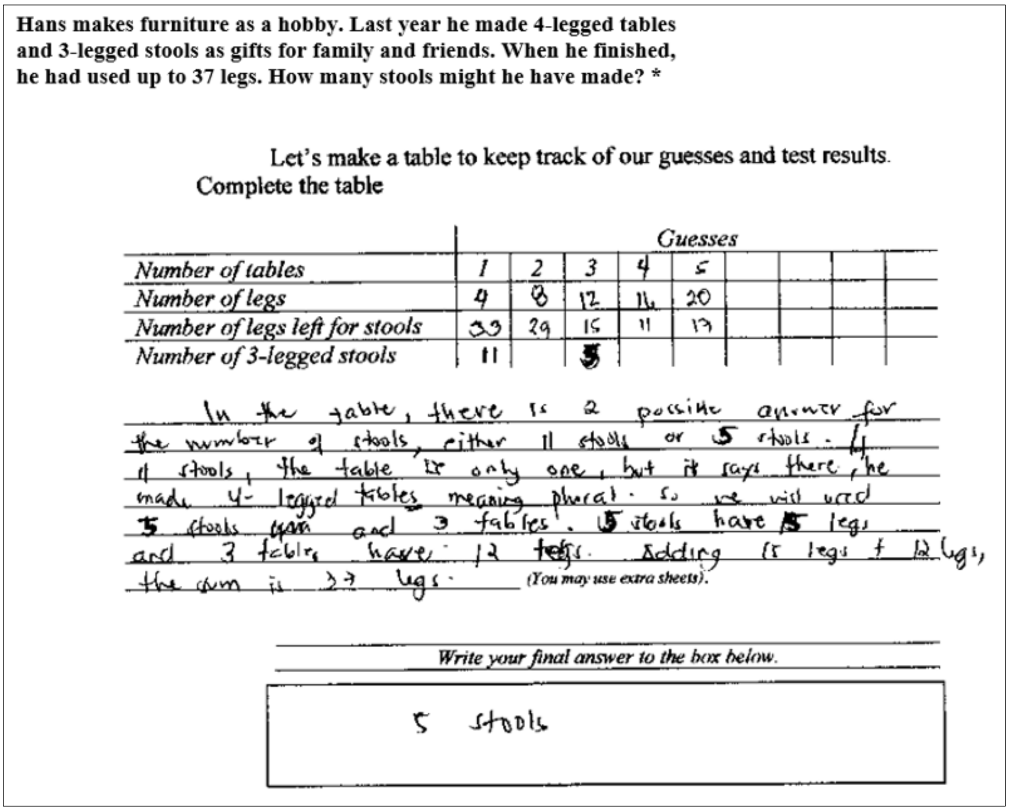
Let's examine a **simpler analogous problem.** We will replace x and y with simple numbers, and see what happens. Suppose the first player made 8 free throws (x) and the second made 12 free throws (y). Then the third player had a score equal to their arithmetic mean, or $\frac{8+12}{2} = \frac{20}{2} = 10$. Now, the fourth player had a score equal to the arithmetic mean of the first three players, namely, $\frac{8+12+10}{3} = \frac{30}{3} = 10$. Similarly, the score made by the fifth player is the arithmetic mean of the scores of the first four players, $\frac{8+12+10+10}{4} = \frac{40}{4} = 10$.

Write your final answer to the box below.

10

Figure 8. Sample Responses on 'simple and analogous problem' and 'intelligent guessing and testing'

On the other hand, Figure 9 illustrates Berto's use of the heuristic "intelligent guessing and testing" in Activity 3.7.b. Berto made an error during the "testing" stage by committing a fundamental addition mistake. Similarly, with only minor variations in their solutions, the other participants also incorrectly applied the suggested heuristics, leading to wrong answers in both the medium and independent problem-solving stages. It was concluded that a common error among participants during the Gradual Release of Assistance (GRA) instruction was the application of the heuristics "solving a simple and analogous problem" and "intelligent guessing and testing" during the medium assistance stage.



Hans makes furniture as a hobby. Last year he made 4-legged tables and 3-legged stools as gifts for family and friends. When he finished, he had used up to 37 legs. How many stools might he have made? *

Let's make a table to keep track of our guesses and test results.
Complete the table

	Guesses								
Number of tables	1	2	3	4	5				
Number of legs	4	8	12	16	20				
Number of legs left for stools	33	29	15	11	17				
Number of 3-legged stools	11		5						

In the table, there is 2 possible anwer for the number of stools, either 11 stools or 5 stools. If 11 stools, the table is only one, but it says there, he made 4-legged tables meaning pheral. So we will used 5 stools and 3 tables. 5 stools have 15 legs and 3 tables have 12 legs. Adding 15 legs + 12 legs, the sum is 27 legs. *(You may use extra sheets).*

Write your final answer to the box below.

5 stools

Figure 9. Sample Responses on 'simple and analogous problem' and 'intelligent guessing and testing'

Affective Domains

In terms of confidence, Alfredo initially expressed a lack of confidence in solving mathematics problems without assistance, rating his confidence as low as "two or three out of 10." However, after the maximum assistance stage, his confidence rating significantly improved. As indicated by his statement in Table 7 under the maximum assistance stage, Alfredo recognized that the support provided during this stage was crucial in encouraging his engagement in problem-solving activities. Reflecting on the skills acquired during this period, he remarked:

> *"It gives me the necessary information, ideas, and strategy that will help me every time I solve a problem."*

According to Gunawan et al. (2022), confidence in problem-solving is fostered through prior knowledge, a solid understanding of concepts, and sensitivity to problem contexts. Hendriana et al. (2018) also note that a key indicator of confidence is the willingness to confront challenges. For Alfredo, the gradually reduced assistance and his experiences in problem-solving enabled him to develop skills that helped him overcome his initial fear of problem-solving. This transformation led him to actively engage in solving problems. Table 7 highlights Alfredo's responses to confidence questions, illustrating his ability to embrace challenges presented by non-routine problems. Notably, this improvement in confidence was evident throughout the stages, from maximum assistance to independent problem-solving. This trend of increasing confidence was observed consistently across all participants.

In terms of attitude toward problem-solving, Maria's responses to attitude questions and observed reactions are summarized in Table 7. Initially, Maria expressed a preference for engaging in activities other than solving non-routine problems, indicating a lack of interest when no assistance was provided. However, her attitude shifted from negative to positive during the maximum assistance stage. At this stage, Maria demonstrated a willingness to tackle problems regardless of whether they were graded, reflecting a growing interest in improving her problem-solving skills. Her statement, "Can you say that you are doing it correctly if you will not try?" suggests that she was motivated to engage in problem-solving as a form of self-assessment.

Table 7. Alfredo's and Maria's Sample Statements in the Affective Domains

GRA Stages	**Affective Domains**	
	Sample *Alfredo's* responses to Confidence	**Sample *Maria's* responses on Attitude**
At the Onset of the Study	"I will be sad… because I'm sure that it will be a hard time for me to solve that problem."	"I'm dead! I will just perform an acrobatic tumble, sir. Haha!"
Maximum Assistance Stage	"I am enlightened sir! (*paused*) …in using different methods or strategies. Then it came to my mind that I need to solve the problems in our future meetings"	"Graded or not, I will still accept the challenge. Because, can you say if you are doing it correctly if you will not try?"
Medium Assistance Stage	"I'll accept the challenge, sir! However, I will carefully figure out how to solve it using those things I have learned from my previous activity."	"Okay sir, let's see my improvement, charot (*charot is a Filipino expression which means 'joke'*) haha! Well, I will feel nervous, can I do it? But on the other hand, how am I going to know that I can do it myself if I don't even try, right, sir?"
Independent Problem-solving	"I have used the things I learned and experienced from the previous activities in maximum and medium assistance stages as bases in problem-solving. The things I learned during those stages made me possible to write each idea that might help me in solving problems"	"Okay, I'll do it."

This positive shift was sustained into the medium assistance stage. Although Maria expressed feelings of nervousness, stating, "I would feel nervous," this was due to concerns about not having resources if she encountered difficulties, rather than a lack of interest in problem-solving. Her hesitation was linked to her perceived skills and fear of making mistakes, not a disinterest in solving problems. This change in attitude highlights her increasing engagement and willingness to confront problem-solving challenges.

Further, a positive reaction to problem-solving, even without available assistance, was evident after the independent problem-solving stage. This finding suggests that when an appropriate mathematics learning model, such as Gradual Release of Assistance (GRA) instruction, is implemented, learners can become engaged, productive, inventive, creative, and autonomous (Harun et al., 2021). The observed positive attitudes towards problem-solving across all six cases indicate that GRA instruction was effective in fostering a constructive learning environment.

The introduction of an appropriate learning practice led to a noticeable shift in students' attitudes, as they exhibited a more favorable outlook towards problem-solving. This positive attitude, in turn, positively impacted their learning processes, consistent with findings by Mutohir et al. (2018). Maria's attitude towards problem-solving evolved from initial reluctance to a more enthusiastic and positive approach by the independent problem-solving stage. This change in attitude was mirrored with minimal variation in the other five cases, demonstrating that the GRA instruction successfully influenced their overall attitude and engagement in problem-solving activities.

The findings from the cases of Berto, Maria, Alfredo, and Nena support Proposition 1 (P1), which posits that "the pre-service teachers' solution processes in solving problems from the maximum assistance stage to independent problem-solving have improved in terms of using different heuristic strategies." The data from these cases demonstrate that the Gradual Release of Assistance (GRA) instruction effectively enhances problem-solving skills among pre-service teachers with both 'high' and 'medium' mathematical abilities, irrespective of their initial attitudes towards problem-solving.

As noted by Surya et al. (2017), early mathematical ability, alongside the learning model, is crucial in building students' problem-solving capacities and confidence. The successful application of GRA instruction in this study highlights its effectiveness in complementing the mathematical abilities of students, thereby improving their problem-solving skills. This underscores that the GRA instruction model is well-suited for enhancing problem-solving strategies among pre-service teachers, particularly those with higher or medium levels of mathematical ability.

However, the findings from the cases of Xavier and Yolly did not support Proposition 1 (P1). Specifically, their performance during the independent problem-solving stage showed minimal improvement compared to their performance in the aptitude test before the GRA instruction began. This lack of improvement in heuristic application suggests that the GRA instruction was ineffective for students with low mathematical ability, despite any positive changes in their attitudes toward problem-solving.

This outcome highlights that students with low mathematical ability may struggle to benefit from GRA instruction without sufficient support. According to Adeniji and Baker (2023), students with limited mathematical schema require full instructional assistance to manage cognitive demands effectively. To address this, Surya et al. (2017) recommend several methods for assessing students' traits and aptitudes before GRA instruction, including reviewing academic records, conducting initial and prerequisite tests, individual communication, and administering surveys. These methods can be crucial for identifying students who might need extended support.

For students with low mathematical ability, it is advisable to provide extended support during the maximum assistance stage until they achieve the desired learning outcomes. Only then should they progress to subsequent stages of instruction or engage in post-GRA activities to ensure they can effectively apply the learned heuristics.

The results from all cases (Alfredo, Berto, Maria, Nena, Xavier, and Yolly) support Proposition 2 (P2). Specifically, "the pre-service teachers' confidence in solving problems improved over time, up to the independent problem-solving stage." This indicates that the GRA instruction effectively enhanced the participants' confidence in solving non-routine problems independently.

The increase in confidence can be attributed to the knowledge and problem-solving skills acquired during the GRA instruction. Hendriana et al. (2018) support this conclusion, noting that problem-solving ability is closely linked to confidence. The GRA instruction, with its provision of gradual assistance, aligns with the contextual learning approach described by Surya et al. (2017), which similarly boosts students' confidence in problem-solving. Additionally, problem-based learning has been shown to improve confidence in problem-solving, reinforcing the effectiveness of the GRA instruction in this regard (Hendriana et al., 2018).

Maria's case, reflecting medium mathematical ability, demonstrates that the gradual release of assistance (GRA) instruction is effective in cultivating a positive attitude towards problem-solving. Although initial responses to the attitude questionnaire showed some adverse reactions, the GRA instruction ultimately led to a positive shift in Maria's attitude. The findings from Maria's case, along with those of the other five participants (Alfredo, Berto, Nena, Xavier, and Yolly), support Proposition 3 (P3). Specifically, "the pre-service teachers' attitudes towards solving problems became more positive over time, up to the independent problem-solving stage." This suggests that GRA instruction effectively transforms attitudes from negative to positive, enabling pre-service teachers to approach non-routine problems with a more constructive mindset.

Furthermore, Mutohir et al. (2018) support this by noting that achievement scores positively affect students' attitudes. The assistance provided through GRA instruction led students to solve problems more effectively, fostering a sense of accomplishment and, consequently, a positive attitude towards problem-solving. However, despite improvements in confidence and attitude among participants with low mathematical ability, there was no corresponding enhancement in their heuristic application performance. This finding contrasts with studies suggesting a direct relationship between confidence, attitude, and problem-solving ability (Novak & Tassell, 2017). Therefore, it is recommended to focus on strengthening foundational mathematical skills and considering additional measures, such as post-GRA skill enhancement activities or extended practice within each GRA stage, to further elevate the problem-solving performance of students with low mathematical ability.

CONCLUSION

The study demonstrated that the Gradual Release of Assistance (GRA) model substantially mitigated students' resistance to solving non-routine problems by effectively imparting problem-solving heuristics. This approach not only improved students' problem-solving capabilities but also enhanced their confidence and positive attitudes toward mathematics. These advancements were evident among students with both high and medium mathematical abilities, indicating the GRA model's efficacy in fostering a more engaging and productive learning environment. By systematically reducing support while teaching heuristic strategies, students developed greater independence in problem-solving, showcasing the GRA model's effectiveness in promoting both cognitive and affective growth in problem-solving skills.

Despite these benefits, the study faced several limitations. Specifically, the GRA instruction did not yield significant improvements in the problem-solving performance of students with low mathematical ability during independent tasks. This suggests that while GRA instruction bolstered confidence and attitudes, these affective gains did not necessarily translate into enhanced problem-solving performance for all students. Furthermore, the study's focus on a specific cohort of pre-service mathematics teachers may limit the generalizability of the findings to other educational contexts or subject areas. The reliance on a singular instructional model and the absence of varied learning environments may also constrain the applicability of the results.

Future research should address these limitations by developing targeted interventions within the GRA framework to better support students with low mathematical abilities. Exploring the application of GRA in other academic disciplines could offer valuable insights into its broader educational impact. Additionally, examining the efficacy of individual types of instructional assistance within the GRA model may reveal which supports are most effective in diverse contexts. Expanding research to include a range of learner profiles and educational settings will further enrich our understanding of how scaffolded activities can be optimized to improve learning outcomes across various domains.

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Declarations

Author Contribution	:	RLC: Conceptualization, Writing - Original Draft, data curation, formal analysis, methodology, investigation project administration, resources, and writing – review & editing. MRCL: Conceptualization, Supervision, writing – review & editing, Formal Analysis, and Methodology, and Funding acquisition.
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