Journal on Mathematics Education
Volume 15, No. 3, 2024, pp. 815-834

Enhancing fraction learning through problem-solving and historical context: A didactic unit approach

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Received: 4 March 2024 | Revised: 14 May 2024 | Accepted: 20 June 2024 | Published Online: 30 July 2024

Abstract

This study examines the outcomes of implementing a teaching unit that integrates problem-solving with the history of mathematics to improve fifth-grade students' understanding of fractions at an educational institution in Yopal, Colombia. Utilizing an educational action research methodology, the study commenced with a diagnostic assessment to identify learning gaps, followed by the design and implementation of a didactic unit. This unit comprised three activities embedded in historical contexts to facilitate the learning of fractions, with its effectiveness assessed through an exit test. Conducted with 32 fifth-grade students, the research employed diagnostic and exit tests to evaluate student performance before and after the activities. The results demonstrated that the teaching unit significantly enhanced students' abilities to understand and apply fractions, as evidenced by their improved performance in the activities within the didactic unit.

Keywords: Didactic Unit, Education Fractions, History of Mathematics, Pólya Method

How to Cite: Torres-Peña, R. C., Peña-González, D., & Ariza, E. A. (2024). Enhancing fraction learning through problem-solving and historical context: A didactic unit approach. *Journal on Mathematics Education, 15*(3), 815-834. http://doi.org/10.22342/jme.v15i3.pp815-834

Mathematics plays a pivotal role in cognitive development and the demonstration of human intellectual capacity. Its foundational importance during the early years of education cannot be overstated, as it establishes the basis for acquiring knowledge, attitudes, and skills essential for logical reasoning. This foundational learning equips students to address real-world problems with efficacy.

Mathematics involves various principles that require reasoning and comprehension of specific contexts for problem-solving. Consequently, it intersects significantly with logical, abstract, and critical thinking. Effective teaching strategies necessitate educators' use of specialized materials and techniques designed to enhance these competencies. Within mathematical content, certain topics, such as fractions, are explored through real-world problems that resonate with students due to their broad applicability. Fractions, a fundamental topic from elementary school onward, involve logical sequencing processes. Students often encounter systematic errors, particularly in understanding and manipulating fractions as part-to-whole relationships (Butto, 2013; Wahyu et al., 2020).

The challenges associated with learning fractions are underscored by reports from the United Nations Educational, Scientific and Cultural Organization (UNESCO), which highlight that the primary goal of mathematics education in basic education is to ensure mathematical literacy for all students.

Understanding fractions is crucial as they are integral to the concepts of percentage, ratio, and proportion, which are essential in subsequent educational stages (UNESCO, 2016). In Latin America and the Caribbean, significant educational challenges persist. By 2015, 36% of children and adolescents in this region did not meet minimum literacy levels, and 52% fell below the required standards in mathematics. Specifically, Colombia's performance, with a score of 391, is below the OECD average of 489. Although Colombia ranks higher than Brazil, Argentina, Panama, and the Dominican Republic, it lags behind Uruguay, Chile, Mexico, Costa Rica, and Peru. This indicates that only 33% of Colombian students are capable of interpreting and representing straightforward mathematical situations.

Addressing this issue within the national context, the Colombian Institute for the Evaluation of Education (ICFES) has published SABER test results since 2015. These tests measure the competencies of students nearing the completion of secondary education. In mathematics, 44% of students fail to achieve the minimum performance levels required at the end of primary education. This trend has persisted, with 72% of students consistently categorized as having insufficient or minimal performance. This data reflects significant challenges in applying operations such as addition, subtraction, and multiplication to solve problems, interpreting numerical sequences from graphical representations, recognizing direct proportionality relationships, and organizing and classifying statistical data.

In analyzing fraction learning in two Colombian public schools, low performance levels were observed, particularly in understanding and applying fraction concepts and executing basic operations. This assessment, based on diagnostic exercises from SABER tests over the past six years, revealed that 72% of students performed at insufficient or minimal levels, highlighting a significant challenge in mastering this fundamental mathematical concept. These identifiable difficulties in learning fractions can be addressed through didactic practice proposals that incorporate diverse strategies to foster motivation and interest, ensuring effective use of class time to cover essential mathematical topics.

Educational theories that inform the design of didactic units for teaching mathematics include Piaget's (1988) cognitive development theory (Babakr, 2019), Vygotsky's (1978) scaffolding theory and its connection to the sociocultural context, and Ausubel's (1976) meaningful learning theory. These theories converge on a student-centered teaching approach, advocating for constructive learning that encourages students to connect new information with prior knowledge. This process is influenced by motivational, affective, and socio-cultural factors.

Significant research contributions to didactic units in mathematics include the works of Silva (2017), Gaviria (2016), Ospina and García (2019), and Vásquez et al. (2019). The topic of fractions in primary education has also been addressed in studies by Díaz (2017), Chotimah and Fesiyed (2020), Loc et al. (2017), Phuong and Loc (2020), Reséndiz and González (2018), and Dominguez et al. (2020), with a focus on various regional contexts.

Integrating these educational approaches into teacher planning underscores the importance of didactic units as crucial tools in curriculum design. Implementing activities that are both practically significant and functional, based on prior diagnostics and student needs, facilitates a curriculum-based teaching-learning process relevant both in and out of the classroom (Ambròs, 2009). Pólya (1969) emphasized the effective teaching of mathematical concepts through problem-solving strategies from a heuristic perspective, advocating for the use of analogies to stimulate conjectures, generalizations, and abstractions (Yayuk & Husamah, 2020).

Engaging students in practical activities is essential for the development of logical-mathematical thinking, fostering skills such as observation, imagination, intuition, and logical reasoning (Arteaga & Macias, 2016; Björklund & Palmér, 2022). Mastery of fractions is closely linked to both mathematical and natural language. However, difficulties in comparison, numerical operations, equivalences, percentages, and decimals hinder problem-solving abilities (Salazar & Diaz, 2009). Thus, incorporating fractions into teaching plans requires cognitive and metacognitive strategies, enabling students to observe, connect mathematically, and reflect on their problem-solving choices.

The use of historical contexts in mathematics education has garnered interest, with studies demonstrating advantages of integrating historical perspectives. Azman and Maat (2021) identified historical contexts as valuable for both teaching and understanding the evolution of mathematics. Vittori (2021) argued that historical integration fosters creativity in mathematical thought, while Schubring (2011) noted a consensus that historical elements can enhance mathematics instruction. Incorporating historical aspects into teacher education programs, as explored by Charalambous et al. (2008), deepens understanding of mathematics content and its developmental trajectory, providing effective strategies for integrating historical materials into instructional practices. Accordingly, a series of activities were developed for fifth-grade students, designed to enhance understanding of fraction concepts through problem-solving and historical perspectives.

METHODS

Type of Research

This study employed a qualitative paradigm utilizing a complex systemic approach. The research methodology was grounded in the Action Research (AR) method, which is designed to implement the theory of didactic situations within contexts of action, formulation, validation, and institutionalization, as outlined by Latorre (2007) and Ronen (2020). This approach is further underpinned by the self-reflective spiral sequence, emphasizing the essential stages of planning, action, observation, and reflection. These phases are intended to be flexible and cyclical (initial reflection → planning → action → observation of action → initial reflection), recurring throughout the research process to ensure continuous improvement and deeper insight.

Population and Sampling

The target population for this intervention consisted of 32 fifth-grade students from a public educational institution in Colombia. An assessment revealed that students who employed resources both timely and effectively demonstrated a greater ability to solve problems with increased speed and accuracy. Conversely, students who adhered to traditional problem-solving methods required more time and exhibited a higher propensity for errors in their solutions. Table 1 presents key characteristics of the intervention group, comprising the 32 fifth-grade students.

Table 1 provides a detailed overview of the demographic and social characteristics of the 32 fifth-grade students who participated in this study, offering insight into the diverse backgrounds represented. The distribution includes 12 students from stratum 1 and 20 from stratum 2, reflecting a range of socioeconomic conditions. Gender representation is relatively balanced, with 19 female students and 13 male students. The students' ages range from 10 to 13 years, and the family structures predominantly consist of nuclear families, with a few single-parent households. A notable disparity in connectivity to virtual classes is observed, with 14 students having access and 18 lacking access. This discrepancy may affect both the accessibility and effectiveness of the proposed educational interventions. Understanding these factors is crucial for comprehending the learning environment and identifying potential barriers to educational access, which could significantly influence the outcomes of the didactic unit and its associated activities.

Table 1. Population Characteristics

Characteristic	Specification	Number of Students
Stratum* 1	1	12
	2	20
Gender	Female	19
	Male	13
Grade	1-	19
	1	12
	2	1
Age	10	11
	11	15
	12	5
	13	1
Familiar	Single Parent	6
	Nuclear	26
Connectivity to virtual class	Yes	14
	No	18

* It is a classification of the socioeconomic condition of the Colombian population, which varies from 1 to 6.

Collection Techniques and Instruments

The data collected throughout the design and implementation of the didactic unit—including class observations, as well as individual and group evaluations—serve as the primary basis for evaluating the effectiveness of the activities. This evaluation encompasses a descriptive analysis of the students' work. A rubric was developed to assess the diagnostic tests, the activities within the didactic unit, and the exit test. The exit test, derived from standardized primary mathematics assessments used in Colombia, facilitates a comparative analysis of the various assessments. Table 2 outlines the types of evaluations conducted prior to, during, and following the intervention.

Table 2. Types and evaluation criteria

Assessments	Type	Criteria
Didactic Unit	Rubric	Qualitative
	Diagnostic test	Quantitative
Students	Activity rubric	Qualitative-quantitative
	Post-test	Quantitative

Description of the Intervention Proposal

Building upon the aforementioned considerations, a didactic unit was designed and implemented to facilitate the intervention. This unit consists of a series of activities structured as remote learning guides. These activities adhere to a format recommended by the University of Magdalena, Colombia, which includes the topic, objective, expected learning outcomes, description, methodology, content, and tasks to be completed. Each activity is organized around phases of expected learning, exploration, structuring, practice, evaluation, and conclusion.

The design of the activities aligns with the first Basic Learning Right (DBA) for fifth grade, which emphasizes the interpretation and application of rational numbers in their fractional representation to formulate and solve both additive and multiplicative problems. The specific learning objectives include:

1. Understanding the part-whole relationship and representing it using fractions, ratios, or quotients.
2. Applying rational (fractional) numbers in various contexts to problem-solving.
3. Identifying the operations required to solve different types of problems.
4. Solving problems that involve recognizing measurement patterns associated with natural or rational (fractional) numbers.

In response to the global impact of the COVID-19 pandemic, which necessitated isolation and confinement as preventive measures, there was an urgent need to develop remote learning guides. These guides were designed for students to complete at home with support from parents or caregivers. To enhance communication and interaction in the learning process, tutorials were provided through virtual tools such as WhatsApp, email, and video conferencing via Zoom. These measures facilitated continuous education and supported the documentation of the project's progress.

It is noteworthy that the teaching unit was developed using George Pólya's problem-solving methodology. This approach includes three activities (see Table 3) characterized by engaging, interactive actions and the presentation of historical mathematical problems adapted to the students' context. Some activities involve hands-on work with manipulative materials, and each activity provides a detailed, step-by-step guide aimed at achieving the objectives outlined in the teaching unit.

Table 3. Description of the didactic unit.

Activity	Time	Expected Learning
1: Designs of the king	Two weeks	Recognizes the fraction as a part - whole relationship and as an operator in its different representations
2: The visit to the zoo	Two weeks	Interpret the fraction as a ratio and as a percentage based on solving mathematical problems.
3: Fractions in history	Two weeks	Recognizes the fraction as a part-whole relationship, as an operator, as a ratio and as a percentage in its different representations, based on the solution of historical problems in mathematics.

Additionally, several tasks included in the teaching unit are outlined as follows.

Activity 1: Strategic Castle Distribution Across the Kingdom

A strategic king ordered the construction of 24 identical castles across his kingdom to mislead potential invaders and obscure the location of his true residence. He specified that the castles be distributed as follows: in the northern, southern, eastern, and western regions of his kingdom.

1. Castle Distribution Strategy:
    - How can the 24 castles be distributed throughout the kingdom?
    - How many castles would be built in each region?
    - What fraction of the total castles is built in each region?
    - Create a graphical representation of your solution.
2. Alternative Distribution Plans:
    - How could the 24 castles be distributed differently, ensuring that each region receives a unique number of castles?
    - How many castles would be built in each region?
    - What fraction of the total castles does each region receive?
    - Create a graphical representation of your solution.
3. Specific Regional Distribution:
    - Is it feasible to distribute the castles such that 5 are built in the northern region and one-sixth of the total in the southern region?
    - What fraction of the total would the 5 castles represent?
    - Create a graphical representation of your solution.

Activity 2: The Farm

Mr. Marcos, the owner of a fruit and vegetable farm, has decided to allocate his land to plant the crops depicted in Figure 1.

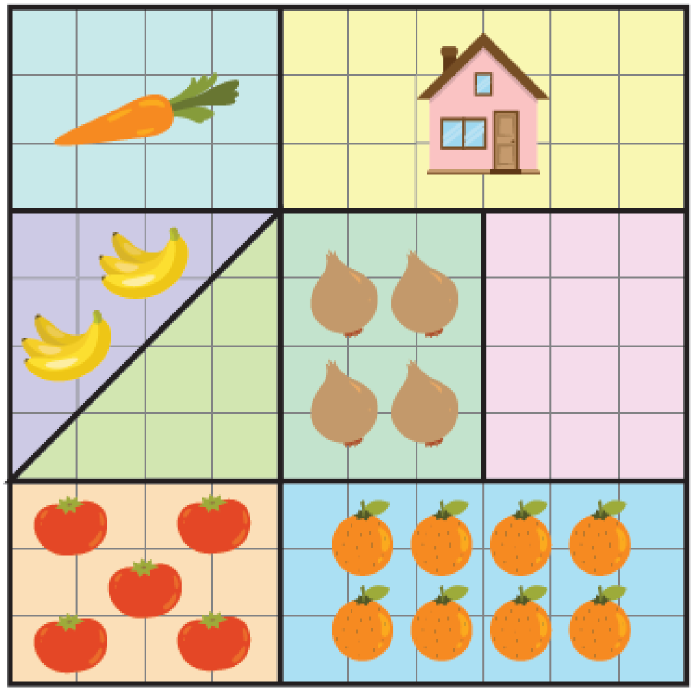

Figure 1. Distribution of Mr. Marcos Farmland for Various Crops and Housing

Calculate the fraction of the total farm that each of the following situations represents and provide justifications for your answers and the methods used**.**

1. The portion of land Mr. Marcos plans to use for building his house.
2. The portion of land designated for banana planting.
3. The portion of land allocated for planting; indicate what percentage of the total land this represents.
4. The portion of land that will remain unused for planting.
5. Using a graph, depict each portion and the percentage of the total land it represents.

Activity 3: Fractions in History

Expected Learning Outcomes: Students will recognize fractions as representing part-whole relationships, operators, ratios, and percentages through various representations (Figure 2). This understanding will be developed by solving historical problems in mathematics.

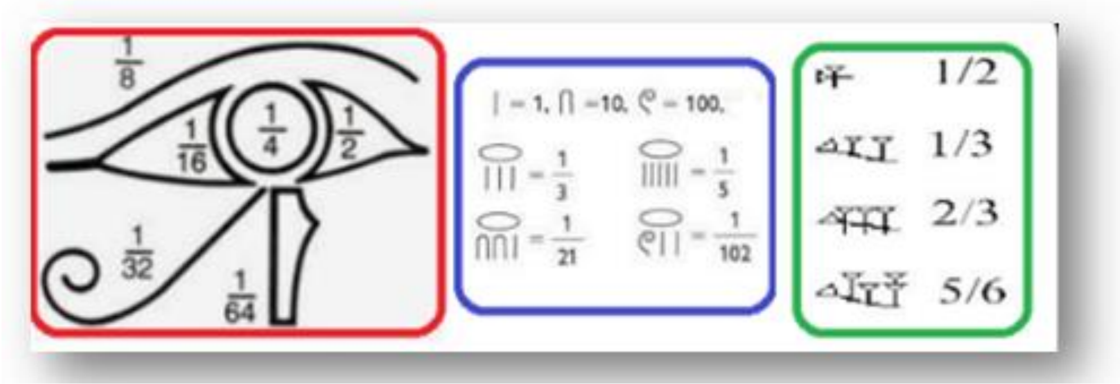

Figure 2. Historical Representations of Fractions Across Cultures

Situation 1: Distribution of Bread

The first six problems of the Rhind Papyrus (problems 1 to 6) involve distributing varying numbers of loaves (1, 2, 6, 7, 8, 9) among ten men. This highlights one of the primary activities that led to the development of fractions and the decimal base, which are considered fundamental for these distributions.

1. *Initial Distribution*: Given three loaves of bread and ten people, how would you distribute the bread among the ten people?
2. *Equal Distribution*: How would you distribute the three loaves of bread equally among the ten people? - Create a graphical representation of your solution.
3. *Historical Context*: How do you think the Egyptians would have solved this problem of distributing three loaves of bread among ten people?
4. *Analysis and Comparison*: Compare your solution with the historical solution found in the Egyptian method. What differences or similarities do you observe?

The validity of the instruments used in this study is primarily established through their careful design and alignment with the educational objectives. The diagnostic test, activity rubric, and exit test were meticulously developed to ensure they accurately measure students' understanding and application of fraction concepts as described in the didactic unit. The tests are rooted in established educational standards and borrow from Colombian standardized primary tests in mathematics, enhancing their validity in assessing the targeted competencies. In addition, the rubrics and tests were applied uniformly across the student sample, and their administration was controlled to maintain consistent conditions.

Moreover, the use of a rubric to evaluate both the diagnostic and exit tests helps standardize the measurement criteria, reducing subjective interpretation and increasing reliability. The comparative analysis of pre- and post-intervention data further supports the reliability of these tools, showing consistent improvements across different assessment moments which suggest that the instruments were effective in capturing genuine learning progressions.

RESULTS AND DISCUSSION

The analysis of responses to the activities outlined in the guide, as recorded on a performance evaluation grid aligned with the specifically designed rubric, is detailed in Table 4. The data presented for the activity "The Designs of the King - Los Designios Del Rey" reveals that 52% of the students have achieved an advanced level of proficiency. These students are adept at interpreting and representing fractions both as part-whole relationships and as operators between positive integers across various situations and contexts.

In the next tier, 22% of the students demonstrated satisfactory performance, showing their ability to recognize and represent fractions in both capacities, though this proficiency is evident only in specific situations or contexts. Conversely, 13% of the students are capable of recognizing fractions in these roles only in limited contexts. The remaining 13% of the students do not demonstrate an understanding of fractions as either part-whole relationships or operators between positive integers.

Table 4. Performance results of activity 1, called The Designs of the King

Performance Levels	Number of Students	%
ADVANCED	16	52
SATISFACTORY	8	22
MINIMUM	4	13
LOWER	4	13
Total	32	100

These results indicate that the majority of students achieved the highest performance level, effectively engaging with the proposed activities. This outcome underscores the effectiveness of the didactic strategies employed, which are closely aligned with Pólya's problem-solving approach. Gaviria (2016) highlights that this approach enables most students to distinguish the roles of the numerator and denominator in a fraction and to solve problems accurately, thereby significantly enhancing their skills across various contexts.

For students demonstrating lower performance levels, their challenges are consistent with the findings of Gómez (2019), Fauzi et al. (2023), and Arenas-Peñaloza (2021), who emphasize the inherent difficulty of fractions as a topic for elementary students. This difficulty explains why, although some students grasp the concept, others require additional time and practice. These students need further instruction to fully understand fractions, both as parts of a whole and as a quantity that represents the relationship between a specific amount (considered as a whole unit) and a segment of that amount, as well as understanding fractions as operators that effect transformations on quantities.

In conclusion, the intervention using the activities developed in "The Designs of the King" effectively enabled students to demonstrate their ability to understand and represent fractions, both as part-whole relationships and as operators on positive numbers. This success was achieved through meticulously designed exercises involving the addition, combination, segregation, separation, repetition, and equalization of fractional parts. The effectiveness of the intervention was further supported by the provision of remote learning materials and the individual and group guidance offered by teachers to facilitate the learning process.

Figures 3 and 4 present examples of student responses. For instance, Figure 3 showcases Student E12's method for solving the problem of distributing 24 castles across various regions of a kingdom. The task required students to determine the number of castles each region should receive, calculate the fraction of the total number of castles allocated to each region, and propose alternative distributions to ensure varied numbers of castles per region.

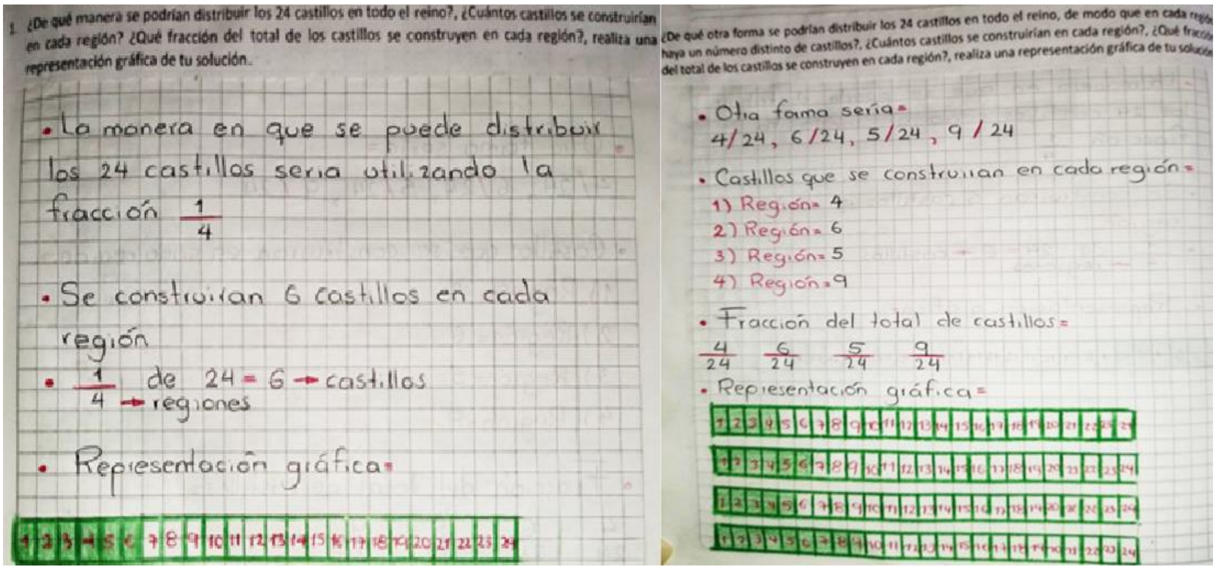

Figure 3. Student E12's Solution to Castle Distribution Problem.

Figure 4 illustrates Student E27's approach to the same problem. This student was tasked with calculating the number of castles for each region, determining the fraction of the total castles allocated to each region, and providing visual representations of these allocations. Additionally, Figure 4 explores alternative distributions, where each region receives a unique number of castles. This exercise involved calculating the number of castles per region and their respective proportions of the total, along with graphical representations of these variations.

The students' grasp of the part-whole relationship is evident from their interpretation of the task to "distribute the 24 castles throughout the kingdom." They understood that these castles should be allocated across four distinct locations within the kingdom, adhering to specific conditions for each scenario. In both parts of the exercise, students created diagrams to illustrate the total number of castles assigned to each location. Additionally, they detailed the number of castles to be constructed at each site and expressed the part-whole relationship through fractions corresponding to each location, demonstrating an understanding of fractions as operators. This reflects the students' ability to interpret and represent fractions both as part-whole relationships and as operators on positive integers across various contexts and situations.

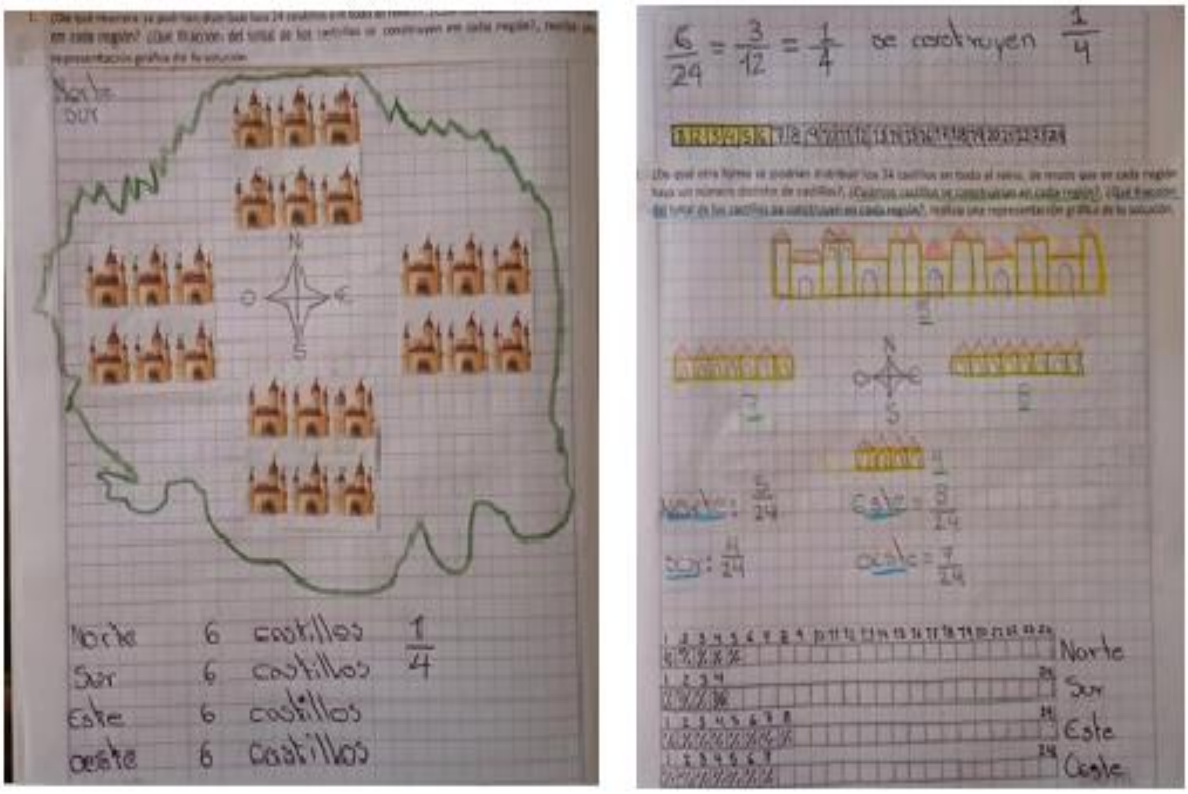

Figure 4. Student E27's Castle Distribution Solution.

The findings presented in Table 5 for the activity "The Visit to the Zoo" indicate that 68% of the students, categorized as either satisfactory or advanced, are proficient in interpreting, recognizing, and representing fractions as both ratios and percentages among positive integers across various situations and contexts. In contrast, 18% of the students are able to identify fractions as ratios and percentages only in specific situations or contexts. The remaining 14% of students do not recognize fractions as either ratios or percentages among positive integers.

Table 5. Performance results of activity 2, “The Visit to the Zoo”

Performance Levels	Number of students	%
ADVANCED	16	50
SATISFACTORY	5	18
MINIMUM	6	18
LOWER	5	14
Total	32	100

In line with the quantitative data presented, fractions were considered as ratios, which allow for the comparison of quantities of different magnitudes by differentiating between the numerator and denominator (Fandiño, 2009; Fauzi et al., 2023). This perspective contrasts with the part-whole interpretation within a measurement context. The exercises proposed in the second activity were designed based on this understanding. The results obtained are corroborated by Salinas (2013) and Hariyani et al. (2022), who suggest that comprehension of the fraction concept is achieved through a series of steps involving multiple interactions and interpretations, which are facilitated by the use of concrete materials with both short- and long-term objectives.

Given these observations, the effectiveness of the specially designed concrete materials in assisting students to recognize fractions as ratios and percentages among positive integers is evident. This effectiveness is illustrated through exercises involving equivalent fractions, proper fractions, percentages, and their equivalences. Additionally, students demonstrated increased engagement and a more positive attitude towards their mathematics tasks when these materials were incorporated.

Figure 5 presents a student's response to the scenarios outlined in Activity 2. Specifically, Figure 5 depicts Student E6’s responses to the problem scenarios related to a farm. The student calculated the fractions representing different situations on the farm, justified his answers, and explained the methods used. This analysis not only showcases the student’s problem-solving approach but also highlights his reasoning skills.

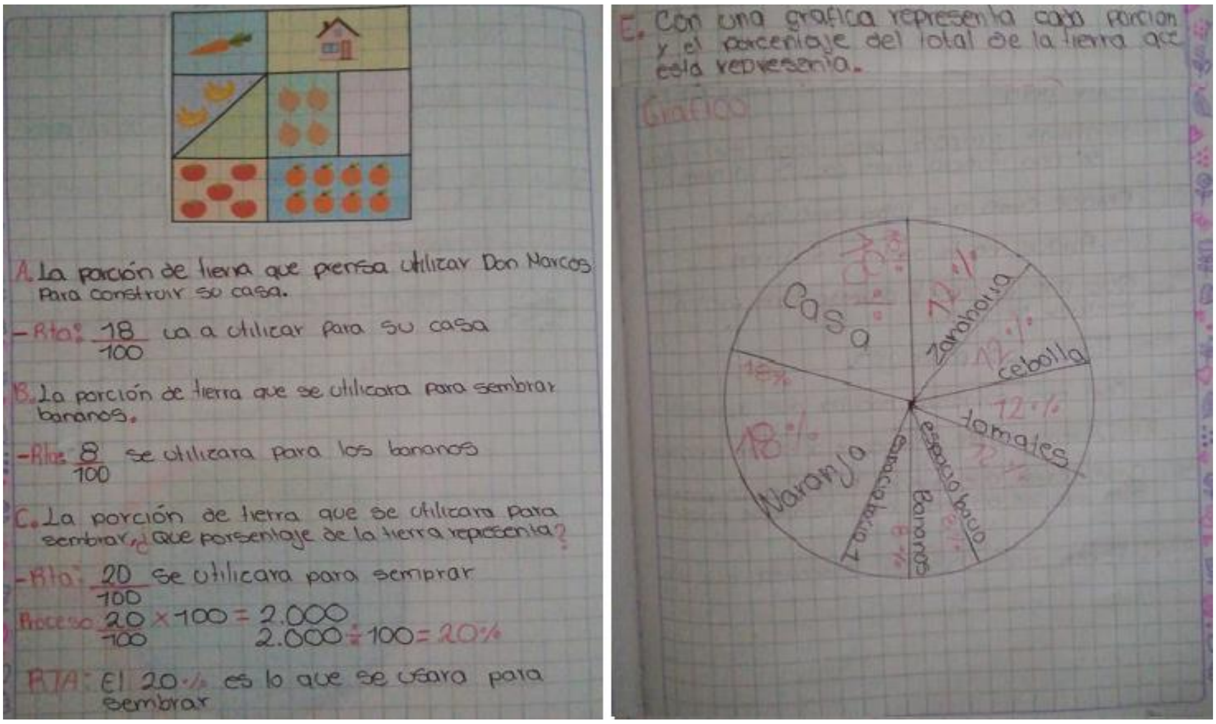

Figure 5. Student E6's Solution to Farm Fraction Problems.

By comparing the number of squares allocated to each crop with the total number of squares representing the farm, the student effectively interprets and represents the concept of a fraction as a ratio, thereby achieving the expected learning outcomes. Additionally, by calculating the corresponding percentages and graphing them in a circular diagram, the student demonstrates an understanding of the situation and represents the fraction as a percentage, further indicating the achievement of the learning objectives.

The data presented in Table 6 for the activity "Fractions in History" show that 83% of the students have attained satisfactory or advanced levels. These students effectively comprehend and apply the concept of fractions in various forms to solve problems across different contexts. In contrast, 10% of the students demonstrate only a partial understanding of fractions and apply the concept effectively in specific contexts. Meanwhile, 7% of the students are unable to recognize or utilize the concept of fractions in any form to solve problems.

Table 6. Performance results of activity 3, “Fractions in History”

Performance Levels	Number of students	%
ADVANCED	19	62
SATISFACTORY	7	21
MINIMUM	4	10
LOWER	2	7
Total	32	100

The results align closely with the frameworks proposed by Lamon (2020), Getenet & Callingham (2021), and the research conducted by Vásquez et al. (2019). These sources suggest that a didactic unit focused on active engagement enables students to successfully complete activities and connect them to their own experiences, thereby enhancing their understanding and mastery of the content covered in each activity facilitated by the guide. This notable improvement is supported by Piaget's theory (1988), which posits that in primary education, educators should make numerical thinking contextually relevant by utilizing real-life scenarios and aligning with students' stages of intellectual development.

The results are also consistent with the findings of Álvarez (2017), Cañabate (2021), and Pearn et al. (2022), which indicate that intervention projects can significantly enhance both performance and a deep understanding of the concepts presented in the didactic unit. The open and flexible nature of these projects fosters the development of numerical thinking among students. As students engage with the activities, they become more aware of their learning journey, reflecting on their progress and understanding. The success observed in Activity 3 was evident, with a high percentage of students achieving advanced and satisfactory levels. This success was attributed to the students' ability to comprehend the problems presented and apply various strategies or solutions to address real-life challenges effectively.

Figures 6 and 7 illustrate a student's responses to the scenarios presented in this activity. Figure 6 presents Student E12’s approach to Situation 1, which involved dividing three loaves of bread among ten people. The student detailed the method for equally distributing the bread and provided a graphical representation of this distribution. This visualization facilitates an understanding of the division method and ensures fairness in the allocation.

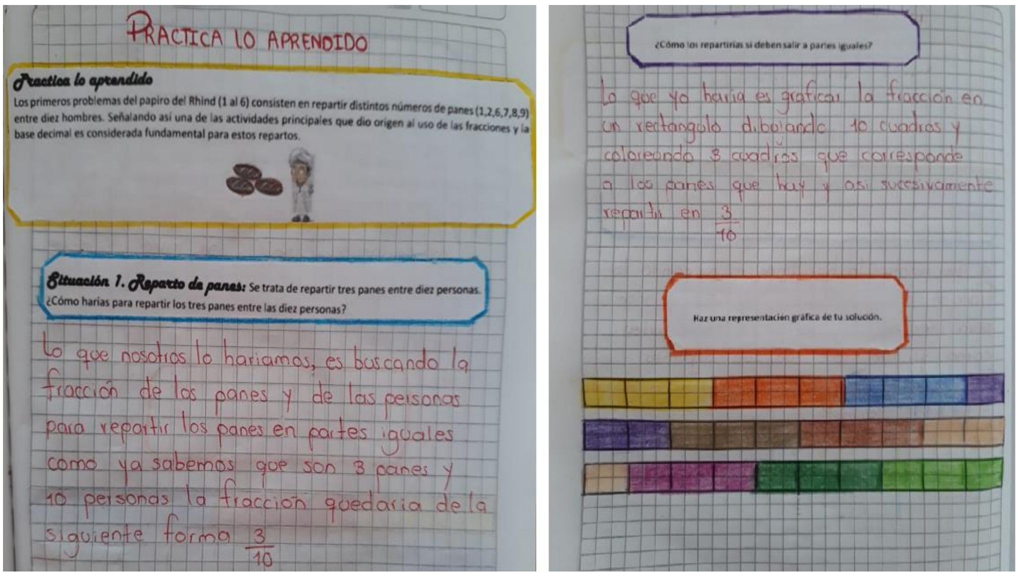

Figure 6. Student E12's Solution to Bread Distribution Task

Figure 7 depicts Student E12's approach to hypothesizing how the Egyptians might have organized animals. The task required the formation of groups with equal numbers of geese and turkeys, and the student explained the grouping process used. Additionally, the student calculated the percentage of animals that are chickens and provided a detailed account of the steps taken to reach this conclusion.

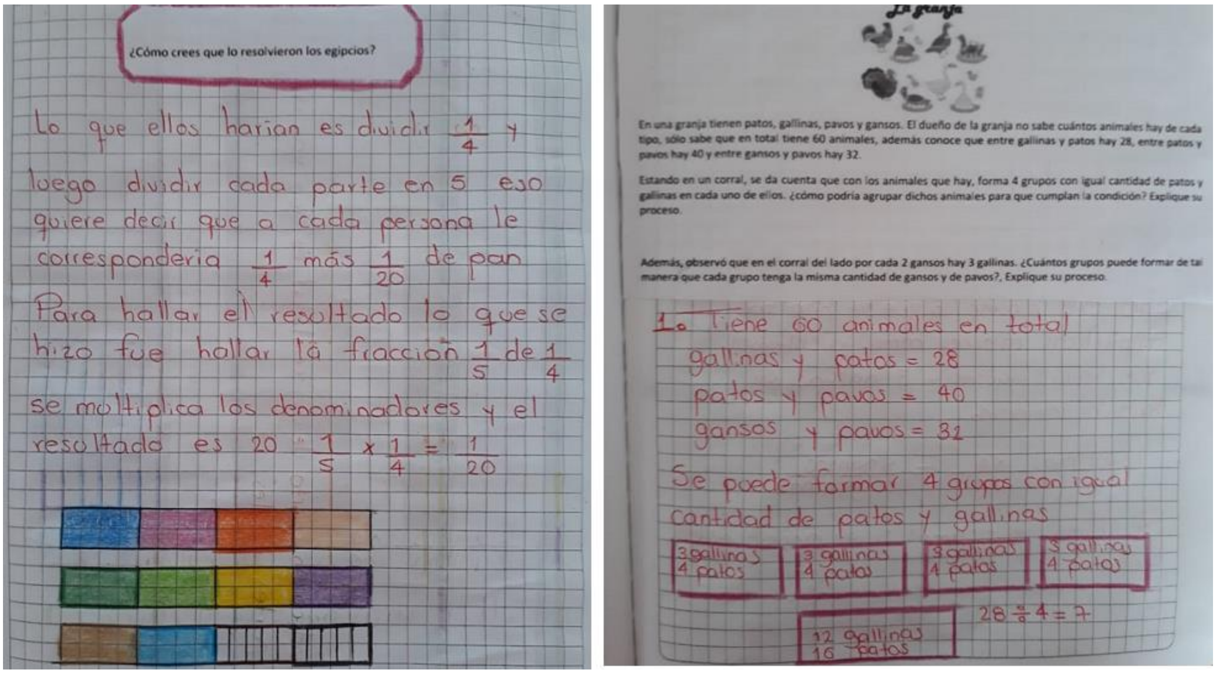

Figure 7. Student E12's Solution to Animal Grouping Task.

The student detailed their reasoning by comparing the quantity of bread available for distribution to the total number of people, resulting in the fraction. This was illustrated graphically, showing that each loaf was divided into 10 equal parts, yielding a total of 30 pieces, with 3 pieces allocated to each individual. The solution was refined using an ancient Egyptian method: each loaf was divided into 4 equal parts, with one part distributed to each person. The remaining two parts were further divided into 5 equal parts each, creating 10 additional pieces to ensure each person received one more piece. This calculation was interpreted as a fraction of a number, computed as $\frac{1}{5} \times \frac{1}{4}$, which equals $\frac{1}{20}$. Thus, in addition to the initial $\frac{1}{4}$ of a loaf, each person would receive an extra $\frac{1}{20}$ of a loaf, totaling $\frac{1}{4} + \frac{1}{20}$ of a loaf per person.

In the subsequent scenario, the student addressed the organization of 28 animals—hens and ducks—into 4 groups with an equal number of each animal type per group. The student determined that it was feasible to form 7 groups, with each group consisting of 3 hens and 4 ducks. Utilizing the condition "for every 2 geese, there are 3 hens," the student calculated the ratio and extrapolated it to estimate the total number of geese and hens on the farm. This led to the verification that the total number of animals equaled 60. The student then calculated the ratio of hens to the total number of farm animals to determine the percentage of hens. By multiplying the number of hens (12) by 100% and dividing by the total number of animals (60), the student concluded that hens constitute 20% of the farm's animal population. This sequence of activities clearly facilitated the students' proficiency in understanding and solving the exercises, thereby enhancing their overall comprehension.

As illustrated in Table 7, the results of the exit test show that 49% of the assessed students achieved advanced performance by effectively using fractions as part-whole relationships, quotients, ratios, and problem-solving tools across various contexts. In contrast, 45% of students demonstrated satisfactory performance, while 6% were categorized as having minimum performance. This indicates that the latter group of students faces difficulties in applying fractions as part-whole relationships, quotients, ratios, or percentages when solving everyday problems.

Table 7. Results of the Exit Test

Performance Levels	Number of students	%
ADVANCED	16	49
SATISFACTORY	15	45
MINIMUM	1	6
LOWER	0	0
Total	32	100

This comparison clearly indicates that the majority of students involved in the intervention showed an improvement in their performance levels, consistent with the findings of Ospina and García (2019) and Polotskaia and Savard (2021). Their research, grounded in Pólya's (1969) methodologies, demonstrated that didactic strategies significantly enhance fraction learning for a substantial percentage of students. Similarly, Valdovinos (2014) found that pedagogical interventions focused on problem-solving enable students to effectively understand and solve fraction problems. These interventions also facilitate connections between daily activities and mathematical concepts through the use of manipulatives and technology, thereby enriching students' comprehension of the problems.

In this context, the didactic unit functioned as a critical interactive component for remote learning, offering content and activities that, through their visual simplicity, added coherence and relevance to students' home-based learning experiences. The small percentage of students who did not achieve high performance levels reflects findings from Reséndiz and González (2018) and Domínguez et al. (2020), which highlight that students often struggle with fraction operations and may resort to converting fractions to decimals. This issue can be attributed to a lack of conceptual clarity or understanding of fractions and their basic operations.

Conversely, Ruíz and Lemos (2018) observed that direct teacher interventions contribute to a more effective learning environment, fostering increased motivation and interest in the timely and appropriate completion of activities. Similarly, López et al. (2016) asserted that enhancing motivation boosts students' self-confidence, which, in turn, amplifies their potential for mathematical thinking. It is crucial to recognize that teaching practices guiding students through problem-solving with fractions significantly impact their development of mathematical reasoning. This influence arises from the teacher's approach to the explanatory process, which can determine whether students engage in mechanical procedures or demonstrate genuine comprehension (Ebby et al., 2020).

Observations from the diagnostic phase revealed attitudinal challenges among students, including apathy and a lack of motivation towards mathematics. However, after implementing the educational guides, it was possible to manage and mitigate issues such as negativity, fear of making mistakes, and time constraints, leading to improved performance in solving fraction problems. These improvements suggest that only a small percentage of students remained at low performance levels, indicating a broader competency in understanding and applying the concept of fractions, especially when interpreting problem statements presented in everyday language.

The application of the didactic unit, which utilized historical contexts in mathematics to enhance understanding of fractions, proved beneficial for the 32 fifth-grade students. The results of this study demonstrate a significant enhancement in the students' abilities to comprehend and apply fractions, attributed to the educational interventions and historically contextualized problems. The consistency in performance improvements, as evidenced by the data from the exit tests compared to the diagnostic tests, underscores the reliability of these findings. Both qualitative feedback from students and quantitative data confirms the effectiveness of the teaching strategies and assessment methods employed.

The incorporation of everyday language and relatable historical contexts in problem statements likely contributed to increased student engagement and comprehension. This outcome aligns with educational research suggesting that contextualization enhances learning. Furthermore, the structured reflection and continuous assessment process facilitated by the self-reflective spiral sequence of the Action Research methodology ensured that the assessment tools served not only as measurement instruments but also as integral components of the iterative improvement of teaching practices.

CONCLUSIONS

The development of this intervention, which integrates historical elements into mathematics teaching, has demonstrated a significant enhancement in learning outcomes. By engaging students with historical contexts, this approach fosters curiosity and motivation while deepening their understanding, interpretation, and application of rational numbers in fractional representations. This advancement contributes to the field of mathematics education by extending beyond current methodologies.

The design of the didactic unit, grounded in inductive reasoning and analogies aligned with Pólya's problem-solving strategy, successfully utilized story-based activities to improve fraction learning. Through detailed instructions and relatable examples, students were able to grasp the practical applications of fractions, thereby enhancing their understanding of complex mathematical concepts and developing mental calculation skills. This methodology provides a clear scientific justification for the approach, suggesting that its application could be beneficial for other mathematical concepts and educational levels.

Our findings indicate that a structured approach to didactic planning—incorporating phases of exploration, structuring, practice, and assessment—can effectively consolidate knowledge and skills, fostering a comprehensive understanding of fractions. The progressive improvement in student performance underscores the efficacy of this approach and suggests that future research could explore its applicability to other mathematical domains.

Incorporating historical elements into the teaching of fractions at the primary level proves to be a valuable pedagogical strategy, enriching student learning and enhancing academic performance. By contextualizing the origins, development, and applications of fractions through historical perspectives, educators can link mathematical concepts to students' social and cultural realities, offering diverse problem-solving perspectives. Moreover, this approach can ignite students' curiosity and interest by exploring the historical roots of fundamental concepts. Comparing historical and modern solutions to fraction problems provides a powerful means to foster active and meaningful learning.

Student feedback underscores the importance of engaged and motivated learning, facilitated by visual and imaginative elements. This feedback suggests that incorporating such elements into mathematics education significantly enhances the learning experience and points to potential extensions of this research into broader educational contexts.

Future research should investigate the integration of similar didactic strategies across different mathematical topics and age groups to further validate and expand these findings. Ongoing research will focus on the scalability of this approach and its effectiveness in diverse educational settings, aiming to provide a broader empirical basis for the use of historical elements and structured didactic units in enhancing mathematical understanding.

Declarations

Author Contribution	:	RCT-P: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Visualization, Writing – Original draft, and Writing – review & editing. DP-G: Data curation, Formal analysis, Investigation, Methodology, Project administration, Validation, Visualization, Writing - original draft, and Writing - review & editing. EAA: Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Visualization, Writing - Original draft, and Writing – review & editing.
Funding Statement	:	None.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

REFERENCES

Álvarez, E. J. (2017). Diseño de una unidad didáctica para el aprendizaje cooperativo de números racionales en 2º de la ESO. *Master's thesis in Education*. Universidad Internacional de la Rioja.

Ambròs, A. (2009). La programación de unidades didácticas. *Aula de Innovación Educativa*, *180*, 26-32. http://www.ub.edu/dllenpantalla/sites/default/files/3%20%20AU%20188%20Alba%20Ambròs%20programar%20por%20competencias.pdf

Arenas-Peñaloza & Rodríguez-Vásquez. (2021). Enseñanza y aprendizaje del concepto de fracción en la educación primaria: estado del arte. *Cultura Educación Sociedad, 12*(2), 22-60. https://doi.org/10.17981/cultedusoc.12.2.2021.03

Arteaga, M. B., & Macías, S. J. (2016). Didáctica de las Matemáticas en la Educación infantil. *Editorial Universidad Internacional de La Rioja, S. A*. First edition. Spain.

Ausubel, D. P. (1976). Psicología educativa: Un punto de vista cognoscitivo. *Trillas Editorial*. Mexico.

Azman, N. A., & Maat, S. M. (2021). Integration of the history of mathematics in mathematics education: A systematic literature review. *International Journal of Academic Research in Business and Social Sciences*, *11*(4), 1035-1057. http://dx.doi.org/10.6007/IJARBSS/v11-i4/9768

Babakr, Z. H., Mohamedamin, P., & Kakamad, K. (2019). Piaget's cognitive developmental theory: Critical review. *Education Quarterly Reviews*, *2*(3), 517-524. https://doi.org/10.31014/aior.1993.02.03.84

Björklund, C., & Palmér, H. (2022). Teaching toddlers the meaning of numbers—connecting modes of mathematical representations in book reading. *Educational Studies in Mathematics, 110*, 525–544. https://doi.org/10.1007/s10649-022-10147-3

Butto, Z. C. (2013). El aprendizaje de fracciones en educación primaria: Una propuesta de enseñanza en dos ambientes. *Horizontes Pedagógicos, 15*(1), 33-45. https://dialnet.unirioja.es/descarga/articulo/4892957.pdf

Cañabate, D., Bubnys, R., Nogué, L., Martínez-Mínguez, L., Nieva, C., & Colomer, J. (2021). Cooperative learning to reduce inequalities: Instructional approaches and dimensions. *Sustainability, 13*(18), 10234. https://doi.org/10.3390/su131810234

Charalambous, C. Y., Philippou, G. N., & Kyriakides, L. (2008). Tracing the development of preservice teachers' efficacy beliefs in teaching mathematics during fieldwork. *Educational Studies in Mathematics*, *67*, 125-142. https://doi.org/10.1007/s10649-007-9084-2

Chotimah, C., & Festiyed. (2020). Meta-analysis of the effect of using PhET interactive simulations on worksheets on high school students' physics learning outcomes. *Journal of Physics: Conference Series, 1481*(1), 012093. https://doi.org/10.1088/1742-6596/1481/1/012093

Díaz, P. J. (2017). Importance of Phet simulation in teaching and learning equivalent fractions. *Revista Educación y Desarrollo Social*, *11*(1), 48-63. https://doi.org/10.18359/reds.2011

Dominguez, H., Crespo, S., del Valle, T., Adams Corral, M., Coupe, M., Gonzalez, G., & Ormazabal, Y. (2020). Learning to transform, transforming to learn: Children's creative thinking with fractions. *Journal of Humanistic Mathematics*, *10*(2), 76-101. https://doi.org/10.5642/jhummath.202002.06

Ebby, C., Hulbert, E., & Broadhead, R. (2020). A focus on addition and subtraction. *Bringing Mathematics Education Research to the Classroom*. Routledge. https://doi.org/10.4324/9781003038337

Fandiño, M. (2009). Las fracciones: Aspectos conceptuales y Didácticos. *Didácticas Magisterio*. 222 p.

Fauzi, I., Rakhmat, C., & Budiman, N. (2023). Complex thinking: How are students' mathematical problem-solving skills in elementary school?. *Bulletin of Science Education, 3*(3), 228-240. http://dx.doi.org/10.51278/bse.v3i3.916

Gaviria, U. G. (2016). Estrategia didáctica para trabajar el concepto de fracción como relación Parte-Todo en grado quinto, teniendo en cuenta su origen histórico. *Master's Thesis*. Universidad Nacional de Colombia

Getenet, S., & Callingham, R. (2021). Teaching interrelated concepts of fraction for understanding and teacher's pedagogical content knowledge. *Mathematics Education Research Journal, 33*, 201–221. https://doi.org/10.1007/s13394-019-00275-0

Gómez, R. M. (2019). Identificación de dificultades en el proceso de enseñanza-aprendizaje de las fracciones y la resolución de problemas en Educación Primaria. *Master's thesis in Specific Didactics.* Universitat Jaume. http://hdl.handle.net/10234/184791

Hariyani, M., Herman, T., Suryadi, D., & Prabawanto, S. (2022). Exploration of student learning obstacles in solving fraction problems in elementary school. *International Journal of Educational Methodology*, *8*(3), 505-515. https://doi.org/10.12973/ijem.8.3.505

Lamon, S. (2020). *Teaching Fractions and Ratios for Understanding*. Routledge.

Latorre, A. (2007). La investigación-acción: conocer y cambiar la práctica educativa. Barcelona, España. Graó Editorial. https://www.uv.mx/rmipe/files/2019/07/La-investigacion-accion-conocer-y-cambiar-la-practica-educativa.pdf

Loc, N.P., Tong, D.H., & Chau, P.T. (2017). Identifying concept fraction of primary school students: The investigation in Vietnam. *Educational Research and Reviews*, *12*(8), 531-539. https://doi.org/10.5897/ERR2017.3220

López, B. F., Rentería, M. L., & Vergara, M. F. (2016). El aprendizaje de las operaciones básicas matemáticas en educación primaria, mediado por ambientes virtuales de aprendizaje. *Universidad Pontificia Bolivariana Maestría En Ciencias Naturales y Matemáticas*.

Ospina, N., & García, O.J. (2019). Propuesta de una unidad didáctica como estrategia para la resolución de problemas a partir de una estructura aditiva a una estructura multiplicativa para lograr aprendizajes significativos en niños y niñas del grado tercero de la básica primaria. *Master's Thesis in Education*. Universidad del Tolima.

Pearn, C., Stephens, M., & Pierce, R. (2022). Algebraic reasoning in years 5 and 6: Classifying its emergence and progression using reverse fraction tasks. *ZDM Mathematics Education, 54*, 1257–1271. https://doi.org/10.1007/s11858-022-01426-7

Piaget, J. (1988). *Psicologia evolutive de Jean Piaget*. Fourth edition. Mexico: Editorial Paidós Mexicana, S. A.

Polotskaia, E., & Savard, A. (2021). Some multiplicative structures in elementary education: A view from relational paradigm. *Educational Studies in Mathematics, 106,* 447–469. https://doi.org/10.1007/s10649-020-09979-8

Pólya, G. (1969). *How to Solve it?*. Trillas.

Phuong, N.T., & Loc, N.P. (2020). Solving word problems involving “ratio” concept of the fifth-grade students: A study in Vietnam. *Universal Journal of Educational Research*, 8(7), 2937-2945. http://doi.org/10.13189/ujer.2020.080722

Reséndiz, B. E., & González, S. C. (2018). Teaching fractions in third-grade elementary students: Discourse analysis and pedagogical practices. *Revista Internacional de Ciencias Sociales y Humanidades SOCIOTAM,* *28*(1), 109-138. https://www.redalyc.org/jatsRepo/654/65457048006/html/index.html

Ronen, I.K. (2020). Action research as a methodology for professional development in leading an educational process. *Studies in Educational Evaluation*, *64.* 100826. https://doi.org/10.1016/j.stueduc.2019.100826

Ruíz, R. A., & Lemos, G. D. (2018). Recursos didácticos para la enseñanza de resolución de problemas de suma – resta. La experiencia de primero de Primaria de la Institución Educativa Simón Bolívar de Torno Rojo - Puerto Libertador – Córdoba. *Master's thesis*. Universidad Santo Tomás.

Salazar, M., & Díaz, L. (2009). La Actividad de Medir Aporta Significados a Fracciones y Razones. *Comité Latinoamericano de Matemática Educativa*, Acta 22, 207–216.

Salinas, O.D. (2013). Estrategias didácticas para la enseñanza de fracciones en el tercer ciclo de la educación primaria. *Universidad Pedagógica Nacional*, México. http://200.23.113.51/pdf/29998.pdf

Schubring, G. (2011). Conceptions for relating the evolution of mathematical concepts to mathematics learning—epistemology, history, and semiotics interacting: To the memory of Carl Menger (1902–1985). *Educational Studies in Mathematics*, *77*(1), 79-104. https://doi.org/10.1007/s10649-011-9301-x

Silva, H. A. (2017). Propuesta didáctica para el fortalecimiento del aprendizaje de los números racionales en el grado 601 del Colegio Miguel Antonio Caro I.E.D J.M. a través de la teoría de las situaciones didácticas. Facultad de ciencias de la educación Instituto de postgrados Maestría en Ciencias de la Educación. Énfasis en Orientación Educativa. Repository of the Universidad libre. Retrieved from: https://repository.unilibre.edu.co/handle/10901/10270

United Nations Educational, Scientific and Cultural Organization (UNESCO). (2016). *Contributions to the teaching of Mathematics*. Available in: https://unesdoc.unesco.org/ark:/48223/pf0000248073

Valdovinos, S. (2014). Desarrollo de habilidades para el aprendizaje de las fracciones por medio de la manipulación de objetos y las tecnologías en los alumnos de quinto grado de primaria. Master's Thesis in Education. Universidad Pedagógica Naciona, Michoacan - México. Available in: http://bibliotecaupn161.com.mx/tesis/221PROYECTODETITULACION.pdf.pdf

Vásquez, T. E., Armenta, C. M., & Romero, F. C. (2019). The fraction as a measure and as an operator: A design experience of didactic activities. *Acta Latinoamericana de Matemática Educativa*, *32*(2), 322-331. http://funes.uniandes.edu.co/14129/1/Vasquez2019La.pdf

Vittori, D. (2021). Which organization for which party? An organizational analysis of the five-star movement. *Contemporary Italian Politics*, *13*(1), 31-48. https://doi.org/10.1080/23248823.2020.1853373

Vygotsky, L. S. (1978). *Mind in society: The development of higher mental processes*. Harvard University Press.

Wahyu, K., Kuzu, T.E., Subarinah, S., Ratnasari, D., & Mahfudy, S. (2020). Partitive fraction division: Revealing and promoting primary students' understanding. *Journal on Mathematics Education, 11*(2), 237-258. http://doi.org/10.22342/jme.11.2.11062.237-258

Yayuk, E., & Husamah, E. (2020). The difficulties of prospective elementary school teachers in item problem solving for mathematics: Polya`s steps. *Journal for the Education of Gifted Young Scientists*, *8*(1), 361-368. https://doi.org/10.17478/jegys.665833