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Praxeological analysis in Indonesian and Singaporean mathematics textbooks: An understanding geometrical similarity by students

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Abstract

Mathematics textbooks are essential tools for facilitating the achievement of learning objectives, yet their use varies across classrooms, even when the same mathematical concepts are being taught. This study aims to investigate students' understanding of geometrical similarity through praxeological analysis within the framework of the Anthropological Theory of the Didactic (ATD). The analysis identifies differences in the structuring of geometrical similarity tasks between Indonesian and Singaporean textbooks. According to ATD praxeology, mathematical content is analyzed through the "praxis" and "logos" blocks, where praxis encompasses task types (T) and techniques (τ), while logos involves technology (θ) and theory (Θ). The findings highlight distinct priorities in how geometrical similarity is presented in textbooks from both countries, with implications for students' conceptual and procedural understanding. Specifically, the absence of pantograph tasks in Singaporean textbooks and the lack of scale factor exercises in Indonesian textbooks reflect differences in knowledge construction and task emphasis. These variations impact students' ability to relate textbook knowledge to real-world applications. The study underscores the significance of incorporating a diverse range of task designs and practical experiences to enhance students' geometric reasoning abilities. A well-structured textbook not only supports students in achieving learning objectives but also fosters a continuous process of building, connecting, and deepening their mathematical understanding.

Keywords: Anthropological Theory of the Didactic, Comparative Study, Mathematics Textbooks, Praxeology, Similarity

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Textbooks play a crucial role in shaping and enhancing active learning processes while supporting student engagement across diverse educational contexts. Numerous studies emphasize their significance in fostering active teaching practices and driving instructional innovation (Glasnovic Gracin, 2018; Rezat et al., 2021; van den Ham & Heinze, 2018). Furthermore, textbooks serve as pivotal instruments for initiating instructional change and promoting innovative teaching approaches (Rezat et al., 2021). Glasnovic Gracin (2018) highlights the role of textbooks in structuring varied tasks that actively involve students in mathematical activities, such as computation, interpretation, and argumentation. Moreover, the selection of textbooks significantly influences educational practices, as they are integral components of the learning process (van den Ham & Heinze, 2018).

Research also underscores the impact of textbooks on students' cognitive development, particularly in cultivating critical thinking skills through well-designed and purposeful tasks (Takeuchi & Shinno, 2020; Wijayanti & Winslow, 2017). As foundational educational resources, mathematics textbooks are extensively utilized to achieve curriculum-aligned learning objectives (Glasnovic Gracin, 2018). They serve as indispensable tools for both teachers and students, providing structured guidance that shapes teaching and learning practices in classroom settings (O'Halloran et al., 2018).

Mathematics textbooks play a central role in fostering active teaching practices by serving as essential tools that teachers utilize to structure and implement effective classroom learning experiences. Prior studies have investigated the ways in which teachers use mathematics textbooks to organize and deliver instruction in classroom settings (Ulusoy & İncikabı, 2020). Typically arranged by topics, these textbooks present new material through a combination of explanatory content, illustrative examples, and a variety of tasks and activities designed to engage students. The learning process often includes reading and studying new content, analyzing visual representations such as tables and graphs, and completing assigned exercises (Abdullah & Shin, 2019). Teachers act as key facilitators in this process, making critical decisions about when to introduce examples and assign tasks to students (Amaral & Hollebrands, 2017).

Effective textbooks are expected to not only convey knowledge but also stimulate interest, encourage active participation, and facilitate the practical application of mathematical concepts. As foundational resources in mathematics education (Choi & Park, 2013), they must engage students, promote active learning, develop mathematical reasoning, and connect theoretical knowledge to real-world applications (Hong & Choi, 2018). Research further indicates that key instructional elements, such as student assignments, teacher-generated questions, group configurations, and assessment methods, are often derived directly from textbooks (Lepik et al., 2015). A well-designed textbook can profoundly influence the quality of mathematics education by supporting the diverse needs of students and enhancing their learning outcomes (Gravemeijer, 2014).

This study focuses on a comparative analysis of mathematics textbooks, with a particular emphasis on the concept of similarity. Guided by the Anthropological Theory of the Didactic (ATD) (Chevallard & Bosch, 2020a), the research adopts this theoretical framework to explore the similarities and differences between mathematics textbooks used in Indonesia and Singapore. Singapore was selected as a reference point in this analysis due to its exceptional performance in recent international assessments. According to the 2022 PISA results, Singapore ranked first in mathematics with a score of 575, whereas Indonesia ranked 69th with a score of 366 (OECD, 2023). This significant disparity highlights an opportunity for Indonesia to gain valuable insights from Singapore's highly effective educational system, particularly in its approach to mathematics instruction.

Both Singapore and Indonesia implement national curricula designed to meet their respective educational priorities and contextual needs. Indonesia's diverse cultural landscape and regional variations in student abilities significantly shape its approach to textbook design (Yang & Sianturi, 2017). In contrast, Singapore's curriculum emphasizes the development of problem-solving skills as a central focus of mathematics education (Toh et al., 2019). Indonesia's mathematics curriculum leans more heavily on a scientific approach, encouraging the investigation of phenomena, the construction of new knowledge, and the integration of prior understanding.

This study seeks to explore how these distinct curricular approaches are manifested in the textbooks of the two countries, with a particular focus on identifying elements of Singapore's methodology that could be adapted to enhance mathematics learning in Indonesia. By employing the ATD, the study examines the constraints and conditions imposed by each country's curriculum, as reflected in their textbooks (Pansell & Bjorklund Boistrup, 2018). These textbooks, shaped through processes of didactic transposition and influenced by cultural and historical factors, serve as critical empirical evidence to analyze variations in what is considered teachable knowledge across contexts (Kang & Kilpatrick, 1992).

Furthermore, this study also seeks to address a gap in the existing literature by examining not only the effectiveness of textbook usage in the classroom but also the interrelationships between the pieces of knowledge presented within the textbooks under analysis. Focusing specifically on the concept of similarity, the study investigates the structure and content of mathematics textbooks in Indonesia and Singapore—two nations with distinct educational traditions. Geometry, particularly the concept of congruence, is selected as the focal topic due to its fundamental role in mathematics education. Congruence serves as a critical link between various geometry topics and their practical applications in everyday contexts (Yang et al., 2017). The concept of similarity is instrumental in helping students understand the relationships between shape and size, a concept frequently applied in fields such as art, architecture, and science (DeJarnette et al., 2015). Moreover, mastering similarity enhances students' problem-solving and critical thinking skills, which are essential competencies in modern education.

Through this comparative analysis, the study aims to uncover the distinctive characteristics of textbooks in each country, with a particular emphasis on task design. By analyzing the knowledge and its interrelations within the textbooks, this research provides insights that can inform the ongoing development and refinement of teaching materials, ensuring they align with evolving educational demands. Furthermore, the study explores the nature of the knowledge presented in the teaching materials from Indonesia and Singapore. Specifically, it seeks to answer the question: How is the concept of similarity presented in mathematics textbooks in Indonesia and Singapore, particularly in relation to task types as defined by praxeology theory? Finally, the findings will contribute to a deeper understanding of how curricular and pedagogical choices influence the teaching of key mathematical concepts.

METHODS

The theory of didactic transposition, as introduced by Chevallard and Bosch (2020b), identifies four key stages in the process of knowledge transmission: scholarly knowledge, knowledge to be taught, taught knowledge, and learned knowledge. Despite its relevance, there is a notable absence of comparative studies examining the treatment of similarity concepts in Indonesian mathematics textbooks relative to those of other countries. This study aims to fill this gap, particularly focusing on the topic of similarity within the broader framework of textbook analysis.

A qualitative research approach was employed, utilizing a hermeneutic phenomenological design. Phenomenology seeks to uncover the interplay between objectivity and subjectivity inherent in human experiences (Guillen, 2019). Meanwhile, hermeneutics delves into understanding behaviors, both verbal and non-verbal, cultural practices, organizational systems, and the meanings they embody, all while preserving their uniqueness (Guillen, 2019). These two approaches are complementary, with phenomenology providing the foundation for understanding lived experiences and hermeneutics offering tools for interpreting the meaning behind those experiences.

Friesen et al. (2012) emphasize that the phenomenological hermeneutic approach is particularly well-suited for research in education, as it allows for an in-depth exploration of the cultural, structural, and experiential dimensions of teaching and learning. By employing this methodology, the study aims to provide meaningful insights into the design and content of similarity materials in mathematics textbooks, facilitating a deeper understanding of how educational practices and cultural contexts influence the transmission of mathematical knowledge.

Analytical Model: A Reference Epistemological Model

The analytical framework utilized in this study draws from praxeological theoretical analysis, which seeks to examine textbooks originating from diverse countries (Chevallard & Bosch, 2020b). Praxeology within the framework of ATD theory comprises two primary components: praxis and logos. Praxeology entails scrutinizing the mathematical content of a textbook through the lenses of praxis and logos. Each component encompasses task type (T) and technique (τ) for praxis, denoting the nature of the given task and the method of its execution, respectively. For logos, it involves technology (θ) and theory (Θ), wherein technology elucidates and justifies techniques, and theory provides explanations or justifications for technology (Shinno & Mizoguchi, 2021). This praxeological theory offers an alternative perspective for understanding the content composition within textbooks.

Data Collection Procedure

Data collection for this study was conducted through document analysis, a case-based investigative method focusing on written materials, such as notes or documents. This approach is widely used in educational research and, in this study, mathematics textbooks served as the primary data source. The textbooks analyzed were secondary school mathematics textbooks from Indonesia and Singapore, with a particular focus on the tasks they included and the conceptual frameworks they employed.

Additionally, an individual written assessment was administered to 30 ninth-grade students following their instruction on the topic of similarity. The participants, drawn from a single class in a secondary school in Yogyakarta, Indonesia, completed their solutions on answer sheets. To ensure robust data triangulation, interviews were conducted as a supplementary method. This triangulation strengthens the reliability and depth of the research findings by combining insights from document analysis, written assessments, and interviews. Through these methods, the study provides a comprehensive analysis of how the concept of similarity is taught and understood in different educational contexts.

Textbook Selection

The primary data for this study were derived from middle school mathematics textbooks, chosen based on the relevance of geometrical similarity topics in the curriculum rather than the specific grade level. For Indonesia, the selected textbooks include a 9th-grade mathematics book published by Kemdikbud for the 2013 curriculum (issued in 2018) and a 7th-grade mathematics book published by Kemdikbudristek for the Merdeka curriculum (issued in 2022). For Singapore, the study analyzed the New Syllabus Mathematics Textbook 2 and 3.

Textbooks often mirror the national objectives outlined in a country's curriculum, incorporating instructional guidance as part of the implemented curriculum. The differing educational contexts of Indonesia and Singapore may account for structural variations in their mathematics textbooks and how these resources are utilized in classrooms. Textbooks in both countries are developed to align with national standards, serving as key tools for teachers in lesson planning or as direct teaching resources during instruction.

The development process for these textbooks involves collaboration among teachers, researchers, and mathematicians, followed by an evaluation conducted by an editorial team. Each textbook included in this study has been officially approved and published under the oversight of the respective Ministries of Education in Indonesia and Singapore, ensuring adherence to national educational standards. These considerations underscore the significance of the selected textbooks as representations of their respective national curricula and as critical components of the educational frameworks in each country.

Data Analysis

Data analysis, as defined by Islam (2020), is a systematic process of seeking meaning. In qualitative research, this process is iterative and occurs concurrently with other stages, such as data collection and the writing of findings (Creswell, 2018; Flick, 2018). For this study, the analysis began with identifying the types of task designs present in each textbook. This process was carried out collaboratively by all researchers through a forum discussion. Upon reaching a consensus regarding the types and sequences of task designs, individual researchers then examined the associated techniques, technology, and theoretical underpinnings.

The findings from this individual analysis were subsequently brought back to the group, where researchers compared and refined their interpretations until a consensus was achieved. Once agreement was reached, the next phase involved developing questions for 9th-grade students to further validate the findings regarding task design arrangements in the analyzed mathematics textbooks. These questions were carefully reviewed and validated by mathematics experts and classroom teachers to ensure their reliability and alignment with the study's objectives.

The comparison of the main topics in the similarity material from the mathematics textbooks of Indonesia and Singapore is summarized in Table 1, which serves as a foundation for analyzing the organization, presentation, and educational approaches embedded in the respective textbooks. The validated questions and subsequent student responses offer additional empirical support for understanding how the identified task designs are implemented in educational practice.

Table 1. The main topics focused on similarity in textbooks from Indonesia and Singapore

Country	Chapter (Grade, objective, page number)	Objectives
Indonesia	Congruent & Similarity (9^{th} grade, 41 pages) (2013 curriculum)	1. Identifying two similar or non-similar objects. 2. Explaining the conditions/properties of two similar polygons. 3. Testing and proving whether two triangles are similar or not. 4. Determining the length of sides or unknown angles of two similar polygons. 5. Solving everyday problems based on observations related to the application of the concept of similarity of polygons.
	Similarity (7^{th} grade, 26 pages) (Merdeka curriculum)	1. Determine the relationship between angles on intersecting lines and on two parallel lines cut by a transversal. 2. Estimate the size of an angle. 3. Use information about angles (supplementary, complementary, vertical, and opposite angles in flat shapes) to solve problems for unknown angles.

Country	Chapter (Grade, objective, page number)	Objectives
		4. Use the condition of similarity to determine if two triangles are similar. 5. Use the condition of similarity to solve problems.
Singapore	Congruent & Similarity (8th grade, 42 pages)	1. Examine whether two figures are similar. 2. State the properties of similar triangles and polygons. 3. Make simple scale drawings with appropriate scales. 4. Interpret scales on maps. 5. Solve simple problems involving similarity.
	Congruent & Similarity (9th grade, 44 pages)	1. Apply the three-similarity test to determine whether two or more triangles are similar. 2. Solve problems involving similar triangles.

RESULTS AND DISCUSSION

Mathematics textbooks have a substantial influence on mathematics education in schools (Hodgen, 2010; Sherman et al., 2020; Sievert et al., 2019). According to Hodgen (2010), textbooks serve not only as instructional tools but also as frameworks that facilitate the contextual transfer and application of mathematical knowledge within teaching practices. Similarly, Sherman et al. (2020) emphasize that textbooks play a vital role in assisting teachers by organizing tasks that nurture students' mathematical thinking while providing foundational scaffolding for effective classroom instruction. Sievert et al. (2019) further highlight that textbooks support teacher knowledge by offering structured learning opportunities that enhance students' adaptive expertise in mathematics over time.

When designing mathematical tasks for students, textbooks are integral in supporting the delivery of specific mathematical content (Lundberg & Kilhamn, 2018). As such, they are critical for understanding how students engage with and interpret mathematical concepts through the tasks they are assigned. This study leverages the Anthropological Theory of the Didactic (ATD) to analyze the praxeological structure of tasks in mathematics textbooks.

The analysis focused on all questions presented in the main text sections of the textbooks, as well as practice questions typically found at the end of each chapter. These questions were categorized according to the sub-objectives they were designed to achieve. The classification process began with the praxis block, which involves the task (T) and technique (τ). Subsequently, the logos block was considered, interpreting the technological (θ) and theoretical (Θ) components. This systematic classification provides insights into how textbooks organize and present tasks to facilitate students' mathematical understanding, while also aligning with curriculum goals and pedagogical principles.

The Praxis Block

To identify each type of task, the researcher employed four techniques based on the epistemological theory of knowledge (Audi, 2010) namely perceptual (τ_1), memorial (τ_2), introspective (τ_3), and a priori (τ_4). Students begin by acquiring knowledge about similarity material in the mathematics textbook through the perceptual stage. As they read the text and observe illustrations, images, and visual examples depicting these concepts, they can gain initial understanding of what similarity means in the context of geometry. Subsequently, in the memorial stage, students store this information in their memory by reviewing the definitions, properties, and examples provided in the textbook.

The introspective process is a critical stage where students actively reflect on their comprehension of similarity concepts. During this phase, they may formulate questions about the material, compare and contrast related concepts, and explore distinctions to deepen their understanding. Introspection also involves collaborative learning, where students engage in discussions with peers or seek guidance from teachers. These interactions provide opportunities to hear alternative perspectives, gain clarifications, and enhance their conceptual grasp through the exchange of ideas and insights. Furthermore, in the a priori stage, students employ logical reasoning and draw upon their prior knowledge to establish connections between the concept of similarity and other mathematical principles they have previously encountered. This stage encourages the synthesis of new and existing knowledge, fostering a holistic understanding of the material.

By systematically progressing through these stages—perceptual, memorial, introspective, and a priori—students develop a robust and nuanced understanding of similarity in mathematics. This comprehensive approach equips them with the skills and confidence to effectively apply their knowledge to solve complex geometric problems, bridging theoretical understanding with practical application.

In the Table 2, the difference lies in T_4, T_7, and T_8. Unlike the mathematics textbooks in Indonesia, in the Singapore textbooks, the similarity material is reaffirmed by linking similarity as magnification and similarity on a scale image. T_7 and T_8 in the Singapore textbook are given to Grade 9th, which is a continuation of Congruent & Similarity material at Grade 8th. In Indonesian Textbooks, the recognition of similarity is done through the practice of using the pantograph (T_4), while in Singapore textbooks, this is not done.

Table 2. Types and number of assignments on the concept of similarity textbook Indonesia and Singapore

Type of Task	**Description**	**Indonesia (2013 Curriculum)**	**Indonesia (Merdeka Curriculum)**	**Singapore**
		τ	τ	τ
T_1	The Relationship Between Angles		$\tau_1, \tau_2, \tau_3, \tau_4$	
T_2	Identifying Two Similar Objects	$\tau_1, \tau_2, \tau_3, \tau_4$	τ_1, τ_2, τ_3	$\tau_1, \tau_2, \tau_3, \tau_4$
T_3	Determining the Conditions for Similar Polygons	$\tau_1, \tau_2, \tau_3, \tau_4$		$\tau_1, \tau_2, \tau_3, \tau_4$
T_4	Pantograph	τ_1, τ_2, τ_3		
T_5	Terms of Two Similar Triangles	$\tau_1, \tau_2, \tau_3, \tau_4$	$\tau_1, \tau_2, \tau_3, \tau_4$	$\tau_1, \tau_2, \tau_3, \tau_4$
T_6	Solving the problem of similarity in the real world	τ_3, τ_4	τ_3, τ_4	τ_3, τ_4
T_7	Similarity and Enlargement		τ_1, τ_2, τ_3	$\tau_1, \tau_2, \tau_3, \tau_4$
T_8	Similarity and Scale Drawing			$\tau_1, \tau_2, \tau_3, \tau_4$

From Table 2, the technique presented in the Indonesian and Singapore textbook has reflected the epistemology of knowledge to the a priori stage. This certainly provides opportunities for student understanding and teacher presentation to form effective learning and achieve learning objectives.

The Logos Block

Logos block consists of two parts: technology (θ) and theory (Θ). Logos Blok examines the implementation justification (technology (θ)) on the design task (T), and the type of technique (τ) used based on the theoretical basis (Θ) that underlies it. Table 3 and Table 4 depict the praxiological relationships found in the Indonesian and Singaporean textbooks.

Table 3. The relationship between praxis and block logos in the Indonesian mathematics textbook praxeology

Theme	Type of Task	Technique (2013 Curriculum)	Technique (Merdeka Curriculum)	Technology	Theory
Similarity	T_1: The Relationship Between Angles		τ_1, τ_2, τ_3, τ_4	θ_4: Similarity of Plane Figure	Θ_1: transformation
	T_2: Identifying Two Similar Objects	$\tau_1, \tau_2, \tau_3, \tau_4$	τ_1, τ_2, τ_3		
	T_3: Determining the Conditions for Similar Polygons	$\tau_1, \tau_2, \tau_3, \tau_4$			
	T_4: Pantograph	τ_1, τ_2, τ_3			
	T_5: Terms of Two Similar Triangles	$\tau_1, \tau_2, \tau_3, \tau_4$	τ_1, τ_2, τ_3, τ_4		
	T_6: Solving the problem of similarity in the real world	τ_3, τ_4	τ_3, τ_4	θ_6: Connecting similarity concepts in daily activities	
	T_7: Similarity and Enlargement		τ_1, τ_2, τ_3	θ_7: Creating simple drawings and interpreting the scale on the map	

Table 3 shows the relationship between the praxis and logos blocks in Indonesian mathematics textbooks, particularly on the theme of similarity. The table presents an analysis of the types of tasks given to students, the techniques used, the technologies applied, and the theoretical basis underlying each task. There is a difference in the arrangement and presentation of the similarity material in the 2013 Curriculum mathematics textbook and the Merdeka Curriculum textbook. In the 2013 Curriculum mathematics textbook, the topic of similarity is presented in a more structured and gradual sequence, starting with T_2, which introduces the concept of identifying two similar objects, followed by T_3, which discusses the conditions for determining the similarity of polygons. The material then covers the pantograph (T_4) as a tool related to the concept of similarity, before finally studying the terms for two similar triangles (T_5). The book concludes with real-world similarity problems (T_6), providing a direct application of the concepts learned.

In contrast, the Merdeka Curriculum textbook begins the topic of similarity with a discussion on the relationship between angles (T_1), which is not found in the 2013 Curriculum book. It then continues with the identification of similar objects (T_2) and the terms for two similar triangles (T_5), which are also present in the 2013 Curriculum book. However, the material on the pantograph is absent in the Merdeka Curriculum, and real-world similarity problems are only discussed with one question (T_6). The Merdeka Curriculum book then introduces the concept of similarity and enlargement (T_7), which is not found in the 2013 Curriculum book. Table 3 provides insight into how Indonesian mathematics textbooks integrate various praxeological elements in teaching the concept of similarity, as well as demonstrating a systematic approach in introducing and applying mathematical theories to students. Table 3 also highlights the differences in the presentation of material between the 2013 Curriculum and the Merdeka Curriculum textbooks, where both curricula have different sequences and emphases in teaching similarity.

In the 2013 Curriculum mathematics textbook, the concept of similarity is introduced alongside congruence in grade 9, following prior instruction on angles in earlier grades. This sequencing provides a significant advantage, as students already have a foundational understanding of angles, which is essential for grasping similarity, particularly in the context of corresponding angles in similar triangles. This structured progression enables a smoother introduction to similarity concepts, allowing students to connect prior knowledge with new material more effectively. Additionally, the sequential presentation highlights the direct relationship between congruence and similarity—two closely related but distinct concepts—which can often be confusing without a well-established conceptual foundation. According to Clements and Sarama (2020), a systematic and sequential teaching order enhances student understanding by allowing learners to build progressively on existing knowledge, thereby facilitating the comprehension of more complex concepts.

Conversely, in the Merdeka Curriculum, while there is an emphasis on providing teachers with greater freedom and flexibility in delivering material, the topic of similarity appears to be introduced more abruptly. Foundational geometric concepts critical to understanding similarity, such as the relationship between transversal angles and the ratio of side lengths, are not always covered in sufficient depth beforehand. The curriculum assumes that students already possess this prior knowledge, but if these concepts have not been thoroughly taught or fully understood, it can compromise the effectiveness of the instruction on similarity.

This fragmented approach risks causing confusion among students, making it harder for them to grasp the interconnected relationships between angles, side ratios, and the principles of similarity. As Cornelius and Owiny (2024) and Peterson et al. (2018) argue, learning processes must adhere to a logical and coherent sequence, as gaps or disjointed instruction can hinder the development of a deeper understanding of subsequent concepts. A carefully structured and sequential teaching approach, as seen in the 2013 Curriculum, is therefore more conducive to fostering comprehensive learning in mathematics.

The analysis of the tasks reveals that although the Merdeka Curriculum emphasizes flexibility, an improper sequencing of material can hinder the development of a deeper understanding of mathematical concepts. In contrast, the 2013 Curriculum employs a more systematic and structured approach, offering students a gradual progression in their learning. This approach ensures that foundational concepts are thoroughly mastered before introducing more complex topics, creating a solid basis for understanding and application.

While the flexibility provided by the Merdeka Curriculum is undoubtedly beneficial in adapting to diverse classroom needs, it highlights the critical importance of maintaining a logical and well-structured sequence of material. Understanding the interconnections between concepts and presenting them in a cohesive order is essential for fostering an effective and comprehensive learning experience. Therefore, balancing flexibility with a robust conceptual framework and proper sequencing is key to achieving successful outcomes in mathematics education.

Based on the sequence of topics presented in the Singapore mathematics textbook in Table 4, the approach used to teach the concept of similarity is systematic and well-structured. An analysis of this sequence provides insights into how the material is delivered progressively to build students' understanding. The sequence begins with the observation of similar objects (T_2), followed by formal theory ($T_3; T_5$), and concludes with real-world applications and scale drawings ($T_7; T_8$), allowing students to develop a strong and holistic understanding of similarity. This approach emphasizes the connection between mathematical concepts and their applications in daily life, which can enhance students' understanding and skills in applying these concepts (Rodríguez-Nieto et al., 2023).

Table 4. Relation of praxis and block logos in Singapore mathematics textbook praxeology

Theme	Type of Task	Technique	Technology	Theory
Similarity	T_2: Identifying Two Similar Objects	$\tau_1, \tau_2, \tau_3, \tau_4$	θ_4: Similarity of Plane Figure	Θ_1: transformation
	T_3: Determining the Conditions for Similar Polygons	$\tau_1, \tau_2, \tau_3, \tau_4$		
	T_5: Terms of Two Congruent Triangles	$\tau_1, \tau_2, \tau_3, \tau_4$		
	T_6: Solving the problem of similarity in the real world	τ_3, τ_4	θ_6: Connecting similarity concepts in daily activities	
	T_7: Similarity and Enlargement	$\tau_1, \tau_2, \tau_3, \tau_4$	θ_7: Creating simple drawings and interpreting the scale on the map	
	T_8: Similarity and Scale Drawing	$\tau_1, \tau_2, \tau_3, \tau_4$		

Based on Tables 3 and 4, the Indonesian mathematics textbook for the Merdeka Curriculum begins with the introduction of transversal angles (T_1), providing a fundamental understanding of angle relationships before progressing to the topic of similarity. This presentation of the material allows more freedom for students to explore the concepts directly, though it may slightly overlook a deeper understanding of the more complex theories of similarity. In T_2, both in the Indonesian and Singaporean textbooks, students are asked to provide more real-life examples of similarity before defining the concept of similarity. A significant difference is evident in the placement of the similarity material across the curricula. In the Indonesian 2013 Curriculum mathematics textbook, similarity is taught in 9th grade, after students have gained a more mature understanding of fundamental geometry concepts. The material begins with identifying similar objects (T_2), followed by determining the conditions for similar polygons (T_3) and similar triangles (T_4). This approach strengthens students' understanding of the concept of similarity through real-world examples and a more structured theory, focusing on fundamental understanding first. On the other hand, in the Merdeka Curriculum, similarity is taught earlier, in 7th grade. Introducing similarity in 7th grade may accelerate students' understanding of the topic, but it also presents potential difficulties in comprehending the relationships between more complex concepts.

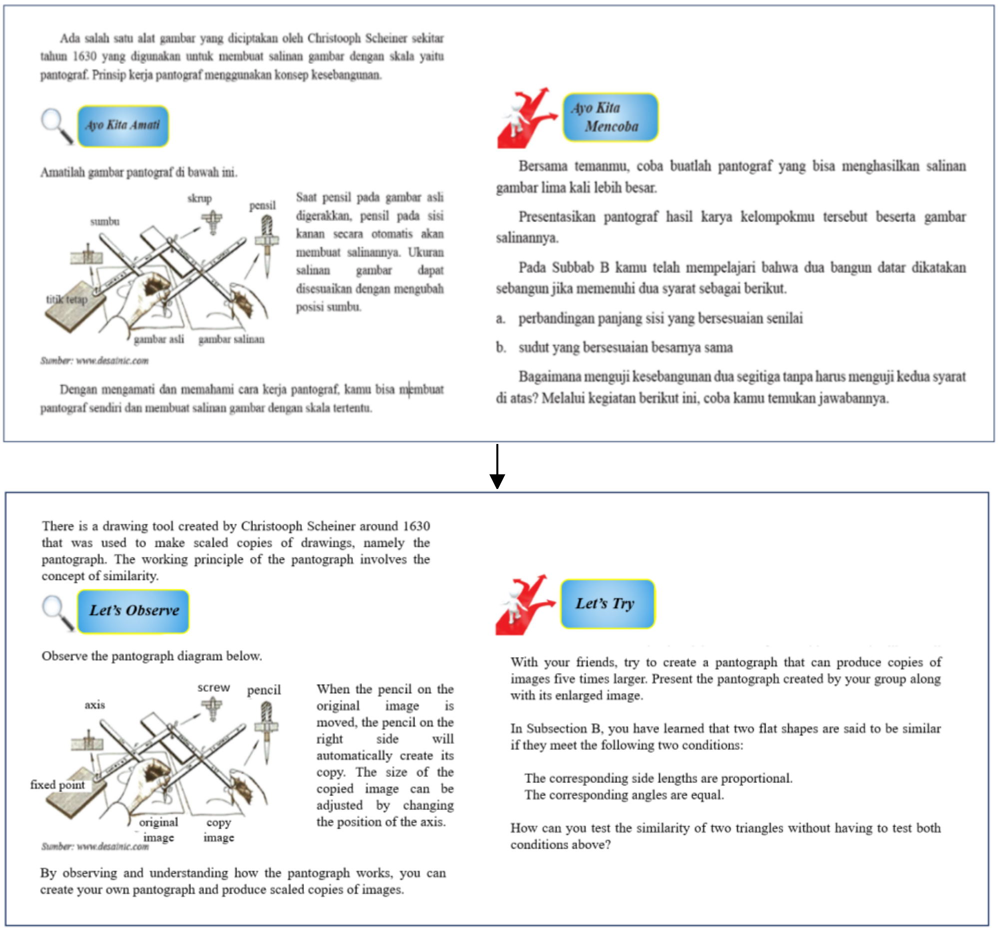
Meanwhile, in Singapore, similarity is taught in 8th and 9th grades with a gradual and in-depth approach. In 8th grade, students are introduced to the concept of similarity through initial steps, such as examining whether two figures are similar (T_2), stating the properties of similar triangles and polygons (T_3), and creating scale drawings and interpreting scales on maps (T_5). This 8th-grade instruction provides a solid foundation for students to understand the application of similarity in real life and develop technical skills in creating scale drawings. In 9th grade, the similarity material continues with a deeper application, such as more practical and analytical uses, allowing students to master higher-level skills in solving similarity problems.

The introduction of the pantograph in T_4 of the Indonesian 2013 Curriculum mathematics textbook aims for students to directly see and experience the concept of similarity in images or objects. In the arrangement of T_4's design, the use of the pantograph also provides practical experience for students. Students can personally feel how to change the size of an image with the tool, aiding them in understanding the concept of similarity in a more concrete way. Practical experience in geometry is beneficial for students. Using concrete manipulatives, has been shown to improve problem-solving performance and reduce cognitive load (Shi et al., 2023). Practical experience with accurate drawings and numerical calculations also can help students understand fundamental principles of geometry and prepare them for logical demonstration (Citrini, 2015).

In Tasks 6 and 7, which are only found in the Singaporean textbook, the concept of a scale factor can be naturally understood by students. The teacher guides students in understanding that enlargement does not always mean the resultant image is larger than the original. The resultant image can be smaller than the original, and the scale factor will be less than one, but it is still considered an enlargement. If the scale factor is one, then the resultant image is similar to the original. Tasks 6 and 7 can establish a connection between the content of similarity, enlargement, and scale drawings. Similarity is the underlying principle that links enlargement, ratio, and scale drawings in geometry.

Focus on the Indonesian 2013 Curriculum and Singapore mathematics textbooks. There are differences between $[T_4/\tau/\theta_4/\Theta_1]$ (see Figure 1) for Indonesian textbooks $[T_7/\tau/\theta_7/\Theta_1]$ (Figure 2), $[T_8/\tau/\theta_7/\Theta_1]$ (Figure 3) for Singapore mathematics textbooks. In this study, the researcher investigates the opportunities and potentials offered by mathematics textbooks in constructing knowledge and the interrelationships of pieces of knowledge that will be taught on the subject of similarity.

The Pantograph $[T_4/\tau/\theta_4/\Theta_2]$, which is not included in the Singapore textbook, is not explained. An introduction to similarity through a pantograph is shown in Figure 1. The pantograph is used to make a scaled copy of an image. The material is in the similarity sub material. The pantograph operates based on the concept of similarity, where the lengths of sides and angles remain proportional. Therefore, the use of the pantograph can assist students in understanding the relationship between similar shapes. Students' understanding of the relationship between similar shapes is a critical aspect of geometric reasoning (Basu et al., 2022). Students can also develop technical skills, such as measurement and tool manipulation skills, which can be useful in comprehending the concept of similarity and working with images or objects in a mathematical context. Participating in technical projects helps students gain exposure to new tools and ways of thinking, as well as develop their technical skills in using tools, creating experiments, analyzing results, and improving their work (Pernia-Espinoza et al., 2020). Losing a pantograph in one of the knowledge nodes conveyed through a textbook will undoubtedly impact the construction and connection of knowledge that students have.


Ada salah satu alat gambar yang diciptakan oleh Christooph Scheiner sekitar tahun 1630 yang digunakan untuk membuat salinan gambar dengan skala yaitu pantograf. Prinsip kerja pantograf menggunakan konsep kesebangunan.

Ayo Kita Amati

Amatilah gambar pantograf di bawah ini.

Saat pensil pada gambar asli digerakkan, pensil pada sisi kanan secara otomatis akan membuat salinannya. Ukuran salinan gambar dapat disesuaikan dengan mengubah posisi sumbu.

Sumber: www.desainic.com

Dengan mengamati dan memahami cara kerja pantograf, kamu bisa membuat pantograf sendiri dan membuat salinan gambar dengan skala tertentu.

Ayo Kita Mencoba

Bersama temanmu, coba buatlah pantograf yang bisa menghasilkan salinan gambar lima kali lebih besar.

Presentasikan pantograf hasil karya kelompokmu tersebut beserta gambar salinannya.

Pada Subbab B kamu telah mempelajari bahwa dua bangun datar dikatakan sebangun jika memenuhi dua syarat sebagai berikut.

a. perbandingan panjang sisi yang bersesuaian senilai

b. sudut yang bersesuaian besarnya sama

Bagaimana menguji kesebangunan dua segitiga tanpa harus menguji kedua syarat di atas? Melalui kegiatan berikut ini, coba kamu temukan jawabannya.

There is a drawing tool created by Christooph Scheiner around 1630 that was used to make scaled copies of drawings, namely the pantograph. The working principle of the pantograph involves the concept of similarity.

Let's Observe

Observe the pantograph diagram below.

When the pencil on the original image is moved, the pencil on the right side will automatically create its copy. The size of the copied image can be adjusted by changing the position of the axis.

Sumber: www.desainic.com

By observing and understanding how the pantograph works, you can create your own pantograph and produce scaled copies of images.

Let's Try

With your friends, try to create a pantograph that can produce copies of images five times larger. Present the pantograph created by your group along with its enlarged image.

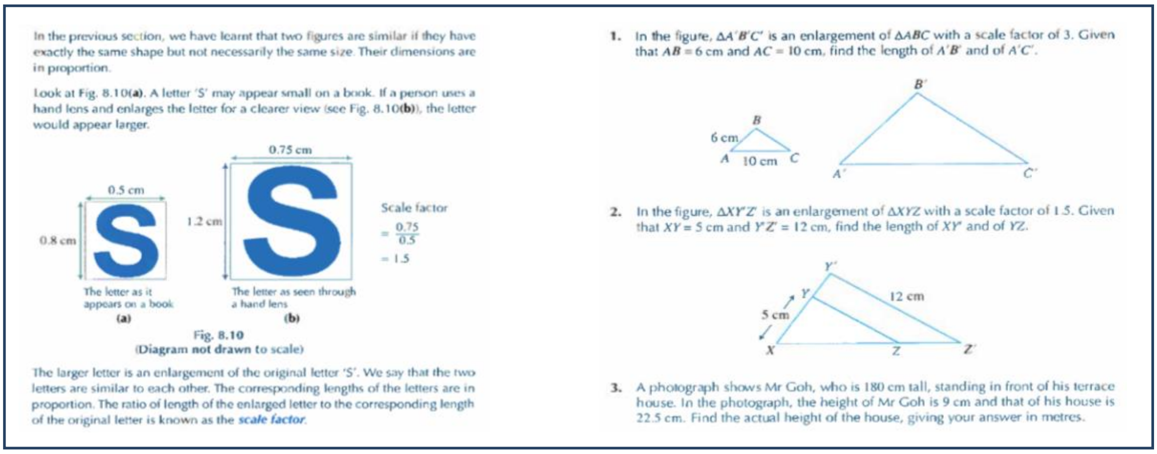
In Subsection B, you have learned that two flat shapes are said to be similar if they meet the following two conditions:

The corresponding side lengths are proportional.
The corresponding angles are equal.

How can you test the similarity of two triangles without having to test both conditions above?

Figure 1. $[T_4/\tau/\theta_4/\Theta_2]$ pantographs in Indonesian textbooks that do not exist in Singapore textbooks (Subchan et al., 2018)

In Figure 2 and Figure 3, the construction of knowledge through design tasks in Indonesian textbooks will reduce the relationship of the material being studied with an example of its use in the real world. This has an impact on the thinking opportunities of students who tend to jump on Indonesian textbooks.


In the previous section, we have learnt that two figures are similar if they have exactly the same shape but not necessarily the same size. Their dimensions are in proportion.

Look at Fig. 8.10**(a)**. A letter 'S' may appear small on a book. If a person uses a hand lens and enlarges the letter for a clearer view (see Fig. 8.10**(b)**), the letter would appear larger.

The larger letter is an enlargement of the original letter 'S'. We say that the two letters are similar to each other. The corresponding lengths of the letters are in proportion. The ratio of length of the enlarged letter to the corresponding length of the original letter is known as the ***scale factor***.

1. In the figure, $\Delta A'B'C'$ is an enlargement of ΔABC with a scale factor of 3. Given that AB = 6 cm and AC = 10 cm, find the length of $A'B'$ and of $A'C'$.

2. In the figure, $\Delta XY'Z'$ is an enlargement of ΔXYZ with a scale factor of 1.5. Given that XY = 5 cm and $Y'Z'$ = 12 cm, find the length of XY' and of YZ.

3. A photograph shows Mr Goh, who is 180 cm tall, standing in front of his terrace house. In the photograph, the height of Mr Goh is 9 cm and that of his house is 22.5 cm. Find the actual height of the house, giving your answer in metres.

Figure 2. $[T_7/\tau/\theta_7/\Theta_1]$ similarity and enlargement in Singapore textbooks that do not exist in Indonesian textbooks (Yeo et al., 2019)

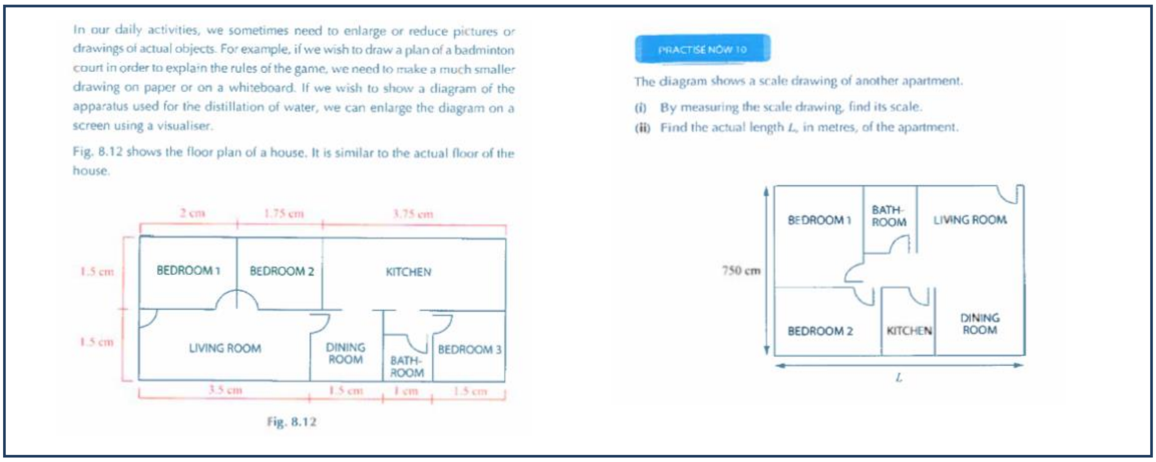


Figure 3. [$T_8/\tau/\theta_7/\Theta_1$] Similarity and scale drawing in the Singapore textbook, which does not exist in the Indonesian textbook (Yeo et al., 2019)

Praxeological-Didactical Analysis

The concept of praxeology serves as a framework for analyzing the reconstruction of similarity material in the textbook (Chaachoua et al., 2019). The praxeological analysis conducted in this research indicates that the organization of tasks in the similarity material is often fragmented. Specifically, certain tasks lack coherence and connections to one another, while some are associated with varying techniques.

An analysis of the techniques (τ) highlights that understanding the influence of epistemological stages on student performance can enable teachers to modify tasks from the textbook to foster persistence and independent learning. The quality of student learning is closely tied to the nature of the tasks presented in the textbook and how teachers interpret and implement this content (Thompson & Senk, 2014). By leveraging the insights from presented techniques, teachers can design tasks more effectively, optimizing learning outcomes in the classroom (Clivaz & Miyakawa, 2020; Miyakawa & Winsløw, 2019; Takeuchi & Shinno, 2020).

The organization of knowledge within task types and the associated techniques used to engage with mathematical ideas can provide a basis for evaluating the diversity and functionality of the tasks (Prediger et al., 2021). By doing so, it becomes possible to assess the complexity and interrelation of knowledge components in the textbook. Praxeological analysis allows for a deeper understanding of how these tasks function within the instructional framework and their potential impact on student learning. For instance, tasks involving proportional dimensions, such as those depicted in Figure 4, provide an opportunity to explore how students engage with and solve problems using ratios, a fundamental concept in similarity.

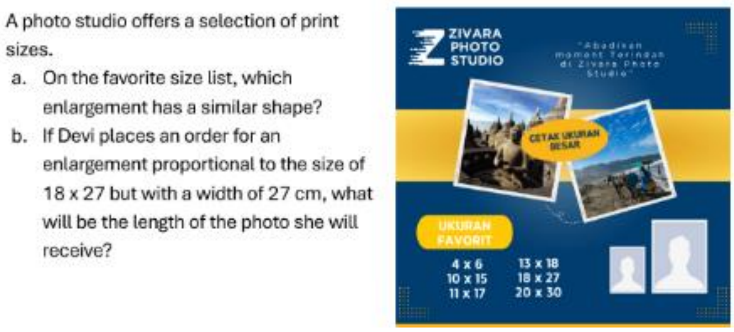


Figure 4. Similarity problem related to ratio

In this problem, understanding ratios and proportions becomes crucial for solving similarity problems. Students are tasked with identifying and measuring specific proportional relationships. This often involves the application of mathematical concepts such as side ratios and relationships between geometric measurements. For problem in Figure 4, students give different responses. Figure 5 displays students' answers to the problem.

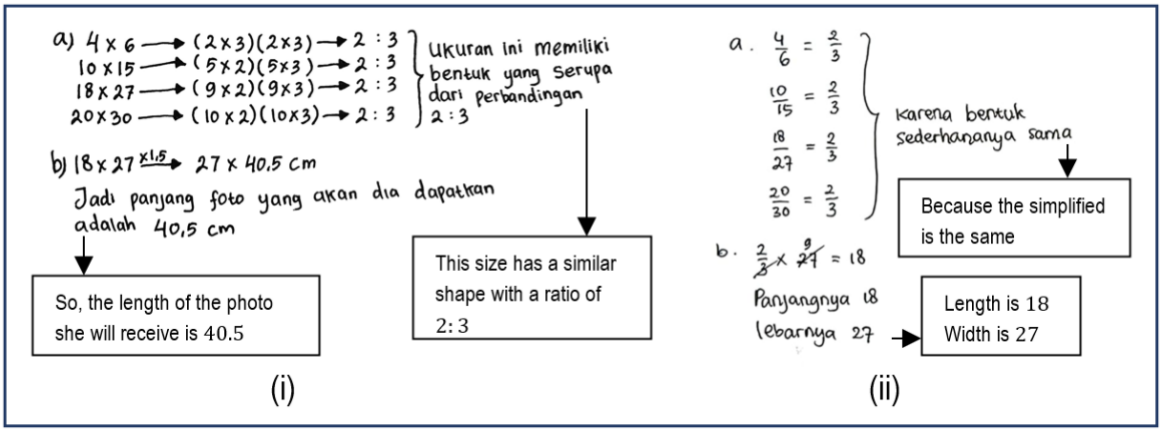


Figure 5. Students answer for similarity problem related to ratio

Figure 5 (i) is the correct answers for points (a) and (b), while for Figure 5 (ii), the student misunderstood point (b) in solving the problem. The question refers to ordering an enlargement proportional to the size of 18 x 27 but with a width of 27 cm. So, students should provide an answer related to the length of the photo Devi will receive, not the width. In Figure 5 (ii), the student provided an answer of 27 for the width, and after clarification through an interview, it was revealed that the student's mistake was in understanding the sequence of the photo's length and width positions. The student misunderstood the question, resulting in an inaccurate response. It's important for students to read the question carefully and ensure that the response given aligns with what is asked in the question (Jupri et al., 2021). The students' limited understanding of the concept of similarity also results in their inability to address more complex challenges. The illustration in Figure 6 serves as an example of this situation.

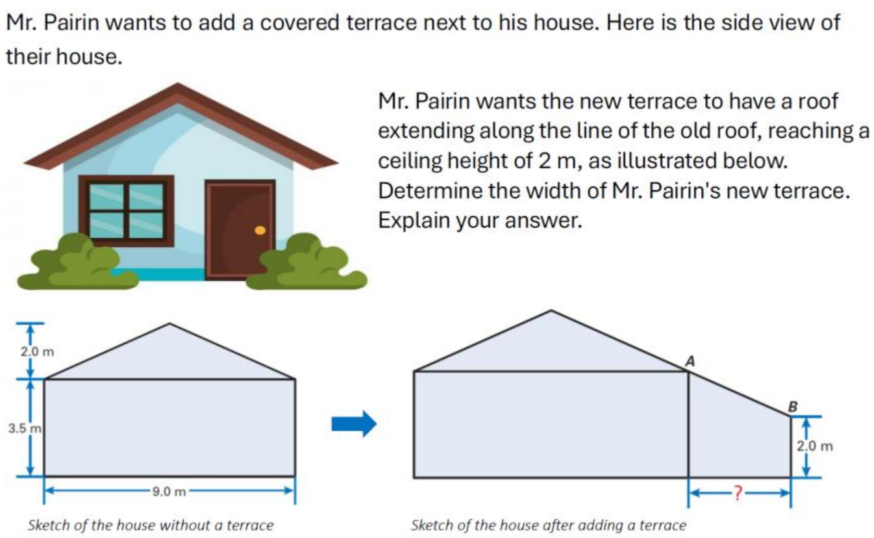


Figure 6. Complex similarity problem

In the given context, a substantial number of students encounter challenges when attempting to accurately address the posed problem. This indicates that the challenges posed in the question can be considered as an obstacle or complexity affecting the majority of students. Figure 7 illustrates the students' responses to the question.

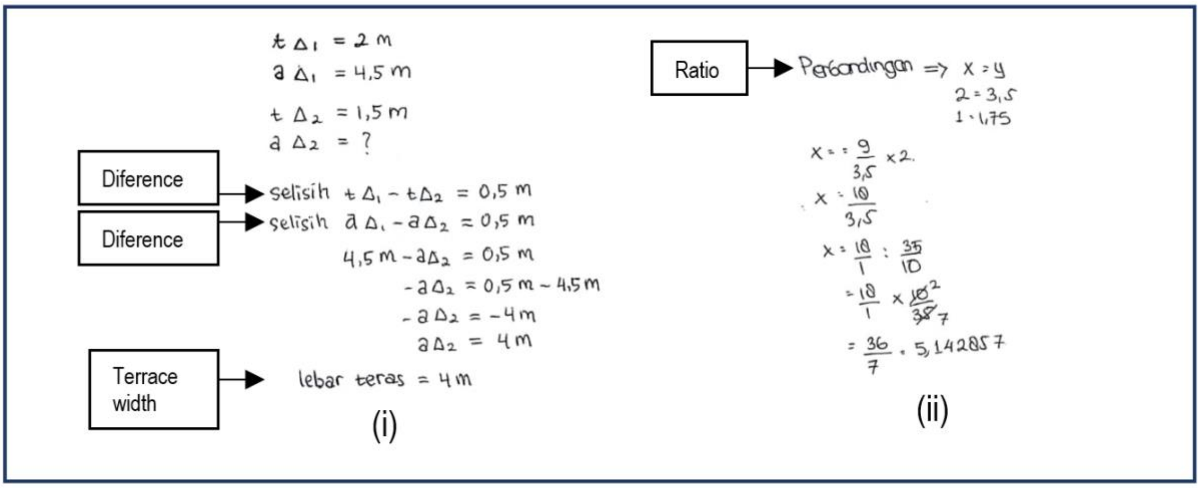


Figure 7. Students answer for complex similarity problem

The analysis of students' responses, as depicted in Figure 7 (i) and (ii), highlights the significant challenges they face in solving a complex similarity problem related to the extension of a house roof. Although some students successfully identified initial steps, such as measuring the ceiling height, none provided correct answers, revealing specific gaps in their understanding and problem-solving processes. These difficulties stem from several interconnected factors, which collectively hinder students' ability to address the posed problem accurately.

One of the primary challenges identified is students' difficulty in connecting the provided information. While some were able to identify certain elements, such as ceiling height, they struggled to integrate this information to calculate the terrace width. This inability to establish a relationship between the given measurements and the desired outcome reflects a disconnect between the problem's visual and mathematical components. Another critical issue is students' uncertainty in applying proportions. Interviews revealed a lack of confidence in using proportional reasoning to relate the geometric relationships of the triangle formed in the problem to the calculation of terrace width. Many students failed to recognize how the given dimensions correspond to the concept of similarity and proportionality, which is fundamental to solving the problem effectively.

Moreover, insufficient practice emerged as a significant factor contributing to students' difficulties. Many students expressed that they had limited exposure to similar types of problems, which hindered their ability to apply learned concepts to more complex tasks. This lack of practice is particularly evident in their hesitation and errors in attempting to connect proportions and measurements. Without adequate reinforcement through diverse problem sets, students are less prepared to generalize their knowledge and tackle challenging questions involving geometric reasoning and proportionality.

These errors are indicative of didactical obstacles, which are learning barriers that arise due to instructional methods or the design of educational resources. In this context, the primary obstacles can be traced back to limitations in the teaching process and textbook design. Students struggled to utilize the visual information provided in the problem effectively, suggesting that the materials or methods used did not sufficiently bridge the gap between visual interpretation and mathematical reasoning. Additionally, the lack of scaffolding in task progression within the textbooks contributed to students' inability to handle complex problems. Without a systematic approach to building foundational skills in proportions and their application, students were ill-equipped to address such tasks confidently.

In the Logos Block, particularly in the technological component, the focus is on understanding how arguments, justifications, and the arrangement of knowledge are constructed using selected techniques (Bosch & Gascón, 2014; Chevallard et al., 2015). This perspective moves beyond focusing on isolated techniques to considering a complete framework for explaining mathematical ideas. These techniques include perceptual (τ_1), memorial (τ_2), introspective (τ_3), and a priori (τ_4) processes. The sequence and organization of material in the textbook play a pivotal role in shaping students' knowledge construction, as they influence how these techniques are utilized. Mathematics textbooks can prioritize practical applications, concrete examples, or problem-solving approaches, all of which contribute to students' understanding of complex concepts. Moreover, the clarity and quality of visual elements in textbooks are essential for students to comprehend problems involving visual representations, such as geometric shapes or graphs (Yilmaz & Argun, 2018). The complexity and relevance of problems also determine students' capacity to apply learned concepts effectively. Examples and illustrations tied to real-world scenarios further enhance the practical understanding and application of mathematical ideas (Zaslavsky, 2019).

One of the most critical factors in textbook design is continuity, which ensures that knowledge is systematically connected and built upon (Amaral & Hollebrands, 2017; Dixon & Brown, 2012). Continuity is often reflected in how design tasks are structured within the textbook, facilitating opportunities for students to learn and organize their knowledge effectively. A well-sequenced textbook enables students to see the logical progression between tasks, fostering a deeper understanding of concepts and their interrelations. Teachers also benefit from this continuity, as it helps them identify the most effective instructional steps to teach mathematical material (Hidayah & Forgasz, 2020; Ma, 2010). When textbooks are well-organized, they become powerful tools that support students in achieving their learning objectives (Collopy, 2003). This organization provides students with the opportunity not only to acquire new knowledge but also to connect and expand upon their existing knowledge. By emphasizing these connections, textbooks facilitate a continuous process of constructing, developing, and organizing knowledge, which helps students form a cohesive understanding of mathematical concepts (Taranto et al., 2020).

This analysis highlights the crucial role that textbooks play in addressing learning barriers and mitigating didactical obstacles. An effective mathematics textbook must support students' progressive understanding by offering tasks that gradually increase in complexity, ensuring that foundational skills are firmly established before introducing more advanced topics. Textbooks should incorporate clear and high-quality visual aids along with explicit instructions to guide students in linking illustrations to mathematical concepts. Additionally, providing a diverse range of practice problems can enhance students' familiarity with concepts, reinforce key ideas, and build their confidence in applying proportional reasoning and other mathematical principles. By addressing these needs, textbooks can equip students with the tools necessary to tackle complex problems and foster a deeper understanding of mathematical ideas. Ultimately, the construction of knowledge within the textbook becomes a dynamic and continuous process, connecting previous learning to new concepts and preparing students for advanced problem-solving and critical thinking.

CONCLUSION

This study unveils distinctions in the structuring of mathematical knowledge, particularly evident in the treatment of similarity material, within Indonesian and Singaporean textbooks. The examination of both educational resources reveals a diversity of mathematical tasks, each dissected through the lens of the Anthropological Theory of the Didactic (ATD) praxeology method. However, this analysis brings to light areas where enhancements can be made to enrich the learning experience for students.

The results of this study reveal that although Indonesian mathematics textbooks contain real contextualization of similarity material, the context presented is still general and not fully in accordance with the experiences of each student. Textbooks published by the Ministry of Education and Culture show efforts to connect the material to real-life situations, for example by asking questions about similarities in everyday life before defining the concept of similarity. However, there are tasks to explore the similar figurative nature of objects they encounter. The use of aids such as pantographs in Indonesian textbooks aims to provide students with direct experience to understand the concept of congruence more concretely, but not all students have direct access or opportunity to experience it. In contrast, textbooks in Singapore emphasize a deep understanding of the concepts of scale and magnification, which helps students understand that magnification does not always mean a larger size can also be smaller. This highlights the need to improve the design of mathematics tasks in Indonesian textbooks by providing more specific and relevant scenarios, to support more effective learning. By expanding the applied context and including tasks related to students' everyday experiences, a richer and more relevant learning environment can be created.

This study focuses on a praxeological analysis of a selected textbook from Indonesia and Singapore, which may not comprehensively represent all educational materials available in the two countries. Furthermore, this study was conducted in the context of a specific educational unit and material, which may affect the generalizability of the research results to other regions or educational systems. This study also relies on document analysis and student evaluations, which may not cover all aspects of students' learning experiences and understandings of similarity. This comparative analysis not only shows the differences that exist, but also highlights the importance of collaboration between educators, policy makers, and curriculum designers to continuously improve educational materials. Such collaboration has the potential to produce educational innovations that are more in line with the diverse learning needs of Indonesian students. Future research could explore the influence of specific textbook characteristics on student learning outcomes through empirical studies of a wider range of educational resources. In addition, investigating the role of teacher training in the effective use of textbooks could provide additional insights. This collaboration has the potential to produce educational innovations that are more in line with the diverse learning needs of Indonesian students.

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Author Contribution	:	KSK: Conceptualization, Writing - Original Draft, Editing and Visualization. DS: Writing - Review & Editing, Formal analysis, and Methodology. JAD: Writing - Review & Editing, Formal analysis, and Methodology. AJ: Writing - Review & Editing, Formal analysis, and Validation and Supervision.
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REFERENCES

Abdullah, A. H., & Shin, B. (2019). A Comparative Study of Quadrilaterals Topic Content in Mathematics Textbooks between Malaysia and South Korea. *Journal on Mathematics Education*, *10*(3), 315–340. https://doi.org/10.22342/jme.10.3.7572.315-340

Amaral, R. B., & Hollebrands, K. (2017). An analysis of context-based similarity tasks in textbooks from Brazil and the United States. *International Journal of Mathematical Education in Science and Technology*, *48*(8), 1166–1184. https://doi.org/10.1080/0020739X.2017.1315188

Audi, R. (2010). *Epistemology: A contemporary introduction to the theory of knowledge*. Routledge. https://doi.org/10.4324/9780203846469

Basu, M., Koellner, K., Jacobs, J. K., & Seago, N. (2022). Understanding Similarity through Dilations of Nonstandard Shapes. *Mathematics Teacher: Learning and Teaching PK-12*, *115*(9), 642–649. https://doi.org/10.5951/mtlt.2021.0284

Bosch, M., & Gascón, J. (2014). Introduction to the Anthropological Theory of the Didactic (ATD). In *Networking of theories as a research practice in mathematics education* (pp. 67–83). Springer. https://doi.org/10.1007/978-3-319-05389-9_5

Chaachoua, H., Bessot, A., Romo, A., & Castela, C. (2019). Developments and functionalities in the praxeological model. *Working with the Anthropological Theory of the Didactic in Mathematics Education: A Comprehensive Casebook*, *41*. https://doi.org/10.4324/9780429198168-4

Chevallard, Y., & Bosch, M. (2020a). Anthropological theory of the didactic (ATD). *Encyclopedia of Mathematics Education*, 53–61. https://doi.org/10.1007/978-3-030-15789-0_100034

Chevallard, Y., & Bosch, M. (2020b). Didactic transposition in mathematics education. *Encyclopedia of Mathematics Education*, 214–218. https://doi.org/10.1007/978-3-030-15789-0_48

Chevallard, Y., Bosch, M., & Kim, S. (2015). What is a theory according to the anthropological theory of the didactic? *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education*, 2614–2620. Retrieved from https://hal.science/hal-01289424

Choi, K. M., & Park, H.-J. (2013). A comparative analysis of geometry education on curriculum standards, textbook structure, and textbook items between the US and Korea. *Eurasia Journal of Mathematics, Science and Technology Education*, *9*(4), 379–391. https://doi.org/10.12973/eurasia.2013.947a

Citrini, C. (2015). The Role of Geometry in Reasoning and Teaching. In *The Visual Language of Technique: Volume 3-Heritage and Expectations in Education* (pp. 77–91). Springer. https://doi.org/10.1007/978-3-319-05326-4_8

Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach*. Routledge. https://doi.org/10.4324/9781003083528

Clivaz, S., & Miyakawa, T. (2020). The effects of culture on mathematics lessons: an international comparative study of a collaboratively designed lesson. *Educational Studies in Mathematics*, *105*(1), 53–70. https://doi.org/10.1007/s10649-020-09980-1

Collopy, R. (2003). Curriculum materials as a professional development tool: How a mathematics textbook affected two teachers' learning. *The Elementary School Journal*, *103*(3), 287–311. https://doi.org/10.1086/499727

Cornelius, K. E., & Owiny, R. L. (2024). How Can I Ensure My Lessons Are Logically Sequenced? In *The Practical Guide to High-Leverage Practices in Special Education* (pp. 173–186). Routledge. https://doi.org/10.4324/9781003525844

Creswell, J. W. (2018). Mixed Methods Procedures. In Research Design: Qualitative, Quantitative, and Mixed Methods Approaches. SAGE Publications Ltd. Retrieved from https://books.google.co.id/books?id=4uB76IC_pOQC&printsec=copyright&hl=id#v=onepage&q&f=false

DeJarnette, A. F., Lausell, S. L. R., & González, G. (2015). Shadow puppets: Exploring a context for similarity and dilations. *The Mathematics Teacher*, *109*(1), 20–27. https://doi.org/10.5951/mathteacher.109.1.0020

Dixon, R. A., & Brown, R. A. (2012). Transfer of Learning: Connecting Concepts during Problem Solving. *Journal of Technology Education*, *24*(1), 2–17. https://doi.org/10.21061/jte.v24i1.a.1

Flick, U. (2018). *Designing qualitative research*. (Vols. 1-0). SAGE Publications Ltd, https://doi.org/10.4135/9781529622737

Friesen, N., Henriksson, C., & Saevi, T. (2012). *Hermeneutic phenomenology in education: Method and practice* (M. Friesen, C. Henriksson, & T. Saevi (eds.); Vol. 4). Sense Publisher. https://doi.org/10.1007/978-94-6091-834-6

Glasnovic Gracin, D. (2018). Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples. *International Journal of Mathematical Education in Science and Technology*, *49*(7), 1003–1024. https://doi.org/10.1080/0020739X.2018.1431849

Gravemeijer, K. P. E. (2014). Transforming mathematics education: The role of textbooks and teachers. In *Transforming mathematics instruction* (pp. 153–172). Springer. https://doi.org/10.1007/978-3-319-04993-9_10

Guillen, D. E. F. (2019). Qualitative research: Hermeneutical phenomenological method. *Propósitos y Representaciones*, *7*(1), 201–229. https://doi.org/10.20511/pyr2019.v7n1.267

Hidayah, M., & Forgasz, H. (2020). A Comparison of Mathematical Tasks Types Used in Indonesian and Australian Textbooks Based on Geometry Contents. *Journal on Mathematics Education*, *11*(3), 385–404. https://doi.org/10.22342/jme.11.3.11754.385-404

Hodgen, J. (2010). Knowing and identity: A situated theory of mathematics knowledge in teaching. In *Mathematical knowledge in teaching* (pp. 27–42). Springer. https://doi.org/10.1007/978-90-481-9766-8_3

Hong, D. S., & Choi, K. M. (2018). Reasoning and Proving Opportunities in Textbooks: A Comparative Analysis. *International Journal of Research in Education and Science*, *4*(1), 82–97. https://doi.org/10.21890/ijres.382937

Islam, M. (2020). Data analysis: types, process, methods, techniques and tools. *International Journal on Data Science and Technology*, *6*(1), 10–15. https://doi.org/10.11648/j.ijdst.20200601.12

Jupri, A., Sispiyati, R., & Chin, K. E. (2021). An investigation of students algebraic proficiency from a structure sense perspective. *Journal on Mathematics Education*, *12*(1), 315–340. https://doi.org/10.22342/jme.12.1.13125.147-158

Kang, W., & Kilpatrick, J. (1992). Didactic transposition in mathematics textbooks. *For the Learning of Mathematics*, *12*(1), 2–7. Retrieved from https://api.semanticscholar.org/CorpusID:58900546

Lepik, M., Grevholm, B., & Viholainen, A. (2015). Using textbooks in the mathematics classroom–the teachers' view. *Nordic Studies in Mathematics Education*, *20*(3–4), 129–156. https://doi.org/10.15388/ActPaed.2015.35.9193

Lundberg, A. L. V., & Kilhamn, C. (2018). Transposition of Knowledge: Encountering Proportionality in an Algebra Task. *International Journal of Science and Mathematics Education*, *16*(3), 559–579. https://doi.org/10.1007/s10763-016-9781-3

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Routledge. https://doi.org/10.4324/9781003009443

Miyakawa, T., & Winsløw, C. (2019). Paradidactic infrastructure for sharing and documenting mathematics teacher knowledge: a case study of "practice research" in Japan. *Journal of Mathematics Teacher Education*, *22*(3), 281–303. https://doi.org/10.1007/s10857-017-9394-y

O'Halloran, K. L., Beezer, R. A., & Farmer, D. W. (2018). A new generation of mathematics textbook research and development. *ZDM*, *50*(5), 863–879. https://doi.org/10.1007/s11858-018-0959-8

OECD. (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education* (PISA). OECD Publishing. https://doi.org/10.1787/53f23881-en

Pansell, A., & Bjorklund Boistrup, L. (2018). Mathematics teachers' teaching practices in relation to textbooks: exploring praxeologies. *The Mathematics Enthusiast*, *15*(3), 541–562. https://doi.org/10.54870/1551-3440.1444

Pernia-Espinoza, A., Sanz-Garcia, A., Martinez-de-Pison-Ascacibar, F. J., Peciña-Marqueta, S., & Blanco-Fernandez, J. (2020). Technical projects with social commitment for teaching-learning intervention in STEM students. *2020 IEEE Global Engineering Education Conference (EDUCON)*, 579–586. https://doi.org/10.1109/EDUCON45650.2020.9125176

Peterson, A., Dumont, H., Lafuente, M., & Law, N. (2018). *Understanding innovative pedagogies: Key themes to analyse new approaches to teaching and learning*. https://doi.org/10.1787/19939019

Prediger, S., Barzel, B., Hußmann, S., & Leuders, T. (2021). Towards a research base for textbooks as teacher support: The case of engaging students in active knowledge organization in the KOSIMA project. *ZDM–Mathematics Education*, *53*(6), 1233–1248. https://doi.org/10.1007/s11858-021-01245-2

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM – Mathematics Education*, *53*(6), 1189–1206. https://doi.org/10.1007/s11858-021-01309-3

Rodríguez-Nieto, C. A., Cervantes-Barraza, J. A., & Moll, V. F. (2023). Exploring mathematical connections in the context of proof and mathematical argumentation: A new proposal of networking of theories. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(5), em2264. https://doi.org/10.29333/ejmste/13157

Sherman, M. F., Cayton, C., Walkington, C., & Funsch, A. (2020). An analysis of secondary mathematics textbooks with regard to technology integration. *Journal for Research in Mathematics Education*, *51*(3), 361–374. https://doi.org/10.5951/jresemtheduc-2020-0005

Shi, L., Dong, L., Zhao, W., & Tan, D. (2023). Improving middle school students' geometry problem solving ability through hands-on experience: An fNIRS study. *Frontiers in Psychology*, *14*, 1126047. https://doi.org/10.3389/fpsyg.2023.1126047

Shinno, Y., & Mizoguchi, T. (2021). Theoretical approaches to teachers' lesson designs involving the adaptation of mathematics textbooks: two cases from kyouzai kenkyuu in Japan. *ZDM - Mathematics Education*, *53*(6), 1387–1402. https://doi.org/10.1007/s11858-021-01269-8

Sievert, H., van den Ham, A.-K., Niedermeyer, I., & Heinze, A. (2019). Effects of mathematics textbooks on the development of primary school children's adaptive expertise in arithmetic. *Learning and Individual Differences*, *74*, 101716. https://doi.org/10.1016/j.lindif.2019.02.006

Subchan, S., Winarni, W., Mufid, M. S., Fahim, K., & Syaifudin, W. H. (2018). *Matematika SMP Kelas IX*. Pusat Kurikulum dan Perbukuan, Balitbang, Kemendikbud. Retrieved from https://buku.kemdikbud.go.id/katalog/Buku-Matematika-Kelas-IX

Takeuchi, H., & Shinno, Y. (2020). Comparing the Lower Secondary Textbooks of Japan and England: a Praxeological Analysis of Symmetry and Transformations in Geometry. *International Journal of Science and Mathematics Education*, *18*(4), 791–810. https://doi.org/10.1007/s10763-019-09982-3

Taranto, E., Robutti, O., & Arzarello, F. (2020). Learning within MOOCs for mathematics teacher education. *ZDM - Mathematics Education*, *52*(7), 1439–1453. https://doi.org/10.1007/s11858-020-01178-2

Thompson, D. R., & Senk, S. L. (2014). The same geometry textbook does not mean the same classroom enactment. *ZDM*, *46*(5), 781–795. https://doi.org/10.1007/s11858-014-0622-y

Toh, T. L., Chan, C. M. E., Tay, E. G., Leong, Y. H., Quek, K. S., Toh, P. C., Ho, W. K., Dindyal, J., Ho, F. H., & Dong, F. (2019). *Problem Solving in the Singapore School Mathematics Curriculum BT - Mathematics Education in Singapore* (T. L. Toh, B. Kaur, & E. G. Tay (eds.); pp. 141–164). Springer Singapore. https://doi.org/10.1007/978-981-13-3573-0_7

Ulusoy, F., & İncikabı, L. (2020). Middle School Teachers' Use of Compulsory Textbooks in Instruction of Mathematics. *International Journal for Mathematics Teaching and Learning*, *21*(1), 1–18. https://doi.org/10.4256/ijmtl.v21i1.227

van den Ham, A.-K., & Heinze, A. (2018). Does the textbook matter? Longitudinal effects of textbook choice on primary school students' achievement in mathematics. *Studies in Educational Evaluation*, *59*, 133–140. https://doi.org/https://doi.org/10.1016/j.stueduc.2018.07.005

Wijayanti, D., & Winslow, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *REDIMAT*, *6*(3), 307–330. https://doi.org/10.17583/redimat.2017.2078

Yang, D.-C., & Sianturi, I. A. (2017). An analysis of Singaporean versus Indonesian textbooks based on trigonometry content. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(7), 3829–3848. https://doi.org/10.12973/eurasia.2017.00760a

Yang, D.-C., Tseng, Y.-K., & Wang, T.-L. (2017). A comparison of geometry problems in middle-grade mathematics textbooks from Taiwan, Singapore, Finland, and the United States. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(7), 2841–2857. https://doi.org/10.12973/eurasia.2017.00721a

Yeo, J., Seng, T. K., Yee, L. C., Chow, I., Meng, N. C., Liew, J., & Hong, O. C. (2019). *New syllabus mathematics 2*. Shinglee Publishers. Retrieved from https://shinglee.com.sg/publications_details.php?BookID=147

Yilmaz, R., & Argun, Z. (2018). Role of Visualization in Mathematical Abstraction: The Case of Congruence Concept. *International Journal of Education in Mathematics, Science and Technology*, *6*(1), 41–57. https://doi.org/10.46328/IJEMST.V6I1.427

Zaslavsky, O. (2019). There is more to examples than meets the eye: Thinking with and through mathematical examples in different settings. *The Journal of Mathematical Behavior*, *53*, 245–255. https://doi.org/10.1016/J.JMATHB.2017.10.001