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Proportional reasoning in the artisan personality type: A case study of high school students in trigonometry ratios

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Abstract

Proportional reasoning is a critical component of mathematical competence that should be developed at the senior high school level, as it fosters both foundational and advanced mathematical understanding. Educators frequently encounter variations in proportional reasoning abilities among students, often influenced by individual personality types. However, limited research has specifically investigated the proportional reasoning capabilities of high school students with artisan personality types. This study aims to examine the strategies and approaches utilized by students with Artisan Personality Types (APT) in solving trigonometric comparison problems. Employing a qualitative descriptive methodology within a case study design, the research focused on high school students identified as having APT. Data were collected using proportional reasoning tasks, the Keirsey Personality Type Questionnaire, and structured interviews. The analysis was conducted qualitatively, with findings categorized based on established indicators of proportional reasoning. Results indicate that APT students demonstrate the ability to address proportional reasoning problems related to covariation, ratios, and proportions, employing distinct strategies and logical reasoning. Nevertheless, instances of both correct and incorrect responses were observed, often stemming from misinterpretations of the problem context. These findings provide valuable insights for future studies aimed at designing targeted instructional strategies and developing learning tools to enhance the proportional reasoning skills of students with APT.

Keywords: Artisan Personality Type, High School Students, Proportional Reasoning, Trigonometric Ratios

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Reasoning plays a fundamental role in mathematics as it enables individuals to draw conclusions and establish cause-and-effect relationships based on prior knowledge (Dong et al., 2020). Moreover, the logical connection between mathematical concepts and other related ideas must be supported by sound reasoning. Each individual's approach to mathematical tasks and problems can be influenced by their personality type. Logical thinking, which encompasses reasoning, is a crucial component of mathematical reasoning, particularly the ability to engage in proportional reasoning (Hjelte et al., 2020). Consequently, students' proficiency in proportional reasoning when solving mathematical problems may be influenced by their personality traits. This variance is expected, as each student possesses a unique personality. Differences in behavior and character can be likened to the clothes individuals wear, serving as external representations of their attitudes and behavior. According to Pambudi et al. (2021), certain personality types are associated with distinct learning preferences: Guardian types favor structured classes with routine procedures and clear instructions, Artisan types thrive in interactive environments with ample discussion, Idealist types prefer collaborative group work, and Rational types tend to favor independent learning. Understanding these personality types provides valuable insights that can enhance the effectiveness of teaching and learning strategies.

Proportional reasoning is a critical concept in middle school mathematics that warrants significant attention, as it is directly linked to mathematical learning and serves as a foundational competency developed at the secondary school level. This skill strengthens students' understanding of both basic and advanced mathematical concepts (Langrall & Swafford, 2000). Additionally, one of the key objectives of mathematics education is to foster a positive perception of mathematics as a systematic and interconnected discipline. This interconnectedness implies that mathematical concepts are not isolated but rather relate to other subjects and the natural world, thereby facilitating organized reasoning (Sholli et al., 2020). Proportional reasoning is essential for the development of mathematical proficiency (Vanluydt et al., 2020). It underpins key mathematical concepts such as value, inverse relationships, comparison, and ratios. The focus of this research is to explore the understanding of ratios, proportions, covariation, and the application of various problem-solving strategies.

According to Lamon (2020), there are four key characteristics or indicators that are essential for proportional reasoning, which align with the instruments utilized in this study. These indicators are: 1) Understanding covariation, where students recognize the relationship between changes in a variable, such as Andi's distance from a flagpole, and corresponding changes in another variable, such as the viewing angle. For instance, as Andi moves farther from the flagpole, the viewing angle becomes smaller, and vice versa, indicating negative covariation between the distance and the angle of view; 2) Understanding ratios, where students are able to compare Andi's distance from the flagpole and his viewing angle using numerical values and symbols; 3) Understanding proportions, where students can identify types of comparisons as equivalent, inverse, or neither using numbers and symbols; and 4) Developing various strategies to solve problems related to addition, subtraction, multiplication, and division.

Different personality types exhibit distinct characteristics in problem-solving. According to Keirsey and Bates (1978), individuals with an Artisan personality type prioritize living in the present, viewing the past as unhelpful and the future as less relevant. They tend to make quick decisions without extensive contemplation. These students favor learning experiences that involve action, allowing them to make and implement plans. Theoretical perspectives suggest that such characteristics can influence their approach to solving mathematical problems. In line with Sunarto's (2015) discussion on the role of the teacher, it is important to acknowledge that each student possesses a unique character. Consequently, their learning techniques and soft skills are shaped by their personality traits. Psychologists believe that these differences reflect underlying personality types.

This study specifically focuses on the Artisan personality type and its relationship with proportional reasoning in problem-solving. Research by Putra et al. (2019) indicates that students with an Artisan personality often fail to complete all indicators of problem-solving. These students tend to overlook writing down problem information and neglect to revisit their answers. Their approach to problem-solving is often rushed, lacking thoroughness, which leads to suboptimal completion of their plans. Such tendencies are likely to impact their learning outcomes. Additionally, there is limited understanding of how secondary school students engage in logical reasoning tasks. To address this gap, Bronkhorst et al. (2020) conducted research exploring the reasoning strategies of 16- and 17-year-old students in both formal and everyday reasoning tasks. Their findings are valuable for raising teachers' awareness of the reasoning strategies employed by students, as well as the reasoning challenges they encounter, which can inform the development of instructional materials to improve students' logical reasoning skills.

Our research aims to explore proportional reasoning among Artisan personality type high school students, providing insights that can benefit teachers. By understanding a student's personality characteristics and tendencies, teachers can better assess their attitudes and behaviors during problem-solving. Recognizing students' personality traits is crucial for effective knowledge transfer, as it helps educators interpret students' responses and reasoning when completing mathematics tasks. Nabila et al. (2023) also found that students with Artisan personalities tend to employ open-ended strategies in problem-solving. Studying personality types is essential, as each individual's unique personality, which is inherent from birth, influences all aspects of life, including education. A student's approach to problem-solving can vary depending on their personality, impacting how they address mathematical challenges.

The target of this study, which investigates proportional reasoning and the Artisan personality type, addresses a novel area of research that has yet to be extensively explored. A review of journal articles published from 2011 to 2022 in the Scopus database yielded 182 articles related to proportional reasoning and 73 articles concerning personality types in the context of mathematics. However, the bibliometric analysis using VosViewer reveals that while proportional reasoning and personality types are both linked to problem-solving, there is no existing research directly connecting proportional reasoning with personality types. This gap in the literature highlights the novelty of this research, as no studies have explicitly examined the relationship between mathematical proportional reasoning and the Artisan personality type.

Mathematics education involves various forms of reasoning, including arithmetic, proportional reasoning, and algebraic reasoning. Proportional reasoning, in particular, plays a critical role in bridging arithmetic reasoning and algebraic thinking (Öztürk et al., 2021). Anggorowati et al. (2024) also note that different personality types exhibit distinct approaches when solving mathematical problems. Personality, defined as an individual's consistent patterns of thoughts, feelings, and behaviors, significantly influences this process. Several studies have investigated varying levels of proportional reasoning (Gea et al., 2023; Izzatin et al., 2021; Sari et al., 2023), but this research specifically focuses on high school students with Artisan personality types and their proportional reasoning abilities in solving problems related to trigonometric comparisons.

This study aims to characterize the proportional reasoning abilities of students with Artisan personality types when tackling trigonometric comparison problems. The findings are expected to provide valuable insights for educators in designing effective teaching strategies that consider students' personality traits. Additionally, these results can serve as a reference for future studies exploring proportional reasoning in relation to other personality types.

METHODS

Research Design

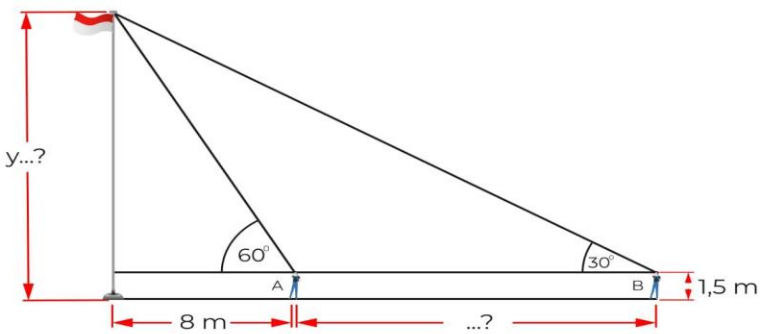
This study employed a descriptive qualitative research design utilizing an intrinsic case study approach. The selection of artisan subjects for solving proportional reasoning tasks was aimed at providing valuable insights into the academic domain. An intrinsic case study is particularly relevant when the case under investigation contains inherent features that merit an in-depth exploration (Hadi et al., 2021). Moreover, case studies offer participants the opportunity to ascribe meaning to their experiences in proportional reasoning, thereby making them well-suited for research requiring detailed explanations (Creswell, 2014).

Participants

The research subjects were students from Class X IA at a high school located in Kolaka Regency. The technique was employed for subject selection, based on specific criteria that included female gender and moderate mathematics ability. The criteria for mathematics ability were established through test scores, with participants scoring between 60 and 80. A total of 89 female students underwent both a mathematics ability test and a personality type assessment, utilizing an instrument developed by David Keirsey. Among these students, one was identified as possessing moderate mathematics ability and classified as having an Artisan Personality Type (APT), which was more prevalent than the Rational personality type. This student consistently exhibited the Artisan personality type across multiple testing sessions and was ultimately chosen as the primary focus of the study.

Research Instruments

The study utilized two primary instruments: the personality-type questionnaire and proportional reasoning tasks. The personality-type questionnaire aimed to collect data specifically related to the Artisan personality type. The results from this assessment provided preliminary data for classifying the subjects. Participants completed a personality instrument adapted from David Keirsey's standardized tool, which had been validated by two linguists and one psychology lecturer. The questionnaire consisted of 70 statements, with no right or wrong answers, and scoring was based on the highest score derived from pairwise comparisons of items corresponding to the dimensions of Extraversion-Introversion (E-I), Sensing-Intuition (S-N), Thinking-Feeling (T-F), and Judging-Perceiving (J-P). Unlike the Myers-Briggs Type Indicator (MBTI), which emphasizes individual cognitive processes, Keirsey's framework focuses on long-term behavioral patterns (Mazni et al., 2010).



A flagpole stands upright on a flat field if Andi stands at position A, which is 8 m from the flagpole, observing the top of the flagpole with an elevation angle of 60°. It is known that the height of Andi's eye position from the ground is 1.5 m. Next, Andi moves back a few meters to position B, and Andi again observes the top of the flagpole with an elevation angle of 30°.

a. How does Andi's distance from the flagpole change concerning the change in the magnitude of the elevation?
b. Determine the height of the flagpole! Explain the steps!
c. If Andi's distance from the flagpole is 8 m, the elevation angle is 60°. How far is Andi from the flagpole so the elevation angle is 30°?
d. Is the comparison direct proportion or inverse proportion, or neither?
e. If Andi's distance from the flagpole is a_1m, then the elevation angle to the top of the flagpole is α_1°. If Andi's distance from the flagpole is a_2m, then the elevation angle to the top of the flagpole is α_2°. Compare Andi's distance from the flagpole and the elevation angle. Determine whether the comparisons are direct proportion, inverse proportion, or neither.

Figure 1. Mathematics Proportional Reasoning Task

In addition to categorizing personality types, participants' proportional reasoning abilities were assessed through a trigonometry-based proportional reasoning test. The test was validated by three experts: two university lecturers from Surabaya and Kendari, and one high school mathematics teacher from Kolaka Regency.

The Mathematical Proportional Reasoning task comprised five questions, designed to assess participants' proficiency in proportional reasoning. These questions are presented in Figure 1. The task instrument was employed to collect data on proportional reasoning, complemented by in-depth interviews. The key aspects investigated in students' proportional reasoning included their understanding of covariation and ratio, as well as the development of various strategies for problem-solving, as adapted from Lamon (2020) and Van de Walle (2013). Questions (a) through (e) in Figure 1 represent students' understanding of covariation, ratio, and proportional reasoning strategies.

Data Analysis Technique

The data analysis process followed a structured approach, involving classification, data reduction, data presentation, and data interpretation (Miles et al., 2013). Classification was based on indicators of proportional reasoning, and the data collected from the two proportional reasoning tests were analyzed using data triangulation. The results from the students' responses were compared and processed through these triangulation techniques.

Data reduction entailed summarizing and focusing on key information, selecting relevant points, and excluding irrelevant data, with particular emphasis on the indicators of proportional reasoning derived from interview results. The data were subsequently presented in the form of narrative text or concise descriptions related to the proportional reasoning of the Artisan students.

Finally, data interpretation involved analyzing the results from student work, interviews, and confirmations provided by the Artisan students. In the final stage, the researcher synthesized the findings and drew conclusions. The indicators of proportional reasoning used to evaluate the students' performance in solving trigonometry problems are outlined in Table 1.

Table 1. Indicators of proportional reasoning in solving trigonometric problems

Indicators of Proportional Reasoning		Proportional Reasoning Activity
Understanding of covariation	1.	Detecting the covariation relationship in the given problem. (K1)
	2.	Providing the reason why the quantity relationship is a covariation relationship. (K2)
	3.	Using covariation in solving the given problem. (K3)
	4.	Providing reasons for using covariation in solving the given problem. (K4)
Understanding of ratio	1.	Using the ratio in solving the given problem. (R1)
	2.	Giving reasons for using ratios in solving the given problem. (R2)
Understanding of proportion	1.	Planning the use of quantity as a proportion relationship of quantity. (P1)
	2.	Giving reasons for using quantity as a proportion relation. (P2)
	3.	Extending the same relationship to other pairs of quantities. (P3)
	4.	Providing reasons for extending the same relationship to other quantity pairs. (P4)
Use of specific strategies	1.	Using a particular strategy or method to solve a given problem. (L1)
	2.	Providing reasons for using the strategy in solving the given problem. (L2)

	3. Revisiting the initial conjecture about the solution obtained by using other strategies in solving the given problem. (L3)
	4. Checking the suitability of the solution obtained with the given problem. (L4)

Notes: Adapted from the characteristics of proportional reasoning (Lamon, 2020; Van de Walle, 2013)

RESULTS AND DISCUSSION

The data collection process commenced following the identification of students who met the established research criteria. One student, exhibiting an artisan personality type (APT), was selected for inclusion in the study. Subsequently, data related to proportional reasoning were gathered through task-based interviews with the selected student. The analysis was based on established indicators of proportional reasoning, which are delineated across three stages: (1) understanding covariation, (2) understanding ratio and proportion, and (3) employing specific strategies. To ensure the reliability and validity of the data concerning the student's mathematical proportional reasoning, a triangulation method was employed. This involved an initial data collection followed by a subsequent round of data gathering. In cases where discrepancies between the datasets were observed, a reduction process was applied. The analysis of proportional reasoning in mathematics was thus framed within the context of the artisan personality type. Each task associated with students' trigonometric comparison material was analyzed using valid indicators specific to mathematical proportional reasoning, tailored to the APT.

Understanding Covariation

Subject APT addressed the problem presented in Figure 1, Part A, by providing both the solution and an explanation regarding the relationship between the change in Andi's distance from the flagpole and the viewing angle. The subject's response is detailed in the interview excerpt provided in Table 2.

Table 2. APT interview excerpt understanding covariation

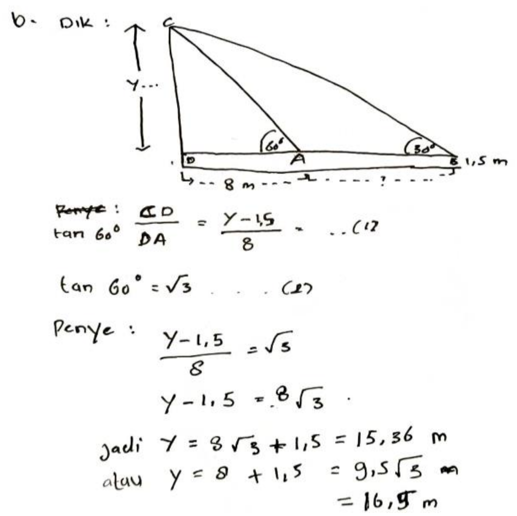
KW (Interview Code)	Conversation	KI (Indicator Code)
PA-17	For information or response. Tell us your answer!	
APT-17	From my answer, the effect of Andi's change in distance on the change in angle of view is that the further away Andi is from the flagpole, the angle will decrease from 60º to 30º.	K1
PA-18	So, what can we conclude about the changes?	
APT-18	The further Andi walks or the longer the distance Andi makes, the smaller the angle. Conversely, the closer Andi walks, the greater the angle.	K2, K3
PA-20	Was there a specific reason for your response?	
APT-20	The specific reason is that what is clearly illustrated from the question is that he will initially move. Moving a distance of 8m, the angle is 60º. When he walks again for about 10 meters, the angle changes to 30º. This means that the further he walks, the smaller the degree will be.	K4

The responses provided by Subject APT, as presented in Table 2, clearly demonstrate the application of proportional reasoning, particularly in the understanding of covariation. This is evident from APT's problem-solving approach, where the relationship between the change in Andi's distance from the flagpole and the corresponding angle of view was accurately determined. In response to question APT-18 (K3), Subject APT recognized the covariation between the change in distance and the angle of view.

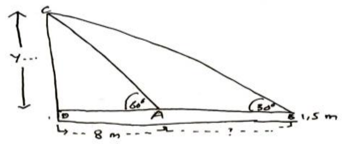
Specifically, the subject identified the relationship between the two variables as one of covariation within the context of a trigonometric comparison problem. The interview results and responses further indicate that APT's reasoning encompasses an understanding of the interplay between the magnitude of Andi's shift at point A and the corresponding elevation angle at point B.

Using a Specific Strategy

Subject APT approached the problem in a systematic manner, leading to the solution for the height of the flagpole, as outlined in the steps of their process. The subject's approach is depicted in Figure 2.



Given:



tan 60º = $\frac{\text{CD}}{\text{DA}} = \frac{y-15}{8}$ … (1)
tan 60º = $\sqrt{3}$ … (2)
Solution: $\frac{y-1,5}{8} = \sqrt{3}$
y – 1,5 = 8$\sqrt{3}$
So y = 8$\sqrt{3}$ + 1,5 = 15,36m
Or y = 8 + 1,5 = 9,5$\sqrt{3}$m = 16,6m

Figure 2. Use of a specific strategy

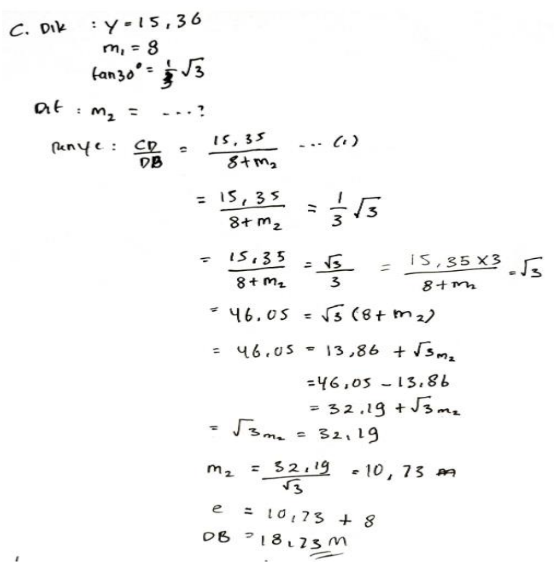
Based on the response provided in Figure 2, it is evident that Subject APT employed a specific strategy involving the cross-multiplication method. The subject began by sketching the problem, then proceeded with two versions of the answer, one of which contained a conceptual error: the incorrect addition of numbers while disregarding the roots. After obtaining the numerical values, the subject then combined the root operations, mistakenly applying the wrong concept in this manner. This is further substantiated by the interview results with APT, as shown in Table 3.

Table 3. APT interview using specific strategy

KW (Interview Code)	Conversation	KI (Indicator Code)
PA-31	Please tell me about the working steps from the beginning.	
APT-31	First, we redraw the structure, and then we are asked how high the flagpole is. y we find the front side of the corner per hypotenuse; the front side here is not there, while the flagpole is less than 1.5m. So, y - 1.5 means the height can be subtracted by 1.5. Then, divided by the side of 8m is the first known. For the second known, tan 60°= 3$\sqrt{3}$.. In the solution, we combine the two equations into y-1, 5/8$\sqrt{3}$. = we change it, we decompose it into y-1.5 fixed for 8. 8 we connect with 8/$\sqrt{3}$.	L1
PA-32	What can you conclude?	

APT-32	So, it can be concluded = 8 $\sqrt{3}$ plus this is why it is added; from the beginning, it was less because there was a displacement of the segment that was previously in the left segment; it became the right segment, so we added it, we changed it to 8$\sqrt{3}$ + 1.5 = 15.36m.	L2
PA-37	Does the flag's height in the real world make sense if it's that size?	
APT-37	No, it's only about a few meters.	L3
PA-38	If it's on a flat field, not in a room?	
APT-38	It could be.	L4
PA-42	What is the name of this step to make it 8$\sqrt{3}$?	
APT-42	I add, combine. If I don't apply the three later, it won't be used if it's an addition model or later, it won't be easy to apply again. It can be eight directly here, but then, the triple root directly like here 8 + 1.5 = 9.5√3m.	L4
PA-45	What is the final result?	
APT-45	Yes, it can be like that or = 8 should increase by 1.5. So, it's 9.5 but has a root $\sqrt{3}$ because it didn't take meters before $\sqrt{3}$ meter.	L2

Subject APT solved the problem by demonstrating an understanding of the given task and arriving at the answer. Specifically, when Andi's distance from the flagpole is 8 m and the elevation angle is 60º, the subject determined Andi’s distance from the flagpole for an elevation angle of 30º. The subject's solution can be observed in Figure 3.



c. Given: y = 15,36
m_1 = 8
tan 30º = $\frac{1}{3}\sqrt{3}$
Question: $m_2 = \ldots$?
Solution: $\frac{\text{CD}}{\text{DB}} = \frac{15{,}35}{8+m_2}$ … (1)
= $\frac{15{,}35}{8+m_2} = \frac{1}{3}\sqrt{3} = \frac{\sqrt{3}}{3} = \frac{15{,}35\ x\ 3}{8+m_2}$ =$\sqrt{3}$
= 46,05 = $\sqrt{3}$(8+m_2)
= 46,05 = 13,86 + $\sqrt{3}m_2$
= 46,05-13,86 = 32,19 +$\sqrt{3}m_2$
=$\sqrt{3}m_2$=32,19
$m_2 = \frac{32{,}19}{\sqrt{3}}$ = 10,73
e = 10,73 + 8
DB = 18,73m

Figure 3. APT's written answer on the indicator of using the specific strategy

Based on the subject’s response in Figure 3, it is clear that Subject APT applies proportional reasoning through the use of a specific strategy. The problem-solving process involves the subject first writing down the relevant information, determining Andi's distance from the flagpole corresponding to an elevation angle of 30º, and performing the necessary calculations between the distance and the angle, utilizing the cross-multiplication strategy. However, the final answer provided by APT is still incorrect due to a lack of precision in the initial calculation step, where the value 15.35 was incorrectly used. The subject did not fully comprehend the nature of the roots, leading to an erroneous result of 10.73 meters. Despite this, the subject adhered to the cross-multiplication strategy, providing reasons for the steps taken. This is corroborated by the interview results with APT, which are detailed in Table 4.

Table 4. APT interview result using the specific strategy

Interview Code	Conversation	Indicator Code
PA-55	Okay, you can explain the work steps!	
APT-55	The way to solve it is like the formula that we applied in point b, which is the front of the angle per hypotenuse; we already got the front of the angle, which is 15.35 from the previous point, and the DB is 8+m_2, m_2 here, it means what is the distance from a to b? Symbolised as m_2, because m_1, the other one has been obtained, which is 8m, then m_2 I look for it, then to get it $\frac{15,35}{8+m_2}$, I enter tan 30, which is = $\frac{1}{3}\sqrt{3}$ because this is the third root formula I round it up $\frac{\sqrt{3}}{3}$ by 1x$\sqrt{3}$ = $\sqrt{3}$, 1x$\sqrt{3}$, 3x$\sqrt{3}$ gets 3 so I formulate it as $\sqrt{3}$/3. So then, I multiplied between 15.35x3 to get 46.05 = I entered it earlier, then I didn't get it, so I just wrote 8 +m_2,, then = 46.05 = 13.86. This 13.86 is obtained from $\sqrt{3}$x8. The result is 13.86 +$\sqrt{3}$ x m_2, it becomes $\sqrt{3}$xm_2.	L1
PA-56	Where did you get 46.05 from?	
APT-56	46.05 is from 15 if 15.35 is multiplied by 3 from the formula sin cos tan, tan 30° and then 46.05-13.86, gives 32.19. Yes, 32.19 plus enter the value of $\sqrt{3}, m_2$, again. Re-enter =$\sqrt{3}$, = m_2 re-enter the value, which is 32.19. Here, we can enter the m_2 = from 32$\sqrt{32,19}$/ $\sqrt{3}$ = 10.73. We have the result, meaning the distance from a to b is 10.73. For the elevation value, we just add 10.73 plus 8 of the value m_1 so that the overall value is from D to B. Eh, the total value from the flagpole to the last point where Andi stood is 18.73.	L1
PA-59	Something was interesting. How did 32.19 + $\sqrt{3}$ m_2 become like this?	
APT-59	Because this is the meaning of 32.19, right here, we sum up this one, this two, and this we don't sum up first, which means we get rid of it first; later, we finish summing it up, and then it reappears.	L2
PA-60	Next?	
APT-60	To make it easier to find the m, we flip it again. It becomes $\sqrt{3}/m_2$. You don't actually have to use this; you can just enter it $m_2$here = 32.19/$\sqrt{3}$.	L2
PA-61	So that's the other way that you thought of?	
APT-61	If this means we add the third root, while this is moving, moving the position too.	L2
PA-62	Can this be added? One has a variable, one doesn't. Can you do it?	
APT-62	No, because it's clear that there's one, it can't be added.	L3
PA-63	Did you say you want to add it?	
APT-63	Mistakenly, to make it easier, if we use it directly, we will mistakenly add this while it should be divided.	L4
PA-64	If it's related to the previous problem until you said moving segments, does this have no effect, or will the sign be changed later?	
APT-64	It doesn't affect it.	L4

PA-65	Why?	
APT-65	Because it actually has an effect, from this, the addition is added, and there is a root. This root is the root m_2 we move, so we have to move the segment first from what was originally 32,19+$\sqrt{3}m_2$ to = 32.19. We have to move it so directly we take them m_2, then enter the amount and divide by $\sqrt{3}$.	L2
PA-69	Oh yes. Besides that, are there any other properties you use when working?	
APT-69	Other properties, such as operations, are also present. In addition to multiplication, there is addition, multiplication, division, and rooting.	L1
PA-70	What did you apply in your answer?	
APT-70	Here, the multiplication for the second one is the same as the cross.	L1
PA-71	Is there still something missing?	
APT-71	Nothing, because I have added from m_1+m_2 probably 18,73.	L2
PA-73	Why must you use the formula or property you mentioned earlier?	
APT-73	To make it easier to ger the result	L2
PA-80	Then, I'm still curious about how the strategy you mentioned this time is different from any other description. Are there other strategies that might be used?	
APT-80	You can use other strategies.	L3
PA-81	What strategy is that?	
APT-81	The problem can be directly solved as $\frac{15.35\text{x}3}{8+m_2}$ = $\sqrt{3}$, because it's the same result. The root still goes in here.	L4

Subject APT concluded by stating that if Andi's distance from the flagpole is 8m and the elevation angle is 60º, then Andi's distance from the flagpole for an elevation angle of 30º would be 18.73m, as seen in the final solution.

Understanding Ratios and Proportions

Subject APT approached the problem by understanding the task, comparing Andi's distance from the flagpole to the elevation angle, and identifying the type of comparison. The subject's solution is shown in Figure 4.

d. Given:

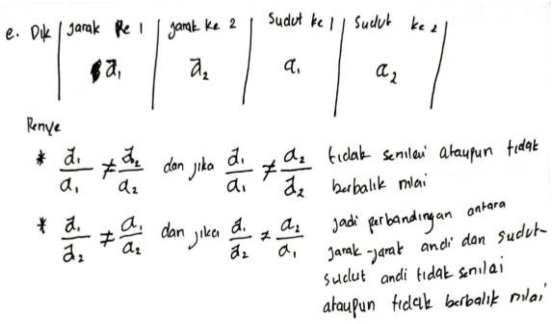
First distance m	Second distance m	First angle°	Second Angle°
8	10,73	60	30

* $\frac{8}{60} \neq \frac{10,73}{30}$ and if $\frac{8}{60} \neq \frac{30}{10,73}$ not equal or inverse

* $\frac{8}{10,73} \neq \frac{60}{30}$ and if $\frac{8}{10,73} \neq \frac{30}{60}$ so the ratio between andi is distance and andi is angle is neither equal nor opposite in value

Figure 4. APT written answer understanding ratio and proportion (Question d)

Figure 4 shows that it is evident that the subject understood ratios and proportions, as illustrated in Figure 5. In this solution, the subject used symbols to make comparisons and adjusted the numbers and symbols accordingly when providing the answers to the types of comparisons.



e. Given:

First distance 1	Second distance 2	First angle	Second angle
a_1	a_2	α_1	α_2

$*\frac{a_1}{\alpha_1} \neq \frac{a_2}{\alpha_2}$ and if $\frac{a_1}{\alpha_1} \neq \frac{\alpha_2}{a_2}$ not equal or inverse

$*\frac{a_1}{a_2} \neq \frac{\alpha_1}{\alpha_2}$ and if $\frac{a_1}{a_2} \neq \frac{\alpha_2}{\alpha_1}$ so the ratio between andi is distance and andi is angle is neither equal nor opposite in value

Figure 5. APT's written answer on the indicator of understanding ratio and proportion (Question e)

As demonstrated in Figures 4 and 5, Subject APT applied proportional reasoning by using ratios to compare Andi's distance from the flagpole with the elevation angle. Additionally, proportions were employed to determine the type of comparison. Despite errors in the initial calculation, the subject correctly identified the comparison as neither equivalent nor inverse. This was achieved through the proper application of proportional reasoning principles, even though the underlying calculations contained errors from the earlier problem. The subject's correct approach in determining the comparison is further supported by the interview results, as presented in Table 5.

Table 5. APT interview excerpt understanding ratio and proportion (Question d)

Interview Code	Conversation	Indicator Code
PA-89	What's the answer problem d?	
APT-89	I used the first distance to the first angle which is 8/60 = the second distance to the second angle which is 10.73/30. If you prove it, it's not equal. If it is reversed, namely the first distance to the first angle = the first angle to the second angle to the second distance, after proving it, it is not reversed. So I conclude that they are not equivalent or not inverse.	R1, P1
PA-96	Are there any other possibilities?	
APT-96	The second possibility is if I compare distances and angles. Like the distance between the first distance and the second distance 8/10.73 = the first angle to the second angle which is 60/30 and when proven it is not equal and if 8/10.73 = the second angle per the first angle which is 30/60 also after proving it is not inverse value. So, I conclude that the comparison between Andi's distances and Andi's angles is neither equal nor inverse.	P3
PA-97	Why, tell me why?	
APT-97	They both have different results when I proved it by finding the same multiplication. So, they are neither equal nor inversely proportional.	R2, P2, P4

Subject APT correctly solved the problem and as shown in Figure 5, compared Andi's distance from the flagpole to the elevation angle, determining the type of comparison by substituting symbols for the distance and angle values. Despite using incorrect data, the subject performed the correct comparison, identifying that the relationship was neither equal nor inverse. This solution was validated through the subject's explanation in the interview as summarized in Table 6.

Table 6. APT interview results indicator of understanding ratio and proportion (Question e)

Interview Code	Conversation	Indicator Code
PA-112	Is it still representative if a symbol that was formerly a number is used?	
APT-112	Yes, it is representative.	R1
PA-113	Do you need to use a scratch paper or calculator for this?	
APT-113	No, because it's not a number but a symbol	R2
PA-114	So, what's the final answer?	
APT-114	Yes, it follows the first answer.	R2
PA-115	How can you be sure that this step was used correctly?	
APT-115	The answer is the same for both points d and e because the problems are nearly identical.	P1
PA-116	Why did you choose this approach?	
APT-116	Because it is easier to solve.	P2
PA-118	Does your answer address the question, and why?	
APT-118	Yes, it does. The question asks whether it is equal or inverse, and I have demonstrated that point e is neither equal nor inverse.	P3, P4

Proportional reasoning encompasses various types of comparisons, including value comparisons and inverse value comparisons, alongside ratios and proportions. According to Castillo and Fernández (2022), comparison in mathematics refers to the process of evaluating two or more objects, which is inherently connected to ratios and proportions. A ratio is a numerical relationship that links two quantities or magnitudes through a multiplicative relationship. In contrast, a proportion is an equality between two ratios or a statement indicating that two comparisons are equivalent. Additionally, a value comparison occurs when two or more quantities share the same value, and when one variable increases, the other also increases. On the other hand, an inverse value comparison refers to a scenario where, as one variable increases, another decreases.

Table 7 presents the valid data on mathematical proportional reasoning for students with an APT, as previously discussed. This analysis confirms the application of proportional reasoning by Subject APT in various aspects of mathematical problem-solving.

Table 7. Description of task and interview results

Indicator of Proportional Reasoning	Proportional Reasoning Ability Written Test Stages	Proportional Reasoning Ability Interview Stage
Understanding covariation	The subject expresses the same answer as the one written about understanding covariation.	Students express their understanding of covariation by suggesting the effect of Andi's change in distance on the change in angle of view is that the further Andi is from the flagpole, the angle will decrease from 60° to 30°.
Understanding ratio and proportion	Students use ratios by comparing values from previous calculations in the table,	The subject used ratios by comparing Andi's distance from the

	namely comparisons between distance and first angle, second distance and second angle, and first distance and second distance with first angle and second angle. In addition, using proportion when mentioning the type of comparison makes the answer correct even though the comparison values used have calculation errors in related problems.	flagpole and the elevation angle with the values from the previous calculations in the table, and proportions were used when determining the type of comparison. The subject concluded that the two are not equivalent or inverse and has proven that when using symbols, they are not equivalent or inverse.
Using a specific strategy	The subject used a particular strategy, which involved the cross-tabulation strategy. The subject provided two versions of answers, one of which contained a concept error: how to add numbers by ignoring the roots after obtaining the number and then combining the root operations. The subject applied the wrong concept in this way.	The APT subject used the crossed times strategy to solve the problem given. The multiplication for the first and second is the same: cross multiplication.

The findings of this study provide valuable insights into the relationship between proportional reasoning and the artisan personality type, as reflected in students' responses to mathematical problems, specifically trigonometric comparison problems. These problems contained indicators of proportional reasoning, including students' understanding of covariation, ratios, proportions, and the application of specific problem-solving strategies. Artisan students demonstrated the ability to solve problems, although their answers sometimes included errors—both correct and incorrect comparisons—when alternative answers were checked. Furthermore, these students were able to provide logical explanations for problems requiring reasoning. However, mathematical calculation errors were prevalent, suggesting a need for greater focus and accuracy in problem-solving, as well as a better understanding of root properties. The subjects in our study exhibited errors related to basic concepts, which indicated a need for further learning and a more meticulous approach.

The personality description of the subjects is useful in tailoring instructional strategies according to individual personality types when solving problems and engaging in learning activities. For example, Subject APT correctly answered the first question and demonstrated an understanding of covariation, even though he did not explicitly mention the relationship between Andi's distance from the flagpole and the viewing angle as covariation. Nonetheless, the interview and answer analysis indicated that the student grasped the concept of covariation. In the second problem, which involved calculating the height of the flagpole, the student employed a particular strategy but made a conceptual error in one of the alternative solutions. This highlighted the student's limited understanding of root operations. In the third problem, the student used a cross-multiplication strategy to determine the distance of Andi from the flagpole at a 30° elevation angle, but the answer was incorrect due to a lack of carefulness and inadequate understanding of the nature of root operations from the previous problem. The logical reasons provided by the student for problems requiring reasoning, such as the assertion that values are not equal and not inversely proportional, were sound.

Research by Kumalasari et al. (2022) suggests that students with artisan personalities tend to be direct in expressing what they observe and prefer to engage in practical, quick solutions, even if these do not strictly follow established rules. According to Putra et al. (2019), students with artisan personality types are often rushed and less thorough in problem-solving, leading to suboptimal planning in completing answers. However, in our study, the subject's initial answers were correct, although errors in procedure or incorrect operational values were present, particularly when the student rushed through the tasks.

Furthermore, Listiawati et al. (2023) explain that observations indicate that most students actively engage in learning, except for those who may struggle to comprehend the teacher's instructions. These slower learners often face comprehension difficulties and require additional support in understanding the concepts presented. The subjects in this study exhibited similar challenges, with their understanding of the material presented by the teacher requiring further development. Nonetheless, students' personalities played a significant role in their ability to solve mathematical problems.

Consistent with previous research, the answers provided by students in this study were influenced by the specific context of the problems. In our research, the context of daily life, familiar to the students, facilitated their ability to solve trigonometric comparison problems, even though some errors still occurred. Although only a few answers were fully correct, the rush to complete the work led to errors in procedural steps.

The characteristics of artisan students, as outlined in Keirsey's theory of personality types, align with active and extroverted behavior, with a desire to be the center of attention and demonstrate their abilities. As noted by Novitasari et al. (2020), individuals with artisan personalities exhibit flexibility in solving problems, applying multiple strategies, such as drawing and using variables, to successfully solve mathematical problems. In contrast, some students in our study exhibited errors in answering questions, reflecting a need for greater attention to detail and a more methodical approach in solving problems.

Regarding the equivalence relationship between ratios, research by Gea et al. (2023) highlights that students make comparisons using multiplication operations and find equivalent ratios. In our study, artisan students used non-equivalent and non-reversible comparisons, producing correct answers from inaccurate data.

In summary, students with specific tendencies based on their personality type, when aware of their strengths and weaknesses, can approach mathematical problems more reflectively. This self-awareness can minimize errors, improve knowledge development, and promote more thorough, accurate problem-solving. The findings of this study underscore the importance of recognizing students' potential, personality types, and areas for improvement to enhance effective knowledge transfer and learning. Consequently, the proportional reasoning of students with artisan personality types in solving trigonometric comparison problems can help foster understanding of specific learning traits, making learning more effective and efficient. Moreover, this research contributes to understanding the proportional reasoning abilities of students with artisan personalities, providing a foundation for future studies aimed at designing instructional strategies and learning media that support proportional reasoning development in artisan-type students.

CONCLUSION

This study has demonstrated that artisan students possess an understanding of covariation, ratio, and proportion, applying specific strategies when working with these concepts. Despite their general understanding, students exhibit conceptual and procedural errors when making comparisons and drawing conclusions. These errors often stem from incorrect initial information, such as when symbols are substituted for numbers, which affects their logical reasoning. Interestingly, when interviewed, students displayed the ability to provide accurate and relevant supporting information, indicating their capacity to approach proportional reasoning tasks with more clarity and insight during reflective dialogue.

The limitations of this research primarily stem from its focus on trigonometric comparison material and the examination of a single personality type. While the study effectively highlighted how proportional reasoning can be assessed through trigonometric problems and provided valuable insights into students' abilities and responses, the narrow scope limits the generalizability of the findings. Furthermore, a single personality type was explored, which constrains the exploration of how varying personality traits might influence students' learning processes and reasoning strategies.

To expand on these findings, future research should include a broader range of learning materials and personality types to gain a deeper understanding of the specific characteristics of learners across different contexts. This will allow for a more comprehensive exploration of how different personality types engage with proportional reasoning and other mathematical concepts. Additionally, further research could leverage the insights gained about artisan students' characteristics to design more tailored educational strategies and learning media, enhancing both the effectiveness and efficiency of teaching approaches for diverse learners.

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REFERENCES

Anggorowati, W., Kariadinata, R., & Widiastuti A., T. T. (2024). Analysis of creative thinking skill in solving mathematical vroblems viewed from the types of personality. *International Conference on Mathematics and Science Education*, *2024*, 275–284. https://doi.org/10.18502/kss.v9i13.15928

Bronkhorst, H., Roorda, G., Suhre, C., & Goedhart, M. (2020). Logical reasoning in formal and everyday reasoning tasks. *International Journal of Science and Mathematics Education*, *18*(8), 1673–1694. https://doi.org/10.1007/s10763-019-10039-8

Castillo, S., & Fernández, C. (2022). Secondary school students' performances on ratio comparison problems. *Acta Scientiae*, *24*(6), 60–88. https://doi.org/10.17648/acta.scientiae.6834

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (p. 273). SAGE Publications Inc. http://dx.doi.org/10.5539/elt.v12n5p40

Dong, A., Jong, M. S.-Y., & King, R. B. (2020). How does prior knowledge influence learning engagement? the mediating roles of cognitive load and help-seeking. *Frontiers in Psychology*, *11*(October), 1–10. https://doi.org/10.3389/fpsyg.2020.591203

Gea, M. M., Hernández-Solís, L. A., Batanero, C., & Álvarez-Arroyo, R. (2023). Relating students' proportional reasoning level and their understanding of fair games. *Journal on Mathematics Education*, *14*(4), 663–682. https://doi.org/10.22342/jme.v14i4.pp663-682

Hadi, Abd., Asrori, & Rusman. (2021). Penelitian kualitatif studi fenomenologi, case study. Pena Persada. https://www.researchgate.net/publication/362371905_Penelitian_Kualitatif_Studi_Fenomenologi_Case_Study_Grounded_Theory_Etnografi_Biografi

Hjelte, A., Schindler, M., & Nilsson, P. (2020). Kinds of mathematical reasoning addressed in empirical research in mathematics education: A systematic review. *Education Sciences*, *10*(10), 1–15. https://doi.org/10.3390/educsci10100289

Izzatin, M., Waluya, S., Rochmad, Kartono, Dwidayati, N., & Dewi, N. (2021). Students' proportional reasoning in solving non-routine problems based on mathematical disposition. *Journal of Physics: Conference Series*, *1918*(4), 042114. https://doi.org/10.1088/1742-6596/1918/4/042114

Keirsey, D., & Bates, M. (1978). *Please understand me: Character and temperament* (Fifth Edit). Prometheus Nemesis Book Company. https://www.researchgate.net/publication/272159871_Please_Understand_Me

Kumalasari, A. D., Karremans, J. C., & Dijksterhuis, A. (2022). Do people choose happiness? Anticipated happiness affects both intuitive and deliberative decision-making. *Current Psychology*, *41*(9), 6500-6510. https://doi.org/10.1007/s12144-020-01144-x

Lamon, S. J. (2020). Teaching fractions and ratios for understanding. In *Teaching Fractions and Ratios for Understanding*. Routledge. https://doi.org/10.4324/9781410617132

Langrall, C. W., & Swafford, J. (2000). Three balloons for two dollars: Developing proportional reasoning. *Mathematics Teaching in the Middle School*, *6*(4), 254-261. https://doi.org/10.5951/MTMS.6.4.0254

Listiawati, N., Sabon, S. S., Siswantari, Subijanto, Wibowo, S., Zulkardi, & Riyanto, B. (2023). Analysis of implementing Realistic Mathematics Education principles to enhance mathematics competence of slow learner students. *Journal on Mathematics Education*, *14*(4), 683–700. https://doi.org/10.22342/jme.v14i4.pp683-700

Mazni, O., Syed-Abdullah, S.-L., & Naimah Mohd Hussin. (2010). Analyzing personality types to predict team performance. *2010 International Conference on Science and Social Research (CSSR 2010)*, *Cssr*, 624–628. https://doi.org/10.1109/CSSR.2010.5773856

Miles, M. B., Huberman, A. M., & Saldana, J. (2013). *Qualitative data analysis* (Third Edit). Arizona State University.

https://www.researchgate.net/publication/272566756_Qualitative_Data_Analysis_A_Methods_Sourcebook

Nabila, C., Sukirwan, S., Setiani, Y., Farooq, S. M. Y., Vereshchaha, V., & Caw, S. (2023). Senior high school students' strategies for solving mathematical problems based on their personality type. *Numerical: Jurnal Matematika dan Pendidikan Matematika*, *7*(2), 287–300. https://doi.org/10.25217/numerical.v7i2.3861

Novitasari, Y., Setianingsih, R., & Novitasari, Y. F. (2020). Flexibility of guardian and artisan selected students in solving fraction problems. *Mathematics, Informatics, Science, and Education International Conference (MISEIC 2019)*, *95*(Miseic), 119–122. https://doi.org/10.2991/miseic-19.2019.28

Öztürk, M., Demir, Ü., & Akkan, Y. (2021). Investigation of proportional reasoning problem solving processes of seventh grade students: a mixed method research. *International Journal on Social and Education Sciences*, *3*(1), 48–67. https://doi.org/10.46328/ijonses.66

Pambudi, D. S., Murtikusuma, R. P., Trapsilasiwi, D., Oktavianingtyas, E., Wiliandani, I., & Ningrum, T. P. (2021). Mathematical representation of the grade 11 of senior high school students in solving linear programming questions based on david keirsey's personality type. *Proceedings of the 1st International Conference on Mathematics and Mathematics Education (ICMMEd 2020)*, *550*(Icmmed 2020), 162–166. https://doi.org/10.2991/assehr.k.210508.059

Putra, R. W. Y., Supriadi, N., Ilmiyana, M., Khoiriah, Fitriani, D., Aziz, A. F., & Solihat, T. (2019). The analysis of the mathematical problem-solving ability of high school students reviewed from personality types of the rational and artisan. *Journal of Physics: Conference Series*, *1155*(1), 012039. https://doi.org/10.1088/1742-6596/1155/1/012039

Sari, Y. M., Fiangga, S., Milla, Y. I. El, & Puspaningtyas, N. D. (2023). Exploring students' proportional reasoning in solving guided-unguided area conservation problem: A case of Indonesian students. *Journal on Mathematics Education*, *14*(2), 375–394. https://doi.org/10.22342/jme.v14i2.pp375-394

Sholli, A. K., Lukito, A., & Setianingsih, R. (2020). Guardian high school student's conception about mathematics as sensible. *Journal of Physics: Conference Series*, *1581*(1), 1–5. https://doi.org/10.1088/1742-6596/1581/1/012031

Sunarto, M. J. D. (2015). Improving students soft skills using thinking process profile based on personality types. *International Journal of Evaluation and Research in Education (IJERE)*, *4*(3), 118. https://doi.org/10.11591/ijere.v4i3.4502

Van de Walle, J. A. (2013). *Elementary and middle school mathematics teaching developmentally*. Pearson Education.

Vanluydt, E., Degrande, T., Verschaffel, L., & Van Dooren, W. (2020). Early stages of proportional reasoning: A cross-sectional study with 5- to 9-year-olds. *European Journal of Psychology of Education*, *35*(3), 529–547. https://doi.org/10.1007/s10212-019-00434-8