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Primary school teachers' knowledge for teaching multiplicative thinking

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Abstract

Multiplicative thinking is important for students' learning of key topics in mathematics such as algebra, geometry, measurement, fractions, statistics, and probability. This study employed an embedded mixed-method approach to investigate primary school teachers' pedagogical content knowledge for developing multiplicative thinking in students. The study participants (n = 62) were primary school teachers in Australia at different levels of teaching experience, from preservice teachers to novice, experienced, and expert teachers. Teachers completed a carefully designed questionnaire, and a model of Teacher Capacity was the framework for instrument design and data analysis. The investigation in this research focused on three key teaching stages for developing multiplicative thinking: transitional, multiplicative, and proportional reasoning. Estimated marginal means, pairwise comparison, and regression analysis statistical tests were conducted using SPSS 2.0. The results highlight specific areas requiring attention in teacher professional development and preparation programs to enhance teachers' capacity to effectively support students' learning and development around multiplicative thinking. For example, there is a need to enhance teachers' capacity for multiplicative thinking during transitional teaching and the need to emphasize the development of teachers' knowledge of students and the design of instruction.

Keywords: Multiplicative Thinking, Pedagogical Content Knowledge, Teacher Capacity, Teaching Experience

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This paper presents research that explored primary school teachers' knowledge for developing the concept of multiplicative thinking in students. The importance of multiplicative thinking in supporting students' learning of key topics and success in further mathematics is widely and clearly stated in mathematics education literature (e.g., Siemon, 2013; Askew et al., 2019). It forms the basis for understanding proportions, patterns, fractions, measurement, rates, percentages, statistical thinking, the development of algebraic thinking and understanding the complex issues in society (Askew et al., 2019). However, there is evidence of low student performance in this area (Siemon, 2013). Multiplicative thinking underpins students' learning of STEM disciplines which is currently an area of focus for many governments globally (Siemon et al., 2018).Teachers' pedagogical content knowledge (PCK) is critical in determining students' learning attainment (Fennema & Franke, 1992). However, research is needed into teachers' PCK for developing multiplicative thinking in students. Past research efforts have largely been directed towards understanding students' conceptual development of multiplicative thinking and development of both diagnostic and teaching materials to support student development in this area, ignoring those aspects to do with teachers. Fennema and Franke's (1992) argument about the role of teachers' PCK in determining student attainment substantiates the need for a balance of research into understanding students' thinking, understanding, development, and difficulties with multiplicative problems and the understanding of teachers' PCK for developing multiplicative thinking in students. Furthermore, Downton et al. (2019) argued that given the complexities and importance of students' multiplicative thinking, effort should be devoted towards promoting teachers' PCK for developing students' multiplicative thinking. This focus on inquiry into teachers' knowledge for multiplicative thinking relates to a study conducted by Matitaputty et al. (2024) who investigated teachers specialised knowledge for teaching permutation and combination through Teacher professional education programs. While some teachers demonstrated pround knowledge of teaching permutations and combinations, the study also identified the need for teacher professional education programs to enhance teachers' instructional strategies on the topic.

Multiplicative thinking represents learners' mental adaptive processing of multiplication and division concepts by using different methods and approaches in various mathematical problems (Singh, 2012). It allows learners' capacity to successfully grapple with mathematical problems across mathematics topics that require understanding and application of multiplicative ideas. Multiplicative thinking is characterised by the following:

1. a capacity to work flexibly and efficiently with extended range of numbers—for instance, whole numbers, decimals, fractions, and percentages.
2. the ability to recognise and solve a range of problems involving multiplication or division, including direct and indirect proportion.
3. the means to communicate this effectively in a variety of ways—for example, using words, diagrams, symbolic expressions, and written algorithms.

(Siemon, 2013, p. 41)

These characteristics allow multiplicative thinking to be summarised as the ability to recognise where to use multiplication, division, and proportional reasoning solution strategies, the ability to communicate and justify the solution strategies, and the capacity to solve problems requiring knowledge of multiplication and division in a broad range of contexts using different strategies. In this study, multiplicative thinking was conceptualised beyond the three characteristics suggested by Siemon et al. (2006). Multiplicative thinking further entails understanding how the knowledge of multiplication, division, and proportional reasoning is connected and applicable to key topics in mathematics. Understanding these characteristics is vital to inform not only the design of curriculum, but also the design of teaching. While the concept of multiplicative thinking is broad and used in many topics in mathematics, such as fractions, probability, trigonometry, patterns, and statistics, this study looks at multiplicative thinking by starting from its basic level of multiplication and division. This position aligns with Hurst and Hurrell (2016) who conceded that the concept of multiplicative thinking is not simple to teach and learn, but understanding the concept from multiplication and division offers an excellent starting point. We should make it clear, however, that Siemon et al. (2006) conceptualisation of multiplicative thinking remained especially useful throughout this study.

Studies have pointed to student underperformance in multiplicative thinking. In South Africa, evidence from research of middle primary students' mathematical progress indicates continued reliance on counting-based strategies when solving multiplicative problems (Venkat & Mathews, 2019). A study conducted in Australia by Seah and Booker (2005) found that Year 8 students' achievement on multiplicative reasoning tasks was low. Furthermore, projects by Siemon et al. (2001) and Siemon et al. (2006) between 2001 and 2006 identified low levels of multiplicative thinking in students, contributing to students' low performance in mathematics. Downton et al. (2019) argued that this low performance can be attributed to teachers' PCK for developing students' multiplicative thinking.

There is substantial research demonstrating the depth and breadth of inquiry into students' understanding, thinking, performance, and developmental hierarchy, as well as ways to support students' development of multiplicative thinking. This claim coheres with Sowder et al. (1998) who maintained that the primary and high school mathematics related to multiplicative structures has undergone scrutiny over the past decade and that researchers have identified the types of reasoning and difficulties students have with concepts of multiplicative thinking. In Australia, the development of the Learning and Assessment Framework (LAF) from the SNMYP (2003–2006) project is evidence of efforts to improve students' learning of multiplicative thinking. The Reframing Mathematical Futures (RMF) project (Day & Hurrell, 2015), which used SNMYP materials with the aim of improving student performance in multiplicative thinking and proportional reasoning in Years 7 to 10, provides evidence of substantial efforts to improve students' attainment in multiplicative thinking.

Research is needed into teachers' PCK for developing multiplicative thinking in students. This study aimed to investigate teachers' PCK for developing multiplicative thinking in students using the Teacher Capacity model (Zhang & Stephens, 2013), with particular attention to three key teaching stages that form the foundation to understanding key topics in mathematics: transitional stage (from additive to multiplicative), multiplicative stage (multiplication and division word problems), and proportional reasoning stage (Malola et al, 2021).

This study sought to answer the following two research questions:

1. Research Question 1. To what extent can teachers orchestrate student learning of multiplicative thinking through the three key teaching stages of transition from additive to multiplicative thinking, multiplicative (multiplication and division of word problems), and proportional reasoning?
2. Research Question 2. How does primary school teachers' PCK for multiplicative thinking compare across the four components of the Teacher Capacity model (mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction)?

METHODS

Study Design

Considering the COVID-19 pandemic environment in which this study was conducted, the study used an embedded mixed-methods research design. The questionnaire data were used for both quantitative and qualitative analysis. "The Embedded Design mixes the different data sets at the design level, with one type of data being embedded within a methodology framed by the other data type" (Creswell et al., 2003, p. 67). A representation of an embedded mixed-methods research design is shown in Figure 1.

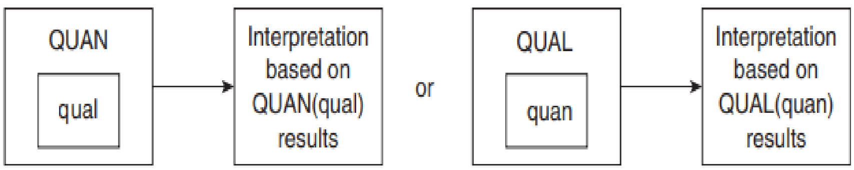


Figure 1. An embedded Mixed-Methods Design (Cresswell et al., 2003, p.68)

The study reported here used a single questionnaire to collect both quantitative and qualitative data. This study was largely quantitative, and the qualitative data played a supportive secondary role to the quantitative data. This was emphasised by Creswell et al. (2003) who explained that in an embedded mixed-methods research design, the secondary data should play a supplemental role to the primary data. This paper reports largely on quantitative data.

Framework

This study used the model of Teacher Capacity (Zhang & Stephens, 2013) as the framework for questionnaire design and data analysis to explore teachers' PCK for developing multiplicative thinking in students across the three key teaching stages. The Teacher Capacity model shown in Figure 2 suggests that teacher capacity for teaching mathematics comprises the following four elements: knowledge of mathematics, knowledge of curriculum, knowledge of students' thinking, and design of instruction (p. 489).

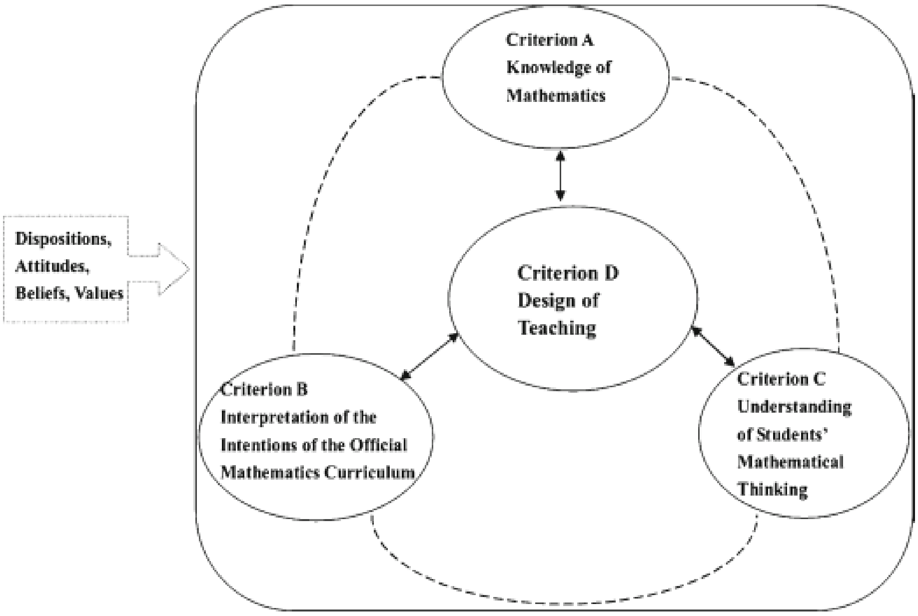


Figure 2. The model of teacher capacity

While this model recognises teachers' own attitudes, beliefs, values, and dispositions to act in the teaching of mathematics as external factors influencing teacher capacity, the current study focused on the four components of teacher capacity for teaching mathematics. Table 1 provides a summary of these four elements of the model and how they were understood, interpreted in terms of multiplicative thinking, and used in this study.

Table 1. Summary of components of the teacher capacity model

Mathematical knowledge	Curriculum knowledge
Mathematical language, multiplicative ideas, the effectiveness of solution strategies, student correct and incorrect responses, variety of teaching/solution strategies, connection to key	Identifying multiplicative thinking in curriculum, knowing where multiplicative thinking begins and aspects of multiplicative thinking suitable for each year level, knowing where multiplicative thinking knowledge is applicable in curriculum, and

topics, knowledge of student future challenges with the use of inefficient strategies.	knowing how multiplicative thinking is connected to other key topics in mathematics.
Knowledge of students	**Design of instruction**
Knowledge of where students "should be," supporting individual students, knowing when to move students forward, knowledge of potential challenges and successes, identifying and correcting errors.	Lesson design, addressing misconceptions, adherence to curriculum standards, variety of resources solution strategies, and choice of teaching.

Questionnaire Development and Validation

A carefully designed questionnaire was used to collect data for this study. In developing the questionnaire items for this study, the four elements of the Teacher Capacity model (Zhang & Stephens, 2013) as discussed informed the structure of questionnaire items according to the mathematical problem presented at each of the three key teaching stages. The items were designed to investigate teachers' PCK in each of the four elements of the Teacher Capacity model that collectively constitute PCK for developing multiplicative thinking in students. The questionnaire had three parts with each part focussing on one of the three key teaching stages (transitional, multiplicative, and proportional reasoning). The questionnaire was reviewed and validated by experts in mathematics education. Experts in science education with special interest on teachers' pedagogical content knowledge for science also participated in the expert validation of the questionnaire. Pilot testing was also conducted involving three primary school teachers before the questionnaire was used in the main study reported here. We note that a similar approach to questionnaire design was employed by Pincheira and Alsinan (2024) to assess teachers' mathematical knowledge for teaching algebra. The questionnaire was useful in revealing areas that required attention in teacher education programs.

Part 1: Transitional Stage

The transitional stage involves a gradual move from reliance on counting of all to emerging multiplicative thinking. Researchers such as Siemon et al. (2011), Ell et al. (2004) and Malola et al. (2020) have pointed out that additive thinking in terms of counting strategies such as counting on, skip counting, counting in groups, and breaking down and building numbers (part–part–whole) form the foundations for developing multiplicative thinking and will appear as children move away from dependence entirely on additive strategies. To understand teachers' basic competencies and skills to support emerging multiplicative thinking in students, the following question suitable for the transitional stage was presented to teachers:

A new theme park has opened in Melbourne, and you and five other friends are going there for your birthday party. Tickets cost $43 each. How much will it cost altogether?

A sample of a student's response referred to as Student C in this study was included in the teacher questionnaire and is shown in Figure 3.

Student C

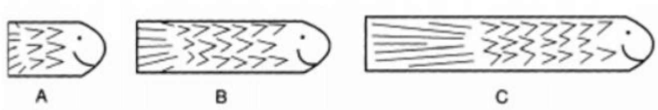
Figure 3. Solution strategy used by student C

This question was presented and discussed at a webinar titled Australian Teachers Using Japanese Lesson Study: A Structured Problem-Solving Lesson on Multiplicative Thinking organised by the Australian Association of Mathematics Teachers in 2020. It encompasses the key features of multiplicative thinking at the transitional teaching stage and adequately provided understanding of teachers' PCK for supporting students at this stage of developing multiplicative thinking. Seven questionnaire items were developed from this question addressing all four components of the Teacher Capacity model (Zhang & Stephens, 2013): mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction. Examples of questionnaire items that were developed include: Can you identify other additive strategies for solving the above problem which are more efficient than used by student C? What are the present and future challenges that students are likely to encounter, if they continue to rely on additive strategies to solve this problem? How will you support students moving forward from relying on additive strategies to using more efficient (multiplicative) strategies? Please provide as many as possible.

Questionnaire Part 2. Multiplicative Stage

Multiplicative stage involves students solving word problems involving multiplication and division without relying on additive strategies (Malola et al., 2021). This stage is a core feature of multiplicative thinking, where students use multiplicative actions without relying on additive ideas. The Piagetian Fish Task adapted from Clark and Kamii (1996) in Figure 4 was used to explore teachers' PCK for supporting students through this stage of developing multiplicative thinking.

The following problem was presented to Year 4/5 students.



Fish A measures 5cm, Fish B is 10cm, and Fish C is 15 cm, and that the length corresponds to the amount of food consumed by each fish. Given that fish C consumes 9 grams of pellets and students are asked to find how many pellets will be required to feed fish A and B respectively.

Figure 4. The fish task

Four questionnaire items were developed from this question focusing on teachers' PCK for developing students' multiplicative thinking at the multiplicative teaching stage addressing the four elements of the teacher capacity model: knowledge of mathematics, knowledge of curriculum, knowledge of students' thinking, and design of instruction. Examples of questionnaire items that were developed and used include: How would you expect a student in Year 5 to solve this problem? What challenges are students likely to experience and how would you support them? What content in the curriculum relates to this problem?

Questionnaire Part 3. Proportional Reasoning Stage

Researchers such as Askew (2018), Harel and Confrey (1994), as well as Siemon et al. (2011) emphasise that teaching multiplicative thinking with a focus on proportional reasoning empowers students to engage successfully with more sophisticated problems. The question in Figure 5 was adapted from Siemon et al. (2011) to explore teachers' PCK for supporting students at this stage of developing multiplicative thinking.

The following problem is suitable for Year 6 students: Coffee is available in three sizes. *Which size represents the best buy if 300-gram jar costs $13.99, then 150-gram jar costs $8.55 and the 50-gram jar costs $2.85?*

Figure 5. Coffee jar

Two questionnaire items from this question addressing two components of the teacher capacity model: mathematical knowledge and knowledge of students' thinking, with emphasis on mathematical knowledge. These questionnaire items are: How could you solve this problem? Suggest one or more strategies. some students in Year 6 may find this a difficult question. Please identify one or more difficulties. How could you help them?

Participants and Data Collection

This study sought participation from Australian primary school teachers at different levels of teaching experience. Grouping teachers based on teaching experience resulted in four categories of teachers: expert teachers (8+ years of teaching experience), experienced teachers (4–7 years of teaching experience), novice teachers (1–3 years of teaching experience), and preservice teachers (those studying towards a teaching qualification). For this specific study, the preservice teacher participants were Master of Teaching (MTeach) students from one institution of higher learning in the Australian state of Victoria. These students already held a first degree in other fields of study at the time they enrolled for the MTeach degree.

Data collection was conducted using an online questionnaire through professional teacher networks. The online questionnaire was developed using a survey software tool called Qualtrics (https://www.qualtrics.com/). Initially, the study targeted an equal number of participants per category of teachers (25 teachers in each category). However, due to several COVID19-related limitations encountered in the conduct of the study, equal sample sizes across teacher categories were not achieved.

Table 2 shows that 33 expert teachers participated in the study, of which 25 completed the whole questionnaire, and eight completed Parts 1 and 2 comprising biographical information and questions related to the transitional teaching stage of developing multiplicative thinking.

Table 2. Distribution of participants according to teaching experience

Teacher category	Complete	Incomplete	Total
Expert	25	8	33
Experienced	8	5	13
Novice	5	0	5
Preservice	8	3	11
Total	**46**	**16**	**62**

Table 2 further shows that 13 experienced teachers participated in the study. Of this number, eight completed the whole questionnaire and five completed Parts 1 and 2. It also shows that all five novice teachers who participated in the study completed the whole questionnaire. Eleven preservice teachers participated in the study, and eight of these completed all sections of the questionnaire while three completed Parts 1 and 2. These numbers bring the total number of participants in the study to 62, comprising of 16 incomplete responses and 46 complete responses.

Data Processing and Analysis

The data analysis was conducted in two phases. Phase 1 focused on individual teaching stages: transitional teaching stage (n = 62), multiplicative stage (n = 46), and proportional reasoning stage (n = 46). Phase 2 focused on teachers' performance across the four elements of the Teacher Capacity model (n = 46). To explore the different teacher groups' (preservice, novice, experienced, and expert) PCK for multiplicative thinking, estimated marginal means were calculated and pairwise comparison analysis was conducted at each key teaching stage. Estimated marginal means (EMMs) and the pairwise comparison of the EMMs of teacher scores at the transitional teaching stage of developing multiplicative thinking in students were calculated using SPSS. The EMMs give an estimate of the means based on a statistical model other than the observed data as in the case of the descriptive mean (van Dooren, 2020). It gives a predicted mean of the dependent variable (in this case, teachers' scores at the transitional teaching stage) considering any adjustments to the independent variables in the model (Coxe et al., 2009).

A pairwise comparison analysis compares the estimated mean differences in scores for paired groups of independent variables (Hinton et al., 2014). In this study, the independent variables were the various categories of teachers (preservice, novice, experienced, and expert). It also shows the standard error of the estimated mean differences and whether the estimated mean difference is significant or not. In this study, the pairwise comparison was useful to demonstrate any estimated mean differences in scores at the transitional teaching stage between groups of teachers according to teaching experience—for example, the estimated mean difference between experienced and expert teachers or between preservice and experienced teachers.

RESULTS AND DISCUSSION

Estimated Marginal Means and Pairwise Comparison

The quantitative data analysis considered 46 complete cases across all three key teaching stages, while the qualitative data analysis at the transitional key teaching stage considered 62 cases, with 46 complete and 16 incomplete cases. The reader is reminded of the practical limitation of this study in obtaining equal sample sizes across the four categories of teachers despite the concerted efforts to achieve equal sample sizes due to challenges associated with COVID-19.

Estimated Marginal Means at Transitional Teaching Stage

The results in Table 3 show that expert teachers had the highest EMM of 17.1 with a standard error of 0.57, and upper and lower bounds of 17.8 and 13.7, respectively, at a 95% confidence interval. The transitional teaching stage part of the questionnaire had a total score of 22 points.

Table 3. Estimated marginal means at the transitional teaching stage according to the teacher capacity model

Experience	Mean	Standard Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Preservice	15.8	1.0	13.7	17.8
Novice	14.4	1.3	11.8	16.9
Experienced	16.1	1.0	14.1	18.2
Expert	17.1	.57	16.0	18.3

This means that if the study is duplicated with the same sample size of expert teachers, we are confident that 95% of them will score between 13.7 and 17.8 of 22 points with a standard error of 0.57. Hinton et al. (2014) stated that a small value of standard error indicates less variability in the predicted means, and a larger value of standard error indicates higher variability in the predicted means, if the study is duplicated. For the expert teachers, a standard error of 0.57 is small, and this means there will be less variability in the EMMs if the study is repeated with a distinct set of participants.

Table 3 further shows that experienced teachers had the second-highest EMM of 16.1 with a standard error of 1.0 and lower and upper bounds of 14.1 and 18.2, respectively, at a 95% confidence interval. Preservice teachers had an EMM of 15.8 with a standard error of 1.0, and 16.0 and 18.2 lower and upper bounds, respectively, at a 95% confidence interval. The novice teachers scored the lowest with an EMM of 14.4 with a standard error of 1.3, and 11.8 and 17.0 lower and upper bounds, respectively, at a 95% confidence interval.

Pairwise Comparisons of Estimated Marginal Means at the Transitional Stage

A pairwise analysis was conducted to establish any estimated mean differences in scores between groups of teachers at the transitional teaching stage for developing multiplicative thinking in students. Table 4 shows that there were significant differences ($p > 0.05$) in EMMs between Experienced and Preservice teachers and between Expert and Novice teachers.

Table 4. Pairwise comparison of teacher scores at the transitional teaching stage

Mean Difference			Standard Error	Sig.[a]	95% Confidence Interval for a. ...
(I) Experience	(J) Experience	(I–J)			Lower Bound
Preservice	Novice	1.350	1.623	.410	−1.924
	Experienced	−.375	1.423	.793	−3.247
	Expert	−1.370	1.156	.243	−3.703
Novice	Preservice	−1.350	1.623	.410	−4.624
	Experienced	−1.725	1.623	.294	−4.999
	Expert	−2.720	1.394	.058	−5.534
Experienced	Preservice	.375	1.423	.793	−2.497
	Novice	1.725	1.623	.294	−1.549
	Expert	−.995	1.156	.394	−3.328
Expert	Preservice	1.370	1.156	.243	−.963
	Novice	2.720	1.394	.058	−.094
	Experienced	.995	1.156	.394	−1.338

* The mean difference is significant at the 0.05 level.

Further to the significant differences in the EMMs between groups of teachers, Table 4 allows for groups of teachers to be ordered based on the EMMs from the highest to lowest. The expert teachers scored the highest, followed by experienced teachers, then preservice, then novice teachers who scored the lowest.

Estimated Marginal Means at the Multiplicative Stage

The EMMs of scores for different groups of teachers based on their teaching experience are illustrated in Table 5. The questionnaire had a total score of 11 points at the multiplicative stage.

Table 5. Estimated marginal means at the multiplicative teaching stage

Experience	Mean	Standard Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Preservice	10.1	.544	9.027	11.223
Novice	8.2	.688	6.811	9.589
Experienced	5.8	.544	4.652	6.848
Expert	7.5	.308	6.899	8.141

Table 5 shows that at the multiplicative teaching stage, the preservice teachers had the highest EMM of 10.1, with a standard error of 0.544. This means that if the study was to be repeated with the same number of preservice teacher participants who shared the same characteristics as those who participated in this study, it is highly likely that the EMM will be the same. The novice teachers were second highest with an EMM of 8.2 and a standard error of 0.688. The expert teachers followed with an EMM of 7.5 and a standard error of 0.308. Experienced teachers had the lowest EMM of 5.8, with a standard error of 0.544. A pairwise comparison analysis was conducted to establish any differences in the EMM scores between the groups of teachers at the multiplicative stage of developing multiplicative thinking in students.

Pairwise Comparisons of Estimated Marginal Means at the Multiplicative Stage

Table 6 shows that the mean difference of the EMMs was significant between the preservice teachers and novice teachers ($p = 0.034$). However, there was a strong difference in the EMM between preservice teachers and expert teachers, and between preservice teachers and experienced teachers ($p < .001$). This table further shows that the mean difference was significant between the experienced and expert teachers ($p = 0.007$), and between the novice and experienced teachers ($p = 0.008$). These significant results are circled only once in Table 6.

Table 6. Pairwise comparison of teacher scores at the multiplicative teaching stage

Mean difference			Standard error	Sig.[b]	95% confidence interval for b. …
(I) Experience	(J) Experience	(I–J)			Lower Bound
Preservice	Novice	1.925*	.877	.034	.155
	Experienced	4.375*	.769	< .001	2.823
	Expert	2.605*	.625	< .001	1.344
Novice	Preservice	−1.925*	.877	.034	−3.695
	Experienced	2.450*	.877	.008	.680
	Expert	.680	.754	.372	−.841
Experienced	Preservice	−4.375*	.769	< .001	−5.927
	Novice	−2.450*	.877	.008	−4.220
	Expert	−1.770*	.625	.007	−3.031
Expert	Preservice	−2.605*	.625	< .001	−3.866
	Novice	−.680	.754	.372	−2.201
	Experienced	1.770*	.625	.007	.509

* The mean difference is significant at the 0.05 level.

These results interpreted simultaneously with the results in Table 6 mean that, at the multiplicative stage of developing multiplicative thinking in students, the preservice and novice teachers performed better than the expert and experienced teachers despite the significant mean difference in scores between the expert and experienced teachers. It is important to state that the questionnaire items at the multiplicative stage emphasised more the knowledge of students' thinking than the other three components of the teacher capacity model.

Estimated Marginal Means at the Proportional Reasoning Stage

The EMMs of teachers' scores at the proportional reasoning stage were calculated out of 8 total points. Table 7 shows that the preservice teachers had the highest EMM score of 6.1 with a standard error of 0.436. Novice teachers were the second high-scoring group with an EMM of 5.8 and a standard error of 0.552. The expert teachers' group was second from the low-scoring group with an EMM of 5.2 and a standard error of 0.247. Of the four groups of teachers, the experienced teachers' group had the lowest EMM of 4.9 with a standard error of 0.436. The standard errors of EMMs for each group of teachers were nearly 0.5 or less. This means that it is highly likely to obtain the same EMM scores at the proportional reasoning stage if the study were to be replicated using the same sample sizes. It is evident from the results that the preservice and novice teachers performed better than the experienced and expert teachers.

Table 7. Estimated marginal means at the proportional reasoning teaching stage

Experience	Estimated Marginal Means	Standard Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Preservice	6.1	.436	5.245	7.005
Novice	5.8	.552	4.687	6.913
Experienced	4.9	.436	3.995	5.755
Expert	5.2	.247	4.662	5.658

It is important to highlight that the questionnaire items related to the proportional reasoning teaching stage gave more emphasis to teachers' own knowledge of mathematics than the other three components of the teacher capacity model (curriculum knowledge, knowledge of students' thinking, and design of instruction). To establish whether the differences in the EMMs between groups of teachers were significant or not, a pairwise comparison analysis was conducted.

Pairwise Comparisons of Estimated Marginal Means at the Proportional Reasoning Stage

Table 8 shows that the mean difference of the EMMs was significant only between the preservice teachers and experienced teachers ($p = 0.049$). This means that at the proportional reasoning stage, preservice teachers performed much better than experienced teachers. However, the mean difference of the EMMs was not significant between the rest of the groups of teachers ($p > 0.05$).

Table 8. Pairwise comparison of teacher scores at the proportional reasoning stage

Mean Difference			Standard Error	Sig.[b]	95% Confidence Interval for b. ...
(I) Experience	(J) Experience	(I–J)			Lower Bound
Preservice	Novice	.325	.703	.646	−1.094
	Experienced	1.250*	.617	.049	.005
	Expert	.965	.501	.061	−.046
Novice	Preservice	-.325	.703	.646	−1.744
	Experienced	.925	.703	.196	−.494
	Expert	.640	.604	.296	−.580
Experienced	Preservice	−1.250*	.617	.049	−2.495
	Novice	−.925	.703	.196	−2.344
	Expert	−.285	.501	.573	−1.296
Expert	Preservice	−.965	.501	.061	−1.976
	Novice	−.640	.604	.296	−1.860
	Experienced	.285	.501	.573	−.726

* The mean difference is significant at the 0.05 level.

Summary of Teacher Performance at Each Teaching Stage

This section summarises the EMM scores of individual groups of teachers (preservice, novice, experienced, and expert) at each key teaching stage (transitional, multiplicative, and proportional reasoning) for developing multiplicative thinking in students. The EMM scores are presented in Figure 6.

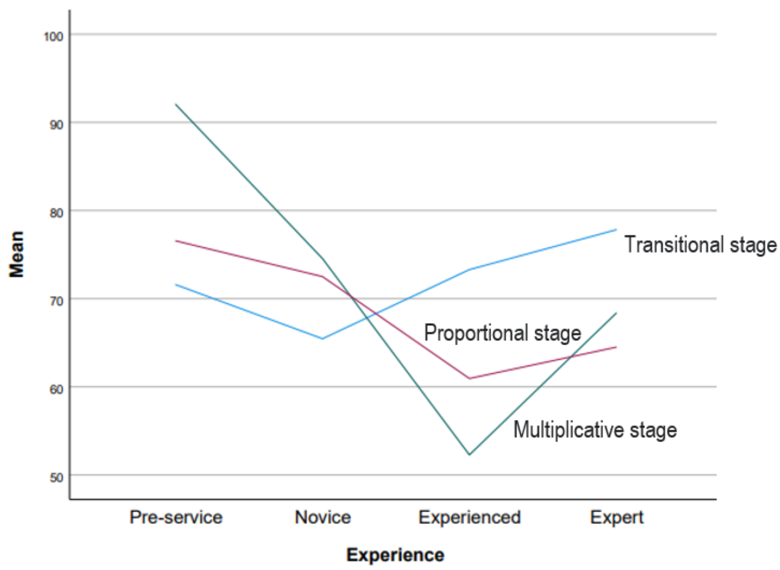


Figure 6. Summary of the estimated marginal means

Figure 6 shows that at the transitional stage, expert teachers had the highest EMM score of 78%. This group was followed by experienced teachers with an EMM of 74%. Preservice teachers came third with an EMM of 71%. Novice teachers had the lowest EMM (66%) compared to the other three groups of teachers. Looking at the figure and with a focus on the in-service teachers (novice, experienced, and expert) only, we can see that at the transitional teaching stage, teachers' PCK of multiplicative thinking increased with experience. This may suggest that expert and experienced teachers are better able to support students' transition from additive to multiplicative thinking than are novice and preservice teachers. The questionnaire items at the transitional teaching stage emphasised the knowledge of curriculum and design of instruction. The results may also suggest that the expert and experienced teachers have a better knowledge of curriculum and design of instruction to support student transition.

Figure 6 shows that at the multiplicative and proportional reasoning stages, the preservice and novice teachers on average scored higher than the expert and experienced teachers. It should be stated that the questionnaire items at the multiplicative stage emphasised the knowledge of students' thinking more than the other three components of the teacher capacity model, while the items related to the proportional reasoning teaching stage emphasised more the knowledge of mathematics. This predictive model of analysis (EMM) and the results suggest that preservice and novice teachers have a higher knowledge of mathematics and of students' thinking than do expert and experienced teachers. This may also mean that preservice and novice teachers are better able to support students' development of multiplicative thinking at the multiplicative and proportional teaching stages.

Mean Scores according to Components of Teacher Capacity Model

Table 9 shows the mean scores on the questionnaire of the four groups of teachers across the four components of the teacher capacity model (Zhang & Stephens, 2013): mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction.

Table 9. Mean scores of teacher groups across the four components of the teacher capacity model

Experience		MK Percentage	CK Percentage	KS Percentage	DI Percentage
Preservice	M	91.3	71.0	76.4	74.1
	N	8	8	8	8
	SD	8.3	21.4	9.5	7.5
Novice	M	92.0	57.8	61.8	65.6
	N	5	5	5	5
	SD	4.5	20.2	27.8	22.0
Experienced	M	81.3	76.6	46.5	66.0
	N	8	8	8	8
	SD	17.3	9.2	12.7	20.0
Expert	M	86.4	80.6	59.5	66.7
	N	25	25	25	25
	SD	13.2	12.8	13.8	19.2
Total	M	87.0	75.8	60.4	67.7
	N	46	46	46	46
	SD	12.8	16.2	17.0	17.9

Note, MK=mathematical knowledge; CK= curriculum knowledge; SK= knowledge of students' thinking; DI = design of instruction; *M* = mean score; *N* = number of participants; *SD* = standard deviation.

Table 9 shows that the mean score on mathematical knowledge for all the teachers (*N* = 46) was 87.0%, with a standard deviation of 12.8. The mean score on curriculum knowledge (*N* = 46) was 75.8% with a standard deviation of 16.2. For knowledge of students' thinking (*N* = 46), the mean score was 60.4% with a standard deviation of 17.0. We can also see from the table that the mean score on design of instruction was 67.7% with a standard deviation of 17.9. It is evident from this table that all teachers performed well on mathematical knowledge, followed by curriculum knowledge, then design of instruction, and last, knowledge of students' thinking. It should further be noted from these results that there was a slight variation in teachers' scores on mathematical knowledge (*SD* = 12.8) than the other three components of the teacher capacity model (*SD* > 16.2). This suggests that there were no demonstrated differences in teachers' mathematical knowledge for multiplicative thinking irrespective of teaching experience.

Regression Analysis

Regression analysis was conducted to investigate which of the four components of the teacher capacity model (mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction) contributed the most to the primary school teachers' PCK for developing multiplicative thinking in students. Mills and Gay (2016) explained that stepwise linear regression typically starts with a model that includes all the independent variables and then removes one at a time the variables that do not contribute as strongly to the dependent variable. This way, we have the variables ranked according to their contribution level to the final score. The results in Table 10 present the output of four regression models conducted to establish the component(s) of the teacher capacity model (Zhang & Stephens, 2013) that contributed most significantly to the final score.

Table 10. Stepwise linear regression analysis showing beta coefficients

Model		Unstandardised Coefficients		Standardised Coefficients	*t*	Sig.[a]
		B	Standard Error	Beta		
1	(Constant)	40.399	4.015		10.062	< .001
	DI percentage	.469	.057	.777	8.176	< .001
2	(Constant)	26.221	3.181		8.242	< .001
	DI percentage	.397	.038	.658	10.338	< .001
	KS Percentage	.315	.040	.497	7.808	< .001
3	(Constant)	15.071	2.706		5.570	< .001
	DI percentage	.328	.028	.544	11.644	< .001
	KS percentage	.309	.028	.488	11.148	< .001
	CK percentage	.213	.030	.319	6.998	< .001
4	(Constant)	–.129	1.154		−.111	.912
	DI percentage	.262	.010	.433	27.490	< .001
	KS percentage	.271	.009	.428	30.296	< .001
	CK percentage	.219	.010	.328	22.777	< .001
	MK percentage	.248	.013	.294	19.540	< .001

Note. DI = design of instruction; KS = knowledge of students' thinking; CK = curriculum knowledge; MK = mathematical knowledge.

[a] Dependent variable: overall score.

Model 4, as shown in Table 10, is the first stage of stepwise linear regression and includes all four independent variables (mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction). We can see that the standardised beta coefficient was lowest for mathematical knowledge (beta = 0.294). The standardised beta coefficient measures the effect size of each independent variable on the dependent variable (Muijs, 2011). The results mean that, for all the independent variables combined, mathematical knowledge contributed the least to variability in teachers' PCK for developing multiplicative thinking in students, and this was eliminated from the model.

The elimination of mathematical knowledge from Model 4 resulted in Model 3, which is the second stage of stepwise linear regression, and comprised curriculum knowledge, knowledge of students' thinking, and design of instruction. We can also see that of these three independent variables, curriculum knowledge had the least (beta = 0.319) contribution to the overall score—that is, teachers' PCK for developing multiplicative thinking in students. The elimination of curriculum knowledge from Model 3 resulted in Model 2, which is the third stage of stepwise linear regression and comprised knowledge of students' thinking and design of instruction. Of these two, knowledge of students' thinking was found to contribute less (beta = 0.497) to variability in teachers' PCK for developing multiplicative thinking in students. The elimination of knowledge of students' thinking from Model 2 resulted in Model 1, the final stage of stepwise linear regression with only design of instruction (beta = 0.777).

These results suggest that design of instruction is the most significant predictor and determinant of teachers' PCK for developing multiplicative thinking in students. This is followed by knowledge of students' thinking, then knowledge of curriculum. The results also reveal that mathematical knowledge is the least powerful predictor or determinant of teachers' PCK for developing multiplicative thinking in students. However, it should be stated that, from the results in this table, the contribution of each element of the teacher capacity model to the overall teachers' PCK for developing multiplicative thinking in students was statistically significant ($p < 0.001$) at a significance level of 0.01.

Textual qualitative analysis was performed to give further understanding of quantitative results. This analysis focused on the depth, quality, and conciseness of responses. As discussed, this study was largely quantitative, and little qualitative results will be provided in this paper. A sample of this analysis in reference to Item 1.2 on the questionnaire is shown in Table 11. Item 1.2 was chosen as a sample because of its uniqueness as it probes teachers' knowledge of the importance of multiplicative thinking to the learning of other topics in mathematics and responses from a few teachers are selected.

Table 11. Summary of teachers' responses to Item 1.2.

Teacher ID		Responses to Item 1.2: What are the present and future challenges that students are likely to encounter if they continue to rely on additive strategies to solve this problem?
Teacher Expert	8	In K-6 additive strategies restrict mental computation, fluency, and flexibility. It restricts thinking in looking at fractions as divisions and decimals and limits abilities in measurement. So many challenges but mostly using inefficient strategies impacts negatively on a student's self-perception as a mathematician.

Teacher ID		Responses to Item 1.2: What are the present and future challenges that students are likely to encounter if they continue to rely on additive strategies to solve this problem?
Teacher Expert	4	When the size of number gets bigger, students will experience work overload. Repeated addition will not work for decimals or fractions such as 1.33 x 2.41.
Teacher Expert	6	Decline of fluency when trying to adapt that strategy to more complex problems. A lack of transference of multiplicative skill to other areas such as calculating area.
Teacher Preservice	55	Inefficient problem solving, inability to work with large numbers, struggle with division.
Teacher Expert	2	Inefficient time management, not using (hopefully) known skip counting/times tables knowledge.
Teacher Preservice	52	Takes longer, as numbers grow bigger, more likely to make mistakes. Not a suitable strategy for multiplying two double-digit sets.
Teacher Experienced	37	Repeated addition when working with tens of hundreds becomes very inefficient.
Teacher Novice	48	Numbers get too big, so they'd have to add too many times.

Table 11 presents a summary of teachers' responses to Item 1.2. This item was chosen as a sample only to demonstrate how qualitative analysis was conducted. These responses are arranged in order from the strongest response at the top to the weakest response at the bottom. On the one hand, we can see that most teachers were concerned about student inefficiency in solving the problem when larger numbers are involved and that this was a concern of all teachers from preservice to expert teachers. On the other hand, while the first four responses demonstrated awareness of the connection between multiplicative thinking and other topics in mathematics, the first two responses are more specific in that they mention the mathematics topics. These two responses came from the two expert teachers and supports the quantitative findings that expert teachers are more capable of supporting students at the transitional stage of developing multiplicative thinking.

Discussion

This study showed that teaching experience is important for effectively teaching some complex concepts in mathematics. The complexity of transitioning students from additive to multiplicative thinking and its pedagogical challenges is well emphasised in mathematics education literature (Clark & Kamii, 1996; Malola et al., 2020; Siemon et al., 2005). In this study, we saw expert and experienced teachers demonstrating high pedagogical competency at the transitional teaching stage (students transitioning from additive to multiplicative thinking). At this teaching stage, teachers support the students in moving forward from heavy reliance on additive strategies such as repeated addition and skip counting to solve multiplicative problems to using more efficient emerging multiplicative strategies such as partitioning, arrays, and area model. In this study, expert and experienced teachers demonstrated higher pedagogical competence in supporting students through this process. Their capacity to effectively support students' transition from additive to multiplicative thinking can be attributed to more professional development opportunities, engagement with peers and curriculum resources, and several opportunities for self-reflection on the lessons taught. This study recommends that expert and experienced teachers be allocated for teaching in the early years' classes (Foundation to Year 3) in primary school. However, it should be stated that some novice and preservice teachers demonstrated higher PCK compared to some experienced and expert teachers.

The teacher capacity model has four components (mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction) that constitute MKT. While all these components were found to be important contributors to teachers' PCK for multiplicative thinking, this study further revealed that design of instruction and knowledge of students' thinking are powerful predictors of teachers' PCK for multiplicative thinking. A similar finding appeared in Zhang and Stephens's (2013) study of teacher capacity. This finding suggests that explanation teachers with limited knowledge about students' challenges with multiplicative thinking may not design effective instruction to support students' development of multiplicative thinking. Teachers need to know their students' conceptions and misconceptions around the concept of multiplicative thinking to effectively support their learning through carefully designed and implemented lessons.

The lower performance of teachers on the design of instruction and knowledge of students' thinking components of the teacher capacity model used in this study suggests the need for teacher professional learning programs to focus on promoting teachers' knowledge of these two components. It has been demonstrated in this study that each of the four components of the teacher capacity model (mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction) is an essential component of teachers' PCK for multiplicative thinking. However, teachers' knowledge of students' thinking, and design of instruction around multiplicative thinking were shown in this study to be limited. Hill and Chin (2018) argue that teachers' knowledge of students' thinking is important to effective instruction. This position corroborates with Malola et al. (2020) who maintain that teachers who are aware of their students are more likely to design and stear instruction towards effective student learning.

PCK for multiplicative thinking was evident in many preservice teachers participating in the study. Some preservice teachers demonstrated higher PCK than some expert, experienced, and novice teachers; these were preservice teachers from one institution emphasising multiplicative thinking in its teacher preparation program. This draws attention to the need for initial teacher preparation programs in all institutions across Australia and beyond to emphasise multiplicative thinking in their teaching. A similar recommendation was made by Pincheira and Alsinan (2024) in their study that assessed preservice teachers' PCK for algebra. We should mention it that the preservice teachers in this tended to display more limited curriculum knowledge and mathematical vocabulary.

CONCLUSION

The importance of multiplicative thinking in supporting students' learning of key topics and success in further mathematics is widely and clearly stated in mathematics education literature. Several studies have been conducted to assess students' performance in multiplicative thinking, and many resources have been developed to support teaching and learning in this area. Empirical evidence points to the continued low performance of students in multiplicative thinking. While teachers' PCK is assumed to be critical in determining students' learning, little explicit attention has been paid by prior studies to teachers' PCK for multiplicative thinking.

This study used the teacher capacity model as the framework for questionnaire design and data collection to explore primary school teachers' PCK for multiplicative thinking. Teachers' PCK for multiplicative thinking was explored across the three key teaching stages: transitional, multiplicative, and proportional reasoning. The results highlight limited knowledge of the connection between multiplicative thinking and other key topics in mathematics among primary school teachers. Understanding these connections is a powerful aspect of multiplicative thinking that supports students' learning and success in further mathematics. The results also indicate high PCK for multiplicative thinking at the transitional teaching stage among most expert and experienced teachers. Furthermore, the results point to a high PCK for multiplicative thinking at the multiplicative and proportional reasoning stages among many novice and preservice teachers.

The results further revealed that all four components of the teacher capacity model (mathematical knowledge, curriculum knowledge, knowledge of students' thinking, and design of instruction) significantly contribute to teachers' PCK for multiplicative thinking. However, the results showed design of instruction to be the most critical determinant of teacher PCK for multiplicative thinking, followed by knowledge of students' thinking, then curriculum knowledge, and last, mathematical knowledge. High curriculum knowledge for multiplicative thinking was more evident among the expert and experienced teachers than among the novice and preservice teachers. There was no significant difference in mathematical knowledge among the four groups of teachers (expert, experienced, novice, and preservice).

While the existing body of literature on multiplicative thinking emphasises assessing students' performance in this area and the development of teaching and learning resources to support both students and teachers, the current research adds new knowledge around multiplicative thinking with an emphasis on teachers' PCK to effectively support students' learning. While existing research and literature on multiplicative thinking underscore students continued low performance without inquiring about teachers' capacity to support students' understanding of multiplicative thinking, the current research opens a new area of focus that will potentially direct research on multiplicative thinking to identify the kind of support teachers require to be empowered to teach multiplicative thinking effectively. The role of teachers' PCK in determining students' learning has been discussed throughout this article.

The limitation of this study is that sample sizes among the four groups of teachers with distinct experience levels were unequal. This was due to the self-selected-sample nature of the study and the COVID-19 conditions discussed in some points above. The difficulty associated with the uneven sample sizes is the generalisation of the study results across and within the four teacher groups (expert, experienced, novice, and expert). However, the quantitative tools and methods used to analyse the data allowed room to control the associated generalisation limitations.

Another limitation was that the qualitative data, while valuable under better circumstances, could have been probed and further elaborated had there been an opportunity to interview teachers. This was impossible because there was an embargo on contacting teachers and schools directly during the COVID-19 pandemic lockdown period during the years in which this study was undertaken (2020–2022).

Further research on teacher PCK for multiplicative thinking should interrogate the connections between teachers' PCK for multiplicative thinking and their students' performance. Further investigation of qualitative factors of teacher PCK for multiplicative thinking, such as the number of workshops attended by teachers, breadth of teaching experience, and the location of schools, is highly recommended. Specifically, the content of the workshops and duration, number of years taught at each year level, and location of schools versus the location of teacher residence would be worthy of further inquiry. The findings also suggest that future research investigates how much multiplicative thinking is emphasised in initial teacher preparation programs across institutions. Future research should explore the link between teacher PCK for mathematics including multiplicative thinking and students' performance in large national studies such as TIMMS and NAPLAN and explore connections between students' multiplicative thinking and their learning in other subjects such as financial literacy and STEM subjects. This study suggests that further research should consider exploring teachers' PCK for multiplicative thinking at an international scale.

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Declarations

Author Contribution : MM: Preparation, creation and/or presentation of the published work, specifically writing the initial draft, including further editing, and revising the article based on reviewers' recommendations.

WTS: Preparation, creation and/or presentation of the published work by those from the original research group, specifically critical review, supervision of research project (PhD), and revising the article based on reviewers' recommendations.

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