Journal on Mathematics Education
Volume 16, No. 1, 2025, pp. 91-110

Numeracy skill development in prospective mathematics teachers: Challenges and opportunities in real-world contexts

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Received: 29 September 2024 | Revised: 12 November 2024 | Accepted: 6 January 2025 | Published Online: 1 March 2025

Abstract

Numeracy skills are essential for prospective mathematics teachers as they bridge mathematical concepts with real-life applications. However, many prospective mathematics teachers face challenges in applying these concepts to practical situations. This study aims to analyze the conceptual and procedural errors made by prospective mathematics teachers when solving numeracy problems within the context of "Save Our Water." A descriptive research design was employed, utilizing a numeracy test adapted from the Minimum Competency Assessment (MCA) and semi-structured interviews as research instruments. The study involved 30 prospective mathematics teachers from the University of Jambi, Indonesia. The findings revealed that conceptual errors primarily stemmed from reliance on rote memorization of formulas without a deeper conceptual understanding. Procedural errors were attributed to difficulties in unit conversion, incorrect formula application, and improper manipulation of formulas. To address these issues, the study recommends incorporating contextual approaches, problem-based learning, and project-based learning strategies that connect mathematical concepts to real-world contexts. Additionally, the use of visual aids, such as diagrams and 3D models, is suggested to enhance conceptual understanding and strengthen the connection between abstract concepts and practical applications. Future research should investigate the effectiveness of these instructional approaches in improving numeracy skills and enhancing the teaching readiness of prospective mathematics teachers.

Keywords: Numeracy Misconceptions and Errors, Prospective Math Teachers, Real-World Contexts

How to Cite: Winarni, S., Kamid, Marzal, J., & Asrial. (2025). Numeracy skill development in prospective mathematics teachers: Challenges and opportunities in real-world contexts. *Journal on Mathematics Education, 16*(1), 91–110. https://doi.org/10.22342/jme.v16i1.pp91-110

Numeracy skills are critical for prospective mathematics teachers, as they involve both the comprehension of mathematical concepts and their application to real-world scenarios. These skills encompass the ability to reason, analyze data, and make informed decisions based on quantitative information (Bjälkebring & Peters, 2021; Dole & Geiger, 2018; Faragher, 2023; Tout, 2020). Prospective mathematics teachers with strong numeracy skills are better positioned to develop effective instructional methods that bridge mathematical concepts with their practical applications in everyday life (Groves, 2013; Haigh, 2016). Such competencies are fundamental for helping students appreciate the relevance of mathematics in their daily experiences (Haigh, 2016), thereby enabling prospective mathematics teachers to create teaching strategies that emphasize the direct benefits of mathematics (Geiger et al., 2015). These competencies also allow prospective teachers to connect mathematical theories to practical contexts, which, in turn, enhances students' understanding of the real-life applications of mathematics

(Maralova, 2024). However, many prospective mathematics teachers continue to face challenges in translating theoretical knowledge into practical applications, particularly when confronted with complex real-world problems (Pourdavood et al., 2020; Toklu & Hursen, 2021; Yesseikyzy et al., 2022).

Despite possessing a strong foundational knowledge of mathematics, research indicates that many prospective mathematics teachers encounter difficulties in linking mathematical concepts to real-life contexts (Choi & Park, 2022; Sellings et al., 2018). These prospective mathematics teachers often report that their prior education in mathematics focused primarily on procedural learning and memorization rather than the application of concepts to real-world situations (Sa, 2020; Sellings et al., 2018). Furthermore, they struggle with developing contextually relevant, problem-based teaching materials due to insufficient training on the practical application of numeracy in everyday life (Toklu & Hursen, 2021). This gap frequently undermines their confidence in presenting real-world examples that can help students make connections between mathematics and its practical use (Groves, 2013; Maralova, 2024).

This study examines the conceptual and procedural errors made by prospective mathematics teachers when solving numeracy problems themed "Save Our Water." Conceptual errors arise when participants fail to fully grasp the underlying mathematical concepts, such as associating volume with the appropriate units. In contrast, procedural errors occur when participants incorrectly execute steps in solving the problem, such as unit conversions or the application of formulas. Although the discussion of such errors in the context of environmental themes is scarce in the literature, analyzing errors within the "Save Our Water" context is highly relevant given contemporary environmental challenges. By linking mathematics to real-world issues like efficient water usage, this study aims to enhance mathematical literacy in a contextual setting, thereby helping prospective teachers recognize the practical application of mathematical concepts in everyday life.

Both conceptual and procedural errors are common in mathematics education. According to Skemp (1976), conceptual errors occur when students rely solely on memorization of procedures or formulas without understanding the foundational concepts. This lack of understanding leads to an inability to determine when and why a specific concept should be applied, as exemplified by misconceptions such as incorrect use of units or inappropriate conceptual connections. Conversely, procedural errors arise when students understand the basic concepts but make mistakes in the execution of steps, often due to faulty algorithms or erroneous rules derived from prior experiences (Brown & Burton, 1978). Research by Star and Rittle-Johnson (2008) demonstrates that students who depend on rote memorization of formulas without understanding their contextual application are often confused when faced with problems that require modifications or adaptations of these formulas. This impedes students' ability to solve more complex or context-driven mathematical problems, emphasizing the necessity for a profound understanding and the flexibility to apply concepts effectively.

Research on numeracy skills within real-life contexts, such as the "Save Our Water" theme, remains limited. However, several studies have demonstrated that embedding real-world contexts in learning can enhance conceptual understanding and enable students to bridge theoretical knowledge with practical applications (Boaler, 2016; Hamidi et al., 2022). This approach, which integrates real-life situations into mathematical learning, helps students recognize the relevance of mathematical concepts in everyday life and strengthens their numeracy skills. Furthermore, several studies have shown that the incorporation of real-world contexts in mathematics education not only boosts numeracy skills but also increases student motivation (Akperov, 2023; Hasanah & Retnawati, 2022). This is due to students' ability to immediately observe the practical benefits of what they are learning. Thus, developing learning materials that integrate real-life contexts is crucial for enhancing students' mathematical understanding, especially in addressing real-world challenges.

Previous research indicates that many students continue to face difficulties in understanding and applying mathematical concepts, particularly within contextual problems. This is consistent with findings by Nelson and Powell (2017) and also Samosir et al. (2024), who identified that students often struggle due to their unfamiliarity with connecting mathematical theory to real-life situations. Students who are accustomed to mechanistic or procedural problem-solving approaches tend to encounter challenges when adapting their knowledge to more complex and applicable contexts, which require deeper understanding and flexible problem-solving skills (Star et al., 2022; Ye et al., 2023).

Additionally, previous studies have identified various conceptual and procedural errors in mathematics education. Roza and Maimunah (2023) categorized errors in integer operations and highlighted the need for improved teaching methods to enhance students' conceptual understanding. Baharuddin et al. (2021) observed procedural errors in slope calculations and conceptual misunderstandings related to slope representation. Law et al. (2015) noted that students often memorize steps without fully understanding the underlying concepts, particularly in trigonometry. Wiest and Amankonah (2019) and also Sperafico et al. (2013) stressed the importance of strong conceptual understanding to avoid procedural errors and misapplications in mathematics. Mathaba et al. (2024) analyzed students' errors in algebra learning, categorizing them based on cognitive levels. While much of the existing research focuses on procedural errors, such as mistakes in the sequence of steps during problem-solving, fewer studies have examined the conceptual foundation necessary for a thorough understanding of mathematics. Moreover, research on error analysis within the context of numeracy related to issues such as "Save Our Water" is still sparse. Exploring this context could enhance students' ability to apply mathematical concepts to real-world problems and improve their numeracy skills, particularly in solving problems related to critical natural resources.

Therefore, analyzing conceptual and procedural errors in real-life contexts like "Save Our Water" is of significant importance, as such errors can obstruct the understanding and application of mathematical concepts. This study seeks to address this gap by examining the errors made by prospective mathematics teachers when solving numeracy problems related to environmental issues. Through this error analysis, the study will identify and explore the conceptual and procedural errors made by prospective mathematics teachers in numeracy problems related to the "Save Our Water" issue. Additionally, the study will propose pedagogical strategies aimed at helping these teachers overcome these errors and enhance their numeracy skills.

METHODS

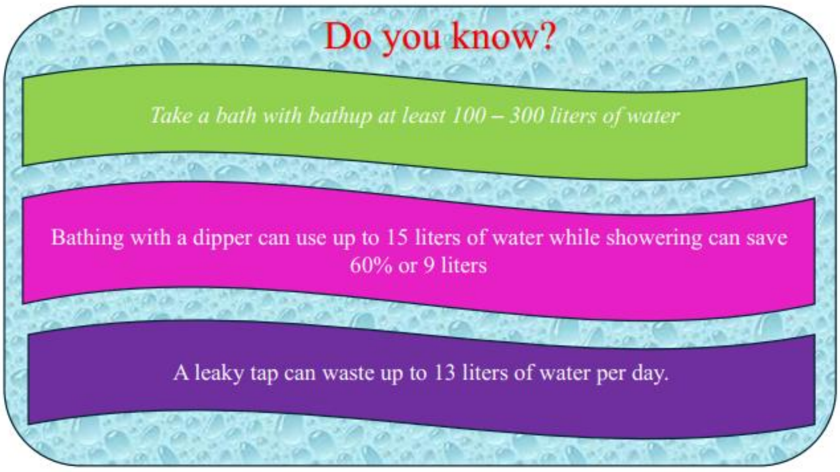
This study employed a qualitative descriptive design to provide a detailed account of the phenomenon under investigation (Moser & Korstjens, 2018) and to develop a comprehensive understanding of the participants' experiences (Bradshaw et al., 2017). The design was selected due to its suitability for analyzing the conceptual and procedural errors of prospective mathematics teachers in solving problems set within real-world contexts. The results of this study aim to inform the development of instructional strategies that can enhance prospective mathematics teachers’ abilities to apply mathematical concepts to practical situations.

Participants

The participants in this study consisted of 30 prospective mathematics teachers from the University of Jambi, Indonesia, who were enrolled in their fifth semester. The participants were selected through purposive sampling to ensure they possessed a relevant background in mathematics education, aligning with the objectives of the study. Furthermore, the participants were preparing to engage in the "Introduction to School Field" program, a field practice component of the Bachelor of Education program, designed to provide practical school-based experience outside of higher education settings.

Instrument

The numeracy test used in this study was adapted from the Minimum Competency Assessment items and designed to assess numeracy indicators such as the ability to analyze data and information, draw conclusions, and interpret more complex contexts (Susanto et al., 2021). The test focused on the theme "Save Our Water" and included questions related to water usage for activities such as bathing with a bathtub, dipper, and shower, as well as the consequences of water wastage from leaking faucets. The test was validated by two mathematics education lecturers to ensure content and construct validity, confirming that each question accurately incorporated mathematical reasoning and problem-solving processes applicable to real-world contexts. The numeracy test instrument, centered on the "Save Our Water" context, is depicted in Figure 1.



SAVE OUR WATER

Excessive water use in household activities is one factor that can cause a clean water crisis in Indonesia in 2025. Each person needs 60 liters of water for bathing and washing every day. The size of the dipper used for bathing is generally a tube with a diameter of 12 cm and a height of 13.5 cm.

1. The volume of water in a filled ladle is... liter. (phi = 3.14)
2. Based on the information above, is the following statement true or false by providing reasons?
   a. The water wasted by a leaking tap is equal to the volume of a dipper every hour.
   b. The amount of water used for bathing in a bathtub is equivalent to the amount used for bathing by 20 people using a dipper.
3. Fikri has a cylindrical bucket filled with water, with a diameter of 35 cm and a height of 46 cm. Based on the information on water usage when bathing using a dipper, the water in Fikri's bucket can be used for several baths.
4. Mr. Umar has a bathtub that is a block with a width of 50 cm, a base diagonal of 78.10 cm, and a height of 60 cm. Based on the information on water usage when bathing using a shower, the water in Mr. Umar's bathtub can be used by several people to bathe.

Figure 1. Numeracy test questions related to the context of saving our water

Semi-structured interviews were conducted to explore the conceptual and procedural errors made by prospective mathematics teachers during problem-solving, to gain insights into their strategies, and to better understand their thought processes in addressing numeracy problems.

The analysis of prospective mathematics teachers' conceptual and procedural errors in solving the "Save Our Water" numeracy problem was informed by theoretical frameworks on types of errors and difficulties. Table 1 outlines the key error types.

Table 1. Indicators of error types

Error type	Definition	Indicator
Conceptual	Conceptual errors arise when students follow procedures without understanding the underlying concepts, failing to recognize the relationships between mathematical ideas. Skemp (1976) refers to this as "instrumental understanding," where students focus solely on procedures without a deep comprehension of the interconnections among concepts.	• Students misunderstand basic mathematical concepts • Students are unable to connect related concepts correctly. • Students use incorrect procedures due to inadequate understanding of the concept.
Procedural	Procedural errors occur when students incorrectly apply steps or rules to solve a problem. Brown and Burton (1978) refer to these as "buggy algorithms," where students follow incorrect procedures due to faulty rule creation.	• Students start the problem with the wrong steps or without following the procedure. • Students do not follow the correct sequence of steps in solving problems. • Students incorrectly apply the rules or formulas that correspond to the questions. • Students make mistakes when calculating or manipulating numbers in the steps provided.

Data Collection

This research was conducted in two distinct phases, as illustrated in Figure 2.

Data Analysis

Data analysis in this study followed the interactive model outlined by Miles and Huberman (1994), focusing on the conceptual and procedural errors encountered by prospective mathematics teachers while solving problems in the context of saving water. The analysis process was organized into three stages:

1. Data Condensation: The research team organized and synthesized data from both the test results and interviews. During this phase, the team identified specific conceptual and procedural errors made by each participant.
2. Data Display: The data were presented in tabular and graphical formats to provide a clear overview of the types of errors made by participants and the problem-solving strategies they successfully employed.
3. Conclusion Drawing and Verification: The researcher drew conclusions based on the errors made by prospective mathematics teachers in solving numeracy problems within the "Save Our Water" context. These conclusions were verified through a thorough review of the test results and interviews.

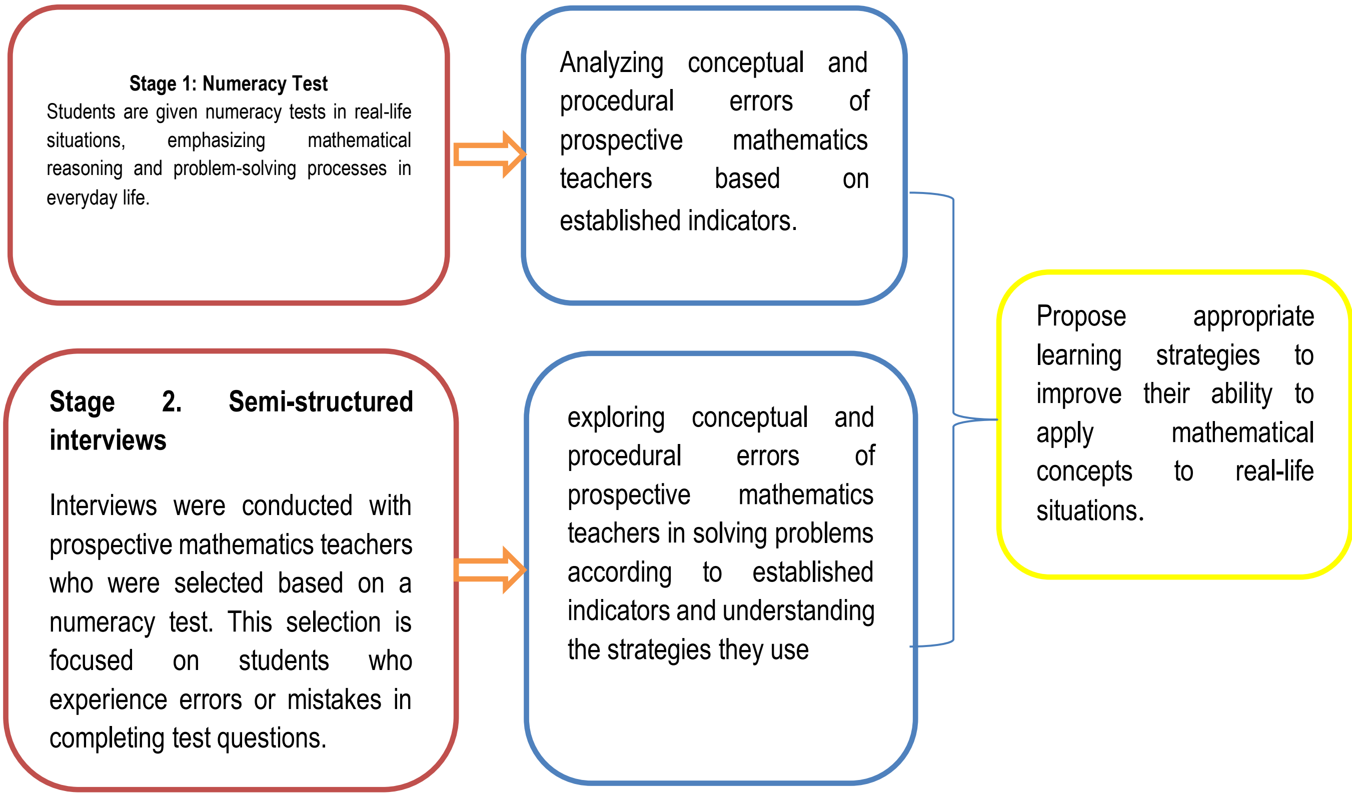


Figure 2. Data collection

RESULTS AND DISCUSSION

This study primarily aims to analyze the conceptual and procedural errors made by prospective mathematics teachers when solving numeracy problems within the "Save Our Water" context. In the initial phase, the prospective teachers were administered a numeracy test, the results of which were used to identify the types of errors encountered. Subsequently, interviews were conducted to further investigate the challenges faced by the participants in problem-solving, to understand the strategies employed, and to gain insight into their cognitive processes while tackling numeracy problems within the "Save Our Water" context.

The findings from both the test results and observations made during the test indicate that nearly all prospective mathematics teachers were able to comprehend the problems presented. However, they required approximately 8 to 10 minutes to fully understand the problem. Despite this, some participants continued to make errors during the problem-solving process. The subsequent sections provide a detailed description of the types of errors observed in each specific question.

Results of the Prospective Mathematics Teacher Test for Solving Question No. 1

The test results revealed that 76% of prospective mathematics teachers were able to accurately and correctly calculate the volume of the dipper. However, 34% of participants made errors in solving this particular problem. The types of errors varied among the participants, particularly in Question No. 1. Table 2 presents a detailed breakdown of the errors and difficulties encountered by the prospective mathematics teachers when addressing this problem.

Table 2. Errors and difficulties encountered by prospective mathematics teachers in solving problem no. 1

No	Error Description and Error Type
1	ALE's Error in Unit Conversion ALE initially calculated the volume of a cylinder by converting the units from centimeters (cm) to meters (m) and then applied the formula for volume. The final result was expressed in liters. Figure 3 illustrates the error encountered during this process. ALE correctly used the volume formula but erroneously assumed that 1 m³ was equivalent to 1 liter, whereas 1 dm³ equals 1 liter. The interview revealed that ALE misunderstood the conversion, mistakenly equating 1 m³ to 1 liter due to a lapse in recalling the proper unit conversion. This error is categorized as both a conceptual error and a conceptual difficulty. Conceptual errors arise when ALE fails to connect related mathematical concepts, and conceptual difficulties occur due to a lapse in recalling the proper relationships between these concepts.

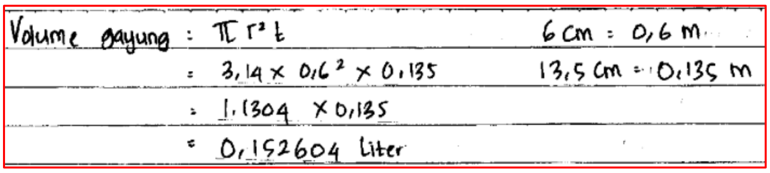


Figure 3. Type 1 error for solving problem no. 1

No	Error Description and Error Type
2	APR's Error in Converting Units APR was provided with the diameter (d = 12 cm) and applied the correct volume formula. However, APR made errors during the unit conversions from cm³ to dm³ and subsequently from dm³ to liters. Figure 4 presents an example of this error. APR's mistake arose from misunderstanding that the given value was the diameter, not the radius. Additionally, APR incorrectly performed the unit conversions, erroneously dividing by 1000 when converting from cm³ to dm³ and again from dm³ to liters. During the interview, APR admitted to confusing the diameter with the radius and also misunderstanding the conversion process. These errors are categorized as procedural errors (incorrect unit conversion) and conceptual errors (misunderstanding of the underlying concepts). Additionally, APR exhibited conceptual difficulties related to the relationships between volume units and procedural difficulties in performing the conversion process, despite understanding the formula and steps involved.

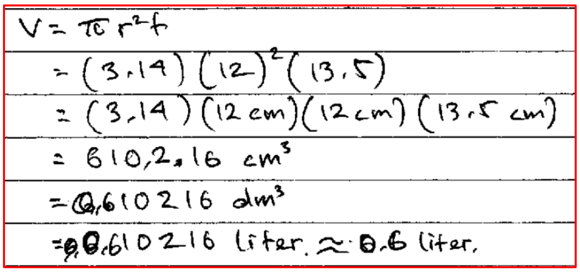


Figure 4. Type 2 error for solving problem no.1

No	Error Description and Error Type
3	DKA, ULH, ANC, and JEP's Error in Formula Application DKA, ULH, ANC, and JEP used the incorrect volume formula, V = πrt, and then attempted to convert the calculated results into liters. Figure 5 displays an example of this error. Although their final calculations were accurate according to the formula they used, they incorrectly applied cm³ as the unit of volume, when it should have been cm² according to their formula. They correctly converted from cm³ to liters. In the interview, they acknowledged their incorrect use of the volume formula and explained that they had relied on their memory of the volume unit being cm³, without considering the context of the formula's application.

No	Error Description and Error Type
	This represents both procedural errors (incorrect formula application) and conceptual errors (failure to understand the conceptual basis of the formula). These errors highlight a tendency to memorize formulas without a deep understanding of the underlying principles.

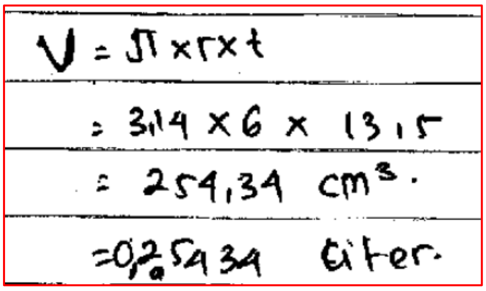


Figure 5. Type 3 error for solving problem no.1

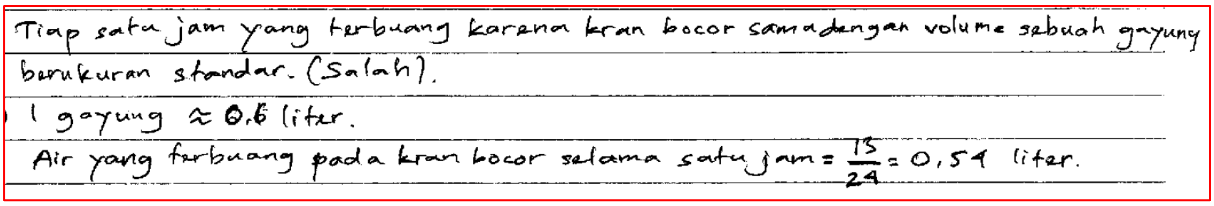
This study revealed that the majority of prospective mathematics teachers made various errors while solving numeracy problems related to the calculation of the volume of a dipper. These errors can be classified into two primary categories: conceptual and procedural errors, which are often interrelated and occur concurrently. The findings indicated that some prospective teachers, such as ALE, committed conceptual errors by assuming that 1 m^3 is equivalent to 1 liter, when, in fact, 1 dm^3 is equal to 1 liter. This reflects a common misconception regarding the metric system of units. In addition, procedural errors were observed in the performance of prospective teachers like APR, who made mistakes in unit conversions, specifically from cm^3 to dm^3 and from dm^3 to liters. These errors suggest that the participants tended to rely on memorization of formulas, rather than accurately understanding the context of unit conversion.

This finding aligns with the results of studies by Merenluoto and Lehtinen (2004) and also Niss and Højgaard (2019), which show that students frequently encounter difficulties in connecting basic mathematical concepts, particularly in the areas of volume and unit conversion. Furthermore, Merenluoto and Lehtinen (2004) emphasized that these misconceptions often stem from an instructional focus that prioritizes procedural memorization over in-depth conceptual understanding. Additionally, research by Star and Rittle-Johnson (2008) found that students accustomed to rigid procedural methods without flexibility often make errors when confronted with problems that require the adaptation of formulas.

Some participants, such as DKA, ULH, ANC, and JEP, encountered difficulties in applying the correct volume formula. Although they correctly recalled that the unit of volume is cubic (cm^3), they failed to recognize that the calculation results should have been expressed in area units (cm^2) according to the formula employed. Consequently, their calculations and conclusions were incorrect. These findings are consistent with the observations of Nelson and Powell (2017) and also Nuraini et al. (2018), who noted that calculation errors are frequently caused by misunderstandings of fundamental concepts. A weak conceptual foundation can hinder procedural fluency and lead to additional errors in calculations (Makonye & Fakude, 2016). This also supports the notion that students often memorize formulas without fully understanding their underlying principles and applications (Boaler, 2016; Suastika & Suwanti, 2019).

To address these challenges, a more focused approach to teaching conceptual understanding, coupled with the use of real-world contexts, is strongly recommended. Rosada and Luthfiana (2022) and also Hamidi et al. (2022) advocated for the use of contextual approaches to enhance conceptual understanding, as these approaches enable students to relate theoretical knowledge to practical, real-world problems. Such strategies not only reinforce mathematical concepts but also empower students to apply these concepts effectively in everyday situations, which is crucial for mastering volume calculations and unit conversions. Furthermore, Project-Based Learning (PBL) has been recognized as an effective teaching strategy for improving students' conceptual understanding, particularly through problem-solving experiences that address real-world challenges (Nayak et al., 2016; Yen et al., 2023). By engaging in experiential learning within real-world contexts, PBL deepens students' understanding of concepts, thereby fostering greater flexibility in applying formulas based on the context of the problem.

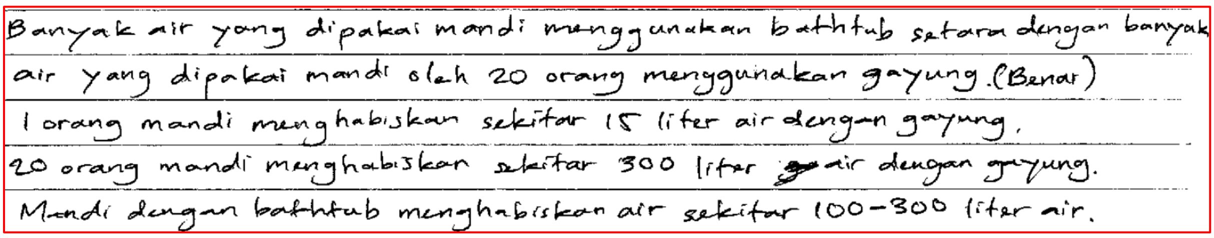
In conclusion, the results of this study underscore the importance of strengthening fundamental concepts in mathematics education for prospective teachers, especially in topics related to volume and unit conversion. A learning approach focused on understanding, coupled with contextual applications, will help mitigate common conceptual and procedural errors and better equip prospective mathematics teachers to become competent educators in the future. Finally, the results of the prospective mathematics teacher test for solving Problem No. 2 were influenced by their responses to Problem No. 1. This dependency occurred because participants were asked to compare the amount of water wasted in one hour with the volume of a dipper. Consequently, errors made in calculating the dipper's volume in Problem No. 1 directly impacted the comparison results in Problem No. 2a. An example of such an error is illustrated in Figure 6.


Tiap satu jam yang terbuang karena kran bocor sama dengan volume sebuah gayung berukuran standar. (Salah).
1 gayung ≈ 0,6 liter.
Air yang terbuang pada kran bocor selama satu jam = $\frac{15}{24}$ = 0,54 liter.

Every hour wasted due to a leaky faucet is equal to the volume of a standard-sized dipper (Wrong)
One dipper = 0.6 liters.
Waste of water due to a leaky faucet for one hour = 15/24 = 0.54 liters

Figure 6. Example of error for solving problem no. 2a

The test results revealed that all participants successfully solved Problem No. 2b without encountering any errors or difficulties. They correctly understood the information regarding water use for bathing with both a bathtub and a dipper, allowing them to accurately solve this part of the problem. Despite their earlier mistakes in calculating the dipper's volume in Problem No. 1, they were able to complete Problem No. 2b correctly. An example of a solution to Problem No. 2a is shown in Figure 7.


Banyak air yang dipakai mandi menggunakan bathtub setara dengan banyak air yang dipakai mandi oleh 20 orang menggunakan gayung. (Benar)
1 orang mandi menghabiskan sekitar 15 liter air dengan gayung.
20 orang mandi menghabiskan sekitar 300 liter air dengan gayung.
Mandi dengan bathtub menghabiskan air sekitar 100-300 liter air.

The amount of water used for bathing in a bathtub is equivalent to the amount used for bathing by 20 people using a dipper (true). One person taking a bath uses about 15 liters of water with a dipper, 20 people taking a bath uses 300 liters of water with a dipper, and taking a bath with a bathtub uses about 100 - 300 liters of water.

Figure 7. Example of answer for solving problem no. 2a

Results of the Prospective Mathematics Teacher Test for Solving Problem No. 3

The results of the test indicate that 56.67% of prospective mathematics teachers successfully solved Problem No. 3, while 43.33% made errors. The errors varied, and Table 3 presents a detailed breakdown of the types of errors and difficulties encountered.

Table 3. Errors and difficulties in solving problem no. 3

No	Error Description and Error Type
1	DVA and MEW's Error in Calculation DVA and MEW used the correct formula to calculate the volume of a cylindrical bucket, but an error in the calculation process led to an incorrect conclusion. Figure 8 illustrates an example of this error. Based on the test results, both DVA and MEW correctly applied the formula, but a mistake in multiplication led them to conclude that the bucket's volume was 13,949.50 cm³ (equivalent to 14 liters). Considering that one bath using a dipper requires about 15 liters of water, the 14-liter bucket could only accommodate one bath. During the interview, they acknowledged the error but were unable to explain why they obtained a volume of 13,949.50 cm³. They speculated that the error occurred due to time pressure and possibly by reading other answers on their written paper. This represents a procedural error in the calculation process.

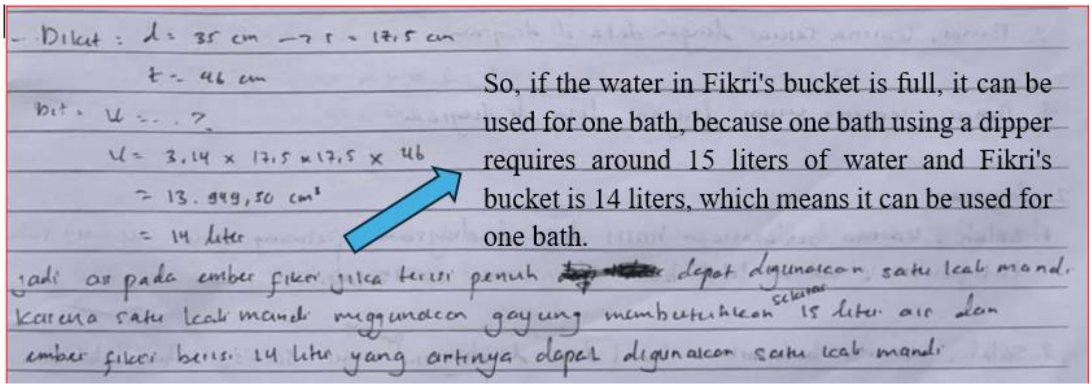


Figure 8. Type 1 error for solving problem no. 3

No	Error Description and Error Type
2	ANC and ULH's Error in Formula Application ANC and ULH determined that d = 35 cm and height = 46 cm, and they used the formula V = πrt to calculate the volume. However, they incorrectly assigned the values for the radius (r) and height, using r = 46 cm and height = 35 cm. This error is illustrated in Figure 9. Similar to the errors in Problem No. 1, ANC and ULH made an error by inputting the incorrect values into the formula, resulting in incorrect conclusions. The types of errors and difficulties they experienced in Problem No. 3 were consistent with those in Problem No. 1.

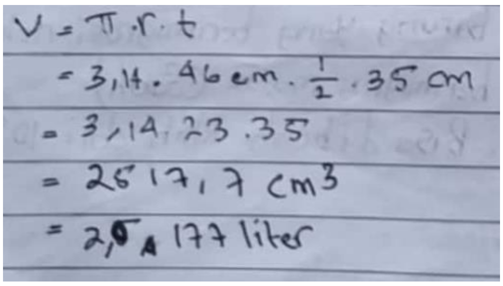


Figure 9. Type 2 error for solving problem no. 3

3 AGM and SUT's Error in Formula Manipulation

AGM and SUT used the formula V = 1/2 πd²t, even though the correct formula is V = 1/4 πd²t. Figure 10 shows an example of this error. The test results indicate that AGM and SUT used an incorrect formula, but their volume calculation was correct, as was the conversion from cm³ to dm³ and from dm³ to liters. In the interview, they explained that they derived their formula from πr²t by substituting r = d/2, since the diameter was given in the question. They correctly squared d, but failed to square the factor of 2, leading to an incorrect formula. This represents a procedural error in formula manipulation. Despite this, AGM and SUT did not experience significant difficulties in solving the problem, as they felt confident in their understanding of the formula and the steps, they took to solve it.

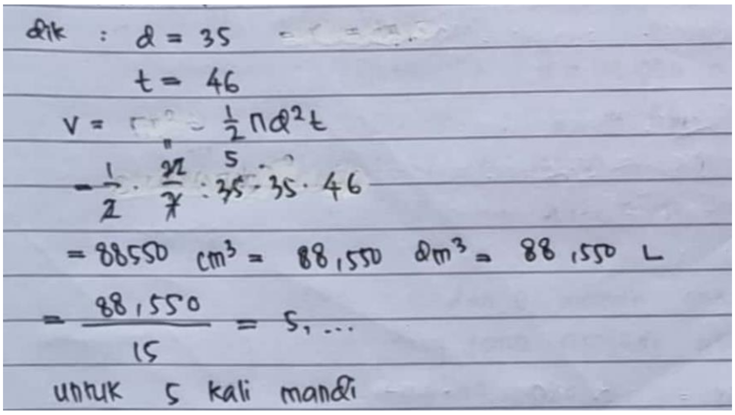


Figure 10. Type 3 error for solving problem no. 3

4 ALE, DKA, FDJ, JEP, KNY, and UCL's Omission

These participants did not attempt to solve Problem No. 3. The interview results revealed that they struggled to understand the previous questions, and as a result, by the time they attempted Problem No. 3, time had expired.

In Problem No. 3, approximately 56.67% of prospective mathematics teachers successfully solved the problem, while 43.33% made errors. The analysis revealed that the errors primarily consisted of procedural errors in calculations, incorrect use of formulas, and mistakes in inputting known values into the formulas.

The majority of participants, such as DVA, MEW, ANC, and ULH, made procedural errors in which they applied the correct formula but made mistakes in the calculation process or in the assignment of variable values. For instance, they incorrectly determined the radius and height of the cylinder, treating the diameter as the radius. Similar errors were found among participants like AGM and SUT, who manipulated the formula without verifying its correctness, although their final calculations remained accurate due to proper unit conversion. These findings suggest a tendency to memorize formulas without understanding the underlying context, which often leads to errors in formula application and variable manipulation.

These results align with the findings of Star and Rittle-Johnson (2008) and Rittle-Johnson and Schneider (2015), who reported that students frequently rely on memorized procedures without fully comprehending when and how to apply the steps of a calculation correctly. They noted that a superficial understanding of variable definitions and the logic behind formulas makes students prone to errors when they need to adapt or modify procedures based on the specific information provided in a problem. Furthermore, Boaler (2016) highlighted that students often face greater challenges when dealing with contextual problems, especially when such problems require the interpretation and application of complex concepts.

To address these issues, it is recommended that teaching methods prioritize understanding the foundational concepts and the logic behind formulas, rather than merely memorizing them. Visual strategies, such as diagrams or 3D models, can enhance students' understanding of the relationships between variables. This is consistent with the work of Herrera et al. (2024), who argue that spatial visualization tools, such as 3D models, can significantly improve students' comprehension of complex mathematical concepts and relationships. Moreover, visual representations like diagrams can enhance problem-solving abilities by helping students visualize the connections between concepts, thereby deepening their understanding (Jitendra & Woodward, 2019).

In addition, the implementation of Problem-Based Learning (PBL) is recommended as an effective pedagogical approach. PBL has consistently been recognized as an effective method for enhancing complex problem-solving skills and developing procedural flexibility in students (Amir et al., 2022; Folomieieva et al., 2024). PBL also fosters students' ability to address real-world challenges in a systematic manner, enabling them to apply knowledge flexibly and develop higher-order skills, which are essential in higher education (Hawamdeh & Adamu, 2021). Furthermore, PBL encourages creativity and improves work-related skills by applying flexible procedures and solving complex problems across various disciplines (Deep et al., 2020). This approach is expected to reduce procedural errors and assist prospective mathematics teachers in developing a deeper understanding of formula application.

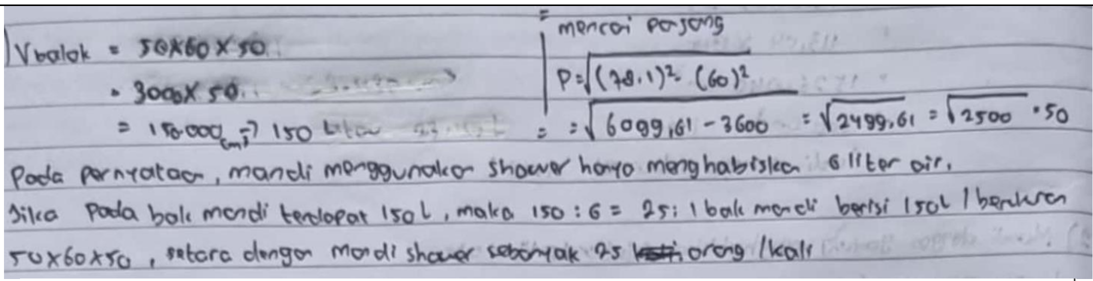
Results of the Prospective Mathematics Teacher Test for Solving Problem No. 4

The results of the test showed that only 13.33% of prospective mathematics teachers solved Problem No. 4 correctly. Another 13.33% only recorded the known information, while 16.67% wrote down the known information and attempted to find the length of the cuboid using the Pythagorean formula. However, 23.33% of the participants did not answer the question, and 33.33% attempted to answer but made errors in their solution. The types of errors varied, and these variations in errors and difficulties faced by the prospective mathematics teachers are summarized in Table 4.

Table 4. Errors encountered by prospective mathematics teachers in solving problem no. 4

No	Error Description and Error Type
1	MKZ, SYA, and VNI's Error in Calculation MKZ, SYA, and VNI began solving Problem No. 4 by calculating the block's length and subsequently used this to find the volume of the bathtub. They then divided the result by the water usage during a shower to determine how many showers could be taken using the available water. However, they made a mistake in calculating the length by subtracting the diagonal of the base from the height, whereas the width of the base should have been subtracted. This error led to incorrect calculations of both the length and the volume, which affected their final conclusions. Figure 11 illustrates this error. In the interview, they acknowledged that the inaccuracy in calculating the length caused the error. This represents a procedural error in calculation, where the participants understood the problem and the steps to solve it, but struggled due to unfamiliarity with numeracy problems at the elementary and secondary school levels.

No	Error Description and Error Type



$V_{cuboid} = 50 \times 60 \times 50$ looking for length

$= 150.000\ cm^3 = 150\ l$ $p = \sqrt{(78,1)^2 - (60)^2} = \sqrt{6099,61 - 3600} = \sqrt{2499,61} = \sqrt{2500} = 50$

In the statement, taking a shower only uses 6 liters. If the bathtub has 150 l, then 150: 6 = 25. One bathtub contains 50 × 60 × 50, equivalent to taking a shower for 25 people/times.

Figure 11. Type 1 error for solving problem no. 4

2 ERN and HAP's Error in Unit Conversion and Data Interpretation

ERN and HAP correctly calculated the volume but failed to convert the volume unit from cm³ to liters. They proceeded to divide the calculated volume by the water usage data for a shower, which was not in the correct unit (liters). Additionally, they misinterpreted the shower water usage data, incorrectly assuming that a shower saves 9 liters of water compared to a dipper, when it should be 6 liters. Figure 12 demonstrates their approach. In the interview, they recognized their mistake but admitted that they did not convert the units and misread the data regarding water usage. These are procedural errors stemming from miscalculations and inaccuracies in reading the available data.

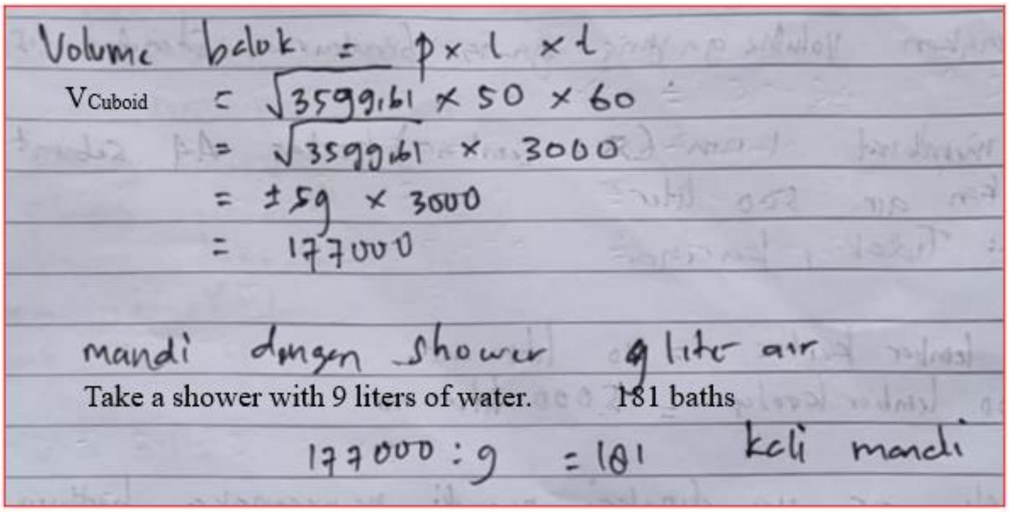
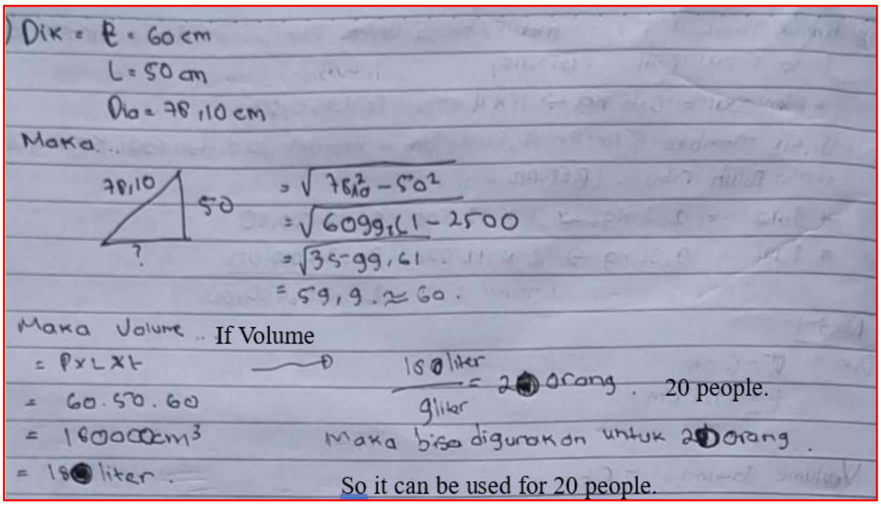


Figure 12. Type 2 errors for solving problem no. 4

3 AGM, SUT, ANC, DNH, ULH, and HDS's Misreading of Data

These participants solved Problem No. 4 by first calculating the length using the Pythagorean formula and then calculating the volume, before dividing it by the water usage for a shower. While they correctly calculated the length and volume, they misinterpreted the shower usage data. The problem stated that a shower saves 60% or 9 liters of water, which should result in a water usage of 6 liters for the shower, not 9 liters. This error led to incorrect conclusions. Figure 13 shows the error made in this approach. AGM,

No	Error Description and Error Type
	SUT, ANC, DNH, ULH, and HDS made procedural errors by misreading the provided data, which affected their final answers, despite understanding the steps required to solve the problem.  **Figure 13.** Type 3 error for solving problem no. 4
4	ANH, BSN, FBN, and SGA's Incomplete Responses These participants only recorded the known information and did not proceed with solving the problem. Similarly, ANR, APR, DMB, and SKY recorded the known information and attempted to calculate the length using the Pythagorean formula, but did not progress further. ALE, DKA, DVA, FDJ, JEP, and MEW did not complete Problem No. 4, as they required additional time to understand the problem. By the time they began to work on it, the time allocated for the test had expired.

The analysis of the responses indicates that misunderstanding of the problem's data was a significant factor leading to errors in solving Problem No. 4. For example, ERN and HAP misinterpreted the data regarding water usage for a shower, which directly impacted their calculations. Misunderstanding the information in a problem is a well-established factor in error formation and can hinder students' ability to accurately interpret and solve the problem. This issue has been discussed in previous studies by Österholm (2007), Ratnaningsih and Hidayat (2021), and Tasman and Yenti (2018), who highlighted the challenges students face when activating prior knowledge and understanding what is required in problem-solving.

Furthermore, some participants failed to progress beyond recording the known information or did not manage to complete the problem due to delays in understanding the question. The speed at which students understand a problem is a crucial skill in mathematics. An efficient understanding of mathematical problems can enhance students' ability to think critically, reduce reliance on mechanical problem-solving, and improve overall problem-solving efficiency (Luo & Yu, 2020). Additionally, quicker comprehension of problems leads to better retention and application of mathematical concepts (None, 2022) and strengthens problem-solving skills (Yanti, 2011).

In conclusion, the results of this study highlight that procedural errors, misinterpretations of data, and slow problem comprehension were the primary challenges faced by prospective mathematics teachers. These findings underscore the importance of teaching strategies that focus on reinforcing procedural skills, improving the ability to read and interpret contextual data accurately, and increasing the speed of problem comprehension to better prepare prospective mathematics teachers.

CONCLUSION

This study analyzed the conceptual and procedural errors made by prospective mathematics teachers within the context of the "Save Our Water" problem. The findings revealed that while most participants demonstrated an understanding of basic concepts, several errors persisted. These included conceptual mistakes arising from the mere memorization of formulas without a deep understanding of underlying concepts, and procedural errors such as improper unit conversions, incorrect use of volume formulas, and flawed manipulations of formulas. These errors suggest that prospective mathematics teachers face challenges in bridging theoretical mathematical knowledge with its practical application to real-world scenarios.

However, this study is not without limitations. Firstly, the sample was restricted to fifth-semester mathematics education students from a single university, which may limit the generalizability of the findings to prospective teachers from other institutions. Additionally, the research focused solely on the "Save Our Water" context, which may not fully represent the diversity of real-world situations relevant to numeracy instruction. Moreover, the instruments employed—numeracy tests and semi-structured interviews—while useful, may not have comprehensively addressed all aspects of numeracy skills required in more complex, real-world situations.

In light of the findings, it is evident that a shift in teaching strategies is necessary, with a greater emphasis on conceptual understanding rather than rote memorization of procedures. The adoption of contextual teaching methods, such as problem-based and project-based learning, is recommended to better connect mathematical concepts to real-life scenarios. Additionally, incorporating visualization tools, including 3D models and diagrams, could enhance students' grasp of mathematical relationships and improve their procedural flexibility. Future research should explore a wider range of numeracy contexts and involve more diverse, heterogeneous samples to gain a broader understanding of prospective teachers' competencies. Furthermore, further investigations into the effectiveness of targeted training programs designed to enhance numeracy skills in practical contexts are crucial to ensure the preparedness of future mathematics educators.

Acknowledgments

We would like to extend our sincere appreciation to the Directorate General of Higher Education, Research, and Technology for providing the 2024 Postgraduate Research-Doctoral Dissertation Research funding. This support, as outlined in Decree Number 0459/E5/PG.02.00/2024 and Contract/Agreement Number PG.02.00.PL/2024, has been instrumental in the completion of this research.

Declarations

Author Contribution : SW: Conceptualization, Writing - Original Draft, Editing and Visualization.
K: Supervision, Formal analysis, and Methodology.
JM: Writing - Review & Editing, and Validation.
A: Writing - Review & Editing, and Validation.

Funding Statement : This research was funded by the Directorate General of Higher Education, Research, and Technology for supporting and funding this research.

Conflict of Interest : The authors declare no conflict of interest.

Additional Information : Additional information is available for this paper.

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