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An enquiry approach to rediscovering sundials as a didactic tool for teaching time

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Abstract

The ability to measure quantities is a fundamental component of primary mathematics education due to its relevance in both real-world applications and mathematical horizontality. However, the concept of time measurement remains one of the most challenging topics for students to grasp due to its abstract nature. Despite the recognized difficulties, there is a lack of effective instructional strategies that integrate constructivist approaches to enhance students' conceptual understanding of time. Addressing this gap, this study presents the design and implementation of a constructivist didactic sequence based on active learning within an Inquiry-Based Learning (IBL) framework. The study involved 31 pre-service teachers in their final year of training, aiming to enhance their pedagogical competence in teaching time measurement through the use of sundials. The research explores how these future educators conceptualize time and how they interpret sundials as a means to represent its passage. To evaluate their assimilation and comprehension of the topic, a phenomenographic analysis was conducted, comparing their depth of knowledge before and after the intervention. The findings indicate a significant improvement in both conceptual understanding and didactic application. The results underscore the effectiveness of sundials as instructional tools, not only for illustrating the passage of time and calendar cycles but also for highlighting the social and historical contexts associated with timekeeping. This study contributes to the field of mathematics education by providing empirical evidence supporting the integration of inquiry-based, constructivist methods in the teaching of time measurement, ultimately enhancing pre-service teachers' instructional competencies and students' conceptual grasp of temporal concepts.

Keywords: Didactics of Time, Phenomenography, Primary School Student Teachers, Sundial

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Time is an integral aspect of human existence, as individuals continuously experience, utilize, and attempt to quantify it. However, its perception remains inherently subjective, with individuals often expressing concerns about having either an excess or a deficiency of time and acknowledging its influence on various dimensions of life. This complexity has made time a subject of extensive philosophical inquiry by renowned thinkers such as Plato, Aristotle, Kant, Heidegger, Piaget, and Foucault, each of whom has examined it from distinct perspectives, yet without reaching a universally accepted definition. Despite the absence of a definitive answer to the question, "What is time?" students encounter and engage with the concept from an early age, both in their daily lives and within educational settings. As noted by Dýrfjörð et al. (2023), children's experiences of time contribute to their overall well-being and inform discussions on quality in early childhood education. Similarly, Thomas et al. (2023) emphasize that students must develop an awareness of time, understand its succession and duration, and acquire the ability to measure it to effectively interpret and utilize time-measuring instruments. Furthermore, time measurement is a fundamental component of mathematics curricula worldwide. The National Council of Teachers of Mathematics (2000) identifies measurement as one of the five key content standards, with time recognized as a fundamental magnitude introduced in early education. Despite its significance both within and beyond the classroom, there is a notable gap in research regarding how time is taught and learned in primary education compared to other mathematical topics (Thomas et al., 2023).

As highlighted by Meaney (2011), understanding measurement—particularly time—poses significant challenges for students in mathematics education. One of the primary difficulties lies in its abstract nature, as time lacks a tangible representation and is subject to variable interpretations depending on an individual's state of mind (van Manen, 1990). Additional challenges include the reliance on non-temporal elements to establish equivalence in durations, the continuous expiration of the present moment (Piaget et al., 1971), and discrepancies between personal temporal rhythms and standardized time systems regulated by clocks (Thomas et al., 2023). Furthermore, students often develop misconceptions about the natural indicators of time passage, such as the daily movement of the sun and the changing seasons, which are frequently rooted in naïve assumptions. Even teachers may hold misconceptions regarding Earth's characteristics and its relationship with the sun (Fulmer, 2013).

These conceptual challenges are further compounded by traditional approaches to scientific inquiry and limited disciplinary competencies, which can impede teachers' professional growth in scientific domains (Murillo et al., 2021). To address these issues, it is crucial to foster pedagogical and methodological innovations informed by research, particularly for pre-service teachers. As future educators, they must be equipped with the skills to engage students in mathematical thinking through authentic and innovative instructional strategies. In this context, the present study examines the effectiveness of sundials as an instructional tool for teaching time in primary education. The research aims to develop a constructivist teaching sequence that enhances pre-service teachers' conceptual understanding and pedagogical competence. Additionally, it evaluates the impact of this approach by analyzing participants' initial and final perspectives on the concept of time.

A persistent challenge in the mathematics teaching-learning process is the disconnect between school-based instruction and real-world applications. This gap hinders students, as future active members of society, from acquiring the cultural and mathematical literacy necessary for informed decision-making (Bonotto, 2001; Vos, 2018). The foundation for these essential skills is established in the early years of education (Euler, 2004). One contributing factor to this issue is the complexity of teacher knowledge, which extends beyond Subject Matter Knowledge (SMK) to encompass Pedagogical Content Knowledge (PCK) (Ball et al., 2008). PCK equips educators with the ability to select appropriate contexts for knowledge application and structure learning tasks within professional teaching settings. Consequently, there is a growing body of research focused on the attributes of learning activities and educational environments within teacher education programs (Tirosh & Wood, 2008).

Specifically, scholars highlight that pre-service teachers often struggle with integrating Content Knowledge (CK) and Pedagogical Knowledge (PK), as limited expertise in these domains impairs their ability to develop PCK effectively (Michelini & Stefanel, 2011). As a result, primary teacher training programs must ensure a strong and cohesive integration between subject-specific knowledge and pedagogical strategies (Schwarz, 2009). Within this framework, the interplay between research and educational practice is particularly fertile (García et al., 2007), as examining specific pedagogical challenges and analyzing the impact of different learning environments generates critical insights into how pre-service teachers acquire knowledge and the factors that facilitate or impede their learning.

The insights gained from these investigations contribute to the advancement of theoretical foundations and facilitate a deeper engagement with research focused on teacher learning (Llinares, 2014). This ongoing inquiry underscores the dynamic relationship between practice in teacher education and research, as studies on teacher learning through teaching experiments reveal a continuous interplay between both domains (Cobb et al., 2003).

One of the most effective pedagogical frameworks for addressing the real needs of both teachers and learners in the instructional process is the Teaching-Learning Sequence (TLS). Méheut and Psillos (2004) define TLS as follows: "A Teaching Learning Sequence is both an interventional research activity and a product, like a traditional curriculum unit package, which includes well-researched teaching-learning activities empirically adapted to student reasoning. Sometimes teaching guidelines covering expected student reactions are also included" (p. 516). This approach has been extensively utilized in research exploring the complexity of teacher knowledge (Da Ponte & Chapman, 2006). Essentially, TLS-based research aims to enhance instructional methodologies by acknowledging the intricacies of classroom interventions. It prioritizes focused analyses at small-scale levels, such as individual classroom sessions, or at medium-scale levels, such as an entire instructional sequence covering one or more curriculum topics. Rather than addressing large-scale curriculum reforms spanning multiple academic years, TLS research concentrates on refining pedagogical strategies through targeted, evidence-based interventions (Psillos & Kariotoglou, 2016).

Guisasola et al. (2021) highlight that the TLS exhibits certain shared characteristics, including the design of instructional activities informed by research findings, the epistemological analysis of curricular content to justify learning objectives, the adoption of a social constructivist approach to learning, and the presentation of empirical evidence on student learning outcomes. However, the same authors emphasize that TLS design proposals lack methodological uniformity due to the broad range of topics they address, such as learning processes, cognitive development, teaching strategies, and classroom interactions, among others. This diversity contributes to variations in research and design methodologies, as different educational phenomena necessitate distinct investigative approaches and epistemological frameworks to effectively understand and implement them (Bell, 2004). Similarly, Ernst et al. (2017) argue that defining instructional methodologies within learner-centered, active pedagogies—such as Inquiry-Based Learning (IBL)—remains a challenging endeavor due to their continuous evolution. Moreover, various contextual factors, including class size, students' prior knowledge, and subject matter complexity, significantly influence a teacher's decision to employ IBL strategies.

Within the framework of Educational Design Research (EDR), Plomp (2013) defines it as "the systematic study of designing, developing, and evaluating educational interventions" (p. 11). Considering TLS as a component of such interventions, it serves as a crucial link between educational practice and theoretical advancement, as it aims to develop both domain-specific learning theories and instructional materials tailored to support effective learning experiences. The effectiveness of TLS is further reinforced through the application of Design-Based Research (DBR) methodology (Zuazagoitia et al., 2021). A pertinent example is the comprehensive review by Guisasola et al. (2021), which examines the impact of TLS within various DBR frameworks. As Bell (2004) asserts, "we should also be open to the possibility that design-based research is a fundamental mode of scholarly inquiry that is useful across fields of the academy" (p. 251), reinforcing the role of DBR in fostering long-term educational innovation.

In contemporary society, characterized by rapid advancements and continuous transformations, science and technology play a crucial role. This underscores the need to enhance students' scientific knowledge and cultivate critical thinking skills through the empirical application of scientific literacy and an understanding of the nature of science (Nuangchalerm, 2010). According to Panasan and Nuangchalerm (2010), students acquire knowledge in a constructivist manner through active interaction with their environment. They further highlight that "the effectiveness index showed that students can gain their knowledge and experiences of scientific conception after learning through project-based and inquiry-based instructions".

This study presents a didactic design implemented in the initial training of pre-service primary school teachers, aiming to deepen their comprehension of the operational mechanism of a sundial. The objective is to equip them with the necessary pedagogical skills to effectively explain the concept of time to future students. The instructional approach is situated within research-based design frameworks and employs a constructivist paradigm, utilizing scientific practices and an inquiry-based teaching-learning strategy. Edelson et al. (1999) strongly advocate for IBL, asserting that direct engagement with scientific inquiry is essential for students to perceive science as an open-ended, question-driven process.

This pedagogical approach underscores the significance of not only acquiring scientific concepts but also actively participating in the construction of knowledge through inquiry, exploration, and reflection. By immersing themselves in this process, students develop a more profound understanding of the dynamic and evolving nature of scientific knowledge. Furthermore, IBL methodologies have been integrated into university-level mathematics courses across various academic levels (Ernst et al., 2017). These approaches aim to adapt the principles of ABL to the mathematical domain, fostering an environment where students engage in exploration and discovery rather than passively receiving information. Ernst et al. (2017) illustrate this through three case studies, the third of which, akin to the present study, focuses on the didactics of mathematics for prospective primary school teachers.

As previously noted in relation to students, this instructional approach holds particular significance in teacher education, as it equips future educators not only with mathematical content knowledge but also with the pedagogical strategies necessary to facilitate active and meaningful learning experiences in their own classrooms. The same authors emphasize that while pre-service teachers possess familiarity with the mathematical concepts they are expected to teach, they may not fully understand why things work the way they do (Ernst et al., 2017). To address this gap, it is essential to encourage future teachers to acquire subject matter knowledge through an investigative process that immerses them in scientific inquiry and mathematical reasoning.

IBL-oriented mathematics courses provide a structured framework that enhances the comprehension of complex mathematical ideas and structures through inquiry-driven methodologies (Ernst et al., 2017). Therefore, initial teacher education programs should be designed to prepare future educators to develop instructional strategies that foster the competencies and skills outlined in the curriculum.

Laursen et al. (2011) highlight that educational research has identified two fundamental principles underlying most IBL approaches, referred to as the "twin pillars": (a) deep cognitive engagement with mathematics and (b) collaborative peer-to-peer learning. The study of time has provided an opportunity to leverage its extensive and interdisciplinary nature in designing the present instructional proposal. Through an IBL framework, the strategies, tools, and methodologies employed are embedded within an active learning paradigm, engaging pre-service teachers in a sequence of exploratory, interpretative, and creative activities.

Similarly, the chosen instrument for studying time—the sundial—exemplifies adaptability and openness, allowing for the integration of diverse disciplines, including history, art, mathematics, and physics. This interdisciplinary approach fosters a multidimensional understanding of knowledge. By incorporating various fields of study, this educational initiative is structured around a series of interwoven pedagogical practices that unfold within a specific socio-historical context. These practices serve both as a record of collective knowledge and as a foundation for broader educational projects.

The mathematical concepts addressed in the construction of the sundial align with the official curriculum for Primary Education teacher training within the Spanish educational system. Specifically, the instructional design focuses on numeration, magnitude and measurement—particularly the measurement of time—and geometry, in accordance with the principles and standards for school mathematics outlined by the National Council of Teachers of Mathematics (2000).

Phenomenography

A primary objective of this study is to evaluate the effectiveness of the TLS in enhancing students' knowledge and to identify the patterns or tendencies they exhibit when responding to questions related to the covered topic. In the context of TLS, learning outcomes are assessed by comparing students' depth of knowledge before and after instruction (Zuazagoitia et al., 2021). To achieve this, the study employs phenomenography, a research approach that facilitates the exploration of learning experiences and the identification of qualitatively distinct ways in which individuals perceive and understand a phenomenon (Sharma et al., 2004). According to Minasian-Batmanian et al. (2006), phenomenographic data provide insights into students' initial comprehension and the progression of their understanding of scientific concepts, a methodology frequently applied in science education (González-Ugalde, 2014). Consequently, this study presents a phenomenographic analysis of pre-service teachers' initial and final conceptions of time and their pedagogical applications of sundials.

Phenomenography operates by formulating questions, categorizing responses, and conducting subsequent analyses to capture variations in individuals' ways of thinking and experiencing a phenomenon. Given the relatively large number of participants (n = 31; of the 40 students enrolled in the course, only 31 provided both pre-test and post-test data), the use of an open-ended questionnaire is particularly advantageous. This method facilitates the efficient handling of responses while enabling the collection of a broad spectrum of experiences related to the phenomenon under investigation (Han & Ellis, 2019).

In designing the questionnaire, it is essential to ensure that the questions enable students to articulate their experiences of the phenomenon while uncovering underlying meanings and intentional attitudes. To achieve this, the study adheres to two key guidelines. The first, as suggested by Akerlind (2005), emphasizes the use of "why" questions rather than "what" questions to explore how students think about and experience the phenomenon. The second guideline involves incorporating monitoring questions that encourage students to engage in deeper reflection on their experiences, thereby providing more meaningful and high-quality responses (Prosser, 2000).

With regard to the categorization of responses, a predetermined classification cannot be applied; instead, categories must emerge organically from the questions themselves, following the patterns or structures that arise from the collected data. Analysts must go beyond evaluating the correctness or explicit content of responses and instead focus on interpreting them, identifying similarities, or recognizing other significant criteria. The diverse ways in which students engage with the phenomenon will ultimately inform the development of descriptive categories.

Once the responses have been collected and an initial independent analysis has been conducted, researchers must reach a consensus on the categorization process. This ensures that responses are systematically assigned to appropriate categories, followed by a subsequent review to reaffirm the classification (Sharma et al., 2004). While this iterative process may be time-consuming, it is essential for obtaining a comprehensive understanding of the range of students' cognitive perspectives.

Phenomenographic research outcomes should include two fundamental components: a detailed description of each category and an extensive collection of representative examples for each category (Bowden, 2000). Since these categories reflect distinct ways in which individuals experience a given phenomenon, they are typically arranged in a hierarchical structure, progressing from more basic to more advanced conceptualizations (Tight, 2016). As noted by González-Ugalde (2014), structuring results in this manner facilitates their practical application in educational development, allowing educators and students to transition from simpler to more complex conceptual understandings.

Didactic Approach of Sundials

The long-standing coexistence of human societies with sundials has left a significant mark on cultural heritage. This historical legacy represents a valuable didactic resource embedded within sundials. As emphasized by the Council of Europe Committee of Ministers, Recommendation No. R (98) 5 to Member States asserts that educational activities in the heritage field are an ideal way of giving meaning to the future by providing a better understanding of the past (Kavelashvili, 2019).

Since effective teaching should be connected to students' social environments and communities and considering the fundamental role of the sun in daily human life, the sundial emerges as an optimal instrument for teaching the concept of time in primary education. By utilizing the simple mechanism of casting a shadow, sundials facilitate the internalization of the abstract notion of time while integrating multiple disciplinary perspectives. As Paola (2019) noted, Sundials, time measurement tools, defined by some scholars as the 'heavens' doors,' constitute a precious interdisciplinary scientific heritage to be interpreted and preserved.

Research Question and Objectives

The research question underpinning this research is whether sundials, or learning how they work, are an effective tool for primary school teachers in teaching the magnitude of time. Therefore, this research has two main objectives:

1. To describe the design of a constructivist TLS contextualized in the study of the magnitude of time, based on sundials, which will promote, on the one hand, the historical-social framing of sundials and knowledge of the factors that show the movement of the shadow in the solar quadrants and, on the other hand, the development of didactic competences with respect to time in future teachers.
2. To verify whether the TLS fosters this progression of ideas through the phenomenographic analysis of the students' initial and final understanding of the concept of time and their didactic proposal.

Justification of the Activity

We aim to justify the implementation of this activity from two perspectives. First, our justification is grounded in the preceding discussion and the key characteristics previously outlined, which we seek to integrate into our instructional sequence. Second, this activity holds particular relevance for our students, given their current stage in their academic journey. As final-year students in the Primary Education degree program, they have successfully completed all mandatory coursework, including subjects related to the didactics of mathematics. At present, they are enrolled in an elective course, Mathematics Workshop, which focuses on the in-depth study of manipulative materials for teaching and learning mathematics in primary school classrooms. Given their advanced level of study and specialized focus, this represents an optimal opportunity to engage them in our didactic sequence.

METHODS

Participants

This study is conducted within the framework of manipulative mathematics, as the participants are enrolled in an elective course focused on instructional materials for teaching mathematics in primary education. The sample consists of 40 final-year students (8 male and 32 female) pursuing a Primary Education Teaching degree at Universitat Jaume I in Castelló, with an average age of approximately 22 years. All participants have successfully completed two prerequisite courses in their second and third years, totaling 16 ECTS credits, which emphasize the use of didactic resources for teaching mathematical concepts in primary education.

Didactic Sequence Design

The didactic sequence was structured into ten sections, incorporating key elements essential to inquiry-based learning. The implementation of this sequence was carried out over four sessions, each lasting 180 minutes.

Section 1: Brief Oral Introduction with an Explanation of the Whole Procedure and Informed Consent

The introductory explanation of the TLS focuses on the concept of time measurement. This explanation is structured into the following sections and is delivered through a slide presentation, which will subsequently be made available in the Virtual Classroom:

1. Refresh students' memories on the content they have covered in the block on Magnitude and Measurement with respect to time.
2. This learning sequence will explore, among other concepts, the way in which previous civilizations have tried to control the passage of time and its recurrence in years, seasons and months; the existence of sundials in their immediate environment; the motivational, didactic and visual abilities of the quadrants to show the passage of time, etc.
3. The spaces for the TLS will be the classroom, outdoors, the Castelló sundials route and Universitat Jaume I's eco-educational garden.
4. Informed consent is obtained for the collection and processing of their responses.

Section 2: Route through Castelló to See Sundials

The sundial route, which begins at the university, provides an opportunity to highlight an often-overlooked aspect of the students' surroundings—the presence of numerous sundials in their environment. Through this activity, students will develop the ability to read sundials and understand their various functions. In certain cases, these sundials serve as calendars, marking specific dates or a singular moment in time, such as the meridian, which was historically used to calibrate the mechanical clock of the cathedral (Figure 1).

Figure 1. Sundial on the campus of Universitat Jaume I (left) and meridian clock (right)

Section 3: Explanation of the Basic Concepts of Astronomy and Geography

A fundamental understanding of key geographical and astronomical concepts is essential for comprehending the use of sundials. This activity includes a review of concepts such as the Earth's axis of rotation and magnetic axis, the system of parallels and meridians that define latitude and longitude, the local meridian, as well as the equinoxes and solstices. These topics are reinforced through a theoretical presentation and a practical demonstration using a representation of the Earth's globe.

Section 4: Recreation of the Motion of the Earth around the Sun

The recreation of this movement illustrated in Figure 2 serves to dispel the common misconception that seasonal changes result from the Earth's varying distance from the Sun (Fulmer, 2013). This activity reinforces the understanding that summer and winter occur simultaneously in opposite hemispheres. Students are required to demonstrate that the true cause of seasonal variation is the 23.5º tilt of the Earth's rotational axis relative to the perpendicular of the ecliptic plane.

Figure 2. Recreation of the incidence of the sun’s rays. On the left, they are more perpendicular in the southern hemisphere and, on the right, in the northern hemisphere

Section 5: Construction of a Horizontal Sundial at the North Pole

To design the simplest possible sundial for construction and conceptual understanding, one could create a horizontal sundial positioned at the North Pole during the period when the Sun remains continuously above the horizon (from March 21 to September 21). In this configuration, the gnomon would be perpendicular to the base and aligned with the Earth's axis of rotation. The shadow cast by the gnomon would rotate uniformly by 360º over the course of a day. Marking the hour lines would be straightforward, as they would divide the quadrant into 24 equal sectors, each with an angular separation of 15 degrees. As part of this activity, students are tasked with attempting to recreate this sundial (Figure 3).

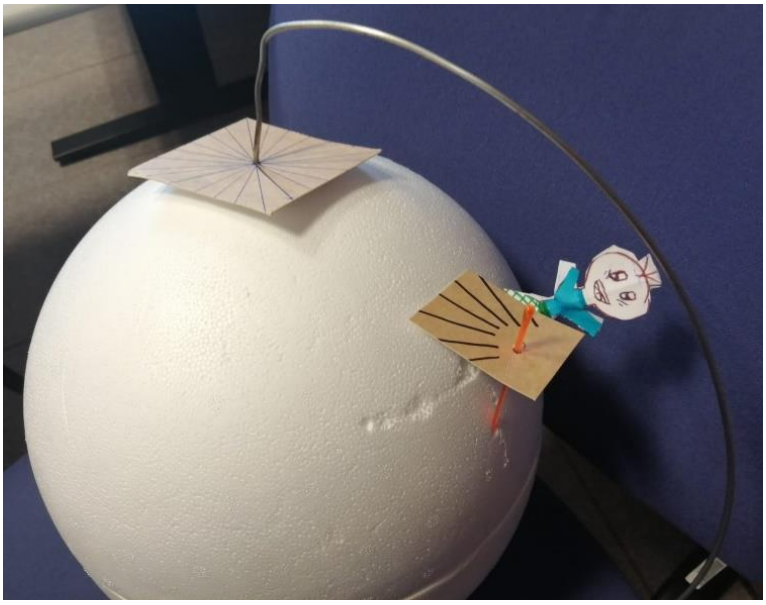

Figure 3. Recreation of the translation of the North Pole sundial to an equatorial one

Section 6: Construction of an Equatorial Sundial at Any Point. Deduction of the Direction of the Gnomon

Building on the understanding of the mechanics of the previous sundial (Section 5), students are asked to use a globe representation to relocate the sundial while maintaining its horizontal orientation. The objective is to determine whether the time indications remain valid at different locations on Earth. Figure 3 illustrates the sundial from the previous section, with an added human figure placed at a random location on the Earth's surface. In front of this figure is an equatorial sundial, whose base remains parallel to both the equator and the original sundial, while its gnomon is aligned with the Earth's axis of rotation. The hour lines of this sundial match those of the initial model, with a consistent separation of 15º, though sunlight will not illuminate all of them continuously for 24 hours. Through this activity, students are expected to deduce that:

1. The gnomon is always parallel to the Earth's axis of rotation.
2. Depending on its geographical position, some hour markings will be exposed to sunlight, while others will remain in shadow.
3. The hour lines remain spaced at 15º intervals, resulting from the division of 360º into 24 equal sectors.
4. By maintaining horizontal alignment, the sundial's plane will always be parallel to the equator.

This adjusted sundial is referred to as an equatorial sundial, as its base remains parallel to the equator. The majority of sundials found in practice are geometric projections of this fundamental model, making it a key reference in sundial construction and interpretation.

Section 7: Instructions for Making a Projection from the Equatorial Sundial to the Horizontal Sundial. Constructing It Physically

Students are then instructed to create a horizontal projection of the equatorial sundial at our specific latitude, effectively constructing a horizontal sundial adapted to our location (Figure 4). To achieve this, they follow the steps outlined in Stirrup's method from 1652, as documented by Palau (1977).

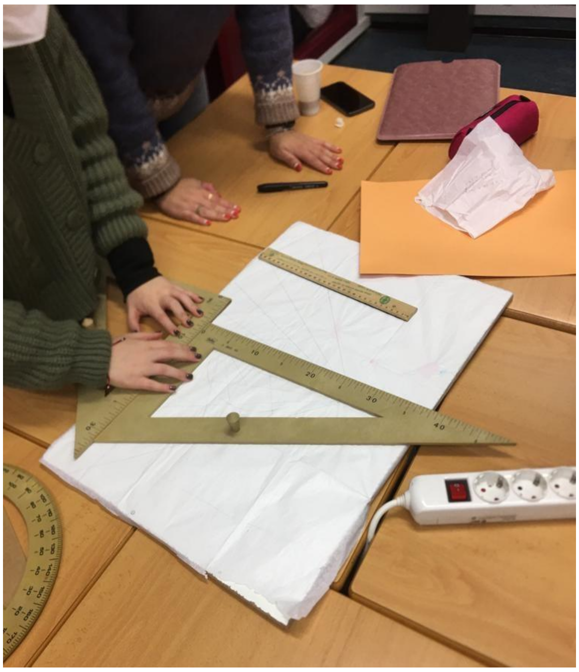

Figure 4. Construction of the horizontal projection of the equatorial sundial at our latitude

Section 8: Exercise to Guess the Time and Calendar based on the Movement of the Shadow around the Sundial

The objective of this activity is to enhance students' understanding of the movement of the gnomon's shadow throughout the day and across different seasons. To facilitate this, students use a torch to simulate the Sun's motion:

1. In one day, moving from east to west. The shadow will show the different hours.
2. In one year, with different heights. That is, the length of the shadow varies between summer and winter. In winter, when the sun reaches a lower elevation above the horizon, it casts a longer shadow, and in summer, it will be shorter. This characteristic results in the calendar function of a sundial.

At this stage, it is essential to clarify a common misconception: although the Earth's orbit around the Sun is elliptical, with the Sun positioned at one of its foci, the variation in Earth-Sun distance does not determine seasonal changes or average seasonal temperatures. Instead, the key factor is the angle at which sunlight strikes the Earth's surface—more oblique in winter and more direct in summer. This effect, already explored in Section 4, is further reinforced through this activity. Notably, in the Northern Hemisphere, summer occurs when the Earth is farthest from the Sun, whereas winter coincides with its closest approach.

Section 9: Explanation of the Equation of Time

After comprehending the significance of the different shadow paths on the solar quadrant, students must analyze the discrepancies between the time indicated by the sundial (true solar time) and the official time. These variations arise due to several factors:

1. The lack of uniformity of the sun passing through the same meridian twice during the year (there is a maximum variation of 16 minutes).
2. The seasonal correction politically imposed in some countries to make better use of daylight hours.
3. Time zone correction (also a political decision).
4. Longitude correction.

These time alterations can be corrected by observing the equation of time, which graphically marks the number of minutes to be added to or subtracted from the time marked by the sundial in order to determine the official time (Figure 5).

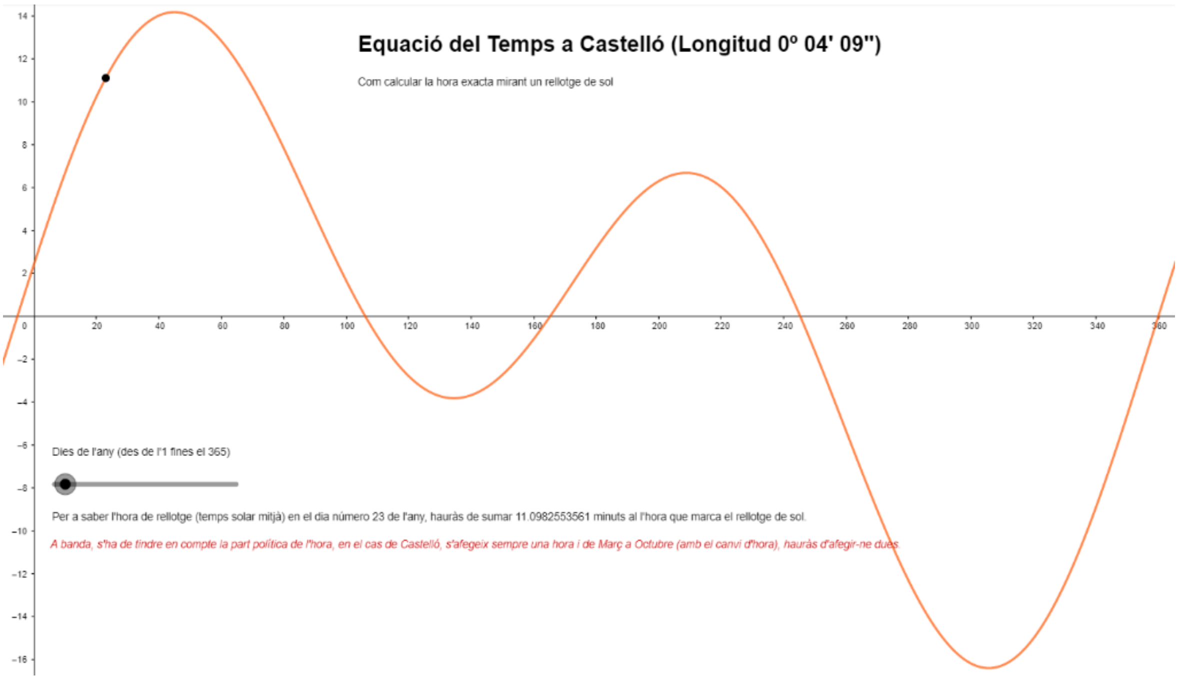

Figure 5. Equation of time for a clock located at our University (Castelló, Spain), taken from https://www.geogebra.org/m/wu7eyjy5

Section 10: Checking that the Sundial and Calendar are Working. We Go Out to the Eco-Educational Garden

The experiment was conducted in the eco-educational garden at Universitat Jaume I, where the sundials were positioned in their correct orientation (Figure 6). By applying the equation of time, the official time was recorded. Each group systematically verified the functionality of their sundials to ensure accuracy. Additionally, observations were made regarding the calendar, particularly in relation to the various sowing dates indicated. Although none of the pre-marked sowing dates coincided precisely with the day of the experiment, a noticeable alignment was observed between the gnomon's shadow and those dates that were temporally closer to the experimental day.

Figure 6. Sundial in the eco-educational garden. Checking operation of the sundial's time and calendar

RESULTS AND DISCUSSION

Phenomenographic Analysis of the Activity

The students (n=31) answered a questionnaire before (Pre-test) and after (Post-test) the implementation of the TLS, in which they were asked about time from two perspectives:

1. Didactic: 'How would you explain time to a primary school child?'
2. Technical about sundials: 'How do you think a sundial shows the passage of time?'

The answers were open-ended, and participants were encouraged to elaborate on their answers as extensively as possible, contributing a diverse range of ideas and insights. They were prompted to justify their responses and were further engaged through follow-up questions designed to foster deeper reflection on their experiences (Prosser, 2000). Examples of such questions included inquiries on how they would demonstrate the concept of time to a student, what instruments they would utilize for the first perspective, and what units of time they could illustrate for the second. For those interested in further exploration, links to the students' responses are provided below; however, please note that the answers are in Catalan.

Pre-test: https://docs.google.com/spreadsheets/d/1rndLc-TpvtuIYwq26q9NX0xuaA9Uwjw5k-DGu-DJqp0/edit?usp=drive_link
Post-test:
https://docs.google.com/spreadsheets/d/1qPmuFYtYd5RMxZDifTpCyfMIhGSwsqehx0sKacVVirw/edit?usp=drive_link

For each time dimension studied, a system of five categories was established (Table 1). Furthermore, to statistically determine whether there was a significant difference between the pre- and post-didactic sequence samples, the Wilcoxon signed-rank test was applied due to the non-normal distribution of the data. The results indicated a statistically significant difference between the pre-test and post-test responses for both dimensions under study, with a p-value of less than 0.0001. This finding, as illustrated in Figure 7, demonstrates a clear improvement in students' depth of knowledge across both time dimensions.

Considering the percentage distributions presented in Table 2, where the second and third categories ("Very Low" and "Low") as well as the fourth and fifth categories ("Average" and "High") were combined for clarity, a substantial proportion of students in the initial questionnaire (pre=11.5%, categorized as "Incorrect") were unable to propose didactic strategies for teaching the concept of time (Figure 7). Many of these students attempted to define time, albeit unsuccessfully, often referencing its units. For example, some responses included: "It is something intangible and irreversible, used to separate events and organize hours, days, weeks, months, and years." Others introduced related concepts such as "day, hour, minute, and year."

Meanwhile, the majority of students (pre=71.5%, categorized as "Very Low" and "Low") provided more accurate definitions of time; however, their didactic explanations remained underdeveloped. Illustrative responses included: "It is a continuous change observable through sunlight, which distinguishes day and night. Clocks allow us to track time based on the sun's position," or "I would explain that time is the process occurring as events in our lives unfold." Lastly, a smaller group of students (pre=17%, categorized as "Average" and "High") demonstrated the ability to provide more substantive didactic explanations, suggesting a more developed understanding of how to teach the concept of time.

Although these students were unable to elaborate extensively on their explanations, they demonstrated an awareness of the fundamental theoretical principles underlying the teaching and learning of measurement. For instance, one student proposed: "*I would explain time using examples, introducing concepts such as the present, past, and future. Initially, I would relate these concepts to familiar events in their daily lives. Gradually, I would expand the explanation by incorporating their daily routines, helping them recognize patterns over time. Step by step, I would introduce the concept of weeks as a collection of days, followed by months and years as larger units encompassing all previously mentioned timeframes*." This response illustrates an understanding of progressive learning, aligning with constructivist approaches to teaching time measurement.

Table 1. Dimensions and categories used in the phenomenographic analysis

Categories	Didactic Dimension of Time	Technical Dimension of Sundials
Incorrect or null	Does not give any didactic indications that support the understanding of the magnitude time. Example: *Time is the distance between one event and another.* *I wouldn't know how to explain it; it's a very difficult concept to define*.	Hardly knows the operation of a sundial and its parts. Example: *Just like the clock that we have at home. With two sticks.*
Very low	Gives vague indications of how she/he would explain the concept of time to her/his future pupils. Sometimes the definition of time is incorrect. Example: *It is the passage of hours, minutes and seconds, if they know them, otherwise I would explain it with days.* *It is a magnitude that cannot be seen, cannot be touched, but can be measured with different instruments. From there I would go on to the instruments, explaining them and so on.*	Recognizes the structure of a sundial and knows its basic operation. Example: *As the day progresses, the sun changes its position and the shadow it casts on the sundial is the indicator of the time.*
Low	Defines time in a little more depth but the didactic proposals are very vague, without being able to show any guarantee of being able to teach future pupils. Example: *Time is the period between two events that take place. And to give examples, I would show you different clocks (digital, analogical, solar, etc.) and we would observe and analyse them for a period of time in order to see the passage of time.*	Understands the movement of the sun and how it affects the shadow of the sundial. Lacks astronomical/geographical notions related to the sundial. Example: *The shadow that is created on the sundial itself indicates the time, if there is no shadow but if there is sun, it is 12 o'clock midday.* *The shadow that we see reflected on the ground or on the support where the sundial is located will change depending on the height and position of the sun. In this way (although it is not possible to use it at night because there is no sun), the shadow will be positioned in one place or another depending on the sun.*

Average	Gives didactic or pedagogical indications with criteria that will help future pupils to understand the magnitude of time, although they could be expanded on. Example: *First, I would explain it from the point of view of their daily life (closest context to them), through the routines they do during the day and little by little introduce the weeks as a set of days. Finally, I would introduce the months and the year as a set of everything mentioned above.*	Understands the movement of the sun and how it affects the sundial's shadow. Knows both units of time that a sundial can give (time and calendar) and knows some astronomical/geographical notions related to the sundial. Example: *With the help of the sun, because as it advances it is one hour of the day or another. And depending on where it arrives, it can indicate the season of the year.*
High	Has a clear concept of how to teach the magnitude of time to pre-school and primary school pupils, showing concrete clarifying examples. Example: *Time is the most abstract magnitude as it cannot be manipulated and, therefore, it is necessary to understand it as a necessity in everyday life. So, in the first place I would situate the need to divide the changes that we observe in nature (physical time) into periods that allow us to adapt our rhythms of life and to ensure that these periods are the same and universal for everyone. From here, we could start brainstorming ideas of periods that are known for dividing time, and we would come up with a specific unit that is international for everyone. I would also mention how measuring instruments have evolved up to the present day. I would therefore begin by mentioning how time measurement began. Then it would be time to talk about sundials, as well as other types of timekeeping: water, sand, etc.*	Understands the movement of the sun and how it affects the shadow of the gnomon. Knows the two units of time that a sundial can give (hourly and calendar) and has astronomical/geographical notions related to the sundial. Knows about the equation of time. Example: *Obviously, a sundial is not extremely accurate, and what's more, in general, the smaller it is, the less accurate it is. That's why we can look at the hours but not the minutes, we simply imagine them. Moreover, the hours with the greatest inclination in the shadow (first and last hours of sunshine) are the most complicated to show. In addition to marking the hours, it also marks the passing of the days thanks to its calendar.*

Regarding students' knowledge of sundials, as expected, their understanding of gnomonics was limited, given that this topic is not explicitly included in any academic curriculum. A significant proportion of students (pre=42%, categorized as Incorrect) were unable to explain the functioning of a sundial and demonstrated minimal familiarity with its characteristics. Some misconceptions included statements such as: "*It is very difficult to misread, but once the sun's shadow disappears, you have to start again from the beginning*," or "*It shows the hours and minutes*."

The majority of the remaining students (pre=56%, categorized as Very Low and Low) exhibited a basic understanding of how sundials work, primarily recognizing their limitations. Their responses often focused on the relationship between the sun's movement and the time displayed, such as: "*As the day progresses, the sun changes position, and the shadow cast on the sundial serves as the time indicator*," or "*Depending on the position of the sun, the shadow projected onto the sundial allows us to determine the hours and minutes at a given moment*." Some students also acknowledged its inaccuracy, stating: "*It cannot provide an exact time, meaning it does not display minutes or seconds*."

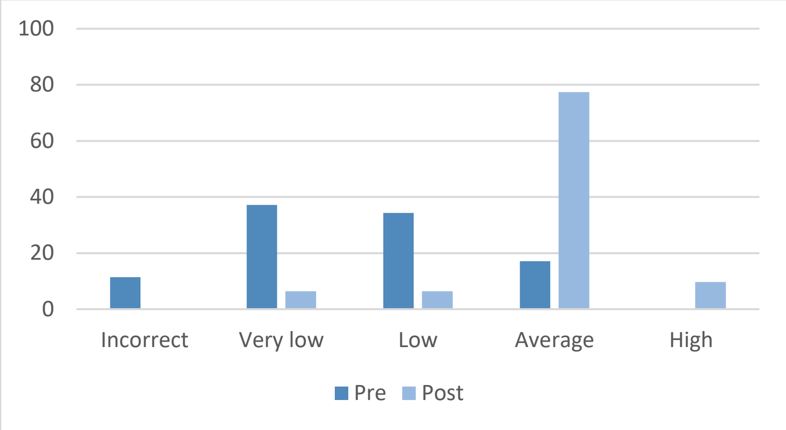

Figure 7. Proposals for teaching what time is (%)

Encouragingly, a small fraction of students (pre=2%, categorized as Average and High) demonstrated a deeper understanding of sundials. Their responses reflected more advanced knowledge, such as: "*With the help of the sun, as it moves, different hours are indicated. Depending on its position, the sundial can also show the season of the year*," or "*It is a functional instrument, but its accuracy depends on its placement and orientation, which can make it difficult to use. Additionally, it lacks precision in indicating the exact time.*" Finally, the phenomenographic analysis of students' responses before the implementation of the TLS indicates that their conceptualizations of both studied aspects—time measurement and sundials—were predominantly situated within the less complex categories (Figure 8).

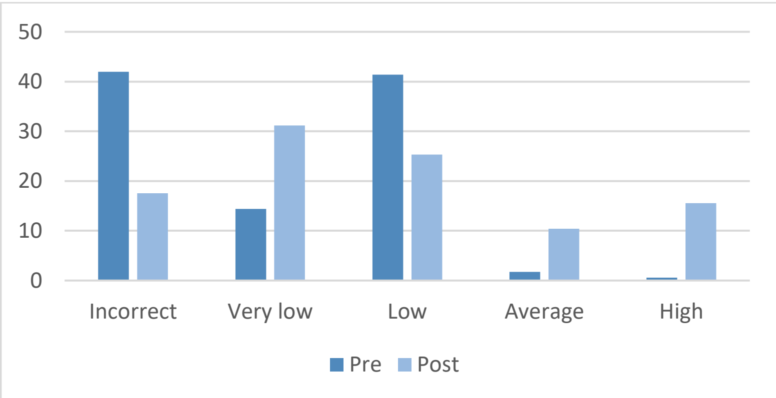

Figure 8. Knowledge about the operation of sundials (%)

After completing the didactic sequence, students' proposals for teaching time became more concrete and descriptive. Notably, all students (post=100%) were able to formulate didactic proposals, marking a significant improvement from the pre-test stage, where a portion of students (pre=11.5%) failed to provide any meaningful suggestions. Only a small percentage (post=13%) presented simple and superficial proposals, such as: "*I would explain that time is the process that unfolds as events occur in our lives, using examples involving clocks and sundials*."

The majority of students (post=87%) demonstrated a considerable improvement by presenting well-structured proposals incorporating didactic principles—an increase from the pre-test results (pre=17%). Their responses reflected a deeper understanding of time measurement and its historical evolution. For instance, one student suggested: "*To the previous definition, I would add an explanation of how measuring instruments have evolved over time. I would begin by discussing the origins of time measurement,* *followed by an introduction to sundials and other historical timekeeping devices, such as water clocks and hourglasses*." Another response emphasized the integration of historical context: "*I would expand on my initial definition by including different types of sundials, explaining why they emerged and their significance to both ancient and contemporary civilizations*." Similarly, another student stated: "*I would incorporate explanations on how time is measured, linking it to the spatial environment and the sun, which we now recognize as another timekeeping system. Additionally, I would include historical insights on the first civilizations that measured time using the sun and mathematical reasoning.*"

The frequent incorporation of historical references in students' post-test responses suggests that the historical narrative embedded in the activity significantly influenced their understanding and engagement. This finding highlights the effectiveness of integrating historical perspectives into the teaching of time measurement, reinforcing the role of contextualized learning in enhancing students' conceptual grasp.

Students' understanding of the technical aspects of sundials, including their operation and interpretation, showed significant improvement following the didactic intervention. The proportion of students experiencing serious difficulties in comprehending sundials decreased notably from 42% in the pre-test to 17.5% in the post-test. However, some students continued to exhibit conceptual gaps, as indicated by responses such as: "*We can observe the shadow's movement to track the passing minutes and hours. Sundials on vertical walls are less accurate and do not display time properly*." Another response revealed misconceptions: "*There are three types of sundials depending on their inclination, all of which must face south. They indicate seconds, hours, days, and even months*." Similarly, another student stated: "*Time is displayed through the sundial's shadow, which marks the passage of time as the sun moves*."

The percentage of students demonstrating only a basic understanding of sundials remained largely unchanged (pre=56% vs. post=56.5%). Their explanations primarily reflected fundamental concepts, such as: "*Sundials function during daylight hours, marking time from sunrise to sunset. They are limited to indicating the solar day*." Another student mentioned: "*If placed correctly, we can observe how the projected shadow shifts as the Earth moves throughout the day*."

Table 2. % analysis table of the responses grouping the second and third categories (Very low & Low) and the fourth and fifth categories (Average & High)

Didactic Proposals regarding Time		
	PRE	POST
% with constraints	11.42	0
% regular	71.42	12.90
% good	17.14	87.09
Technical Knowledge of Sundials		
	PRE	POST
% with constraints	41.95	17.53
% regular	55.74	56.49
% good	2.29	25.97

A key improvement was observed in the proportion of students who fully integrated the concepts studied, which rose from 2% in the pre-test to 26% in the post-test (Average & High). These students provided more in-depth explanations, demonstrating a broader understanding of sundials' function and their mathematical principles. For instance, one student noted: "*The sundial functions through the movement of* *the shadow projected by the gnomon. Its use extends beyond a single solar day; it can also indicate an annual calendar. Understanding its operation requires knowledge of trigonometry and geometry, making it a complex process*." Another response highlighted historical and astronomical insights: "*Sundials can incorporate a calendar, marking significant dates such as solstices (June 21, December 21) and equinoxes (March 21, September 21). Some sundials are equatorial, aligned parallel to the Earth's equator. However, all sundials must face south, a key shared characteristic. It is fascinating how ancient civilizations developed such abstract knowledge with limited resources. This learning has been valuable for my future teaching, as sundials present a compelling tool for instruction in upper primary education*."

The phenomenographic analysis clearly indicates a significant improvement in students' understanding of both concepts. However, the technical dimension of sundials still requires further reinforcement, particularly for students who continue to struggle with fundamental aspects of their operation. This challenge is largely attributed to the humanistic-social orientation of their prior studies, which lack a strong scientific-technical foundation. Despite these gaps, the findings suggest that the intervention effectively enhanced students' conceptual grasp of sundials. While some misconceptions persist, the overall increase in the number of students demonstrating a deeper understanding underscores the success of the didactic approach in fostering meaningful learning. Future instructional strategies should focus on strengthening the technical aspects to ensure a more comprehensive and well-rounded understanding among all students.

CONCLUSION

This research is situated within the framework of initial teacher training, preparing future educators to address the complex and abstract concept of time in their professional practice. The primary contribution of our TLS proposal lies in its advancement of research on the didactics of time in Primary Education. By employing an experiential methodology, we have actively engaged students in hands-on, minds-on learning experiences. These experiences involve a structured sequence of exploratory, interpretive, and creative tasks designed to promote formal thinking through models and interpretive reasoning, aligning with the principles of IBL as defined by Murillo et al. (2021).

Grounded in a constructivist pedagogical approach, this study examines pre-existing conceptions of time while enhancing prospective teachers' didactic competencies through the cognitive development, construction, and practical application of sundials. This approach facilitates a meaningful connection between research and classroom realities. The findings derived from phenomenographic analysis indicate notable progress, both in terms of deepening subject-specific knowledge and fostering innovative reasoning in the didactics of time-related concepts. Furthermore, the initial activities played a critical role in stimulating student motivation and fostering an appreciation for previously overlooked elements in their environment. Modeling the Earth's movement around the Sun enabled students to deduce seasonal variations in daylight duration. The cognitive design of a horizontal sundial at the North Pole served as a foundational model, allowing for its conceptual extrapolation to other latitudes. This approach not only enhanced students' ability to visualize sundials in different geographic locations but also deepened their understanding of their functionality and the significance of gnomon orientation. Subsequently, students were guided in the physical construction of a horizontal quadrant within small groups, which facilitated the interpretation of time and the cyclical movement of shadows throughout the calendar year.

The integration of a marked calendar, specifically identifying planting dates for crops, has effectively established a connection between time concepts and the eco-educational garden. This interdisciplinary approach, facilitated through the use of sundials, represents a key feature of our model, demonstrating the cultural and social relevance of time in an innovative and easily reproducible manner. By engaging with a range of perspectives and experiences, students have developed a more comprehensive and structured understanding of the subject. This process has expanded their conceptual framework, enhanced their confidence, and strengthened their ability to approach the didactics of time effectively. Consequently, we assert that sundials serve as a valuable pedagogical tool for teaching time in primary education, with the school playground functioning as a practical astronomical laboratory due to its accessibility and ease of implementation.

To develop the TLS presented in this study, a systematic process of design, implementation, evaluation, and iterative redesign—based on prior years' experiences—was undertaken. This TLS incorporates an epistemological analysis, ensuring that the learning objectives are aligned with the theoretical underpinnings of the concept of time, along with the appropriate assessment tools to evaluate student progress. Moreover, the didactic strategies employed establish a meaningful connection between conceptual knowledge and practical applications. By challenging students' preconceptions, this approach has led to cognitive contradictions that facilitate conceptual change, enabling learners to interpret and engage with reality more effectively (Duschl et al., 2011).

The teaching of time can be approached through various pedagogical frameworks. For instance, Xirouchaki and Boilevin (2019) propose the use of clepsydras (water clocks) to link time measurement with everyday experiences, while Tesar et al. (2016) advocate for a broader conceptualization of time that extends beyond chronological measurement to include cultural and subjective dimensions, such as cyclical time in indigenous traditions. These diverse perspectives contribute to addressing one of the most well-documented challenges in learning—the comprehension of time measurement (Ravanis & Kaliampos, 2018).

Within this educational framework, sundials serve as a distinctive pedagogical tool, integrating practical-experimental and interdisciplinary dimensions while incorporating cultural perspectives on time. Their didactic potential stems from their dual functionality: as instruments for measuring both time and the calendar, and as a means of connecting time to natural phenomena, particularly the Earth's rotational and translational movements. However, their implementation presents specific pedagogical challenges. First, their functionality is limited by climatic and geographical constraints, making them less effective in regions with minimal sunlight or at extreme latitudes. Second, their conceptual complexity necessitates age-appropriate pedagogical adaptations, such as simplifying the relationship between the Earth's rotation and the apparent motion of the Sun for younger students. Third, their use must be integrated with contemporary timekeeping systems, including time zones, which can be abstract and difficult to grasp for primary-level learners.

To optimize the effectiveness of sundials in education, we propose a gradual increase in conceptual complexity across different educational stages, aligning sundial models with students' cognitive development while reinforcing the connection between theory and practice. In primary education, horizontal sundials should be introduced to familiarize students with fundamental astronomical concepts, such as the Sun's daily movement. At the secondary level, vertical and analemmatic sundials can be incorporated, integrating geometric principles (e.g., angles and projections) and introductory trigonometry to facilitate an understanding of standardized time measurement and to promote critical thinking. In high school, equatorial and spherical sundials should be introduced, providing opportunities to engage with advanced mathematical abstractions, including spherical trigonometry and celestial coordinate systems, which serve as a foundation for further studies in engineering or physics. This progressive framework not only supports the assimilation of increasingly complex concepts but also aligns with the principles of cognitive scaffolding and the development of scientific literacy.

It is essential to acknowledge the considerable level of complexity that the construction of sundials posed for our students, particularly given that their prior mathematical training had not emphasized such intricate concepts. Despite these challenges, the overall assessment of the activity was highly positive. Several key difficulties were identified: (1) in constructing the equatorial sundial, some students struggled with transitioning from understanding its function at the North Pole to comprehending its operation at any location on Earth; (2) the construction of the horizontal sundial involved multiple intricate steps, which posed additional challenges. It is important to note that most participants did not have an academic background in experimental sciences, as their secondary education was primarily in the social sciences, limiting their exposure to technical skills; and (3) the concept and application of the equation of time proved challenging both cognitively and operationally for some students.

The phenomenographic analysis of the data reveals cognitive and procedural gaps in pre-service teachers' ability to teach time, underscoring the need for improved training and a broader range of instructional tools in time education. Nevertheless, the study also demonstrated significant progress in their didactic approaches, leading to more sophisticated conceptual understandings. Notably, students recognized the importance of incorporating the historical and contextual significance of sundials as a motivational foundation for teaching time. Additionally, they highlighted the pedagogical value of using empirical observations of the Sun's movement as an indicator of time and even suggested employing sundials as a classroom-based calendar. These findings affirm that the didactic strategy employed in this study has effectively facilitated the development of mathematical thinking through the use of sundials as a pedagogical tool.

Furthermore, this research contributes to the growing body of literature on TLS by expanding upon existing models (Guisasola et al., 2017). Future research should explore the affective dimension of time as a component of primary school mathematics education for pre-service teachers. Additionally, given the inherent complexity and adaptability of sundials, we propose extending their use to multiple educational stages, allowing for the exploration of diverse mathematical concepts across different levels of instruction.

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