

Elementary school teachers' experiences in engaging with digital technology in teacher professional development: The case of GeoGebra

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Abstract

Technological advances require teachers to be competent in utilizing them in mathematics learning. Teachers' success in teaching mathematics using digital technology cannot be separated from their mathematical and didactic knowledge. This study aims to reveal elementary school teachers' knowledge during their experiences in engaging with digital technology, GeoGebra, in teacher professional development programs. Data was obtained from the implementation of the Professional Development (PD) program in two schools, one in a private school focusing on the design of mathematical learning instruction integrated with GeoGebra and another in a public school focusing on the experimental process of teaching mathematics with GeoGebra. The data were analyzed using the Anthropological Theory of the Didactic (ATD) framework, specifically praxeology. The findings of this study reveal that elementary school teachers are aware of the need for theoretical aspects of a praxeology when designing mathematics learning instruction using GeoGebra in a PD program. Meanwhile, it is challenging for elementary school teachers to make use of GeoGebra instruction to support students' mathematical praxeologies. Therefore, the use of digital technology in teaching mathematics in elementary schools is still a significant challenge due to the mismatch between available technology and teacher needs.

Keywords: Anthropological Theory of the Didactic, GeoGebra, Teacher Professional Development, Teachers' Mathematical and Didactic Knowledge

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The development of technology in the last few decades and the Covid-19 pandemic require teachers to have adequate skills in using digital technology in learning. Particularly in mathematics learning, the challenges of technology-based learning are certainly greater because of the complexities that teachers must pay attention to (Loong & Herbert, 2018). Therefore, the use of technology by mathematics teachers generally only involves courseware and smartboards during classroom instruction and pay less attention to such dynamic mathematics software and interactive mathematics applications (Yao & Zhao, 2022).

Teachers need support to effectively integrate digital technology into their teaching practices. This can be done in the form of a teacher professional development program for teaching mathematics with digital technology. A study conducted by Thurm and Barzel (2020) indicated that a teacher professional development program for teaching mathematics with technology has a significant effect on teacher technology-related beliefs. This means that teachers have increased confidence in the importance of

using technology in mathematics learning when they gain sufficient knowledge and skills in using technology during the teacher professional development program. Meanwhile, Ratnayake et al. (2020) focused their study on a professional development program designed to assist teachers with digital technology task production for implementation in their classrooms. Through this program, teachers develop their knowledge and skills in designing more student-centered tasks using digital technology. Although digital technology allows for new approaches to designing tasks to increase comprehension and mediate mathematical debate, it has not been effortlessly integrated into teachers' teaching activities (Ratnayake et al., 2020). Therefore, supporting teachers to develop knowledge and skills in using digital technology in mathematics learning still really needs to be strengthened in teacher professional development programs, especially for those who have limited access to digital technology-based learning innovations.

In the context of Indonesia, several studies have been conducted to evaluate elementary and mathematics teachers' knowledge and skills in using technology to teach mathematics in the classroom (Mailizar et al., 2020; Mailizar & Fan, 2019; Putra et al., 2022). A survey conducted by Mailizar and Fan (2019) found that mathematics teachers have a severely insufficient understanding of ICT and its application in the classroom. Teachers' knowledge of mathematical software (e.g., Dynamic Mathematics Software (DMS), Dynamic Geometry Software (DGS)) was lower than their knowledge of general software (e.g., Ms. World, Ms. PowerPoint). However, teachers' knowledge of GeoGebra was the highest one among DMS and DGS software (Mailizar & Fan, 2019). Therefore, the present study aims to examine teachers' mathematical knowledge of the use of GeoGebra in designing mathematical learning instruction during teacher professional development programs. The study focuses on supporting elementary school teachers to engage with GeoGebra because the use of digital technology in teaching mathematics in elementary school is still very limited. The use of digital technology in elementary schools in Indonesia could reveal several challenges that education policymakers must pay attention to in the future.

Enhancing professional skills for mathematics teachers is one of the important solutions for reducing the issues in teaching and learning. Developing and refining teachers' mathematical, didactic, and technological knowledge is essential, and PD programs enhance teachers' mathematical knowledge and teaching practice (Lee & Vongkulluksn, 2023). The PD program is designed to connect teachers' learning and teaching practice through integrating aspects of the teaching profession and teacher reflection. Therefore, targeting teachers' knowledge could be a good way to change teachers' behaviors and improve student learning experiences (Lee & Vongkulluksn, 2023).

Due to the complexity of teacher knowledge and practice, some factors hinder the success of PD programs. Hill et al. (2018) explained some possible reasons for unsuccessful PD programs to develop teachers' knowledge and skills, especially in teaching mathematics, and those are categorized by Karsenty and Brodie (2023) into three aspects; teachers, school systems, and PD program. The first reason relates to teachers including a lack of willingness to implement PD materials, sense-making of interpreting the PD program, insufficient resources available for teachers to implement, and challenge to implementing an ambitious learning instruction proposed in the PD program. The second reason comes from school systems including less support from policy and curriculum, and conflict between school policy and PD ideas. The last reason relates to the PD program itself including less powerful from the PD facilitators to ask teachers to implement the PD ideas and the mismatch between the PD program and what the teacher needs. On the other hand, Zehetmeier and Krainer (2011) explain three factors influencing the success of the PD program, namely content, community, and context. Content is closely related to the material needed by the teachers to apply it in schools. The community is concerned with

exchanging and discussing experiences with other teachers and PD facilitators. Then, context is related to the supportive environment both inside and outside school. Thus, the demand for using digital technology in mathematics learning will certainly add to the challenges for teacher professional development.

In the current teacher PD program, teaching models enriched with innovative educational technologies should primarily be used effectively by teacher educators. Therefore, some previous studies have researched the use of digital technology in teacher education and professional development (Bennison & Goos, 2010; Putra et al., 2021; 2023; Ratnayake et al., 2020; Thurm & Barzel, 2020). The use of digital technology, such as GeoGebra, in teacher education has the potential and challenge to support pre-service elementary school teachers' mathematical knowledge (Putra et al., 2021). Pre-service elementary school teachers have a positive attitude toward the use of digital technology in the classroom because it can help students visualize abstract mathematical content. However, a lack of mastery of mathematical content is a challenge for them to be able to use digital technology appropriately. For example, Putra et al. (2023) tried to encourage pre-service elementary teachers to carry out an informal mathematical proof of the volume of 3-D shapes using GeoGebra, and only a few of them could come to deductive argumentation for a formula of the volume of a 3-D shape.

Similarly, the use of digital technology, such as GeoGebra, in teacher professional development programs also provides pros and cons. Some studies have shown that GeoGebra is a dynamic mathematical software that has the potential to support teacher PD (Bozkurt & Ruthven, 2017; Ziatdinov & Valles, 2022). It is because GeoGebra has a wide variety of mathematical tools combining many aspects of different mathematical packages and dynamically joins geometry, algebra, and calculus (Hohenwarter & Fuchs, 2005).

The use of digital technology, such as GeoGebra, in teacher PD programs has a positive impact on teachers' mathematical and didactic knowledge (Martinovic & Manizade, 2020; Santos-Trigo et al., 2021; Verhoef et al., 2015), attitude (Zengin, 2017), belief (Kul, 2012), and collaboration (Mavani et al., 2018; Misfeldt & Zacho, 2016). Teachers' perspectives regarding the use of GeoGebra-supported learning tools during in-service training reveal changes in their role in the classroom, benefits to students' cognitive and affective development, teaching quality; difficulties and disadvantages in using learning tools supported by GeoGebra; and their intention to use GeoGebra-supported learning tools in the future (Açıkgül, 2022). The teachers stated that they learned to use visualization and saw the value in student participation. Meanwhile, teachers found that incorporating GeoGebra into derived methods allowed them to think about how students make sense of learning activities in general (Verhoef et al., 2015). In addition, collaboration between teachers is a useful step to reduce the gap between having access to technology and adapting it for successful use in mathematics classes (Mavani et al., 2018). In addition, Misfeldt and Zacho (2016) also found that teachers' collaboration in designing and using technology-based scenario, GeoGebra, support teachers' competence with technology and their creativity for teaching mathematics.

Mathematical and didactic knowledge is an important consideration when teachers employ digital technology in the classroom. The gap between teachers' mathematical knowledge and their didactic knowledge related to developing mathematical learning instruction using digital technology will be a challenge in the teaching experiment in the classroom. Therefore, we must examine this issue through a theoretical lens; in this situation, researchers employ the Anthropological Theory of Didactic.

The theoretical framework for our study of teacher professional development is based on the Anthropological Theory of the Didactic (ATD) (Chevallard, 1992). ATD utilizes an epistemological and institutional approach to study "didactic phenomena" such as teachers' mathematical and didactic

knowledge (Rasmussen, 2016). Teachers' knowledge developed during the teacher PD program reflects the extent to which teachers can support students in learning mathematics. To evaluate it, ATD proposes a model to study human action in the notion of praxeology. A praxeology consists of two elements, praxis (practice) and logos (knowledge). The practical block is expressed by a type of tasks (T) and a technique (τ). The type of task is broadly understood, but could be explicitly stated (Rasmussen, 2016). An example of a type of mathematical task is how to add two fractions. A mathematical task of the type given to the students can be represented in several forms, such as symbolic and diagram representation, contextual situation, or presentation in digital technology. In the last case, one may represent the task on the dynamic software such as GeoGebra. To respond to the mathematical task, students need to use techniques to solve it. For a task of adding two fractions, $\frac{2}{3} + \frac{1}{2} = \dots$, a student can change the fractions into a common denominator and then add the numerators. To go beyond the practical block, the theoretical block aims to provide a rational discourse to justify and explain the technique. The first unit is known as technology (θ), in which one should distinguish it to the digital technology. A technology is mathematical argumentation of reasoning regarding to the proposed technique. For instance, to clarify the procedure of solving the task of adding two fractions is appropriate, one may justify based on the equivalent value of two fractions. Then, it comes a theory (Θ) to cover the explanation of the technology. However, the theory sometimes hardly observes especially in mathematics learning in a basic education level (elementary school). For example, the formal theory of rational numbers, i.e., two pairs, (a, b) and (c, d) in which $b, d \neq 0$, are called equivalent if and only if $ad = bc$, may not be introduced in an elementary school in the formal explanation, but the teachers should have sufficient knowledge of that theory.

Mathematics teaching is closely related to disseminating mathematical praxeologies (Miyakawa & Winsløw, 2019). Therefore, a mathematical praxeology goes beyond the notion of content knowledge because it describes not only the practical block but also the theoretical one. Meanwhile, content knowledge does not emphasize the difference between them. For instance, a teacher mostly starts the teaching process by giving a certain mathematical task to students, and at least a technique is needed by the students to solve such a task. Besides, a possible technology or theory may appear during the interaction between a teacher and students or among students. Therefore, the teacher is supposed to have sufficient knowledge to organize specific mathematical praxeologies, and this is known as didactic praxeologies. Some researchers may consider that a didactic praxeology is similar to pedagogical content knowledge. It goes beyond the idea of how to teach content to students, but to organize a didactic approach to support students in understanding both a practice and a theory. Therefore, teacher knowledge consists of didactic praxeologies that are inseparable from mathematical praxeologies.

Teachers do not only exercise and develop their mathematical and didactic praxeologies in the classroom or when otherwise interacting with students, but they develop those praxeologies while preparing their teaching and participating in various professional meetings and courses (Miyakawa & Winsløw, 2019). The development of teachers' mathematical and didactic praxeologies depends on several factors, including the education system of the teacher PD. For instance, Lesson Study in Japan is a forum for teachers to design and discuss lesson plans involving not only teachers but also experts from higher education institutes (Miyakawa & Winsløw, 2013). In Indonesia, a teacher is used to collaborate with others during *Musyawah Guru Mata Pelajaran Matematika* (Mathematics Teacher Forum) for junior and secondary school and *Kelompok Kerja Guru* (KKG, Teacher Working Group) for elementary school, without the presence of experts in that forum (Loeneto et al., 2022). Therefore, the intervention of the experts from a teacher education institution in Indonesia into a teacher PD program

could have an impact on the construction of teachers' mathematical and didactic praxeologies. Based on the theoretical framework of ATD, the research questions addressed in this paper are as follows:

1. What mathematical and didactic praxeologies are developed by elementary school teachers' collaborative work on designing mathematics learning instruction using GeoGebra in a PD program?
2. What mathematical and didactic praxeologies are shared by elementary school teachers during their teaching experiences of mathematics learning using GeoGebra?

METHODS

Research Design

This study was based on ATD, specifically, praxeologies applied as a content analysis technique within a qualitative approach. According to Lincoln and Denzi (2003), a qualitative study seeks to preserve and analyze the situated form, content, and experience of social action. The purpose of qualitative study is to describe and interpret issues or phenomena systematically from the point of view of an individual or population being studied and to generate new concepts and theories. Thus, we describe the purpose of the study to portray, analyze, and interpret elementary school teachers' mathematical and didactic knowledge in the PD program. The focus of this study is to analyze and report what mathematical and didactic praxeologies are constructed and discussed during the PD program.

To support teachers' experiences in designing and implementing mathematics instruction using GeoGebra, workshops were conducted with teachers from two schools. The first workshop was a day workshop (approximately 6 hours) conducted in a private elementary school in Pekanbaru, Riau Province, Indonesia. This workshop consisted of a lecture, teachers' experiences constructing a mathematical task using GeoGebra, exploring some mathematical tasks on GeoGebra, constructing a lesson plan, and presenting, and discussing their ideas with other participants. The second workshop was a one-and-a-half-day (approximately 9 hours) workshop conducted in a public elementary school in Dumai, Riau Province, Indonesia. The first day of the second workshop was the same as the first workshop, and the second half-day was the teachers' experience of teaching students in their classes using GeoGebra and teachers' reflections.

This study was conducted in two schools, one in Pekanbaru and one in Dumai, to collect more representative data. The primary goal of this study is to investigate the praxeologies that teachers share when collaborating. The workshop in Dumai took longer because the teachers wanted to implement the design in their classrooms.

Participants

Participants of the first study were 16 female teachers from a private elementary school in Pekanbaru, Riau province, Indonesia. Most teachers have educational backgrounds on education in other subjects, and only 3 teachers have mathematics/mathematics education background who teach mathematics in upper grades of elementary school (Table 1). More than 70% of the participants had teaching experiences more than 16 years. Meanwhile, the participants of the second study were 25 teachers (only 1 male teacher) from a public elementary school in Dumai, Riau province, Indonesia. Most teachers have a background in education in other subjects and elementary education, and several teachers graduated from senior high school. The range of teaching experience of the teachers in this second study was between 6 - 20 years.

Table 1. Teacher demography in Pekanbaru and Dumai

Participants Background	Demography Character	Elementary School in Pekanbaru		Elementary School in Dumai	
		Number	Percentage	Number	Percentage
Gender	Female	16	100%	24	96.00%
	Male	0	0%	1	4.00%
Educational Background	Elementary Education	1	6.25%	8	32.00%
	Mathematics/Mathematics Education	3	18.75%	1	4.00%
	Education in Other Subjects	9	56.25%	9	36.00%
	Others	3	18.75%	7	28.00%
	5 years or less	0	0.00%	2	8.00%
	6 - 10 years	1	6.25%	6	24.00%
Teaching Experiences	11 - 15 years	1	6.25%	7	28.00%
	16 - 20 years	6	37.50%	7	28.00%
	21 - 25 years	6	37.50%	1	4.00%
	26 years or more	2	12.50%	1	4.00%

In constructing mathematical tasks using GeoGebra, the participants in the first study were divided into four groups, and the groups were based on teaching grades, except for group 4 were mathematics teachers teaching upper grades of elementary school (grade 4 to 6). Each group consisted of 4 teachers. Meanwhile, the participants of the second study were divided into 6 groups, consisting of 3-5 teachers. The groups were based on teaching grades because mathematics subjects were taught by classroom teachers.

Data Collection

Data from this study were obtained from the two workshops conducted with two different school types in Riau province, Indonesia. The data collection techniques consisted of focus groups and observation. The focus groups were conducted during the workshops. Meanwhile, the observation was conducted during teachers' teaching experiences. The data were collected in August 2022. The first data, called episode 1, was from a workshop in a school in Pekanbaru focusing on teachers' activities in designing learning instruction using GeoGebra. The second data, called episodes 2 and 3, were from the workshop and teaching experiment in a school in Dumai focusing on teachers' teaching experiment of their design of learning activity using GeoGebra. We present the case of two classes, each class was taught by two teachers, one was the main teacher, and the other was the assistant teacher (episode 2). The assistant teacher's tasks were to help the main teacher prepare the lesson and to manage students' behaviors during the lesson. Then, the data in episode 3 presents teachers' reflections on teachers' teaching experience. The participants involved in the reflection activity were the teachers from the second study.

Data Analysis

Data analysis was carried out from video recordings when teachers discussed their lesson plans of mathematical learning instructions based on GeoGebra (episode 1), implemented two mathematical learning instructions in classrooms (episode 2), and reflected on the implementation of the teaching experiment (episode 3). The analysis was carried out using a grounded theory approach where mathematical and didactic praxeology was not hypothesized previously (Rasmussen, 2016). The analysis



focused on passages that presented mathematical and didactic praxeologies that emerged from discussions and learning carried out by the teachers in the workshops. Therefore, the praxis and logos aspects that appear in each episode become the guideline for interpreting the data from this study.

RESULTS AND DISCUSSION

The results of this study are presented in three sections. The first section presents and discusses the results of teachers' design of learning instruction using GeoGebra from a workshop that has been done. Then, the next section provides two examples of teachers' teaching experiences after participating in a workshop on designing mathematical instructions using GeoGebra. The last section presents the discussion during the reflection on the implementation of teaching experiments by the teachers in their classrooms.

Teachers' Design of Learning Instruction Using GeoGebra

Episode 1: Teachers' Presentation and Discussion of Learning Design

The main aim of the workshop is to support teachers in developing and designing mathematics learning instruction using GeoGebra. After introducing GeoGebra to the teachers, and letting them explore some GeoGebra applets, we asked them to work in small groups. We had four groups. Group 1 to 3 were classroom teachers from grade 1 to 3, and group 4 was mathematics teachers for the upper grades of elementary school (grade 4 to 6). The teachers discussed the mathematical contents to be taught in their classes and considered how they could develop learning instruction using GeoGebra. We present two cases from group 2 and 4.

The first group presented their work from a group of teachers teaching second-grade students. The teachers chose the topic of subtraction of natural numbers. Because they did not find the GeoGebra applet about it, they took the initiative to use the GeoGebra applet about subtracting integers as a medium for teaching this topic (Figure 1). The teachers in group 2 chose this topic because students still tend to use fingers even though they have been triggered by several learning strategies such as using games in learning. This was expressed by Mrs. Isra during her presentation and discussion at the workshop.

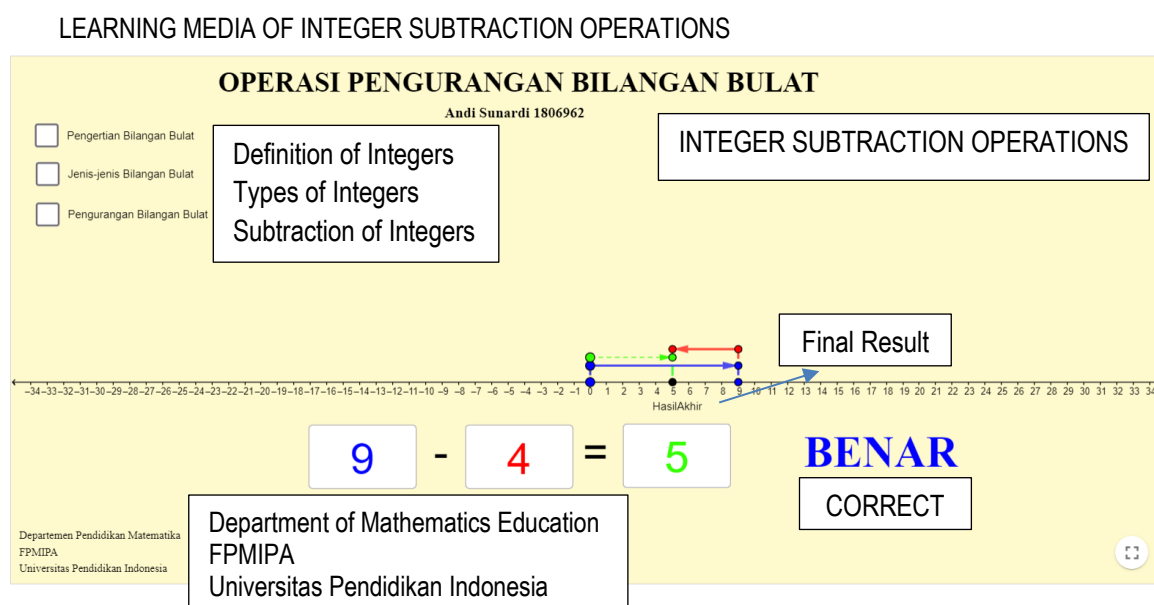


Figure 1. A GeoGebra applet of subtraction of integers

... The background, we chose the operation of subtracting integers using GeoGebra, is because after we evaluated it with the second-grade teacher working group (KKG), students were still subtracting using their fingers. [Therefore], we tried to teach this operation of subtracting integers using a number line. Here, we will see something like this (while moving the cursor in the GeoGebra applet which is displayed on the projected screen). We have tried all this time with subtraction, either through games, we look for alternatives by arranging the children [in line], then, for example, some children are taken away like that, but when it comes to a [written] task, the students return to counting using their fingers. So, we try to treat it again using a number line, like this (while moving the cursor in the GeoGebra applet as seen in [Figure 1](#)) ... (Mrs. Isra, A second-grade teacher).

While Mrs. Isra continued her explanation, Mrs. Rina, a mathematics teacher for sixth grade, protested regarding the incompatibility between the content in the curriculum and the content presented in the GeoGebra applet. Second-grade students have not studied subtraction of integers yet instead of studied natural numbers. Then, it was clarified by Mr. Isra. She said that the GeoGebra applet chosen was tailored to the needs of second-grade students, namely subtracting natural numbers only. This was finally accepted by other teachers. Then, Mrs. Isra continued her explanation of how the group organize the learning activities on that topic.

... We introduce students using real objects. After that, we try, to modify it. We use this application to increase students' level of understanding. Then we use the thematic textbook and/or mathematics textbooks, even though the [mathematics] textbook may be [from] another country (mathematics textbooks in the new Indonesian curriculum are adapted from Japan). ... We combine 3 learning methods, with the concrete objects, then we use it (the activity on the GeoGebra in [Figure 1](#)) with a lot of visualization, and then we apply it in solving mathematical tasks in the textbook. So, we hope that the combination of 3 learning methods can improve students' understanding in the field of mathematics learning. We will adjust it because we are still in the addition and subtraction topics and will move to multiplication [in the second semester], but students must memorize multiplication, but not yet. (Mrs. Isra).

Mrs. Isra emphasizes that learning of subtraction of natural numbers in second grade is carried out in 3 strategies, that support each other. Digital technology, in this case, the GeoGebra applet, is used as a bridge for the students to understand mathematical concepts from concrete to formal mathematics. On the other hand, digital technology acts as a tool in helping students visualize abstract mathematical concepts.

When the researcher, the first author, tried to clarify the link between the curriculum in the second grade to the mathematical contents to be taught, Mrs. Isra explained the choice of the topic to be taught.

The subtraction in the curriculum has reached hundreds, but the concept is that students must understand from the start, namely from units to tens, which is gradual. Why are we in the KKG team a bit slow now to catch up on the topic we understand that students are in a pandemic in the first grade, so we never blame the first-grade teachers. Some of the students cannot add, let alone subtract, but we are trying, the four of us, to find a solution together. We start from the bottom so that later the students understand the mathematics concepts.

So, why is this an example of an easy number, if we give a higher number, for example, 320 is minus 27, right, 320 is a jump of 20, that's the concept, those are the steps. (Mrs. Isra)

Mrs. Isra provided her explanation for choosing small numbers for the subtraction task because she focused more on students' conceptual understanding of the meaning of subtraction. Then, Mrs. Maam, from first grade, confirmed what was conveyed by Mrs. Isra, namely the need for connections between mathematics lessons in the first grade, the second grade, and so on. She emphasized the vital mathematical contents to be taught in each grade, so teachers can focus on them.

Mathematics learning in the second grade connects with the first grade, and the third grade connects with the second grade. Maybe we need to add more essential basic competencies. That's what we want to know, namely the essential competencies that we must teach in the first semester and the following semesters. (Mrs. Maam, a first-grade teacher)

The second group presented a topic on measuring the volume and surface area of a cuboid. The reason is choosing this topic is because cuboid is the foundation for students to understand other 3D shapes. Group 4 used the GeoGebra applet developed by Brzezinski (2022) as presented in Figure 2. The presentation of the discussion was led by Mrs. Rina, a mathematics teacher in sixth grade.

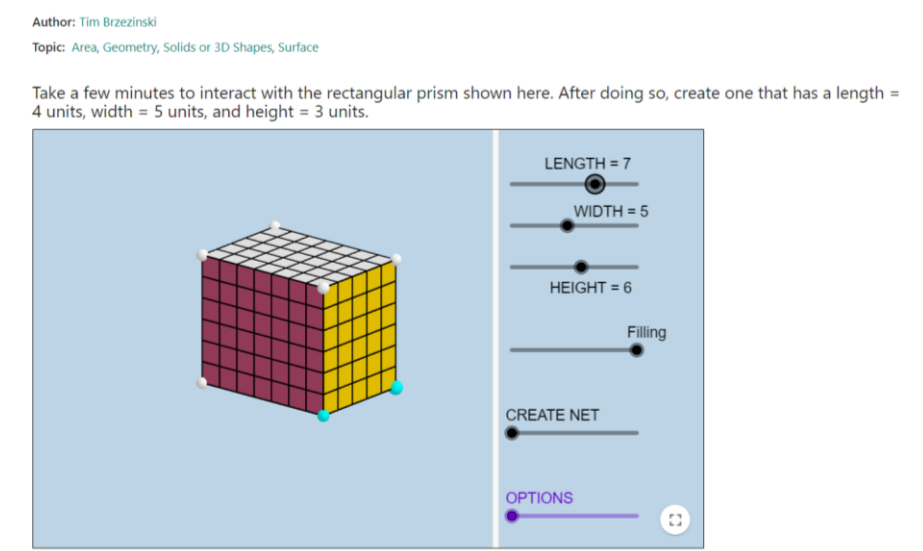


Figure 2. A GeoGebra applet of cuboid

... in the fifth grade, it has been explained that volume is the area of the base times the height, so whatever a 3D shape is, it is the volume of the base times the height. We chose a cuboid, the base is a rectangle, so here we use the area of the base, which is length times width, so we only multiply with the height. We only multiply 7 times 5 times 6, 210, that's the volume.... The problem is the surface area. We need to understand the surface area of a cuboid. A cuboid is a geometric figure whose three opposite sides are the same area, which means we just multiply these 3 areas, and we can multiply by 2, or add the same area twice. That is what students need to know. Like we are in this room, the top and bottom are the same, the left and right are the same, and the front and back are the same. The students

had difficulty when the cuboid did not have a cover. But this [applet] is cool. We can immediately open it. We can show directly to the students the nets of cuboids (Figure 3). The colors have been differentiated; the pink ones are the same, so we only look for this one area.... What we need to emphasize is, that there are students who write 5 times 6, and there are students who write 6 times 5. Two times 5 times 6.... The product of two numbers can be reversed, or it is called commutative property. ...Well, there was a student who did it separately, the pink one first, the yellow one, then the white one, then he added more. We do not blame him. The important thing is that he gets the concept. (Mrs. Rina)

Author: Tim Brzezinski

Topic: Area, Geometry, Solids or 3D Shapes, Surface

Take a few minutes to interact with the rectangular prism shown here. After doing so, create one that has a length = 4 units, width = 5 units, and height = 3 units.

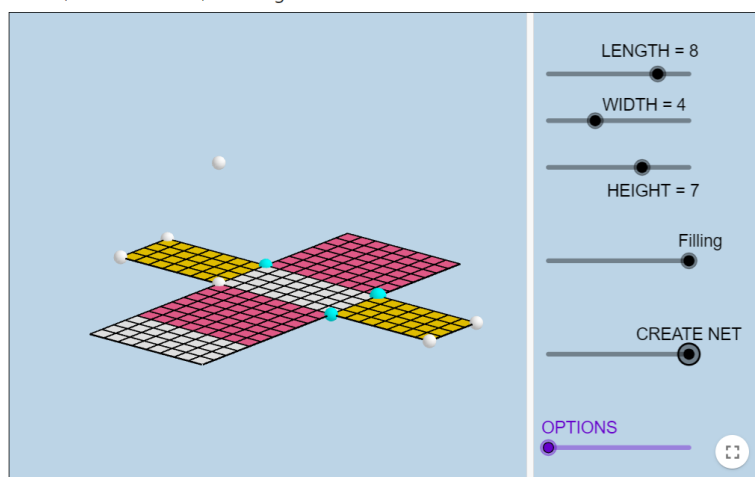


Figure 3. The nets of cuboid

Mrs. Rina explained the reason why her group chose the topic of volume and area of cuboids. It is the foundation for students to study other 3D shapes. She considered that the visualization using the GeoGebra applet could help students to evaluate how to find the surface area of a cuboid.

A third-grade teacher, Mrs. Noni, asked whether the selected GeoGebra applet could be used to explain the edges and sides of the cuboid. Mrs. Rina explained that it could not be used to explain the edges because when it was opened into the nets, the number of edges increased. This can cause students to experience misconceptions regarding the edges of a cuboid. Then, another teacher, Mrs. Desi, asked a question as follows:

...in the application that occurs when the volume is known, looking for one of the edges, for example looking for the height or length. This has been a dilemma for us all this time. By using media like this, how can we increase students' understanding, when one side is to be found? (Mrs. Desi, a third-grade teacher)

Then, Mrs. Rina provided the answer to that question, but she did not connect the answer to the media, the GeoGebra applet, that had been chosen. She tended to explain using inverse multiplication to find the unknown variable of the tasks as explained below:

When we know the results, we must first know the factors shaping the results. So that is what students need to know. Some teachers tend to teach like this, when 5 times 3 equals 15, when one edge is unknown, they immediately teach 15 divided by 3. This is what we should not teach. We must ask the students first, how many times 3 equals 15. This is what should be done. Let students work first, and do not interchange the formula first. Later, they will find the concept, and they will immediately know what is missing. ... Well, some teachers, when l (length) is asked, it is the same as v (volume) divided by w (width) times h (height). Knowing this makes students lazy to think. (Mrs. Rina)

Mrs. Rina emphasized that teachers should not give the students a formula to find an unknown variable instead of giving them a chance to find the connection among variables. The students need to understand the concept rather than just the procedure.

Praxeological Analysis of Episode 1

The summary of a praxeological analysis of episode 1 is presented in [Table 2](#). A type of didactic task presented by this group was to subtract two natural numbers using a number line. The reason to teach students to use number lines is that many students still use fingers to subtract two natural numbers. This technique cannot be used by students when they are subtracting tens, so it can slow them down in carrying out number subtraction operations.

Table 2. Summary of mathematical and didactic praxeologies in episode 1

Cases	Codes	Mathematical Praxeology	Didactic Praxeology
Second Grade	Type of Task	To subtract two natural numbers	To teach students the subtraction of two natural numbers
	Techniques	Subtraction by tens and ones	Number lines using GeoGebra
	Logos	Place value	To move from counting by fingers to flexible counting
Sixth Grade	Type of Task	To find the volume and surface area of a cuboid	To teach students to understand the meaning of the volume and surface area of a cuboid
	Techniques	The relationship between edges and the volume, and sides and the surface area	Visualize the relationships between edges and the volume, and sides and the surface area of a cuboid using GeoGebra
	Logos	Commutative law of multiplication	To move from mastering the procedure to the meaningful conceptual

A group of mathematics teachers who teach in upper grades discuss didactic praxeology related to the volume and surface area of cuboids ([Table 2](#)). GeoGebra helps students to understand the relationship between edges and the volume, and sides and the surface area of a cuboid. In line with this praxis, the teacher emphasized students have meaningful concepts rather than just using procedural techniques that students have to memorize. The teacher expected students to develop their logos of the commutative law of multiplication.

Teachers' Teaching Experiences Using GeoGebra

Episode 2: Teachers' Teaching Experiences

In a workshop conducted in the school in Dumai, we had a chance to observe teachers' teaching experiences using GeoGebra. We present two examples from two teachers who teach first and fourth-grade students.

The first-grade teacher teaches students numbers up to 10. This topic was chosen because the students began to learn to count numbers up to ten and to write the symbolic representations of those numbers. The teacher applied the designed instructional task with their colleagues from the previous day of the workshop. She used a finger addition task designed by Pfaffle (2021) to explore students' number sense (Figure 4).

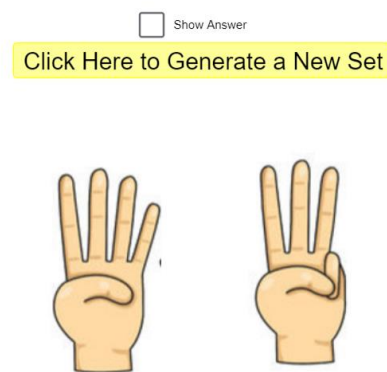


Figure 4. The finger representation on the GeoGebra applet

The teacher, Mrs. Mia, started the class by presenting two examples of finger representation on the GeoGebra applet and asked all students to answer how many fingers there were. She said “Let’s we start the learning process for today. Let’s look at it together, let’s count together, okay?”, while she showed the seven fingers on the screen (Figure 4). All the students started to count from 1 to 7. Then, Mrs. Mia asked the students “How many fingers?” and all students said seven. It was followed by another example of 3 fingers with a similar didactic technique.

Mrs. Mia continued the class by asking some students to come to the front and try to figure out how many fingers were presented on the screen. Most students could find the correct answers for the finger representation using counting one by one. A student (Adi) did a skip counting from 5 directly to 7. The teachers asked him to recount them, and he finally could count correctly.

The second example comes from a fourth-grade teacher, Mrs. Rani, who teaches multiplication of natural numbers. She chose a task of multiplication as a repeated addition and used the GeoGebra applet designed by Reddy (2021). Mrs. Rani started the class by introducing some mathematical symbols of operation (multiplication, addition, subtraction, and division), gave an example of $2 \times 3 = \dots$, and explained its meaning as twice the number of 3. She wrote those symbols and the example of the task on the whiteboard. Then, she informed the students that she would explain the multiplication of natural numbers using picture representations on GeoGebra.

Typical Indonesian teachers, Mrs. Rani first presented the task of multiplication of natural numbers on GeoGebra and asked students to answer altogether. The following is an excerpt from their discussion.

Mrs. Rani : Continue with the next task. Now, let's look at this, 8 times 7, which means 8 times the number 7, which means there are 8 boxes, how many boxes?

Students : 1, 2, 3, 4, 5, 6, 7, 8 (All students counted the boxes altogether while Mrs. Rani pointed them one by one)

Mrs. Rani : 8 is the number of boxes. Now, what is 7? What's this?

Students : Balloons

Mrs. Rani : Well, colorful balloons. How many balloons does each box contain?

Students : 1, 2, 3, 4, 5, 6, 7 (All students counted the balloons altogether while Mrs. Rani pointed them one by one)

Mrs. Rani : Okay seven. Seven plus seven plus seven plus seven plus seven plus seven plus seven plus seven. Who can count? Who would like to come to the front? OK, first, try counting how many? Calculate first, how many [balloons]?

A student, Rudi, came to the front of the class and began to count the balloons one by one (Figure 5), while other students also started to count by pointing their fingers at the objects on the screen.

Rudi : 55

Mrs. Rani : Still wrong. Does anyone know the answer?

Dodi : 54

Mrs. Rani : [Is it] right? Still wrong. Does anyone know the answer? Come on, try it.

Danu : 48

Mrs. Rani : It is still wrong. Try to pay attention [and] just count it slowly.

Anggi : 54

Mrs. Rani : A little more. Come on, who can?

Joko : 56

Mrs. Rani : 56. [Is it] right? Let's check first. Okay. (The teacher clicked the check bottom, and it informed that was correct)



Figure 5. Students try to find the results of the multiplication task presented on the GeoGebra applet

The excerpt above shows the interaction between the teacher and the students to find the correct answer of 8 times 7. The students struggled to find the answer for the task due to the mathematical technique used by the students suggested by the teacher. The students still used to count one by one to find the correct answer to the multiplication task. Besides, the objects presented on the GeoGebra are not well structured (for instance, 7 balloons can be represented as 5 balloons and 2 balloons), so the students could come to more flexible mathematical techniques. On the other hand, the teacher also did not try to guide the students to come to more advanced mathematical techniques to figure out the total balloons on the task. She seemed to focus on finding the correct answer rather than supporting students' mathematical reasoning.

Praxeological Analysis of Teachers' Teaching Experiences

The summary of mathematical and didactical praxeologies from Episode 2 is presented in [Table 3](#). The main didactic task of the teaching experiment in the first grade is to find out how many objects are represented by fingers in the GeoGebra applet. The didactic technique is asking students to count the number of fingers one by one. Therefore, the didactic praxeology that appears during first-grade learning is still in the praxis part and has not touched the logos one.

Table 3. Summary of mathematical and didactic praxeologies in episode 2

Cases	Codes	Mathematical Praxeology	Didactic Praxeology
First Grade	Type of Task	Addition up to 10	To know number facts up to 10 using finger representation in GeoGebra
	Techniques	Counting one-by-one	To count fingers one by one
	Logos	-	-
Fourth Grade	Type of Task	Multiplication of whole numbers	To teach students multiplication of whole numbers using balloon representation in GeoGebra
	Techniques	Counting and adding numbers	Counting many balloons in each group and then adding them all together
	Logos	Multiplication as a repeated addition	Direct instruction of multiplication as a repeated addition

The didactic task of the teaching experiment in the fourth grade is the basic multiplication of whole numbers ([Table 3](#)). Counting the objects one by one is the main mathematical technique proposed by the students as well as the teacher to solve the task. This makes many students have difficulty finding the correct answer to the given task because they make mistakes in counting many objects, namely skip counting. There is no stimulus to students with questions that can reveal their technological discourse in choosing the right technique to find answers to the given mathematics task. In the end, the teacher tried to provide direct instruction of multiplication as a repeated addition to their students.

Teachers' Reflection on GeoGebra Workshop

Episode 3: Teachers' Reflection

To evaluate the GeoGebra workshop that has been conducted with teachers from the elementary school, we organize a discussion with the teachers and the school principal. The discussion aims to reflect what



the teachers have learned from their teaching experiences using GeoGebra in the classroom. The discussion was attended by the researcher (first author), some teachers, and the school principal. The teachers reflected on their teaching experiences on the use of technology in general rather than focusing on how it was used regarding the mathematics praxeologies to be learned by the students. We start with an argument of a sixth-grade teacher.

"For me as a sixth-grade teacher, the obstacles I faced were probably only the first time I tried it. For the next time, I can do it better. That is all in my opinion. For other teachers, the obstacle may be their first time teaching [using technology]. If it is about teaching skills, maybe the teachers are used to doing it, but the facilities and [digital] infrastructures make us stiff, because we usually use a blackboard, this time we use a laptop. That's all, sir, in my opinion from grade 6." (Mr. Umar)

Mr. Umar reflects on the general challenges he faces when teaching using technology. This was his first experience using digital technology, especially GeoGebra, with students. He has confidence that teachers will be able to do it for the next time. The same thing was also expressed by Mrs. Mia, a first-grade teacher, that she was still hampered by using digital technology in teaching as stated below:

"Regarding the learning carried out this morning, honestly it was very good. Maybe I do have shortcomings, in terms of lacking IT. In the future, I might try to study it again. I feel very good teaching with technology like this. So, children quickly grasp the learning material. Hopefully, in the future, I will try to learn more and be enthusiastic." (Mrs. Mia from first-grade class)

Mrs. Mia has a positive attitude toward the use of technology in the classroom although she does not have sufficient competencies in using technology. She thought the students learned the topic, in this case, numbers up to ten, faster than the traditional approach.

To encourage the teachers to reflect on what had been implemented in the class today, the researcher, the first author, asked the question "Today students learned using GeoGebra, how did students respond to learning natural numbers?" This question was responded by Mrs. Rita, who partnered with Mrs. Mia, as follows"

"Students understand more quickly because they see how many fingers are upright, so they count the upright fingers, so they know more quickly how many digits or numbers are on the upright fingers. So far, we only write on the blackboard, the number 5 for example, and the number 6, sometimes they are not interested. They can memorize 1 to 10, but sometimes if we point to an object at random, they do not know what number it is. But if you use your fingers, from what they see, the fingers are upright, and at the same time they count, so they will know quickly, that's all, sir." (Mrs. Rita)

Mrs. Rita compared the learning carried out today using finger visualization in the GeoGebra applet with what is usually done in class, namely writing numbers on the blackboard. She realized that the learning carried out was much better than what had usually been done so far.

Praxeological Analysis of Teachers' Teaching Experiences

Teachers reflect on learning in general rather than specifically on the mathematics learning process that has been implemented in the classroom. Thus, we do not find them explicitly sharing the mathematical and didactic praxeologies of what has been done in the classes. Even though Mrs. Rita reflected that learning to recognize numbers with fingers was more helpful for students than what was usually done in the class, she was not able to justify how visualization with fingers was able to develop students' number sense in a better direction. Therefore, teachers assume that learning that focuses on mathematical praxis, using procedural techniques to solve mathematical tasks, has been considered successful in supporting students' mathematical learning. So, the theoretical block is very difficult to appear in the classroom because teachers are still unable to orchestrate mathematics learning to trigger that block.

Discussion

This study has provided insights into how elementary school teachers' experiences in designing, experimenting, and reflecting mathematics learning using GeoGebra. The current study analyzed mathematical and didactic praxeologies that were developed collectively by elementary school teachers during their participation in the teacher PD programs.

Findings from episode 1 demonstrate mathematical and didactic praxeologies developed by elementary school teachers in collaboration by integrating GeoGebra in mathematics learning in elementary schools. A mathematical type of task proposed by both groups are fundamental problem for students to solve more advanced problems. Compared to some previous studies, such as Miyakawa and Winsløw (2019), the mathematical task proposed by Indonesian teachers still concerns basic mathematical competencies rather than problem-solving and mathematical reasoning. Therefore, to solve such a task, students may not need technological discourse to justify the technique. Meanwhile, a mathematical task proposed in a Japanese professional development program, lesson study, focuses on mathematical problem-solving tasks that require students' technological and theoretical discourse (Miyakawa & Winsløw, 2019). Apart from that, the Covid-19 factor is a rationale for teachers to lower mathematics learning standards by focusing on the core of fundamental mathematical competencies in teaching mathematics content. This is in line with what was expressed by Mrs. Isra many students were not yet able to add two natural numbers, so they still tended to do calculations using their fingers as a mathematical technique for solving the tasks.

Mathematical tasks discussed by the teachers are closely related to the didactic tasks that will be taught to elementary school students. Since the PD program directed them to integrate GeoGebra in producing didactic tasks, some teachers faced a dilemma when the activities in GeoGebra were not able to fully help them visualize mathematical tasks and associated techniques. Therefore, a lack of available learning activities in GeoGebra results in a mismatch between the technology used and the learning goals to be achieved. This shows that the shortcomings of teachers in implementing plans in PD programs can be attributed to the inability of the technology chosen to support teachers in developing learning (Hill et al., 2018; Karsenty & Brodie, 2023).

Compared to the explanation and discussion of Mrs. Isra, Mrs. Rina, a sixth-grade mathematics teacher, provided more detailed mathematical and didactic praxeologies regarding the volume and area of cuboid. She did not only focus on the mathematical task and techniques to solve such task, but she also gave technological discourse to justify a mathematical technique. For instance, she mentioned that "*a cuboid is a geometric figure whose three opposite sides are the same area*". Through this definition, students can realize how to find the surface area of a cuboid. We may say that Mrs. Rina's proposed

didactic technique supports students gain meaning of the surface area of a cuboid and realize the meaning of the procedure to find its area. Besides, she also considers the usefulness of GeoGebra to visualize the surface area of the cuboid. Therefore, we may argue that digital technology, such as GeoGebra, has the potential to support teachers with sufficient mathematical knowledge. These findings support the study conducted by Putra et al. (2022) that educational background influences teachers' mathematical and didactic knowledge. Teachers with a strong mathematics background could design mathematics lessons better than general teachers. As found by Bozkurt and Ruthven (2017), teachers' success in managing a variety of aspects of classroom teaching related to GeoGebra use is closely related to their ability to incorporate technology-mediated tasks in line with their pedagogical goals, prepare their students to use technology efficiently, adapt the format for classroom activities, and expand the curriculum script for the topics studied. However, due to limited experience with employing digital technology in learning, the teachers in this study do not yet own these characteristics.

The findings of episodes 2 and 3 relate to the RQ2 regarding mathematical and didactic praxeologies shared by elementary school teachers during their teaching experiences of mathematical learning using GeoGebra. Both teachers, first and fourth-grade teachers, proposed mathematical types of tasks related to the operation of natural numbers. The first-grade teacher chose finger representation as a didactic technique to support students' number sense. Mrs. Mia is only concerned about how students find the correct answers to the given mathematical tasks. Therefore, the mathematical technique that emerged was just counting one by one. There was no technological discourse that appeared while the teacher, Mrs. Mia was conducting the teaching process. The same thing was also found in teaching multiplication of natural numbers in fourth grade of elementary school. The visual representation presented in GeoGebra is not yet capable of developing mathematical techniques, let alone technological discourse to justify it.

Many factors make it difficult for teachers to implement technology-based mathematics learning, including the challenge of implementing the learning instruction designed in the PD program (Hill et al., 2018; Karsenty & Brodie, 2023). The teacher and the facilitator do not participate in some possible scenarios that may appear in the classroom. Besides, the context of using technology may not be so relevant to the school environment (Zehetmeier & Krainer, 2011) as mentioned by some teachers during the reflection that using digital technology in teaching is their first experience. Although the Covid-19 pandemic has changed the learning mode from face-to-face to online learning in most countries, in many schools in Indonesia, especially at the elementary school level the learning process still does not use digital technology (Pramana et al., 2021).

CONCLUSION

The teacher PD program is believed to be a forum for teachers to develop their professionalism and continue their knowledge and skills. Implementing digital technology in learning is not a simple thing for teachers, especially in mathematics learning in elementary schools. The present study developed the teacher PD program to stimulate teachers to be able to use digital technology. However, in practice, teachers have not been able to escape from the mechanistic approach to teaching students. They still focus on the procedural technique to solve the given mathematical tasks. This can be understood because of the limited mathematical praxeologies that teachers have, so they find it challenging to develop mathematical learning in a more advanced mathematical direction. However, teachers' experiences using digital technology motivate them to use it again in the future. This certainly needs to

be followed up so that teachers can independently construct more meaningful digital technology-based mathematics learning in the future.

The findings of this study also practically contribute to the design of teacher PD programs in the future. We found that teachers from the first study demonstrated better mathematical and didactic praxeologies than those from the second one. Therefore, the findings highlight the need for appropriate PD program models that can meet the needs of targeted teachers. As mentioned by Hill et al. (2018), it is essential to design a PD program that matches teacher needs.

The study has several limitations that need to be acknowledged. The key limitation of this study was the data presented in the first and second studies come from two different sources, therefore it is not possible to compare similarities and differences in teachers' mathematical and didactic praxeologies from public and private schools. However, a study conducted by Putra et al. (2022) showed that the type of school has a significant influence on teachers' technological knowledge, and educational background also influences teachers' mathematical and didactic knowledge. It could be seen indirectly where teachers from the private school were better at expressing their mathematical and didactic knowledge in proposing and discussing their mathematical learning instruction. Further studies should analyze and compare teachers' teaching experiences using digital technologies, especially GeoGebra, from both types of schools to provide more insight into the effectiveness of the PD program.

Another limitation of this study lies in the time of the PD program. One-to-two-day PD program seems to be a short period for teachers to learn a new digital technology for teaching mathematics in elementary schools, especially teachers who have limitations with digital technology. In reality, the schools do not yet have technological facilities for learning, and teachers have limited technological pedagogical and content knowledge (TPACK), so this needs to be a concern for policymakers in implementing various PD program activities by facilitating learning technology so that it can improve the quality of teachers' TPACK. In addition, it is important to provide schools with adequate technology and ensure that teachers have TPACK because some teachers from the public school in this study have mentioned that it was their first experience using laptops in teaching students. It indicates that teachers have limited knowledge despite general technological knowledge.

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