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The potential problem to explore students' functional thinking in mathematical problem-solving

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Abstract

Many studies have reported that functional thinking plays a crucial role in mathematical problem-solving, particularly in fields requiring analytical reasoning, such as maritime studies. However, existing research has yet to comprehensively explore the specific task characteristics that effectively stimulate functional thinking in mathematical problem-solving, particularly among maritime students who must apply these skills in solving safety-of-life problems at sea. Addressing this gap, the present study investigates the potential of mathematical tasks in fostering functional thinking among second-semester students enrolled in the Deck Officer Program in Indonesia. The study involved three students with different mathematical abilities, who were given problem-solving tasks. Their responses were observed, recorded, and analyzed based on their written work. The findings reveal that non-routine problems involving functional situations—where students generalize relationships between varying quantities to determine function rules—effectively promote functional thinking. This is evidenced by the emergence of key functional thinking components, including problem identification, data representation, pattern recognition, covariational and correspondence relationships, and the evaluation of generalization rules. These results contribute to the development of research instruments in mathematics education and provide valuable insights for researchers and educators seeking to enhance functional thinking through task design.

Keywords: Functional Thinking, Mathematical Problem-Solving, Research Instruments, Task

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Research on functional thinking in mathematics education has been evolving since 1905 and continues to develop in contemporary scholarship (Tarida et al., 2024; Team et al., 2021; Vollrath, 1986). This progression underscores the critical role of functional thinking in enhancing students' structural reasoning in mathematics, thereby contributing to their success in solving mathematical problems in school and applying mathematical concepts to real-world contexts (Frey et al., 2022; OECD, 2023; Wei et al., 2023). As a fundamental gateway to algebraic thinking, functional thinking facilitates the comprehension of more advanced mathematical concepts by analyzing relationships between varying quantities (Blanton & Kaput, 2011; Smith, 2008; Taguma et al., 2023). Although functional thinking is a crucial area of investigation across all educational levels, recent research has predominantly focused on its development among primary, middle, and high school students (Ding et al., 2023; Martins et al., 2023; Pang et al., 2022; Sun et al., 2023). However, studies explicitly examining functional thinking among specific student populations remain limited, particularly in the context of maritime students (Tarida et al., 2024; Yuniati et al., 2022). Functional thinking is recognized as an essential competency for maritime students, as outlined in the International Maritime Organization (IMO) Model Course 7.03 and the International Association of Maritime Universities (IAMU) curriculum. It enables students to construct representations based on the generalization of relationships between two or more varying quantities (Pittalis, 2023; Smith, 2008; Tanişli, 2011). For maritime students, functional thinking is particularly significant in ensuring navigational safety by facilitating the analysis of interdependent factors that may contribute to or mitigate the risk of maritime accidents.

Recent advancements in maritime research have introduced numerous alternative solutions for sailors to mitigate the risk of ship accidents through the application of mathematical concepts (Huang, 2023; Jamaliyatul et al., 2023; Jamil & Bhuiyan, 2021; Mycroft & Sriraman, 2021). While these studies involve analyzing relationships between various quantities, they have yet to explore the development of functional thinking processes as a means of deriving solutions. Functional thinking is characterized by the ability to recognize and analyze relationships between quantities (Frey et al., 2022; Martins et al., 2023; Stephens et al., 2017). For instance, Jamaliyatul et al. (2023) developed a more precise wind speed estimation model by applying a linear functional relationship based on an analysis of wind speed data collected on Langkawi Island during the southwest monsoon season in 2019 and 2020. An understanding of wind speed enables sailors to regulate ship maneuvers effectively, thereby enhancing vessel stability and minimizing the risk of shipwrecks. Similarly, the *Komite Nasional Keselamatan Transportasi* (KNKT, 2018) reported that shipwrecks in Indonesia were often caused by miscalculations in ship stability due to improper cargo distribution. The frequent occurrence of maritime accidents has prompted Huang (2023) to develop a more precise mathematical model for optimizing shipping routes by minimizing travel distance and rudder angle. This model accounts for interrelated factors such as wave conditions, wind speed, ocean currents, engine power, hull design, and cargo capacity, all of which influence ship velocity and fuel efficiency.

Building on previous research findings, maritime students require functional thinking skills to effectively represent relationships between key navigational variables, such as speed, ocean currents, and fuel consumption—factors that are critical to ensuring both navigational safety and operational efficiency. This study aims to bridge the gap between mathematics and maritime education by integrating functional thinking processes into maritime students' problem-solving approaches concerning shipping safety. It is anticipated that strengthening their functional thinking competence will enhance their ability to become professional and proficient sailors.

According to Smith (2008), functional thinking involves generalizing relationships between two or more varying quantities. These relationships can be understood through recursive, correspondence, or covariational reasoning (Pittalis et al., 2024; Smith, 2008; Stephens et al., 2017) and require an understanding of patterns, independent and dependent variables (Pitta-Pantazi et al., 2020). The conceptualization of these relationships can be facilitated through representational and linguistic tools, including visual representations, verbal descriptions, algebraic variables, tables, and graphs (Blanton & Kaput, 2011; Pitta-Pantazi, 2020; Stephens et al., 2017). Synthesizing insights from various experts, functional thinking can be broadly defined as the process of generalizing variable relationships. Accordingly, this study identifies six key aspects of functional thinking: problem identification, data representation, pattern recognition, covariational reasoning, correspondence relationships, and evaluating generalizations.

Functional thinking processes can be stimulated through tasks that require students to analyze relationships between varying quantities in real-world contexts (Aguilar & Telese, 2018; Blanton & Kaput, 2011; Smith, 2008; Wakhata et al., 2023). Problem-solving tasks necessitate the integration of real-world knowledge and the application of mathematical operations to derive conclusions. These tasks share similarities with non-routine mathematical problems, which cannot be resolved using standard procedures and thus demand higher-order cognitive processes, including functional thinking (see Khusna et al., 2024; Zhang et al., 2023). Numerous studies have been conducted to foster functional thinking through the exploration of non-routine mathematical tasks across different educational levels.

In primary education, functional thinking is developed by incorporating tasks that involve functional relationships (Pittalis et al., 2024), pattern recognition in real-world contexts (Pang et al., 2022), arithmetic operation sequences, one-to-one correspondence exercises, and pre-functional tasks (Ding et al., 2023). Additionally, caterpillar tasks and function machine tasks have been identified as effective strategies for enhancing functional thinking at this stage (Wilkie, 2014). At the middle school level, problem-solving tasks that facilitate the recognition of numerical and figural patterns, identification of independent and dependent variables, derivation of function formulas, and construction of diagrams and graphs play a crucial role in the development of functional thinking (Pitta-Pantazi et al., 2020). Among high school students, functional thinking is further explored through tasks that emphasize mapping, correspondence, covariation, and functional relationships, such as object-thinking tasks (Martins et al., 2023).

At the tertiary level, research by Yuniati (2022) examined functional thinking among mathematics education students through the development of a figural pattern sequence task incorporating three variables. Given the significance of functional thinking in mathematical reasoning, designing appropriate tasks is essential for effectively investigating students' cognitive processes in this domain. The accuracy of task design directly influences the reliability and validity of the functional thinking data obtained (see Jamil et al., 2023; Khusna et al., 2024). Furthermore, Martins et al. (2023) emphasize the need for further research on task characteristics that can enhance students' functional thinking abilities. A synthesis of previous research findings suggests a common criterion for tasks that effectively promote functional thinking—namely, tasks involving non-routine problem-solving that require students to generalize functional rules.

Previous research indicates that the development of tasks designed to explore functional thinking in maritime students remains limited. This study seeks to address this gap by developing non-routine mathematical tasks aimed at fostering functional thinking within the maritime context. The tasks integrate maritime safety scenarios that require solutions using linear interpolation, thereby creating a meaningful connection between maritime studies and mathematics. Linear interpolation is a mathematical technique used to estimate unknown values between two known data points connected by a straight line (Chapra & Canale, 2011; Noor et al., 2015; Wahab, 2017).

Existing studies on tasks that stimulate functional thinking primarily utilize mono-representational approaches with constant data addition patterns. In contrast, this study introduces a multi-representational approach incorporating a non-constant data addition pattern. This variation encourages students to employ multiple representations, such as tabular and graphical forms, to interpret and analyze data. Specifically, students will translate tabular data into graphical representations to derive the concept of linear interpolation. The dataset used in the task represents the relationship between a ship's average speed and its stopping distance. By engaging in this problem-solving process, students are expected to enhance their ability to mitigate the risk of ship collisions, thereby strengthening their professional competencies as future maritime practitioners. Additionally, this study's findings have the potential to contribute to maritime training programs and inspire mathematics educators to develop similar tasks aimed at improving students' functional thinking skills.

Functional thinking can be fostered through problem-solving tasks that guide students through structured cognitive processes (Sibgatullin et al., 2022; Stephens et al., 2017; Windsor, 2011). Problem-solving skills are essential across various professional domains, including maritime industries (OECD, 2023; Taguma et al., 2023). This study employs Schoenfeld's (1985) problem-solving framework, which consists of five stages: analysis, design, exploration, implementation, and verification. The adaptation of this framework follows the recommendations of Rott et al. (2021), who advocated for incorporating mathematical topics beyond geometry to deepen students' problem-solving experiences. In this study, the application of problem-solving stages is distinguished by the specific mathematical content—linear interpolation—and the involvement of maritime students as research participants.

Non-routine problems play a critical role in activating each phase of the problem-solving process. The resolution of such problems requires cognitive strategies such as guess-and-check, working backwards, and pattern recognition, all of which align with key aspects of functional thinking (Schoenfeld, 2016). Despite the relevance of functional thinking in maritime education, tasks designed to cultivate this cognitive process remain scarce (Tarida et al., 2024). However, maritime professionals must develop strong functional thinking skills to effectively address critical safety concerns, including navigation risk management. Therefore, this study aims to explore the potential of non-routine mathematical tasks in stimulating maritime students' functional thinking abilities, particularly in the context of maritime navigation safety. The necessity of developing such problem-solving tasks to enhance functional thinking is illustrated in Figure 1.

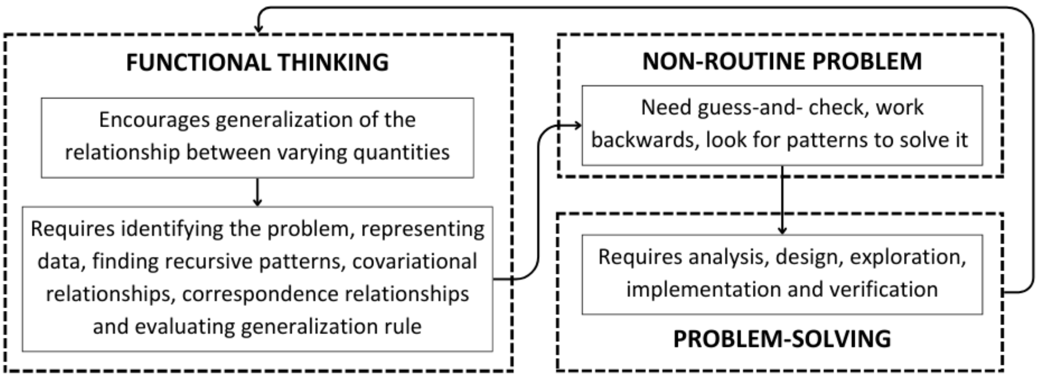


Figure 1. The urgency of developing non-routine problems to trigger functional thinking

Smith (2008) described functional thinking as a form of representational thinking that emphasizes the relationship between two or more varying quantities. This type of thinking enables individuals to transition from recognizing specific relationships in individual instances to generalizing these relationships across multiple occurrences. The foundational framework for functional thinking was first introduced by Confrey and Smith (1991), who identified three key components: (1) recursive reasoning, (2) covariational thinking, and (3) correspondence relationships. Furthermore, Stephens et al. (2017) further defined functional thinking as the ability to identify and describe correspondence relationships using words or symbolic variables. Similarly, Kaput (2008) conceptualized functional thinking as the process of recognizing systematic variations across instances, incorporating key ideas such as causality, growth, and continuous joint variation.

Building upon these theoretical perspectives, this study proposes a comprehensive framework for functional thinking, encompassing six critical aspects: (1) identifying the problem, (2) representing data, (3) recognizing recursive patterns, (4) identifying covariational relationships, (5) determining correspondence relationships, and (6) evaluating generalization rules. The coding scheme for these functional thinking aspects is presented in Table 1. Functional thinking extends beyond the traditional concept of mathematical functions taught in schools; it also involves modeling functional situations in real-world contexts that integrate multiple disciplines (Frey et al., 2022; Lichti & Roth, 2018; Martins et al., 2023; Warren et al., 2011; Wei et al., 2023).

Table 1. Coding scheme of functional thinking

Functional Thinking Code	Description
F1	*Identifying the problem* Noticing the relationship between two quantities
F2	*Representing data* Transforming data from tabular to graphical form
F3	*Finding recursive patterns* Finding changes in each variable value based on the graph
F4	*Finding covariational relationships* Finding the relationship between two variables based on a graph to predict an unknown value based on a graph
F5	*Finding correspondence relationships* Creating general rules based on the graph
F6	*Evaluating generalization rule* Testing the correctness of the general rule

The development of functional thinking can be fostered through solving mathematical problems that incorporate functional situations. These functional situations belong to a broader category of mathematical problems, as they represent real-world scenarios that may originate outside of mathematics but require mathematical reasoning for their resolution. This study presents functional situations within the real-world context of ship maneuvering, a critical aspect of maritime operations (Oneto et al., 2018; Wu et al., 2021). Problem-solving serves as a key mathematical tool for engaging students in higher-order mathematical thinking, including functional thinking processes (see Schoenfeld, 2016). The problem-solving framework utilized in this study follows Schoenfeld's (1985) stages, which include analysis, design, exploration, implementation, and verification. These structured stages provide a systematic approach to solving mathematical problems that align with the contextual challenges developed in this study. Finally, Table 2 further delineates the functional thinking processes involved in solving this problem.

Table 2. Coding scheme of functional thinking in problem-solving

Functional Thinking in Problem-Solving Code	Description
PF1	*Analysis* Noticing the existence of a relationship between two quantities with the identification of quantity information and the aim of predicting unknown data values
PF2	*Design* Transforming data from tabular to graphical form as a strategy design for predicting unknown data values
PF3	*Exploration* Finding changes in each variable value based on graphs by trying various problem-solving strategies
PF4	Finding the relationship between two variables to predict unknown values by trying various problem-solving strategies

Functional Thinking in Problem-Solving Code	Description
PF5	*Implementation* Creating general rules used to predict unknown data values by applying linear interpolation/other possible methods
PF6	*Verification* Testing the correctness of general rules by identifying possible errors and discrepancies between the solution and the established problem conditions

Previous studies have successfully explored students' functional thinking at various educational levels through problem-solving tasks. For high school students, Martins et al. (2023) designed tasks that required students to analyze examples and non-examples of different types of functions (e.g., injective functions) to develop conceptual understanding. In this task, students primarily engaged in functional thinking through correspondence reasoning and conceptualizing functions as mathematical objects.

At the college level, Yuniati et al. (2022) extended Wilkie's (2014) framework by developing tasks involving three interrelated quantities, compared to Wilkie's original two-quantity tasks (Figure 2). Additionally, Oliveira et al. (2021) designed tasks specifically for prospective elementary mathematics teachers, incorporating functional thinking through activities such as "geometric pattern" tasks, "representations" tasks, and "deposits" tasks, each targeting key aspects of functional reasoning. These studies highlight the diverse approaches to fostering functional thinking through carefully structured mathematical tasks.

Task Sheet Instrument (Wilkie, 2014)	Test Sheet Instruments in this Research
Use two quantities	Use three quantities

1st flower	2st flower	3st flower	1st image	2st image	3st image

Figure 2. Examples of tasks exploring functional thinking in university

Based on previous research, the criteria for tasks that effectively stimulate functional thinking include the following: the incorporation of non-routine problems, the presence of functional issues and real-world contexts, and the integration of key aspects of functional thinking with a clear problem-solving objective. In alignment with these criteria, this study develops problem tasks designed to guide students toward discovering general rules of linear interpolation within the context of maritime navigation safety.

METHOD

This qualitative study investigates the potential of mathematical tasks in stimulating students' functional thinking processes when solving mathematical problems. To achieve this objective, the researchers designed non-routine problems involving covariational reasoning within the context of linear interpolation. Prior research by Pang et al. (2022) has demonstrated that non-routine mathematical problems, such as those incorporating growing numeric patterns and input-output structures, can enhance students' functional thinking. These tasks cannot be solved through conventional procedural methods; instead, they necessitate the use of multiple representations, including graphs, tables, and numerical patterns.

Solving such tasks aligns with the characteristics of non-routine problems, which require advanced cognitive engagement, including functional thinking (see Khusna et al., 2024; Zhang et al., 2023). Consequently, this study adopts non-routine problem-solving as a strategy to foster students' functional thinking abilities. The selection of non-routine problems is grounded in their requirement for specific problem-solving strategies, such as guess-and-check, working backwards, and pattern recognition (Schoenfeld, 2016). These strategies support the sequential stages of problem-solving proposed by Schoenfeld (1985), including analysis, design, exploration, verification, and implementation, which are instrumental in activating students' functional thinking processes (see Kriegler, 2004; Pang & Sunwoo, 2022; Stephens et al., 2017; Windsor, 2011).

The use of non-routine covariational reasoning problems has significant potential in fostering students' functional thinking, as these problems necessitate the analysis and generalization of relationships between variables (see As'ari et al., 2019; Blanton & Kaput, 2011; Ding et al., 2023; Pitta-Pantazi et al., 2020; Rolfes et al., 2021; Smith, 2008). Oliveira et al. (2021) demonstrated that tasks incorporating functional relationships can effectively enhance students' functional thinking skills. In the present study, functional relationships are explored within the framework of covariational reasoning, which involves comprehending how variations in one quantity influence changes in another, a fundamental aspect of functional thinking (Blanton & Kaput, 2011; Rolfes et al., 2021; Smith, 2008). Thompson et al. (2017) emphasized that covariational reasoning serves as a foundational component in understanding mathematical concepts related to functions and variation, which in turn influences the development of functional thinking. Furthermore, findings from Rolfes et al. (2021) suggest that covariational reasoning facilitates students' ability to employ diverse representations of quantitative relationships, thereby promoting the advancement of functional thinking. Similarly, Pittalis et al. (2024) highlighted the role of covariational reasoning in deepening students' comprehension of quantitative relationships. Given its significance, covariational reasoning is employed as a conceptual framework for functional thinking in this study.

The topic of linear interpolation was selected due to its integral role in the study of functions, as function-related tasks have been shown to foster students' functional thinking (see Chimoni et al., 2023; Günster & Weigand, 2020; Pang & Sunwoo, 2022; Tanişli, 2011). Furthermore, linear interpolation is a fundamental concept that maritime students must master to develop competencies essential for safe navigation. According to the International Maritime Organization (IMO) Model Course 7.03, which outlines the curriculum for the Officer in Charge of a Navigational Watch, proficiency in linear interpolation is crucial for students enrolled in the Deck Officer Program. In light of these considerations, the criteria for problems designed to enhance students' functional thinking are summarized in Table 3.

Table 3. Problem criteria to explore students' functional thinking

Problem Criteria	Problem
Purpose of Problem	Problem to Find
Material	Linear Interpolation
Approach	Table Function Analysis
Stage to Solve	Problem-Solving
Aspect of Mathematical Thinking	Functional Thinking
Type of Problem	Non-routine Covariational Reasoning Problem

The researcher provided a draft of the problem along with a solution guide to the validators for assessment. The validation process aimed to evaluate the validity of the developed problem. This validation was conducted by four experts, all of whom were lecturers in Mathematics Education. Among them, one validator was a professor of Mathematics Education at the State University of Malang, while the remaining three were lecturers from the State University of Surabaya. Of these, one specialized in research on algebraic thinking, particularly functional thinking, while the other two were instructors in numerical methods courses, with expertise in linear interpolation. The validation process yielded several recommendations from the validators, based on which the researcher revised the draft task. The suggested modifications and corresponding improvements are detailed in Table 4.

Table 4. Task improvement based on validator suggestions

<table>
<tr><th>No.</th><th>Validator Suggestions</th><th>Task Improvement</th></tr>
<tr><td>1.</td><td>Use more straightforward sentences in the instructions/ task questions to make it easier for students to understand. This understanding could trigger students' functional thinking to the maximum.</td><td>What is the estimated stopping distance required for a ship to stop sailing if the ship is sailing at an average speed of 45 Knots? What general rule do you use to estimate the stopping distance? Explain!</td></tr>
<tr><td>2.</td><td>Explain the correlation between problems, solutions and aspects of functional thinking.</td><td rowspan="3">Examples of improvements in the aspect of finding covariational relationships (FT4):
<table>
<tr><th>Problem</th><th>Solution</th><th>Aspect of Functional Thinking</th></tr>
<tr><td>What is the estimated stopping distance required for a ship to stop sailing if the ship is sailing at an average speed of 45 Knots?</td><td>(1) Estimation using data in a table
Students find the relationship between speed and stopping distance through table data, namely "every 10 times the average speed increases, the stopping distance increases by no more than 35"
(2) Estimation using a function graph which is a representation of table data
• Students find the relationship between speed and stopping distance through the pattern formed on the graph
• The relationship between speed and stopping distance forms several similar triangle patterns and a straight-line equation that passes through two points.</td><td>Finding covariational relationships (FT4) is characterized by finding the relationship between speed and stopping distance to estimate the unknown stopping distance among known data. Covariational relationships are found through data in tables and function graphs which are representations of table data.</td></tr>
</table></td></tr>
<tr><td>3.</td><td>The alternative answers still only represent one pattern (based on the researcher's assumptions) and have yet to reveal other alternative answers that might emerge from students.</td></tr>
<tr><td>4.</td><td>Use more communicative sentences in the alternative answers.</td></tr>
</table>

The revised task based on the validator's suggestions is presented in Figure 3. The ship-stopping distance task is designed to engage students in formulating a general rule using a linear interpolation function to predict the ship's stopping distance at a given average speed, based on seven data points presented in a table. The problem-solving process begins with students identifying the problem, prompting them to recognize the relationship between stopping distance and average speed—namely, that as speed increases, the stopping distance extends (F1). Following this initial observation, students work toward understanding the problem's requirements. The solution cannot be determined in a single step but necessitates intermediate steps that are not explicitly outlined in the task instructions.

Students are not directly instructed to represent the tabular data as a graph (F2). However, if they encounter difficulties in identifying a recursive pattern (F3) through the numerical data, they may choose to first visualize the table as a graph (F2). This approach is particularly useful since the pattern of change in stopping distance is not a uniform numerical increment, as seen in the pattern of speed changes. The graphical representation enables students to detect a recursive pattern (F3), which can manifest as a similar triangle pattern, where the base represents changes in average speed and the height represents changes in stopping distance. Additionally, a function graph may prompt students to derive a straight-line equation by considering variations between pairs of speed and distance data points.

By recognizing this recursive pattern, students are expected to establish the relationship between speed and stopping distance. For instance, understanding how an increase in speed affects stopping distance can serve as the key to estimating the stopping distance at an average speed of 45 knots. This process aligns with the aspect of identifying covariational relationships (F4). The requirement to derive a general rule fosters the identification of correspondence relationships (F5), as students formulate an equation based on their prior analysis of covariational relationships. Finally, the inclusion of the instruction "Explain" is strategically designed to prompt students to evaluate their generalization results (F6).

SHIP STOPPING DISTANCE

A ship stops sailing in a berthing area (anchor berth) to wait for the berthing queue at the destination port. The ship anchors seven times in one month. The ship anchors safely because it has paid attention to factors that affect the ship's movement (maneuver), such as internal factors (ship conditions) and external factors (water conditions). When stopping steering, it is assumed that there is no influence from within and outside. The following table provides data on the ship's average speed and the stopping distance required for the ship to stop sailing in the berthing area (anchor berth) for one month.

Average speed (Knots)	10	20	30	40	50	60	70
Stopping distance (Nm)	12	21	46	65	90	111	148

What is the estimated stopping distance required for a ship to stop sailing if the ship is sailing at an average speed of 45 Knots? What general rule do you use to estimate the stopping distance? Explain it!

Figure 3. Tasks developed to trigger functional thinking

The researcher then implemented the validated task on the participants. Participants in this study were 2nd-semester students in the Deck Officer Program at the Nusantara Maritime Academy in Cilacap. Participants were selected based on the category of mathematical ability level. Mathematical ability is essential because it is one of the predictors of student success in problem-solving (Schoenfeld, 2006). Mathematics Test score obtained the participants' mathematical ability (Khusna et al., 2024). Mathematical ability is divided into three categories based on the rules in place of research. The categories are high mathematical ability if the student gets a score (x) that is in the range $(80 < x \leq 100)$, medium if the score (x) is in the range $(60 < x \leq 80)$ and low if the score (x) is in the range $(0 \leq x \leq 60)$. Participants in this study were selected based on the most significant difference in scores between categories of mathematical ability so that they have the potential to produce different functional thinking processes that can be applied to broader learning. To enrich the research findings, participants were also selected based on the emergence of the most functional thinking intensity in each category of their mathematical ability. Participants with these categories are possible in every regular class. Participants were given tasks at different times, so the research findings are expected to contribute more widely. Based on these selection criteria, three out of thirty-six students were obtained as participants with the pseudonyms Niko (high), Dani (medium) and Yoga (low). Participants with these categories are possible in every regular class. The results of the answers of 33 students were not analyzed because there were no variations in answers that indicated the emergence of similar functional thinking. Varied solutions help sailors develop adaptability and make the right decisions to improve navigation safety. The small number of participants in this study aims to explore the students' functional thinking processes more deeply (Pitta-Pantazi et al., 2020). On the other hand, this number is a limitation in this study. Further research is expected to involve more participants to analyze the effectiveness of tasks in significantly improving students' functional thinking. Participants with these categories are possible in every regular class.

Participants completed the tasks at different times, with each session lasting 100 minutes in a comfortable and conducive environment. They were permitted to use a calculator provided by the researcher but were not given any scaffolding before beginning the task. Additionally, they were not allowed to seek clarification from the researcher during the task to ensure that no external influence biased the emergence of their functional thinking. Each participant recorded their answers on a worksheet provided by the researcher, following specific instructions. Notably, they were instructed not to erase incorrect responses but instead to cross them out, allowing for a more comprehensive and in-depth analysis of their thought processes. Upon completion, the worksheets were collected and coded according to the scheme outlined in Tables 1 and 2, enabling the analysis of the intensity and frequency of functional thinking manifestations.

This research method was adapted from previous studies that developed problem-solving tasks to explore students' cognitive development (Fatmanissa et al., 2024; Khusna et al., 2024). To ensure the credibility of the findings and mitigate potential bias arising from subjectivity in qualitative research, the study employed prolonged engagement, persistent observation, and triangulation. Triangulation was achieved through the analysis of students' work on tasks and task-based interview transcripts (Creswell & Poth, 2016; Denzin & Lincoln, 2011; Lincoln & Guba, 1985). The interviews provided deeper insights into students' functional thinking processes while solving mathematical problems (Pitta-Pantazi et al., 2020). While the study does not aim to generalize its findings, its approach holds potential applicability in broader learning contexts with similar characteristics, such as students with varying mathematical abilities and instructional settings that incorporate real-world mathematical applications using tables and graphs.

RESULTS AND DISCUSSION

This section examines the potential challenges encountered in fostering students' functional thinking within problem-solving contexts. The analysis of these challenges is conducted by comparing the problem-solving activities of two participants, aligning their responses with established problem-solving stages. The participants' activities are systematically presented using the coding scheme detailed in Tables 1 and 2. In accordance with research ethics protocols, the data utilized in this study are publicly accessible, as all participants provided informed consent, ensuring confidentiality through the anonymization of their identities and other personal information.

The findings of this study reveal the emergence of various aspects of functional thinking, identified through an analysis of task responses from 36 students categorized into high, medium, and low mathematical ability groups. Notably, none of the students demonstrated all aspects of functional thinking. Five functional thinking aspects (F1–F5) were observed in three students, comprising two with high mathematical ability and one with medium mathematical ability. Additionally, seven students exhibited four functional thinking aspects (F1–F4), including four students from the high-ability group, two from the medium-ability group, and one from the low-ability group. The most common occurrence was the emergence of three functional thinking aspects (F1–F3), identified in 26 students, with a distribution of 10 students from the high-ability group and 16 from the medium-ability group. Notably, no students demonstrated only two (F1–F2) or a single (F1) functional thinking aspect. The data distribution is presented in Table 5. To gain a deeper understanding, further qualitative analysis was conducted through interviews with three participants—Niko, Dani, and Yoga—who exhibited the highest functional thinking emergence within their respective mathematical ability categories (highlighted with a red circle in Table 5).

Table 5. Distribution of the emergence of participant functional thinking aspects based on mathematical ability

Emergence of Functional Thinking Aspects	Mathematical Abilities			Sub-Total of Students
	High	Medium	Low	
All aspects (F1-F6)	0	0	0	0
Five aspects (F1-F5)	2	1	0	3
Four aspects (F1-F4)	4	2	1	7
Three aspects (F1-F3)	10	16	0	26
Two aspects (F1-F2)	0	0	0	0
One aspect (F1)	0	0	0	0
Total of Students	16	19	1	36

Niko is one of two students with high mathematical ability who demonstrated the highest occurrence of functional thinking across five aspects. He was selected for further analysis due to his use of a more diverse range of problem-solving approaches. Dani is the only student with high mathematical ability who exhibited functional thinking in all five aspects. Meanwhile, Yoga is the sole student with low mathematical ability who demonstrated the highest occurrence of functional thinking across four aspects.

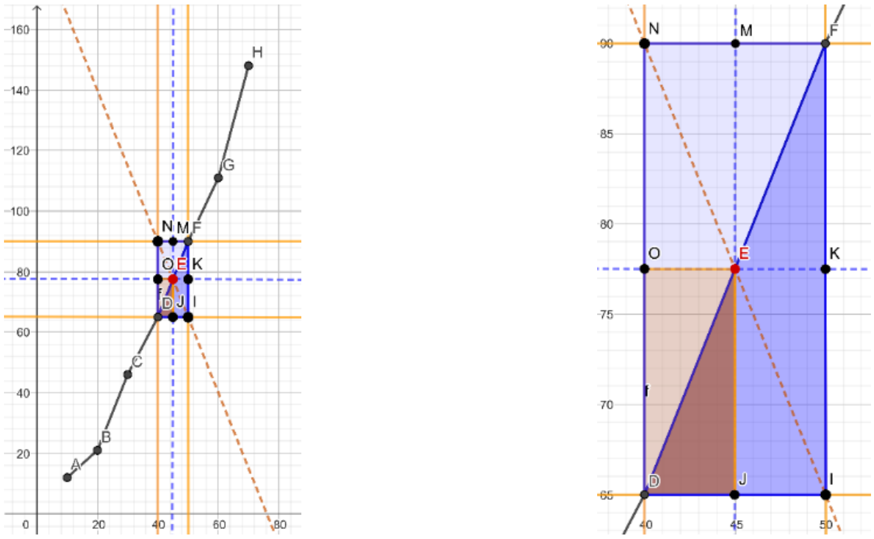
Participants with High Mathematical Ability (Niko)

1. Identifying the Problem
   Niko read the question and began to identify the problem of the ship's stopping distance by noticing the relationship between two quantities **(F1).** These quantities include stopping distance and average speed. Niko noticed that the higher the average speed of the ship, the longer the stopping distance required for the ship to stop completely. Based on this, Niko noticed the existence of a relationship between two quantities with the identification of quantity information and the aim of predicting unknown data values **(PF1).** Niko's method of identifying the problem was not implicitly included in the answer to the task, so the researcher conducted an interview based on the answer to the task. The following is a transcript of the interview that marks the emergence of Niko's activity in identifying the problem.

   Niko : *The relationship is one seen from the first, second, and third anchoring attempts, where if the ship's speed is greater, then the stopping distance will also be greater* ***(F1)****.*
   Researcher : *Does that mean it can be said to be comparable?*
   Niko : *Yes, it does.*

2. Representing Data
   Niko had difficulty seeing the relationship between the average speed and the stopping distance of the ship from the table, so Niko represented the data in the table in a graphical form **(F2).** The function graph can be seen in Figure 4a. The graphical representation occurred at the design stage because Niko transformed the data from tabular to graphical form as a strategy design for predicting unknown data values **(PF2).**
3. Finding Recursive Patterns
   Niko noticed changes in the average speed and stopping distance values on the graph **(F3).** These changes are represented by dots on the graph. From these dots, Niko found three recursive patterns,

including straight lines, rectangles and triangles. Niko found these patterns by trying various methods. In the first method, Niko connected every two closest points with a straight line. Niko focused on the straight-line DF connecting point D (40,65) and point F (50,90) to estimate the stopping distance when the average speed was 45 knots (Figure 4b). Niko chose to focus on the straight-line DF because it is the closest line that passes through the 45 knots point, so it has the potential to produce a more reasonable stopping distance estimate. In the second method, Niko made auxiliary lines at point D (40,65) and point F (50,90) to find a rectangular pattern. For example, in the DIFN rectangle, the length of DI represents the change in average speed and the width of IF represents the change in stopping distance (Figure 4b). Niko also found a triangular pattern. In the DIF triangle, the base DI represents the change in average speed and the height IF represents the change in stopping distance (Figure 4b). Niko discovered the recursive pattern during the exploration stage because he tried various ways to find changes between variable values **(PF3)**.



(a) (b)

Figure 4. (a) Function graph representation by Niko, (b) Recursive pattern finding by Niko

4. Finding Covariational Relationships

In the table data, Niko found that "every ten increase in average speed means that the stopping distance does not increase more than 37". The non-constant increase in stopping distance prompted Niko to find the relationship between speed and stopping distance using a graphical approach. In the previous answer, Niko found three recursive patterns based on the graph. The recursive pattern prompted Niko to find three covariational relationships, including several straight lines passing through two points, several rectangular images and several triangular images. Niko highlighted the straight-line DF passing through point E when the average speed was 45 knots to estimate the ship's stopping distance (Figure 4b). The points on the graph appear to be increasing upwards, representing that the relationship between the quantity of average speed and stopping distance is increasing **(F4).** Furthermore, Niko found two similar quadrilaterals that were closest to point E, namely the small quadrilateral DJEO and the large quadrilateral DIFN (Figure 4b). In both quadrilaterals, a relationship was found that every time the length increases, the width increases (F4). Niko also found two similar triangles that are closest to point E, namely the small rectangle DJE and the large rectangle DIF (Figure 4b). In both triangles, a relationship was found that each time the base increases, the height increases **(F4)**. The discovery of the covariational relationship occurred at the exploration stage because Niko found the relationship between two variables to predict unknown values by trying various problem-solving strategies **(PF4)**.

5. Finding Correspondence Relationships

Niko finds the correspondence relationship based on the previous covariational relationship. Niko obtains a general rule based on the correspondence relationship by representing the average speed as the variable x and the stopping distance as the variable y. Furthermore, Niko assigns an index to each variable. Niko obtains the same estimate by applying the three general rules (marked with red rectangles in Figure 5). The three general rules lead to the linear interpolation rule. The general rule is obtained because Niko finds the relationship between two variables based on a graph to predict an unknown value based on a graph/correspondence relationship **(F5)**. Niko finds the correspondence relationship at the implementation stage because Niko creates general rules used to predict unknown data values by applying linear interpolation **(PF5)**.

$$\begin{aligned} &\leftrightarrow y - y_1 = m(x - x_1) \\ &\leftrightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\ &\leftrightarrow y - 65 = \frac{90 - 65}{50 - 40}(45 - 40) \\ &\leftrightarrow y = 77{,}5 \end{aligned}$$

(a)

$$\begin{aligned} &\frac{l_2}{l_1} = \frac{p_2}{p_1} \\ &\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \\ &\leftrightarrow y - y_1 = \frac{x - x_1}{x_2 - x_1}(y_2 - y_1) \\ &\leftrightarrow y = y_1 + \frac{x - x_1}{x_2 - x_1}(y_2 - y_1) \\ &\leftrightarrow y = 65 + \frac{45 - 40}{50 - 40}(90 - 65) \\ &\leftrightarrow y = 77{,}5 \end{aligned}$$

(b)

$$\begin{aligned} &\frac{t_2}{t_1} = \frac{a_2}{a_1} \\ &\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \\ &\leftrightarrow y - y_1 = \frac{x - x_1}{x_2 - x_1}(y_2 - y_1) \\ &\leftrightarrow y = y_1 + \frac{x - x_1}{x_2 - x_1}(y_2 - y_1) \\ &\leftrightarrow y = 65 + \frac{45 - 40}{50 - 40}(90 - 65) \\ &\leftrightarrow y = 77{,}5 \end{aligned}$$

(c)

Figure 5. (a) General rules for equations of straight lines by Niko, (b) General rules for comparison of two similar quadrilaterals by Niko, (c) General rules for comparison of two similar triangles by Niko

6. Evaluating General Rule

The results of the generalization are in the form of general rules found when analyzing the correspondence relationship. The evaluation of the results of the generalization is not implicitly included in Niko's task answers, so the researcher dug up data through interviews with the following transcripts:

Researcher	:	*Please calculate the estimated stopping distance from speeds of 41, 45 and 46.*
Niko	:	*At 41, the stopping distance is 67.5. At 45, the stopping distance is 77.5. At 46, the stopping distance is 80.*
Researcher	:	*What can be concluded from these results?*
Niko	:	*The estimated stopping distance is reasonable because it is between 65 and 90 Nm. So, the general rule can be used for all unknown data. If we enlarge the image here (GeoGebra), we can also see the results **(F6)**.*

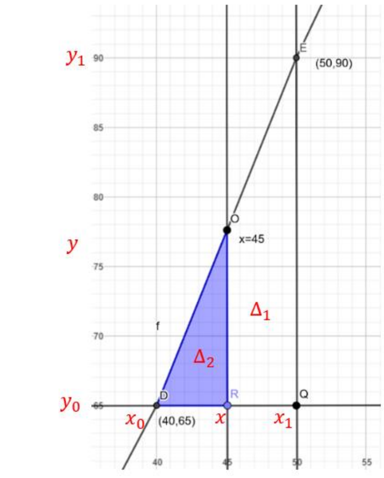
Based on the transcript, Niko evaluated the general rules created by checking other stopping distance data between 40 and 50 knots **(F6)**. From the check results, Niko concluded that the three general rules can be used to estimate unknown data among known data. This general rule evaluation occurs at the verification stage because Niko tests the truth of the general rules by identifying possible errors and inconsistencies between the solution and the established problem conditions **(PF6)**.

Participant with Medium Mathematical Ability (Dani)

1. Identifying the Problem
   Dani concluded that there is a relationship between speed and the stopping distance of the ship **(F1).** The emergence of the F1 aspect is marked by the following interview transcript. Dani identified the problem by identifying the information, objectives and conditions provided **(PF1)**.

   Researcher : *What information did Dani get after reading the question?*
   Dani : *After the engine is turned off, the ship does not stop immediately; there is a distance until the average speed is 0. If the ship is getting faster, it means the stopping distance is also getting further **(F1)***

2. Representing Data
   After knowing that the stopping distance is affected by the average speed, Dani changes the data in the table into a function graph **(F2)**. Dani represents the data in the form of a graph to analyze changes between quantities so that the results of the analysis can be used in solving problems **(PF2)**.
3. Finding Recursive Patterns
   Dani found a recursive pattern in the graph representation results. Dani found changes in each variable value based on the graph **(F3).** The changes are in the form of a triangle pattern, where the change in average speed is represented as the base of the triangle, while the change in stopping distance is represented as the height of the triangle. The discovery of the recursive pattern occurred at the exploration stage because Dani found changes in each variable value based on graphs by trying various problem-solving strategies **(PF3)**.
4. Finding Covariational Relationships
   In the previous answer, Dani found that the change in speed is a representation of the base of the triangle and the change in stopping distance is a representation of the height of the triangle so that the relationship between the change in speed and stopping distance forms a right triangle. So Dani found a covariational relationship in the form of several similar right triangles **(F4)**. This discovery occurred at the exploration stage because Dani found the relationship between two variables to predict unknown values by trying various problem-solving strategies **(PF4)**.
5. Finding Correspondence Relationships
   The correspondence relationship found by Dani is in the form of a comparison of the base and height of two similar triangles. Dani symbolized the base and height of the triangle so that he found a general rule that can be seen in Figure 6 (marked with a red rectangle). The general rule of comparison of two similar triangles leads to the linear interpolation rule. This general rule was obtained because Dani found the relationship between two variables based on a graph to predict an unknown value based on a graph **(F5)**. This general rule occurs at the implementation stage because Dani creates general rules used to predict unknown data values by applying linear interpolation **(PF5)**.



For example x =speed dan y =stopping distance,
$\Delta_1 = \Delta DQE$ is a right triangle with base $= a_1 = x_1 - x_0$ and heighti$= t_1 = y_1 - y_0$
$\Delta_2 = \Delta DRO$ is a right triangle with base$= a_2 = x - x_0$ and height$= t_2 = y - y_0$
Because Δ_1 and Δ_2 are similar triangles, the equation is obtained

$$\frac{y - y_0}{y_1 - y_0} = \frac{x - x_0}{x_1 - x_0}$$

$$\leftrightarrow y - y_0 = \frac{x - x_0}{x_1 - x_0}(y_1 - y_0)$$

$$\leftrightarrow y = y_0 + \frac{x - x_0}{x_1 - x_0}(y_1 - y_0)$$

$$\leftrightarrow y = 65 + \frac{45 - 40}{50 - 40}(90 - 65) = 77{,}5$$

Figure 6. Correspondence relationship findings by Dani

6. Evaluating General Rule

In Dani's task answer, the evaluation of the generalization results was not implicitly included, so the researcher dug up data through interviews using the following transcript.

Researcher	:	*Why are you confident in the general rules that have been discovered?*
Dani	:	*If you look at the graph, that's the most likely option. It can also be used for other data.*
Researcher	:	*Try to work out the meaning of your statement.*
Dani	:	*(start working) I estimated the stopping distance at an average speed of 48 knots. The result is 85 Nm **(F6)**.*
Researcher	:	*What do you think of the results?*
Dani	:	*The results show that the general rules created can be used for any data, ma'am, as long as there is data among the known ones **(F6)**.*

Dani evaluates the general rule by checking other stopping distance data between 40 and 50 knots **(F6).** Dani concludes that the general rule of comparison of two similar triangles is suitable for use in estimating unknown data among known data. This general rule evaluation occurs at the verification stage because Dani tests the validity of the general rule by identifying possible errors and inconsistencies between the solution and the established problem conditions **(PF6)**.

Participants with Low Math Ability (Yoga)

1. Identifying the Problem

The researcher collected data on yoga problem identification by confirming the task answers through interviews using the following transcript.

Yoga	:	*This average speed has the same effect on the stopping distance. If the ship is faster, the stopping distance will also be longer **(F1)**.*
Researcher	:	*Is it possible that as the ship speeds up, its stopping distance will be shorter?*
Yoga	:	*Based on this data, it is impossible.*

Based on the transcript, Yoga began to identify the problem by recognizing the changes in each quantity, namely the quantity of the average speed of the ship, which was getting faster with the quantity of the stopping distance getting longer. This change prompted Yoga to notice/realize that there was a relationship between the average speed and the stopping distance of the ship **(F1)**. Based on this, Yoga identified the problem at the analysis stage by noticing the existence of a relationship between two quantities with the identification of quantity information and the aim of predicting unknown data values **(PF1)**.

2. Representing Data

   Yoga tries changing the data from tabular to graphical **(F2)**. Graphical representation occurs at the design stage because, at this stage, Yoga transforms data from tabular to graphical form as a strategy design for predicting unknown data values **(PF2)**. Furthermore, Yoga represents the average speed of 45 knots, with point H as the initial step in estimating the unknown stopping distance.

3. Finding Recursive Pattern

   Yoga did not find a recursive pattern based on the previously created graph, even though he had marked the average speed of 45 knots with point H. Yoga found that the average speed increased by 10 knots with each anchoring attempt **(F2).** Yoga also found that the stopping distance increased but could not determine the number of additions with each anchoring attempt **(F2)**. The discovery of a recursive pattern occurred at the exploration stage because Yoga found changes in each variable value by trying various problem-solving strategies, even though not through the graph **(PF3)**.

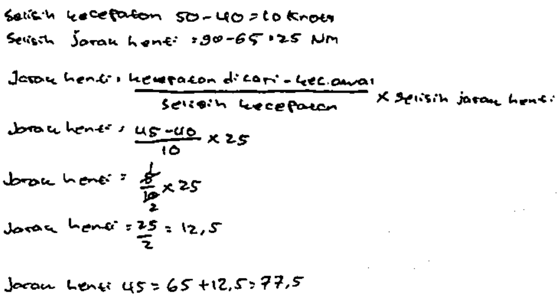
4. Finding Covariational Relationships

   Yoga found a relationship that every 10-knot increase in average speed increases the stopping distance by a maximum of 37 Nm **(F4)**. This discovery occurred at the exploration stage because Yoga found the relationship between two variables to predict unknown values by trying various problem-solving strategies **(PF4)**.

5. Finding Correspondence Relationships

   Yoga found a correspondence from the previous statement that "every average speed increases by 10 knots, the stopping distance increases by a maximum of 37 Nm". The relationship is then arranged using mathematical calculations to produce a general rule **(F5)**. The general rule found by Yoga does not reach mathematical symbolization (Figure 7). This general rule is obtained at the implementation stage because the activity of creating general rules used to predict unknown data values **(PF5)** occurs.

Yoga Answers (Indonesian)



Yoga Answers (English)

$$Speed\ difference = 50 - 40 = 10\ knots$$
$$Stopping\ distance\ difference\ = 90 - 65 = 25\ Nm$$
$$Stopping\ distance = \frac{speed\ sought - initial\ velocity}{speed\ difference} \times stopping\ distance\ difference$$
$$Stopping\ distance = \frac{45 - 40}{10} \times 25$$
$$Stopping\ distance = \frac{5}{10} \times 25$$
$$Stopping\ distance = \frac{25}{2} = 12{,}5$$
$$Stopping\ distance\ 45 = 65 + 12{,}5 = 77{,}5$$

Figure 7. Correspondence relationship finding by Yoga

6. Evaluating General Rule

   The evaluation data of the generalization results were confirmed by the researcher to Yoga through an interview based on the task answers. Yoga evaluated the general rules by checking the known stopping distance data in the question table **(F6)** with the following transcript.

Researcher	:	*Can the general rule estimate every unknown data among the known data?*
Yoga	:	*Yes, it can. This can be checked from the data that is already known.*
Researcher	:	*How do I check the data?*
Yoga	:	*(start working) this uses data when the speed is 30 knots, ma'am if the question is the stopping distance is 46 Nm. If this rule is used, the distance becomes 43 Nm, ma'am. Speed difference* $= 40 - 20$*, stopping distance difference* $= 65 - 21 = 44$*, stopping distance* $= \left(\frac{30-20}{20}\right) \times 44 = 22$*, stopping distance* $= 21 + 22 = 43$ ***(F6).***

Based on the results of the check, Yoga concluded that the general rule is suitable for estimating unknown data among known data. Evaluation of this general rule occurs at the verification stage because Yoga tests the truth of the general rule by identifying possible errors and inconsistencies between the solution and the established problem conditions **(PF6)**.

The findings of this study indicate that participants across different mathematical ability categories exhibited functional thinking aspects; however, variations emerged in the manner in which these aspects developed. Niko employed multiple strategies to identify recursive patterns (F3), covariational relationships (F4), and correspondence relationships (F5). These strategies involved graphical representations, including straight lines, similar right triangles, and similar rectangles, to derive the general rules of linear interpolation. In contrast, Dani relied on a single approach for F3, F4, and F5, using a comparison of two similar triangles to determine the general rules of linear interpolation. Both Niko and Dani successfully derived these general rules through graphical representation. Meanwhile, Yoga was unable to establish the general rules of linear interpolation, despite modifying tabular data within the graph. Furthermore, the general rules identified by Yoga lacked formal mathematical notation, yet he consistently attempted to verify their validity. These results suggest that tasks incorporating non-routine mathematical problems have the potential to foster the development of functional thinking in maritime students with varying mathematical abilities.

The study further revealed that functional thinking emerged in all participants, regardless of their mathematical ability levels, through engagement with mathematical tasks. These tasks included representations illustrating how a ship's average speed influences its stopping distance, necessitating students' application of functional thinking. This finding aligns with the research conducted by Pittalis et al. (2024), which demonstrated that task-based approaches could effectively develop students' functional thinking in relation to quantitative relationships. Similarly, Smith (2008) emphasized that representations of relationships between quantities encourage students to formulate general function rules. Differences in mathematical ability levels appear to influence variations in the approaches employed across different aspects of functional thinking. These differences begin with the identification of recursive patterns, ultimately leading to variations in the discovery of general function rules. Both Niko and Dani successfully formulated general rules of linear interpolation by analyzing recursive patterns within graphical representations. However, Yoga encountered difficulties in interpreting graphs, preventing him from deriving general rules of linear interpolation. The cognitive processes underlying students' mathematical problem-solving are influenced by their respective levels of mathematical ability. According to Sanjaya et al. (2018), students with high mathematical ability tend to employ a wider range of problem-solving strategies, whereas students with moderate ability adopt a more intuitive approach but demonstrate limited depth in justifying their solutions. Meanwhile, students with lower mathematical ability often struggle to devise effective solution strategies.

The findings also highlight that the use of task-based instruments alone is insufficient for comprehensively exploring students' functional thinking. Therefore, supplementary instruments, such as task-based interview protocols, are necessary to gain deeper insights into students' cognitive processes beyond what is captured during problem-solving activities. Interviews revealed that when participants experienced uncertainty regarding their answers, they tended to repeat the problem-solving stages. Variations in mathematical ability appear to influence the frequency with which students revisit these stages. According to Schoenfeld (2006), mathematical ability is a crucial factor in students' success in problem-solving, as it determines how they comprehend problems, make decisions, and apply relevant knowledge in appropriate contexts. The frequency of repetition in problem-solving stages reflects the extent of participants' engagement in the problem-solving process, thereby shaping differences in their approaches. Figure 8 illustrates the sequence of task completion by each participant, highlighting the problem-solving stages that facilitated the emergence of functional thinking aspects. Notably, Yoga perceived that he had completed the exploration stage only once, leading to his difficulty in interpreting graphical representations. His inability to derive general rules suggests an avenue for further research focused on designing tasks that incorporate diverse representations beyond tables and graphs. Such instructional enhancements would enable the development of learning approaches tailored to students' varying mathematical ability levels, thereby optimizing their functional thinking in mathematical problem-solving.

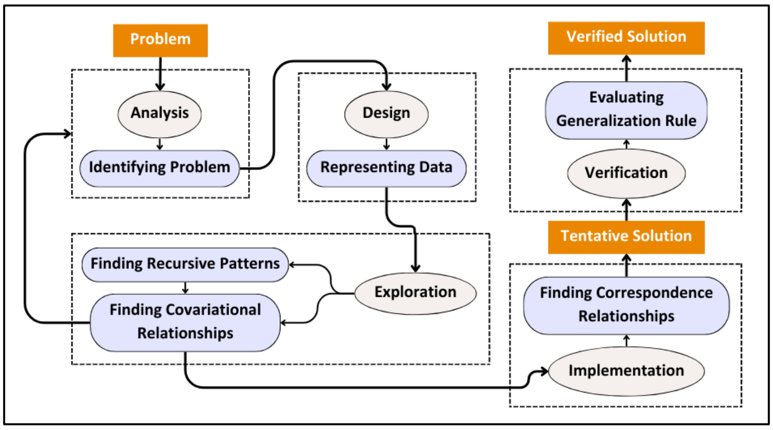


Figure 8. Functional thinking flow in mathematical problem-solving

Previous studies have employed interviews as a data collection technique to examine functional thinking based on students' responses (Günster & Weigand, 2020; Sun et al., 2023; Utami et al., 2023). Analyzing interviews based on student responses provides deeper insights into the development of functional thinking in higher education students. The design of tasks that align with functional thinking aspects plays a crucial role in fostering students' functional thinking skills. This aligns with the research findings of Günster and Weigand (2020), Sun et al. (2023), and Utami et al. (2023), who developed tasks based on a functional thinking framework. Moreover, the present study extends the findings of previous research by incorporating numerical and multi-representational methods, thereby enriching the existing body of knowledge. Other studies have also emphasized the potential of interviews to provide deeper insights into findings derived from students' written responses to tasks (Blanton et al., 2015; Pang et al., 2022; Pitta-Pantazi et al., 2020). While previous research has primarily focused on analyzing the development of functional thinking in primary and middle school students, the present study contributes to the literature by offering insights into the development of functional thinking among students in higher education. Additionally, this study establishes a connection between mathematical concepts and navigation safety, supporting the enhancement of graduate competencies for maritime students.

The findings of this study indicate that tasks incorporating non-routine mathematical problems, which involve representations of relationships between quantities, can enhance students' functional thinking and contribute to reducing the risk of maritime accidents. These tasks serve as problem-solving simulations aimed at mitigating ship collision risks by enabling students to generalize the relationship between average speed and stopping distance. By integrating navigation safety with linear interpolation, these tasks bridge the gap between maritime education and mathematics, equipping students with the necessary analytical skills to become proficient sailors. Furthermore, these tasks can be incorporated into the syllabus and semester lesson plans as part of a teaching strategy in the Applied Mathematics Course, in accordance with the International Maritime Organization (IMO) Model Course 7.03 and the International Association of Maritime Universities (IAMU) curriculum. The implementation of such tasks is not limited to maritime education; educators in other disciplines with similar mathematical learning characteristics may also benefit from adopting these approaches. Additionally, training programs can be organized to support educators in developing and implementing similar tasks, thereby extending their application to a broader range of educational contexts.

CONCLUSION

This study demonstrates that non-routine mathematical problem-solving tasks serve as effective instruments for fostering students' functional thinking through diverse cognitive processes. The findings reveal that each aspect of functional thinking emerged as students engaged with the structured problem-solving stages embedded in the task. Unlike conventional exercises that rely on fixed procedural approaches, the problem presented in this study required students to analyze quantitative relationships characterized by non-fixed addition patterns. Consequently, students were encouraged to employ multiple representations to construct meaningful solutions. Furthermore, the integration of linear interpolation within the context of navigation safety provided an authentic simulation of real-world problem-solving, thereby strengthening the connection between mathematical concepts and maritime applications. By incorporating these tasks into mathematics education, particularly within maritime institutions, students can develop essential problem-solving skills that are fundamental to their future professional competencies. Additionally, this study contributes to the body of research on task design by introducing an innovative approach to representing functional relationships, thereby expanding the range of problem-solving strategies available to educators and researchers.

Despite its contributions, this study is subject to several limitations. One primary constraint is the relatively small sample size, which restricts the generalizability of the findings. A larger participant pool would provide a more comprehensive understanding of the extent to which students develop functional thinking when engaging with non-routine mathematical tasks. Furthermore, while task-based interviews were employed as a supplementary instrument to explore students' thought processes, relying solely on students' written responses may not have fully captured the depth of their reasoning. Although structured interview guidelines were utilized to confirm and clarify functional thinking aspects, further research could integrate additional data collection techniques to enhance the validity and reliability of findings. Another limitation pertains to the scope of representational formats used in the study, as it primarily focused on tabular data. Future investigations could expand this scope by incorporating alternative representations such as algebraic equations, graphical models, function tables, and verbal descriptions to provide a more comprehensive assessment of students' functional thinking development. Finally, future research should consider evaluating the effectiveness of non-routine mathematical tasks in fostering functional thinking among a larger and more diverse cohort of students. Employing quantitative research methodologies, including statistical analyses, could provide more robust empirical evidence regarding the impact of these tasks on students' problem-solving abilities. Additionally, refining and expanding task designs by incorporating multiple representations would accommodate a broader range of learners, enabling them to generalize mathematical concepts across different contexts. Furthermore, future studies should explore instructional strategies that maximize the pedagogical benefits of these tasks, ensuring their applicability in both mathematics education and professional maritime training. By addressing these areas, future research can contribute to the ongoing development of innovative pedagogical approaches that enhance mathematical problem-solving and functional thinking in real-world applications.

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Declarations

Author Contribution	:	LT: Conceptualization, Writing - Original Draft, Methodology, Editing and Visualization. MTB: Conceptualization, Writing - Review & Editing, Formal Analysis, Methodology, Validation and Supervision. AL: Conceptualization, Writing - Review & Editing, Formal Analysis, Methodology, Investigation, Validation and Supervision.
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Conflict of Interest	:	The authors declare no conflict of interest.

REFERENCES

Aguilar, J., & Telese, J. A. (2018). Mathematics understanding of elementary pre-service teachers: The analysis of their procedural-fluency, conceptual-understanding, and problem-solving strategies. *Mathematics Teaching-Research Journal*, *10*(3–4), 24–37. https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2019/04/v10n34-Mathematics-Understanding-of-Elementary-Pre-Service-Teachers.pdf

As'ari, A. R., Kurniati, D., Maharani, S., & Basri, H. (2019). *Ragam soal matematis untuk mengembangkan disposisi berpikir kritis [Variety of mathematical problems to develop critical thinking disposition]*. Universitas Negeri Malang.

Blanton, M. L., & Kaput, J. J. (2011). Functional thinking as a route into algebra in the elementary grades. In *Early algebraization: A global dialogue from multiple perspectives* (pp. 5-23). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-17735-4_2

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J.-S. (2015). The development of children's algebrai thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education*, *46*(1), 39–87. https://doi.org/10.5951/jresematheduc.46.1.0039

Chapra, S. C., & Canale, R. P. (2011). *Numerical Methods for Engineers* (Vol. 1221). Mcgraw-hill.

Chimoni, M., Pitta-Pantazi, D., & Christou, C. (2023). Two different types of technologically enhanced intervention modules to support early algebraic thinking. *Education and Information Technologies*, *28*(3), 3417–3441. https://doi.org/10.1007/s10639-022-11331-x

Confrey, J., & Smith, E. (1991). A framework for functions: Prototypes, multiple representations, and transformations. *Proceedings of the Thirteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, *1*, 57–63. http://eric.ed.gov/?id=ED352274

Creswell, J. W., & Poth, C. N. (2016). *Qualitative inquiry and research design: Choosing among five approaches.* Sage publications.

Denzin, N. K., & Lincoln, Y. S. (2011). *The Sage handbook of qualitative research*. Sage.

Ding, R., Huang, R., & Deng, X. (2023). Multiple pathways for developing functional thinking in elementary mathematics textbooks: A case study in China. *Educational Studies in Mathematics*, *114*(2), 223–248. https://doi.org/10.1007/s10649-023-10237-w

Fatmanissa, N., Yuli Eko Siswono, T., Lukito, A., & Ismail, I. (2024). Utilizing decision-making task: Students' mathematical justification in collaborative problem solving. *Avances de Investigación En Educación Matemática*, *in press*, 1–21. https://doi.org/https://doi.org/10.35763/aiem26.5341

Frey, K., Sproesser, U., & Veldhuis, M. (2022). What is functional thinking ? Theoretical considerations and first results of an international interview study. *Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*.

Günster, S. M., & Weigand, H. G. (2020). Designing digital technology tasks for the development of functional thinking. *ZDM - Mathematics Education*, *52*(7), 1259–1274. https://doi.org/10.1007/s11858-020-01179-1

Huang, L. (2023). A mathematical modeling and an optimization algorithm for marine ship route planning. *Journal of Mathematics*, *2023*(1), 5671089. https://doi.org/10.1155/2023/5671089

Jamaliyatul, N. A. A. H., Badyalina, B., Mokhtar, N. A., Rambli, A., Zubairi, Y. Z., & Ghapor, A. A. (2023). Modelling wind speed data in pulau langkawi with functional relationship. *Sains Malaysiana*, *52*(8), 2419–2430. https://doi.org/10.17576/jsm-2023-5208-18

Jamil, A. F., Siswono, T. Y. E., Setianingsih, R., Lukito, A., & Ismail. (2023). The potential problem to explore metacognitive regulation in ollaborative problem-solving. *Ricerche Di Pedagogia e Didattica*, *18*(1), 57–71. https://doi.org/10.6092/issn.1970-2221/16086

Jamil, M. G., & Bhuiyan, Z. (2021). Deep learning elements in maritime simulation programmes: A pedagogical exploration of learner experiences. *International Journal of Educational Technology in Higher Education*, *18*(1), 1-22. https://doi.org/10.1186/s41239-021-00255-0

Kaput, J. J. (2008). What is algebra? What is algebraic reasoning? In *Algebra In The Early Grades* (pp. 5–18). Routledge.

Khusna, A. H., Yuli, T., Siswono, E., & Wijayanti, P. (2024). Mathematical problem design to explore students ' critical thinking skills in collaborative problem solving. *Mathematics Teaching-Research Journal*, *16*(3), 217–240. https://files.eric.ed.gov/fulltext/EJ1442273.pdf

KNKT. (2018). Laporan Investigasi Kecelakaan Kapal Laut Tenggelamnya KM. Sinar Bangun 4 Tahun 2018. *Kementerian Perhubungan*, 1–53. http://knkt.dephub.go.id/knkt/ntsc_maritime/Laut/2018/FINAL KNKT-18-06-18-03 Sinar Bangun 4-Final PrintA.pdf

Kriegler, S. (2004). Just what is algebraic thinking? *Mathematics Educator*, *8*(1), 139–151. https://www.shastacoe.org/uploaded/SCMP2/Fall_Content_Day_2013/Fall_Content_Day_2013_6-9/SCMP2_Winter_Content_Day_2014/SCMP2_Summer_Institute_2014/M-Algebraic_Thinking_Article_by_Kreigler.pdf

Lichti, M., & Roth, J. (2018). How to foster functional thinking in learning environments using computer-based simulations or real materials. *Journal for STEM Education Research*, *1*(1–2), 148–172. https://doi.org/10.1007/s41979-018-0007-1

Lincoln, Y. S., & Guba, E. . (1985). Naturalistic Inquiry. In *The A-Z of Social Research*. SAGE Publications Inc. https://doi.org/10.4135/9781412986281.n232

Martins, R., Viseu, F., & Rocha, H. (2023). Functional thinking: A study with 10th-grade students. *Education Sciences*, *13*(4), 335. https://doi.org/10.3390/educsci13040335

Mycroft, K., & Sriraman, B. (2021). Mathematics in the Maritime. In *Handbook of the Mathematics of the Arts and Sciences* (pp. 1593-1612). Springer International Publishing. https://doi.org/10.1007/978-3-319-57072-3_137

Noor, N. M., Mustafa, M., Bakri, A., Yahaya, A. S., & Ramli, N. A. (2015). Comparison of linear interpolation method and mean method to replace the missing values in environmental data set. In *Materials science forum* (Vol. 803, pp. 278-281). *Trans Tech Publications Ltd*. https://doi.org/10.4028/www.scientific.net/MSF.803.278

OECD. (2023). Program For International Student (PISA) 2022 Assessment and Analytical Framework. In *OECD (Organisation for Economic Co-operation and Development) Publishing*. https://www.oecd-ilibrary.org/education/pisa-2022-assessment-and-analytical-framework_dfe0bf9c-en

Oliveira, H., Polo-Blanco, I., & Henriques, A. (2021). Exploring prospective elementary mathematics teachers' knowledge: A focus on functional thinking. *Journal on Mathematics Education*, *12*(2), 257–278. https://doi.org/10.22342/jme.12.2.13745.257-278

Oneto, L., Coraddu, A., Cipollini, F., Karpenko, O., Xepapa, K., Sanetti, P., & Anguita, D. (2018). Crash stop maneuvering performance prediction: A data-driven solution for safety and collision avoidance. *Data-Enabled Discovery and Applications*, *2*(1), 1-11. https://doi.org/10.1007/s41688-018-0024-3

Pang, J., Leena, L., & Sunwoo, J. (2022). Task development to measure functional thinking: Focusing on third graders' understanding. *The Korean Society of Educational Studies in Mathematics - Journal of Educational Research in Mathematics*, *32*(3), 351–372. https://doi.org/10.29275/jerm.2022.32.3.351

Pang, J. S., & Sunwoo, J. (2022). Design of a pattern and correspondence unit to foster functional thinking in an elementary mathematics textbook. *ZDM - Mathematics Education*, *54*(6), 1315–1331.

https://doi.org/10.1007/s11858-022-01411-0

Pitta-Pantazi, D., Chimoni, M., & Christou, C. (2020). Different types of algebraic thinking: An empirical study focusing on middle school students. *International Journal of Science and Mathematics Education*, *18*(5), 965–984. https://doi.org/10.1007/s10763-019-10003-6

Pittalis, M. (2023). Young students' arithmetic-algebraic structure sense: An empirical model and profiles of students. *International Journal of Science and Mathematics Education*, *21*(6), 1865–1887. https://doi.org/10.1007/s10763-022-10333-y

Pittalis, M., Sproesser, U., Demosthenous, E., & Odysseos, E. (2024). Enhancing functional thinking in grade 5–6 students through a dynamic mathematics intervention program. In *Education and Information Technologies* (Issue 0123456789). Springer US. https://doi.org/10.1007/s10639-024-12865-y

Rolfes, T., Roth, J., & Schnotz, W. (2021). Mono- and multi-representational learning of the covariational aspect of functional thinking. *Journal for STEM Education Research*, *5*(1), 1–27. https://doi.org/10.1007/s41979-021-00060-4

Rott, B., Specht, B., & Knipping, C. (2021). A descriptive phase model of problem-solving processes. *ZDM - Mathematics Education*, *53*(4), 737–752. https://doi.org/10.1007/s11858-021-01244-3

Sanjaya, A., Johar, R., Ikhsan, M., & Khairi, L. (2018). Students' thinking process in solving mathematical problems based on the levels of mathematical ability. *Journal of Physics: Conference Series*, *1088*(1), 012116. https://doi.org/10.1088/1742-6596/1088/1/012116

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press. https://doi.org/10.1007/BF00305624

Schoenfeld, A. H. (2006). Problem solving from cradle to grave. *Annales de Didactique et de Sciences Cognitives*, *11*, 41–73. https://bibnum.publimath.fr/IST/IST06015.pdf

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *Journal of Education*, *196*(2), 1–38. https://doi.org/10.1177/002205741619600202

Sibgatullin, I. ., Korzhuev, A. ., Khairullina, E. ., Sadykova, A. ., Baturina, R. ., & Chauzova, V. (2022). A systematic review on algebraic thinking in education. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(1), 1–15. https://doi.org/10.29333/EJMSTE/11486

Smith, E. (2008). Representational thinking as a framework for introducing functions in the elementary curriculum. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the Early Grades*. Lawrence Erlbaum Associates/Taylor & Francis Group and National Council of Teachers of Mathematics. https://doi.org/10.4324/9781315097435-6

Stephens, A. C., Fonger, N., Strachota, S., Isler, I., Blanton, M., Knuth, E., & Murphy Gardiner, A. (2017). A learning progression for elementary students' functional thinking. *Mathematical Thinking and Learning*, *19*(3), 143–166. https://doi.org/10.1080/10986065.2017.1328636

Sun, S., Sun, D., & Xu, T. (2023). The developmental progression of early algebraic thinking of elementary school students. *Journal of Intelligence*, *11*(12), 1–19. https://doi.org/10.3390/jintelligence11120222

Taguma, M., Makowiecki, K., & Gabriel, F. (2023). OECD learning compass 2030: Implications for mathematics curricula. In *New ICMI Study Series: Vol. Part F776*. https://doi.org/10.1007/978-3-031-13548-4_33

Tanişli, D. (2011). Functional thinking ways in relation to linear function tables of elementary school students. *Journal of Mathematical Behavior*, *30*(3), 206–223. https://doi.org/10.1016/j.jmathb.2011.08.001

Tarida, L., Budiarto, M. T., & Lukito, A. (2024). State of the art of functional thinking, scaffolding, problem solving and self efficacy (A systematic mapping study). *AIP Conference Proceedings*, *3046*(1), 020017. https://doi.org/10.1063/5.0194597

Team, F., License, C. C., & By-sa, C. C. (2021). *Vision document on Functional Thinking Pre-amble*.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of mathematical thinking. *Compendium for Research in Mathematics Education*, 421–456. https://www.researchgate.net/profile/Patrick-Thompson-2/publication/302581485_Variation_covariation_and_functions_Foundational_ways_of_thinking_mathematically/links/583c389208ae502a85e3a224/Variation-covariation-and-functions-Foundational-ways-of-thinking

Utami, N. S., Prabawanto, S., & Suryadi, D. (2023). How students generate patterns in learning algebra? A focus on functional thinking in secondary school students. *European Journal of Educational Research*, *12*(2), 913–925. https://doi.org/10.12973/eu-jer.12.2.913

Vollrath, H.-J. (1986). Search strategies as indicators of functional thinking. *Educational Studies in Mathematics*, *17*, 387–400. https://doi.org/10.1007/BF00311326

Wahab, M. A. (2017). Interpolation and extrapolation. *Proc. Topics Syst. Eng. Winter Term*, *17*, 1–6. https://doi.org/10.1201/9781315120195-4

Wakhata, R., Balimuttajjo, S., & Mutarutinya, V. (2023). Enhancing the learning of limits of functions using multiple representations. *Mathematics Teaching-Research Journal*, *15*(2), 182–202. https://files.commons.gc.cuny.edu/wp-content/blogs.dir/34462/files/2024/06/10WakhataRwanda.pdf

Warren, E., Miller, J., & Cooper, T. J. (2011). Exploring Young Students' Functional Thinking. *PNA*, *7*(2), 75–84. https://files.eric.ed.gov/fulltext/EJ1057189.pdf

Wei, H., Bos, R., & Drijvers, P. (2023). An embodied approach to abstract functional thinking using digital technology: A systematic literature review. *International Journal of Technology in Mathematics Education*, *30*(2), 75–94. https://doi.org/10.1564/tme

Wilkie, K. J. (2014). Upper primary school teachers' mathematical knowledge for teaching functional thinking in algebra. *Journal of Mathematics Teacher Education*, *17*(5), 397–428. https://doi.org/10.1007/s10857-013-9251-6

Windsor, W. (2011). How problem solving can develop an algebraic perspective of mathematics. *Australian Primary Mathematics Classroom*, *16*(4), 8–13. http://eric.ed.gov/?id=EJ961653

Wu, G., Zhao, M., Cong, Y., Hu, Z., & Li, G. (2021). Algorithm of berthing and maneuvering for catamaran unmanned surface vehicle based on ship maneuverability. *Journal of Marine Science and Engineering*, *9*(3), 289. https://doi.org/10.3390/jmse9030289

Yuniati, S., Nusantara, T., Sulandra, I. M., & Suparjono. (2022). Investigating functional thinking processes that impact on function composition problems in Indonesia. *Acta Scientiae*, *24*(5), 84–118. https://doi.org/10.17648/acta.scientiae.6969

Zhang, J. H., Meng, B., Zou, L. C., Zhu, Y., & Hwang, G. J. (2023). Progressive flowchart development scaffolding to improve university students' computational thinking and programming self-efficacy. *Interactive Learning Environments*, *31*(6), 3792–3809. https://doi.org/10.1080/10494820.2021.1943687