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Exploring three-dimensional geometry using praxeological analysis: Indonesian textbook insights

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Abstract

The integration of three-dimensional geometry in secondary mathematics education plays a crucial role in developing students' spatial reasoning and problem-solving skills. However, textbooks often present limitations in structuring tasks, techniques, and justifications, which may lead to learning obstacles. Despite the importance of well-designed instructional materials, there is a lack of comprehensive studies analyzing Indonesian mathematics textbooks using both the praxeological framework and the learning obstacles perspective in didactic situations. Addressing this gap, this study examines a Grade XII mathematics textbook in Indonesia, focusing on three-dimensional geometry through a structured content analysis. The analysis categorizes tasks based on praxeological components, including types of tasks, solution techniques, technological justifications, and supporting theories, while also identifying potential learning obstacles related to the clarity of visual representations and contextual problem diversity. The findings reveal that the textbook includes 10 types of tasks, solved using 6 techniques, supported by 7 forms of technological reasoning, all grounded in three-dimensional geometry concepts. The presentation of tasks is systematically structured and balances conceptual and procedural aspects, minimizing significant didactic obstacles. However, epistemological obstacles were identified, primarily due to limited visualizations and a lack of diverse contextual tasks, which may hinder students' flexibility in applying three-dimensional geometry concepts. These findings highlight the need for improved task design and enhanced visual representations to foster deeper conceptual understanding and adaptability in problem-solving. This study contributes to mathematics education research by providing empirical insights into textbook design and its impact on students' learning processes, offering recommendations for more effective instructional material development.

Keywords: Didactics, Mathematics Textbooks, Praxeological Framework, Textbook Analysis, Three-Dimensional Geometry

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In numerous countries, including the United States, Japan, Cyprus, Greece, and the Netherlands, mathematics textbooks are regarded as essential instructional resources for educators (Stylianides, 2014). These textbooks serve as fundamental tools that facilitate both teaching and learning processes (Hendriyanto et al., 2023). Given that students engage with textbooks extensively in the classroom (van den Ham & Heinze, 2018), they constitute a critical component of mathematics education (Sievert et al., 2019). As the primary source of instructional content and pedagogical approaches, textbooks significantly shape classroom practices (González-Martín et al., 2013; Haggarty & Pepin, 2002; Pepin et al., 2013).

The structure and use of textbooks directly influence student learning outcomes, demonstrating that variations in textbook content can lead to differences in student performance (Pepin et al., 2013; Rahimah & Visnovska, 2021; Zhang & Savard, 2023). Moreover, the manner in which mathematical concepts are presented—encompassing sequencing, balance, and organization—plays a crucial role in determining student achievement (Hendriyanto et al., 2023; van den Ham & Heinze, 2018; Zhang, 2021). Similar to Japan and China (Fujita & Jones, 2014), mathematics textbooks in Indonesia must align with the national curriculum and obtain formal approval from the Ministry of Education before being legally disseminated and utilized in schools (Utami et al., 2024). Additionally, textbooks not only function as pedagogical tools but also serve as representations of a nation's cultural values (Haggarty & Pepin, 2002).

Given their pivotal role in shaping instructional methodologies, the systematic analysis of textbooks has become a critical area of research, providing valuable insights into the organization and presentation of mathematical content in classroom settings (Céspedes et al., 2022). The diversity of textbooks available offers students varied learning opportunities (Haggarty & Pepin, 2002; Sievert et al., 2019), underscoring the necessity for classroom textbooks to incorporate meaningful and pedagogically rich tasks (Gracin, 2018). Within the global mathematics education research community, the study of mathematics textbooks has been recognized as an essential domain of scientific inquiry (Fan, 2013; Fan et al., 2018). Investigating textbooks is particularly significant due to their profound influence on teaching and learning processes (Hendriyanto et al., 2023), as such analyses contribute to a more comprehensive understanding of curriculum implementation and instructional practices (Gracin, 2018). While Stylianides (2014) previously highlighted the limited scope of research on mathematics textbooks, recent studies indicate an increasing scholarly focus on this subject (Schubring & Fan, 2018). Nevertheless, despite this growing academic interest, certain analytical approaches—such as the praxeological framework—remain relatively unexplored, necessitating further investigation.

Expanding upon this growing body of research, numerous studies have examined mathematics textbooks from various analytical perspectives. For example, Rahimah and Visnovska (2021) investigated the utilization of mathematics textbooks by integrating horizontal, vertical, and contextual analyses, offering a deeper understanding of their role in classroom instruction and their contribution to curriculum implementation. In this context, contextual analysis is particularly valuable for elucidating how textbook content influences mathematics learning within instructional settings. Similarly, Zhang (2021) conducted an analysis of primary school geometry textbooks, specifically focusing on the presentation of three-dimensional shapes. The study revealed that textbooks often structure three-dimensional geometry content in a rigid and linear sequence, emphasizing procedural tasks such as calculating volume and surface area rather than incorporating exploratory activities that could enhance students' spatial reasoning. An excessive focus on procedural aspects may impede the development of a deeper conceptual understanding of three-dimensional geometry. In alignment with these findings, Yunianta et al. (2023) applied a praxeological-didactical framework to analyze the geometry content of Indonesian mathematics textbooks. Their study identified a predominant emphasis on procedural techniques, a hierarchical structuring of tasks, and a lack of substantive conceptual exploration. Additionally, epistemological, ontogenetic, and didactical obstacles were found to hinder students' comprehension of spatial geometry. These findings suggest that to enhance the effectiveness of three-dimensional geometry instruction, modifications in the presentation of textbook content are necessary to better support the development of students' spatial reasoning abilities.

Preliminary findings from an internal study involving 22 students at a private university in Indonesia indicate that many students encounter significant difficulties in understanding three-dimensional geometry, particularly in grasping the relationships between points, lines, and planes. These challenges are reflective of broader difficulties in the teaching and learning of geometry, where abstract spatial concepts present substantial obstacles for both students and educators, especially when compared to more tangible mathematical topics (Kusumah et al., 2020). The inherent complexity of three-dimensional geometry makes it one of the most demanding subjects for high school students in Indonesia, as mastering spatial relationships requires advanced visualization skills and logical reasoning. In light of these challenges, textbooks play a pivotal role in supporting students' learning by offering structured explanations, clear visual representations, and tasks designed to facilitate the attainment of learning objectives.

Geometry has been a fundamental component of mathematics education for centuries (Jablonski & Ludwig, 2023) and continues to be an essential part of mathematics curricula worldwide (Fan et al., 2018). However, it is widely regarded as one of the most challenging mathematical disciplines, often perceived as more difficult than mathematical analysis (Fan et al., 2018; Hardy, 1925). Given its complexity, geometry textbooks serve as critical instructional tools that bridge abstract concepts with real-world applications. Therefore, the analysis of geometry textbooks is essential for gaining insights into how this subject is taught and learned across different educational contexts. As primary empirical sources, textbooks provide valuable perspectives on the core knowledge that must be conveyed in classroom instruction (Takeuchi & Shinno, 2020).

While numerous studies have examined the praxeological structure of mathematics textbooks, research specifically focusing on the treatment of three-dimensional geometry in Indonesian textbooks remains limited. Given the inherent complexity of three-dimensional geometry and the challenges associated with its learning process, it is essential to investigate how this topic is structured through a praxeological framework. Although praxeological analyses have been applied across various mathematical domains, previous studies have primarily concentrated on proportions (Wijayanti & Winslow, 2017), rational numbers (Putra, 2020), spatial geometry (Yunianta et al., 2023), and functions (Utami et al., 2024). Consequently, the praxeological structure of three-dimensional geometry in mathematics textbooks remains largely underexplored.

To address this gap, the present study aims to analyze the organization of three-dimensional geometry within Indonesian mathematics textbooks and identify potential learning obstacles. Specifically, this study examines the composition of tasks, the techniques employed to solve them, as well as the underlying technologies (justification of techniques) and theoretical foundations that support these problem-solving approaches. By adopting the praxeological framework—one of the core concepts within the Anthropological Theory of Didactics (ATD)—this study seeks to provide a more comprehensive understanding of the praxeological structure embedded in three-dimensional geometry textbooks, thereby contributing to the broader discourse on mathematics textbook analysis.

Theoretical Framework

The ATD is a conceptual framework in mathematics education initially developed by Yves Chevallard in the 1980s (Bosch & Gascón, 2014). This theory primarily explores the process of didactic transposition (Bosch et al., 2020; Bosch & Gascón, 2014) and conducts analyses within the broader framework of the ecology of knowledge (Chevallard & Bosch, 2019). ATD serves as a tool for examining didactic knowledge as it emerges within social practices where teaching and learning take place (Chevallard & Bosch, 2019). Didactics, in this context, is defined as the scientific study of the dissemination and acquisition of knowledge within society (Chevallard & Sensevy, 2014; Suryadi, 2023).

A central issue in didactics has traditionally been reduced to two primary components: a knowledge object O and a human subject x who is assumed to "learn" O. This issue is encapsulated within ATD by the relationship $R(x,O)$, which represents the interaction between the learner and the knowledge object (Chevallard & Sensevy, 2014). A fundamental premise of ATD is that the knowledge produced in academic institutions, such as universities, differs from the knowledge presented to students in classroom settings. For students to effectively understand and internalize this knowledge, it must undergo a transformation and simplification process known as didactic transposition (Pansell, 2023). Over time, the concept of didactic transposition has become a recognized sub-theory within the broader ATD framework (Chevallard, 2019).

The concept of didactic transposition underscores the importance of analyzing how mathematics is introduced and restructured within educational contexts to achieve an accurate understanding of school mathematics (Bosch & Gascón, 2014). This process refers to the transformation of knowledge from its original creation and application into a form suitable for teaching and learning in specific educational settings (Bosch & Gascón, 2014; Chevallard & Bosch, 2014). In disciplines such as mathematics, the content taught in schools originates from scientific knowledge produced in universities and research institutions, often integrating elements from various related social practices. To ensure that this knowledge is effectively adapted for classroom instruction, significant modifications are required to render it teachable, meaningful, and applicable to students' learning experiences (Chevallard & Bosch, 2014).

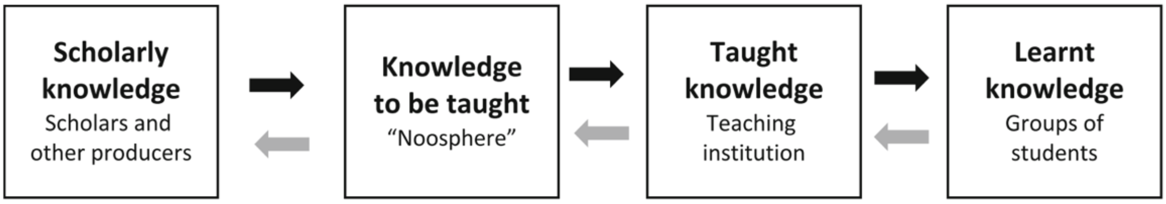


Figure 1. Diagram of the process of didactic transposition

A simplified model illustrating the didactic transposition process is depicted in Figure 1 (Bosch & Gascón, 2014; Chevallard & Bosch, 2014). This process begins with mathematicians generating mathematical knowledge, which is subsequently adapted for instructional purposes within educational institutions. The adaptation process is influenced by multiple stakeholders, including the Ministry of Education, curriculum developers, textbook authors, and policymakers—collectively referred to as the noosphere. The refined knowledge is then presented through school textbooks, with teachers playing a critical role in its dissemination. Educators may either adhere strictly to the textbook or modify the content based on their pedagogical experience and instructional strategies. Ultimately, the knowledge acquired by students represents the final stage of the didactic transposition process (Artigue & Bosch, 2014; Hendriyanto et al., 2023; Putra, 2020).

To ensure clarity in each stage of transposition, the body of knowledge being transmitted must be well-defined (Putra, 2020). Within this framework, the ATD introduces an epistemological model for analyzing human knowledge, known as praxeology (Chevallard, 2006; 2007). ATD conceptualizes both mathematical and didactic practices as praxeologies, which serve as analytical tools for understanding various human activities beyond mathematics (Artigue & Bosch, 2014). The praxeology theory, a sub-theory of ATD, provides insight into the relationship between individuals and educational institutions. This concept facilitates a deeper examination of human activities, including mathematical practices (Chevallard, 2019).

According to the anthropological principles of ATD, all human activities can be interpreted through praxeologies, which exist on different scales—point, local, and regional (Bosch et al., 2020). The praxeology framework proposed in ATD thus offers a structured approach to analyzing diverse forms of human knowledge and practices, particularly in the context of mathematical learning (Bosch, 2015).

The term "praxeology" originates from the Greek words "praxis" and "logos" (Bosch et al., 2017; 2020). Within the framework of didactic transposition, praxeology serves as the fundamental analytical unit for examining human knowledge (Putra, 2020). This concept encompasses not only individuals' actions and methods but also their cognitive processes and reasoning (Chevallard, 2006). The praxis component pertains to the practical dimension of activities, encompassing both "doing" and the associated knowledge. The practical block consists of two interrelated components: the type of task (T) to be learned and the technique (τ) employed to solve it (Putra, 2020). While praxis focuses on execution, logos pertains to conceptual elaboration, which involves describing, explaining, and justifying techniques, as well as structuring task types and their corresponding techniques (Bosch et al., 2020). The logos block comprises two interdependent elements: technology (θ), which provides justification for techniques, and theory (Θ), which offers broader theoretical support (Putra, 2017). By applying the praxeological framework, this study classifies the various types of student tasks related to three-dimensional geometry, analyzes the techniques employed to solve these tasks, and investigates the underlying justifications (technology) and theoretical foundations that structure the knowledge presented in textbooks. This analysis facilitates an evaluation of whether the tasks in the textbook predominantly foster procedural learning or contribute to the development of conceptual understanding (see Table 1).

Table 1. A praxeology

Praxis Block		**Logos Block**	
Type of Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
Problems of a given type.	A way of performing this type of task	A way of explaining and justifying (or designing) the technique	To explain, justify, or generate whatever part of the technology that may sound unclear or missing
In the context of mathematical textbook analysis			
Types of student tasks given in the textbook (problems that students need to solve)	Possible ways for students to solve the type of task given (solution of the problem given in the textbook)	Justification of the ways in which students complete the tasks in the textbook.	Reliable and reasonable basis/ reference for justification of the ways in which students complete the tasks in the textbooks

(Adapted from Takeuchi and Shinno (2020) and Yunianta et al. (2023)

The culmination of the didactic transposition process results in the formation of knowledge to be taught, which is subsequently implemented within a structured learning situation designed by the teacher. Within the dynamic interactions between students and this learning environment, both the emergence of knowledge and potential learning obstacles may arise. Various factors can contribute to students' difficulties in comprehending particular concepts, making it inappropriate to attribute learning challenges to a single component of the instructional system (Brousseau, 2002). However, as a fundamental element within the learning process, textbooks can be systematically analyzed to identify potential sources of learning obstacles. The theoretical framework for learning obstacles adopted in this study is derived from the Theory of Didactic Situations (TDS), originally developed by Guy Brousseau (Torres, 2023).

TDS examines classroom situations that facilitate students' development of mathematical knowledge. A central concept in designing these instructional settings is the didactic milieu, which encompasses the environment, problem situations, and learning artifacts with which students interact to acquire new knowledge. Teachers play a critical role in constructing a didactic milieu that fosters the progressive development of students' mathematical understanding (Jessen et al., 2023). Within TDS, four primary types of learning situations are identified: action situations, formulation situations, validation situations, and institutionalization (Yenil et al., 2023). These stages represent the progression of the learning process. Learning begins with the action situation, where students engage with problem-solving activities using their prior knowledge and experiences, fostering the perception of their environment and enabling meaningful interactions. As students construct new mental representations, the process transitions into the formulation situation, in which abstraction occurs through cognitive engagement. The subsequent interaction between students and teachers facilitates the negotiation of meaning, leading to argumentation, reasoning, and representational activities that support both internal and external validation processes (Suryadi, 2019).

The learning process does not always proceed as anticipated and frequently encounters both didactic and pedagogical challenges. According to Brousseau (2002), learning obstacles can be classified into three categories: ontogenic, epistemological, and didactic. Ontogenic obstacles stem from students' developmental limitations, including neurophysiological factors (Brousseau, 2002), and are closely linked to their cognitive readiness for learning (Suryadi, 2019). Epistemological obstacles, on the other hand, emerge due to constraints within the contextual framework employed during instruction. Brousseau (2002) highlights that these obstacles manifest in students' errors when responding to specific tasks and questions, often resulting from the restricted scope of concepts introduced, which compels students to rely solely on previously demonstrated examples (Yunianta et al., 2023).

Conversely, didactic obstacles arise from the instructional approach itself, shaped by the organization and sequencing of lesson materials (Suryadi, 2019). By examining the concept of didactic obstacles, this study seeks to determine the presence of epistemological, ontogenic, and didactic obstacles within textbooks and assess the extent to which these challenges influence students' comprehension of three-dimensional geometry concepts.

METHODS

This study employs two primary theoretical frameworks: the praxeological framework from the ATD (Chevallard, 2006; 2007; 2019) and the learning obstacle framework derived from the TDS (Brousseau, 2002). The praxeological framework serves to classify the types of student tasks related to three-dimensional geometry, analyze the techniques employed to solve them, and investigate the underlying justifications (technology) and theoretical foundations that shape knowledge organization in textbooks. This analytical approach facilitates an assessment of whether the tasks presented in the textbooks predominantly emphasize procedural learning or actively promote conceptual understanding.

Concurrently, the TDS is utilized to identify potential learning obstacles embedded in the structuring of textbook tasks, specifically epistemological, ontogenic, and didactical obstacles. This analysis seeks to determine the extent to which these obstacles hinder students' comprehension of three-dimensional geometry concepts and to evaluate the influence of textbook content organization on students' learning processes.

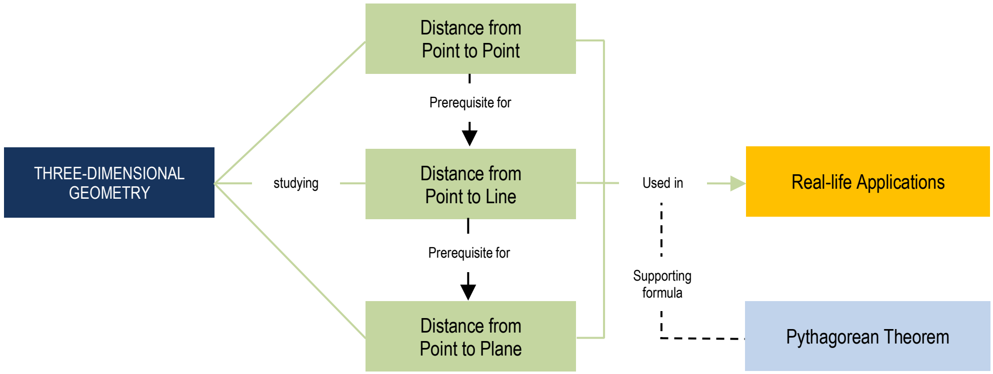


Figure 2. Concept flow diagram of three-dimensional geometry

The textbook examined in this study was published by the Ministry of Education and Culture of the Republic of Indonesia in 2018 and serves as a core instructional resource for implementing the 2013 Curriculum in various high schools across the country. Comprising four chapters, this analysis specifically focuses on the chapter dedicated to three-dimensional geometry. Notably, the textbook encourages feedback for continuous refinement, highlighting its commitment to ongoing revisions aimed at enhancing the quality of mathematics learning. This chapter is presented as a self-contained unit and constitutes the final geometry topic covered in secondary education.

A fundamental prerequisite for understanding the material in this chapter is students' prior knowledge of the Pythagorean theorem and the area of triangles, as these concepts are integral to solving problems involving distances in three-dimensional space. The chapter begins with a reading section on Euclid, followed by a concept flow diagram (Figure 2) that provides a structured overview of the key ideas. This diagram visually represents the interconnections among various sub-topics, ensuring a coherent progression of concepts. Each sub-topic is introduced through a contextual problem designed to foster student engagement. Additionally, the textbook incorporates structured learning activities such as *Let's Observe, Let's Ask Questions, Let's Gather Information,* and *Let's Communicate*, complemented by problem-solving exercises. Worked examples are systematically included within each sub-section to demonstrate effective problem-solving strategies. Furthermore, numerous illustrations are provided to support conceptual understanding, enabling students to visualize spatial relationships more effectively.

The selection of this textbook for analysis was driven by its coverage of three-dimensional geometry, a fundamental component of the secondary school mathematics curriculum. This chapter specifically addresses key topics such as the distance between points, the distance from a point to a line, and the distance from a point to a plane. These topics were chosen due to the prevalent challenges students encounter in grasping the relationships between geometric elements and the difficulties they face in applying these concepts to problem-solving. The objective of this analysis is to assess how the textbook presents these topics and to determine the effectiveness of its instructional approach in facilitating students' conceptual understanding. The learning objectives related to three-dimensional geometry, as outlined in the textbook, are summarized in Table 2.

The textbook analysis conducted in this study employed a systematic praxeological approach to examine the structuring of three-dimensional geometry. The process began with the selection of the textbook, guided by its relevance to the curriculum and the well-documented challenges students encounter in learning geometry. The chosen textbook comprises four chapters, with three-dimensional geometry introduced as the first topic.

The next phase of the analysis involved identifying all tasks embedded within each sub-topic. Tasks were defined as student-engaging activities, including exercises, worked examples, and exploratory questions. These tasks were systematically mapped to identify recurring patterns and subsequently classified into procedural or conceptual categories to evaluate the textbook's emphasis on different learning approaches. In this classification, procedural learning is characterized by students' ability to apply algorithms and formulas without necessarily comprehending their underlying principles, whereas conceptual learning entails a more profound understanding of mathematical concepts, their interrelationships, and their applications in diverse contexts (Arslan, 2010).

Table 2. Learning material and objectives

Topic	Objective
Three-dimensional geometry	1. Observe and describe problems related to the distance between points, the distance from a point to a line, and the distance from a point to a plane in space
	2. Observe and apply the concepts of distance between points, the distance from a point to a line, and the distance from a point to a plane to solve problems in three dimensions.
	3. Construct formulas for the distance between two points and the distance from a point to a line.

After categorizing the tasks, the problem-solving techniques employed in the textbook were analyzed to identify common approaches used across different exercises. Similar techniques were grouped to determine whether the textbook encouraged diverse problem-solving strategies or predominantly relied on a single approach. The subsequent stage involved justifying the identified techniques by examining both their technological (the rationale for employing a particular method) and theoretical (the mathematical principles underpinning the technique) foundations. This process aligns with the praxeological framework in ATD, which distinguishes between *know-how*—comprising task types and solution techniques—and *know-why*, which encompasses the justification and theoretical basis of these techniques (Pansell, 2023).

Finally, the analysis investigated potential learning obstacles that may arise due to the organization and presentation of tasks within the textbook. This included assessing the logical sequencing of exercises, the clarity of explanations, and the role of visual representations in either facilitating or hindering students' comprehension of spatial concepts. The identified obstacles were categorized as epistemological, ontogenic, or didactical, in accordance with the TDS. This step was essential in evaluating whether the textbook effectively fosters conceptual understanding or inadvertently introduces barriers to students' learning processes.

RESULTS AND DISCUSSION

The Praxis: Type of Task

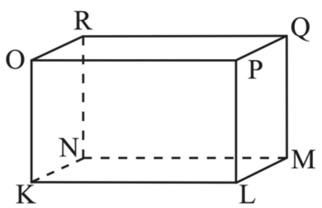
The analysis in this section initially concentrated on the praxis block, which encompasses task types and techniques. Subsequently, the investigation extended to the logos block, which involves the integration of technology and theoretical frameworks. The classification of task types was determined based on the questions (problems) and activities presented throughout the textbook, from the introductory section to the conclusion of the material. The primary objective of analyzing these problems as task types was to examine their sequencing and progression within the classroom setting, thereby identifying potential didactic obstacles (Yunianta et al., 2023). Table 3 provides an overview of the subtopics covered in the three-dimensional geometry chapter of the ninth-grade high school mathematics textbook.

Table 3. Task distribution in the subtopic of three-dimensional geometry

No	Subtopic	Number of Tasks
1	Distance between points	19
2	Distance from a point to a line	12
3	Distance from a point to a plane	11

The three subtopics examined in this study are: Distance between Points, Distance from a Point to a Line, and Distance from a Point to a Plane. The findings indicate that the first subtopic contains the highest number of tasks, with 19 out of the total 42 tasks. To enhance the analysis, these tasks were systematically categorized based on structural and functional similarities. Each task was labeled as T, accompanied by an index representing its specific type. Additionally, the tasks were classified as either procedural or conceptual, distinguishing those that primarily require the application of a procedure from those that involve deeper mathematical reasoning. The analysis identified ten distinct task types in the textbook. This study seeks to elucidate the common characteristics of question sets classified under the same task type, providing illustrative examples, as presented in Table 4.

Table 4. Types and number of tasks for three-dimensional geometry

Type of Task	Description	N	%	Task Classification
T1	Determining the shortest route among a series of interconnected points	1	2,38	Conceptual
T2	Writing important terms based on observations	3	7,14	Conceptual
T3	Generating questions based on observations Tasks involve generating questions based on observations. For example, in one activity, students are asked to reflect on an observation and formulate their own questions. The textbook provides an example question: “What is the definition of the distance between two points?” Students are then encouraged to write additional questions related to their observations in the space provided.	3	7,14	Conceptual
T4	Determining the expression for the distance between two points in the illustration of a three-dimensional object. Example: Given an illustration of a flat-sided solid figure, then the students are asked to determine the correct expression.  a. What is the distance between point P and N?	8	19,05	Conceptual

Type of Task	Description	N	%	Task Classification
T5	Determining the distance between two points Example: Given the regular triangular pyramid T.ABC with an equilateral triangle as the base. TA is perpendicular to the base. If the length of AB = 42 cm and TA = 4 cm, determine the distance between point T and C!	6	14,29	Procedural
T6	Constructing a formula for distance between points	1	2,38	Conceptual
T7	Establishing minimum conditions to achieve minimal rope length	1	2,38	Conceptual
T8	Determining the distance from a point to a line Example: Given the regular pyramid T.ABCD, where the length of edge AB = 3 cm and TA = 6 cm. Determine the distance between point B and the edge TD.	8	19,05	Procedural
T9	Determining the distance from a point to a plane. Example: Given the cube ABCD.EFGH with edge length a cm. Point Q is the midpoint of edge BF. Determine the distance from point H to the plane ACQ	8	19,05	Procedural
T10	Drawing conclusions	3	7,14	Conceptual
	Total	42	100	

The utilization of mathematics textbooks plays a crucial role in shaping instructional approaches in the classroom (Yunianta et al., 2023). Each task type within the textbook contributes distinctively to the way teachers present mathematical concepts to students. The distribution and emphasis placed on different task types establish a structured sequence that influences the progression of content delivery. This predetermined sequence is systematically outlined in Table 5, which provides a comprehensive overview of the task arrangement in the textbook, categorized according to subtopics within the three-dimensional geometry material.

Table 5. Sequences of the task

No	Subtopic	Sequences
1	Distance between points	T1 → T2 → T3 → T4 → T5 → T6 → T10 → T5
2	Distance from a point to a line	T2 → T3 → T7 → T8 → T10 → T8
3	Distance from a point to a plane	T2 → T3 → T9 → T10 → T9

The tasks in the three-dimensional geometry section of the first subtopic are structured to provide students with contextual grounding, fostering an initial understanding before introducing core mathematical concepts. T1 serves as the starting point, guiding students in visualizing the shortest route among multiple interconnected paths. In this task, students are required not only to identify various possible routes between two points but also to analyze and determine the most optimal path based on length. Given its emphasis on conceptual exploration, T1 is classified as a conceptual task, as it encourages students to comprehend the structure of route networks, consider multiple alternatives, compare options, and evaluate solutions before reaching a conclusion.

Building upon this foundation, T2 is introduced to strengthen students' understanding of the definition of distance. This task involves observation and documentation, serving as a bridge to a more in-depth exploration of distance in three-dimensional space. Subsequently, T3 is provided to encourage students to formulate questions based on their observations from T1 and T2, further reinforcing their conceptual grasp of the topic. With the insights gained through these tasks, students then proceed to T4, which requires them to determine the appropriate mathematical expression for distance within a three-dimensional object. This sequential task arrangement is categorized as conceptual, as it extends beyond mere computations or formula application. Instead, it necessitates a deep understanding of fundamental distance concepts in three-dimensional space, fostering students' analytical and reasoning skills.

Building upon these foundational concepts, T5 introduces a contextual problem related to the distance between two points. This task guides students through a structured problem-solving process using previously acquired methods. Subsequently, T6 prompts students to construct a formula for calculating the distance between two points in the Cartesian coordinate system, encouraging them to apply the fundamental principles of the Pythagorean Theorem. As a culminating step, T10 requires students to synthesize their understanding by formulating a general conclusion on determining the distance between two points.

In terms of task classification, T5 is categorized as a procedural task, as it primarily involves the direct application of a given formula without necessitating an in-depth conceptual understanding of the underlying process. In contrast, T6 and T10 are classified as conceptual tasks, as they engage students in constructing their own formulas and deriving conclusions based on observed mathematical patterns. By fostering critical thinking and encouraging connections between mathematical concepts, T6 and T10 support students in developing a deeper conceptual comprehension of distance in three-dimensional space.

In the second and third subtopics, three additional task types—T7, T8, and T9—are introduced. T7, akin to T1, requires students to conduct experiments to determine the necessary conditions for measuring distance and to provide justifications for their conclusions. Due to its emphasis on conceptual exploration and reasoning, T7 is classified as a conceptual task. Meanwhile, T8 and T9 focus on problems concerning the distance from a point to a line and the distance from a point to a plane, respectively. Both tasks involve the straightforward application of established formulas and methods, classifying them as procedural tasks.

The analysis of task classification within the three-dimensional geometry material reveals a relatively balanced distribution between conceptual and procedural tasks. Among the 42 tasks examined, 20 (47.62%) were identified as conceptual, while 22 (52.38%) were procedural. This balance suggests that the textbook not only emphasizes procedural fluency in mathematical problem-solving but also provides ample opportunities for students to develop a deeper conceptual understanding. Procedural tasks facilitate mastery of mathematical techniques through formula application, whereas conceptual tasks encourage students to explore mathematical ideas, refine their reasoning skills, and establish connections among various concepts in three-dimensional geometry.

The Praxis: Technique

Each assigned task inherently possesses at least one corresponding solution technique. In this study, the term technique refers to a strategy or practical approach that students can employ to solve problems within the three-dimensional geometry material. This section aims to systematically identify the possible techniques applicable to each task and classify them based on their distinct solution approaches. To facilitate this classification, each technique is denoted by the symbol τ, accompanied by an index that represents its specific type. The findings related to these techniques, along with their descriptions and categorizations, are presented in Table 6.

Table 6. Techniques for each type of task for three-dimensional geometry

Type of Task		Description	Type of Technique
T1	$\tau 1$	Calculating the length of the path based on the provided information	Measurement and Calculation
	$\tau 2$	Analyzing and comparing possible routes to reach other points	Analysis and Comparison
	$\tau 2$	Conducting travel simulations to test various routes and determine the shortest one.	Analysis and Comparison
T2	$\tau 3$	Identifying and recording key terms found from observations.	Observation and Terminology
	$\tau 3$	Defining and elaborating on the observed key terms.	Observation and Terminology
T3	$\tau 3$	Formulating relevant and informative questions based on observations	Observation and Terminology
	$\tau 3$	Identifying key issues arising from observations and transforming them into questions	Observation and Terminology
T4	$\tau 2$	Selecting the appropriate expression based on the provided diagram.	Analysis and Comparison
T5	$\tau 1$	Utilizing the Pythagorean theorem	Measurement and Calculation
T6	$\tau 4$	Utilizing the Pythagorean theorem to construct the distance formula between two points by exploiting the relationship of right-angled triangles	Analysis and Construction
	$\tau 4$	Using the concept of vectors to construct the distance formula between two points	Analysis and Construction
T7	$\tau 1$	Using a ruler to measure the minimum required length	Measurement and Calculation
	$\tau 4$	Utilizing the approach of comparing the areas of triangles to determine the distance from a point to a line by comparing the areas of the triangles formed	Analysis and Construction
T8	$\tau 1$	Utilizing the fact that diagonals in a square plane intersect at right angles to determine the distance from a point to a line in the context of a square plane	Measurement and Calculation
	$\tau 1$	Utilizing the approach of comparing the areas of triangles to determine the distance from a point to a line by comparing the areas of the triangles formed	Measurement and Calculation
T9	$\tau 1$	Employing the approach of comparing the areas of triangles to determine the distance from a point to a plane by comparing the areas of the triangles formed	Measurement and Calculation
T10	$\tau 5$	Drawing conclusions based on alternative solutions	Drawing Conclusions

The classification of solution techniques employed in solving tasks highlights a diverse range of approaches that engage students in exploring three-dimensional geometry concepts. Among these, Measurement and Calculation (τ1) techniques are predominantly utilized in procedural tasks, where students are required to apply established formulas—such as the Pythagorean theorem—to determine distances between points, lines, and planes. This technique is commonly observed in tasks T1, T5, T7, T8, and T9, where students perform distance calculations based on given geometric structures.

In contrast, Analysis and Comparison (τ2) techniques are frequently applied in conceptual tasks, particularly those requiring the evaluation of multiple possible solutions. Tasks like T1 and T4 necessitate that students analyze diagrams, compare alternative routes or expressions, and select the most appropriate solution within the given context. These tasks foster critical thinking by encouraging students to develop reasoning skills and justify their choices.

Another essential approach is the Observation and Terminology (τ3) technique, which supports conceptual understanding by prompting students to identify key terms and generate relevant questions based on their observations. This technique, as seen in T2 and T3, enables students to construct a deeper comprehension of three-dimensional geometry concepts before engaging in problem-solving. Furthermore, Analysis and Construction (τ4) techniques are fundamental in tasks requiring the derivation of new formulas or the construction of mathematical relationships. For example, in T6 and T7, students are guided to develop the distance formula by utilizing the Pythagorean theorem or vector representations. These tasks emphasize knowledge construction rather than the mere application of pre-existing formulas. Lastly, the Drawing Conclusions (τ5) technique, exemplified in T10, requires students to synthesize their findings and formulate general rules based on observed patterns. This step reinforces conceptual understanding and ensures that students can articulate their mathematical reasoning effectively.

For instance, in T7, students engage in an experiment to determine the minimum rope length connecting point C to line AB. Two primary techniques can be employed to solve this problem. The Measurement and Calculation (τ1) technique allows students to directly measure the rope length using a ruler, providing an empirical approach to the problem. Alternatively, the Analysis and Construction (τ4) technique applies the concept of triangle area comparison to determine the minimum distance from a point to a line, mathematically proving that the minimum rope length corresponds to the perpendicular distance from point C to line AB. The integration of these techniques offers a more comprehensive understanding of the concept of distance in three-dimensional geometry.

The Logos: Technology and Theory

The logos block comprises two fundamental components: technology (θ) and theory (Θ). One section of the study elucidates that these components function as essential tools for justifying the application of specific techniques. A classification was conducted to determine the rationale behind employing these techniques and their underlying foundations. Furthermore, theory serves as the argumentative framework or reference supporting the utilization of technology. Within the scope of the book under discussion, the analysis was confined to a single theoretical perspective, specifically three-dimensional geometry.

Table 7. Praxeology of Three-Dimensional Geometry in Indonesian Textbooks

Type of Task	Technique	Technology	Theory
T1 : Determining the shortest route among a series of interconnected points	τ1 : Measurement and Calculation τ2 : Analysis and Comparison	θ1 : Comparison can identify mathematical objects or concepts' patterns, differences, and similarities.	Θ1 : Three-dimensional geometry
T2 : Writing important	τ3 : Observation and	θ2 : Key terms help understand	

Type of Task	Technique	Technology	Theory
terms based on observations	Terminology	something as the basis for asking questions and drawing conclusions.	
T3 : Generating questions based on observations	τ3 : Observation and Terminology	θ2 : Key terms help understand something as the basis for asking questions and drawing conclusions.	
T4 : Determining the expression for the distance between two points in the illustration of a three-dimensional object	τ2 : Analysis and Comparison	θ3 : Comparison can identify mathematical objects or concepts' patterns, differences, and similarities.	
T5 : Determining the distance between two points	τ1 : Measurement and Calculation	θ4 : Using the Pythagorean theorem.	
T6 : Constructing a formula for distance between points	τ4 : Analysis and Construction	θ4 : Using the Pythagorean theorem.	
T7 : Establishing minimum conditions to achieve minimal rope length	τ1 : Measurement and Calculation	θ5 : The minimum length of the rope can be obtained by repeatedly measuring its size and comparing it.	
T8 : Determining the distance from a point to a line	τ1 : Measurement and Calculation	θ6 : The diagonals of a square intersect at right angles. θ7 : Comparing the areas of triangles by choosing different bases can result in different height-to-base ratios for the triangles.	
T9 : Determining the distance from a point to a plane	τ5 : Drawing Conclusions	θ7 : Comparing the areas of triangles by choosing different bases can result in different height-to-base ratios for the triangles.	
T10 : Drawing conclusions		θ2 : Key terms help understand something as the basis for asking questions and drawing conclusions.	

For instance, as illustrated in Table 7, the elements of praxeology (T1/ τ2/ θ1/ Θ1) encompass tasks that require students to determine the shortest path between two points when multiple routes are available. Students can employ a comparative method to evaluate the possible routes and identify the shortest one. This approach is feasible because mathematical objects or concepts can be analyzed through patterns, differences, and similarities. Furthermore, a single technological component can serve as justification for multiple techniques. For example, our findings indicate that τ3 and τ5—techniques associated with T2, T3, and T10—utilize the same technological component, θ2. This alignment suggests that fundamental mathematical terms facilitate comprehension, serving as a basis for formulating questions and drawing conclusions.

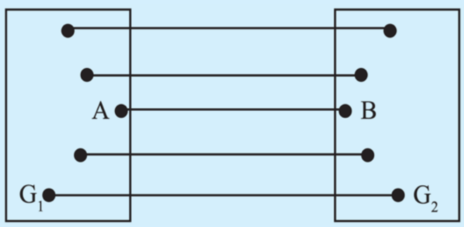
Beyond analyzing the types of tasks and techniques employed in the textbook, this study also seeks to identify potential learning obstacles arising from the textbook's presentation of material. The learning process must anticipate and address these challenges to ensure effective knowledge acquisition. In this context, two primary categories of learning obstacles emerge: discrepancies in instructional practice and the gap between students' prior mathematical knowledge and the expected learning outcomes (Leung & Bolite-Frant, 2015). These obstacles, commonly classified as didactic and epistemological barriers, originate from different sources. Didactic obstacles stem from instructional methods and pedagogical approaches, while epistemological obstacles arise due to students' limited understanding and prior knowledge of mathematical concepts (Sari et al., 2024).

Praxeological analysis, as outlined by Bosch et al. (2017), serves as a valuable framework for examining the manner in which mathematics is presented in textbooks and how these presentations address didactic challenges. The sequencing and staging of material, along with the selection of instructional designs, can contribute to the emergence of didactic obstacles (Suryadi, 2019). In contrast, epistemological obstacles pertain to knowledge gaps that students must bridge through the construction of new understanding (Leung & Bolite-Frant, 2015). Such obstacles often arise when previously acquired knowledge, which was once effective, proves to be insufficient or incorrect in a new context (Schneider, 2014).

In this study, learning obstacles are integral to understanding the difficulties students may encounter while engaging with the textbook. Identifying these obstacles enables a deeper analysis of the challenges stemming from both the structure of tasks and the way mathematical concepts are introduced. The textbook presents the material in a structured manner, requiring prior knowledge of fundamental concepts such as the Pythagorean theorem, right triangles, and spatial figures. Given that students have previously studied these topics, the progression of three-dimensional geometry in the textbook does not introduce significant didactic obstacles. However, potential epistemological obstacles were identified, particularly in the visual representations utilized in the textbook, as elaborated in the subsequent sections.

Problem 1.2

Consider the following problem



If G1 and G2 are geometric figures, then G1 and G2 can be thought of as sets of points. From G1 and G2, a one-to-one pairing can be made between the points in G1 and G2. If AB is the shortest among all line segments connecting these points, then the length of line segment AB is called the distance between figures G1 and G2.

Figure 3. Examples of problems found in the mathematics textbook

To illustrate how these obstacles emerge in actual textbook tasks, we examine two specific examples (T2 and T7) that highlight potential challenges. The problem presented in Figure 3 corresponds to T2, in which students are required to identify key terms based on a given image. This task is designed to introduce the concept of distance between points, where the shortest path length between two points represents the measured distance. However, ambiguity in the use of symbols within the task may lead to misinterpretation. In the explanation section, G1 and G2 are denoted as sets, implying that they contain multiple elements, including G_1 as an element of G1 and G_2 as an element of G2. The notation employed may cause confusion, as G_1 and G_2 might be mistakenly perceived as specific points within the sets G1 and G2, rather than the sets themselves. Consequently, students may incorrectly assume that the distance between G1 and G2 is merely the segment AB, whereas the author's actual intention is to describe the shortest distance between two sets. To mitigate this confusion, it may be necessary to revise the notation or provide additional clarification to explicitly convey the intended meaning of these symbols.

The obstacle arising from this issue is classified as an epistemological obstacle, as it originates from limitations in the contextual information provided (Suryadi, 2019). Such constraints can lead students to develop a restricted or incomplete understanding of the underlying mathematical concept (Prihandhika et al., 2020).

Another potential obstacle that may arise from the mathematics textbook analyzed also appears in task 7, as depicted in Figure 4. Problem 1.4 in Figure 4 is T7, where students are asked to determine the minimum length of a rope from one point to a line segment. Based on the analysis, this activity can be done by directly measuring the rope using a ruler and comparing various rope positions to the line segment to obtain the minimum length (τ1). The task serves as an introduction for students to understand the concept of the distance from a point to a line. Based on the presentation, the author does not explicitly indicate where points A, B, and C are located. Perhaps students must use their reasoning to determine them using the concept of the Pythagorean triple independently. However, the task may lead to different perceptions among students:

1. Regarding the presented figure, the former triangle appears to be isosceles, allowing students to create different point positions than those intended by the textbook author.
2. For students who need to remember the concept of the Pythagorean theorem, the determination of points A, B, and C can be anywhere. Significantly, if accustomed to using the standard agreement/convention in writing point symbols in a flat shape counterclockwise, the positions intended by the author will differ from those made by the students. An alternative presentation could clarify the point positions, thus conveying the purpose of introducing the concept of the distance from a point to a line more clearly.
3. Attention should be paid to using symbols on the flat shape, indicating that if referring to the commonly agreed-upon method when writing symbols, as also presented in every symbolization of other shapes in this book, they should be written sequentially counterclockwise.

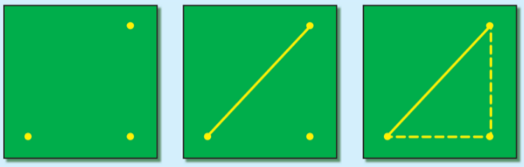
The emergence of these varied student perceptions can be classified as epistemological obstacles, as they stem from the limitations of the contextual information provided by the textbook author. These limitations may restrict students' ability to generalize their understanding and apply mathematical concepts to different contexts (Hariyani et al., 2022; Miftah et al., 2022). Addressing such obstacles requires the inclusion of 'rich' tasks in mathematics textbooks—tasks that actively engage students, challenge their thinking, and enhance the quality of learning by fostering deeper conceptual understanding (Gracin, 2018). By integrating complex and thought-provoking problems, textbooks can create a dynamic learning environment in which students not only memorize information but also establish meaningful connections with their prior knowledge.

A key concern identified in the analyzed mathematics textbook is the lack of diversity in the types of tasks presented. Specifically, as shown in Table 3, tasks categorized as T4, T8, and T9 constitute the highest proportion, each accounting for 19.05% of the total. These tasks primarily focus on determining distances—between points, between points and lines, and between points and planes in three-dimensional space. However, all questions in these categories follow a uniform format: they require students to determine a specific measurement based on a given diagram or description. The absence of variation, such as real-world problem-solving scenarios or higher-order thinking challenges, limits opportunities for students to develop deeper mathematical reasoning.

Consequently, the potential learning barriers identified include both epistemological and didactic obstacles. The epistemological challenges arise from the lack of diverse task presentations, which may hinder students' ability to generalize mathematical concepts beyond structured textbook exercises. Meanwhile, the didactic obstacles are related to the way the learning materials are sequenced and organized, potentially limiting students' engagement and critical thinking development (Suryadi, 2019). To enhance learning effectiveness, it is essential to introduce a wider range of task formats, incorporating contextualized and inquiry-based problems that encourage students to apply mathematical concepts in varied and meaningful ways.

Problem 1.4

Three nails are driven into a board to form the vertices of a right-angled triangle (see Figure 1.8.a). A rope is attached to two of the driven nails (see Figure 1.8.b). Let's denote these nails as points A, B, and C as shown in Figure with AC = 6 cm, BC = 8 cm, and AB = 10 cm.



Through a small experiment, determine the minimum length of the rope that connects nail C (point C) to the rope attached to nails A and B (line segment AB). What conditions must be met to obtain the minimum rope length? Provide reasons for your answer.

Figure 4. Examples of problems found in the mathematics textbook

If all questions within a textbook share similar or identical context, students may struggle to grasp the broader relevance and applicability of mathematical concepts in diverse real-world situations. A uniformity in question contexts also restricts the range of solution methods students can employ. When all tasks require the same problem-solving approach, students may develop a rigid way of thinking, limiting their ability to adapt mathematical reasoning to new and unfamiliar problems. This confinement can hinder the development of flexible problem-solving strategies and critical thinking skills, which are essential for a deeper mathematical understanding.

To mitigate these obstacles, mathematics textbooks should undergo revisions that address both didactic structuring and conceptual clarity. Didactic structuring involves designing a sequence of tasks that progressively build students' understanding, incorporating a variety of problem types that encourage exploration and reasoning beyond rote procedures. Additionally, ensuring conceptual clarity means presenting mathematical ideas in ways that minimize ambiguities and support students in making meaningful connections between abstract concepts and their practical applications. By incorporating diverse tasks and problem contexts, textbooks can foster more effective learning experiences, particularly in helping students develop a comprehensive and adaptable understanding of three-dimensional geometry.

CONCLUSION

This study indicates the presence of ten distinct types of tasks in three-dimensional geometry, with a relatively balanced distribution between conceptual and procedural tasks. Conceptual tasks facilitate students' comprehension of the concept of distance in three-dimensional space, fostering deeper mathematical understanding, while procedural tasks enable them to apply acquired methods and formulas effectively. Furthermore, the study identified various problem-solving techniques employed in these tasks, including measurement and calculation, analysis and comparison, observation and terminology, analysis and construction, and drawing conclusions. These techniques play a critical role in reinforcing students' conceptual grasp and enhancing their mathematical reasoning and problem-solving skills. The balanced integration of conceptual and procedural tasks, along with diverse problem-solving techniques, contributes to a more comprehensive and structured learning experience in three-dimensional geometry.

Furthermore, this study also has certain limitations that should be acknowledged. One notable limitation pertains to the constraints in visualization and contextual diversity within the textbook exercises, which may lead to epistemological obstacles for students. Specifically, the symbolic representation used in tasks related to the concept of distance, coupled with the limited variation in task formats across specific subtopics, may hinder students' ability to generalize and flexibly apply their understanding. Additionally, while the overall sequence of material presentation does not pose significant didactic challenges, certain aspects of task design could be further refined to better support students in overcoming conceptual difficulties. These limitations highlight the need for further refinement in instructional materials to optimize students' learning experiences. Therefore, future research could explore the impact of modified task designs and enriched visualizations on students' comprehension and problem-solving abilities. Additionally, further studies may investigate the role of technology, such as dynamic geometry software, in facilitating a more interactive and adaptive learning environment. By addressing these aspects, subsequent research could contribute to the development of more effective pedagogical strategies for teaching three-dimensional geometry, ultimately fostering a deeper and more meaningful understanding of spatial concepts among students.

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