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Appendix A

Designing a Conditional Probability Teaching Process based on RME

Based on the basic theories of RME such as mathematization, Guided reinvention, Emergent models heuristics, Didactical phenomenology, and lessons designed based on RME, we design the teaching process of conditional probability topic including the following 5 steps:

1. Step 1. Problem setting: The teacher presents the first situation in a real context including problem 1, called the conditional probability experience situation, as the starting point of the learning process.
2. Step 2. Experience: Students study problem 1, analyze the situation and data based on personal knowledge; mobilize necessary knowledge and come up with specific strategies and solutions to solve the problem (building a "model of"). The teacher organizes a few groups to present their products for the whole class to discuss; The teacher comments and evaluates the students' products.
3. Step 3. Rediscovering conditional probability: The teacher presents a second scenario that includes problem 2, called the conditional probability rediscovery scenario. In this scenario, students develop specific strategies and solutions to move from the "model of" problem 1 to the "model for" problem 1 under the teacher's guidance. The teacher comments and evaluates the students' products.
4. Step 4. Forming conditional probability: Teachers organize students to formalize mathematical results from the "model for", and from there teachers institutionalize knowledge of conditional probability.
5. Step 5. Application: The teacher presents two situations 3 and 4, which include problems 3 and 4, respectively. In situation 3, called the situation of applying knowledge of conditional probability, students will apply the concept and formula of calculating conditional probability to solve practical problem 3. In situation 4, called the situation of rediscovering the total probability theorem and Bayes' theorem, students construct two "models of" problem 4 by solving questions 4d/ and 4f/, and develop them into two "models for" problem 4 by solving questions 4e/ and 4g/ under the teacher's guidance. The teacher organizes a few groups to present their products for the whole class to discuss; The teacher comments and evaluates the students' products.

Experimental Content

Teaching method used for experiment: Using model of RME-based learning with four situations: Situation 1. Conditional Probability Experience; Situation 2. Rediscovering Conditional Probability; Situation 3. Applying knowledge of conditional probability; Situation 4. Rediscovering theorem of total probability and Bayes' Theorem. In those situations, situation 1 corresponds to steps 1 and 2; situation 2 corresponds to step 3; situations 3 and 4 are part of step 5. Step 4 (formalization and institutionalization) takes place after step 3 to consolidate students' understanding before application.

Situation 1. Conditional Probability Experience

In this situation, the teacher presents the first real-life problem. Through solving this problem, students will have their first rediscoveries of conditional probability under the guidance of the teacher.
Problem 1. A pharmaceutical laboratory conducts tests on 800 patients suffering from a disease. Some are treated with drug M, others with drug N. The statistical table of test results is as follows

	Drug M	Drug N	Total
Cured	383	291	674
Not cured	72	54	126
Total	455	345	800

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One patient is randomly selected. Let M be the event "The patient is treated with drug M", N is the event "The patient is treated with drug N", and C is the event "The patient is cured".
a/ Calculate the probability that the patient will be treated with drug M.
b/ Calculate the probability that a patient has been cured.
c/ Calculate the probability that a patient is cured and is treated with drug M.
d/ Calculate the probability that the patient will be treated with drug M, knowing that the patient is cured. Explain how to calculate it. We denote the probability to be calculated as $\mathrm{P(M/C)}$.
e/ Calculate the probability that the patient will be cured, given that the patient was treated with drug N. Explain how to calculate it. We denote this probability as $\mathrm{P(C/N)}$.
Express the probability as a decimal rounded to three decimal places.
(Monka, Y. (2004/2025). Probabilités conditionnelles. *Maths et tiques*. https://www.maths-et-tiques.fr/)

The correct answers we expect from students include
a/ To calculate the probability $\mathrm{P(M)}$, the student takes the total number of patients treated with drug M, which is 455, and divides it by the total number of patients, which is 800, and gets

$$\mathrm{P(M)} = 455/800 = 0.569.$$

b/ To calculate the probability P(C), the student takes the total number of cured patients, which is 674, and divides it by the total number of patients, which is 800, and gets

$$\mathrm{P(C)} = 674/800 = 0.843.$$

c/ To calculate the probability $\mathrm{P(C \cap M)}$, the student takes the number of cases in the cell of intersection of the row of Cured and the column of Drug M, which is 383, and divides it by the total number of patients, which is 800, and gets

$$\mathrm{P}(C \cap M) = 383/800 \approx 0.479.$$

d/ To calculate the probability $P(M/C)$, the student takes the number of cases in the cell of intersection of the Cured row and the Drug M column, which is 383, and divides it by the total number of cured patients, which is 674, and gets

$$P(M/C) = 383/674 \approx 0.568.$$

e/ To calculate the probability $P(C/N)$, the student takes the number of cases in the cell of intersection of the Drug N column and the Cured row, which is 291, and divides it by the total number of patients treated with drug N, which is 345, and gets

$$P(C/N) = 291/345 \approx 0.843.$$

Situation 2. Rediscovering Conditional Probability

In this situation, through solving problem 2, students rediscover the formula for calculating the conditional probability of an event and realize that conditional probability is not commutative under the guidance of the teacher.
Problem 2. Given two events A and B. $\overline{\mathrm{A}}$ and $\overline{\mathrm{B}}$ are the opposite events of A and B respectively. a, b, c, d, $a+c$, $a+b$, $b+d$, $c+d$ are respectively the number of occurrences of events $A \cap B$, $\bar{A} \cap B$, $A \cap \bar{B}$, $\overline{\mathrm{A}} \cap \overline{\mathrm{B}}$, A, B, $\overline{\mathrm{A}}$, $\overline{\mathrm{B}}$ and are shown in the statistical table as follows

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	A	$\overline{A}$	$Total$
B	a	b	$a+b$
$\overline{B}$	c	d	$c+d$
$Total$	$a+c$	$b+d$	$a+b+c+d$

a/ Calculate $P(A)$, $P(B)$.
b/ Calculate $P(A\cap B)$.
c/ Calculate the probability of event A, knowing that event B has occurred before and is denoted by $P(A/B)$, and prove that $P(A/B)=P(A\cap B)/P(B)$.
d/ Calculate the probability of event B, knowing that event A has occurred before and is denoted by $P(B/A)$, and prove that $P(B/A)=P(A\cap B)/P(A)$.
e/ Compare $P(A/B)$ and $P(B/A)$. Explain why?

The correct answers we expect from students include
For question a/, to calculate $P(A)$, students take the total number of cases of event A, which is $a+c$, and divide the number of elements of the sample space as $a+b+c+d$, to get

$$P(A)=\frac{a+c}{a+b+c+d}.$$

Similarly, to calculate $P(B)$, students take the total number of occurrences of event B as a + b and divide the number of elements of the sample space as $a+b+c+d$, to get

$$P(B)=\frac{a+b}{a+b+c+d}.$$

For question b/, the student takes the number of cases in the cell of intersection of column of event A and row of event B as a and divides it by the number of elements of the sample space as $a+b+c+d$, to get

$$P(A\cap B)=\frac{a}{a+b+c+d}.$$

For question c/, the student takes the number of cases in the intersection cell of the row of event B and the column of event A, which is a, and divides it by the total number of cases of event B, which is a + b, to get

$$P(A/B)=\frac{a}{a+b}.$$

To prove the formula $P(A/B)=P(A\cap B)/P(B)$, students divide the expression on the right side of $P(A/B)=a/(a+b)$ by the number of elements of the sample space, and get

$$P(A/B)=\frac{a}{a+b}=\frac{a}{a+b+c+d}:\frac{a+b}{a+b+c+d}=\frac{P(A\cap B)}{P(B)}.$$

For question d/, students take the number of cases in the intersection cell of column of event A and row of event B as a and divide by the total number of cases of event A as $a+c$, to get

$$P(B/A)=\frac{a}{a+c}.$$

To prove the formula $P(B/A) = \frac{P(B\cap A)}{P(A)}$, students divide the expression on the right side of $P(B/A) = \frac{a}{a+c}$ by the number of elements of the sample space, and get

$$P(B/A) = \frac{a}{a+c} = \frac{a}{a+b+c+d} : \frac{a+c}{a+b+c+d} = \frac{P(B\cap A)}{P(A)}.$$

For sentence e/, Students realize that $P(A/B)$ is often different from $P(B/A)$, or conditional probability is not commutative, because the order of occurrence of events is different.

Situation 3. Applying Knowledge of Conditional Probability

In this situation, students apply their knowledge of conditional probability to solve problem 3 given by the teacher.

Problem 3. Statistics from an insurance company show that 62% of health insurance buyers are women, and 41% of health insurance buyers are women working in foreign companies. Randomly select a health insurance buyer. Let W be the event "The health insurance buyer is a woman", and F be the event "The health insurance buyer works in a foreign company". Calculate the probability that the health insurance buyer works in a foreign company, knowing that the person is a woman. Express the probability as a decimal rounded to three decimal places.

The correct answers we expect from students include

Since 62% of health insurance buyers are women, we have $P(W) = 0.62$.

Since 41% of health insurance buyers are women working in foreign companies, we have $P(W \cap F) = 0.41$.

The probability that a health insurance buyer works in a foreign company, knowing that she is a woman, is

$$P(F/W) = \frac{P(W\cap F)}{P(W)} = \frac{0.41}{0.62} \approx 0.661.$$

Situation 4. Rediscovering Theorem of Total Probability and Bayes' Theorem

In this situation, through solving problem 3, students will rediscover theorem of total probability and Bayes' theorem under the guidance of the teacher.

Problem 4. At a certain university, 4% of men are over 1,80 m tall and 1% of women are over 1,80 m tall. The total student population is divided in the ratio 3:2 in favour of women. Randomly select a student from among all those taller than 1.80 m. We want to calculate the probability that the selected student is female. (Mathematics Learning Support Centre. (2008). Workbook 35: Sets and Probability. *Helping Engineers Learn Mathematics-Workbooks*. Loughborough University. https://www.lboro.ac.uk/departments/mlsc/student-resources/helm-workbooks/)

Let M be the event "The student is male", F be the event "The student is female", T be the event "The student is taller than 1.80 m".

Note that $M \cap F = \emptyset$ and $M \cup F = \Omega$, where Ω is the sample space.

a/ What is the problem's requirement?

b/ Calculate the probabilities $P(M)$, $P(F)$, $P(T/M)$, $P(T/F)$.

c/ In the following tree diagram with the root at point O, called a probability tree, fill in all the probability values in the place of the question marks.

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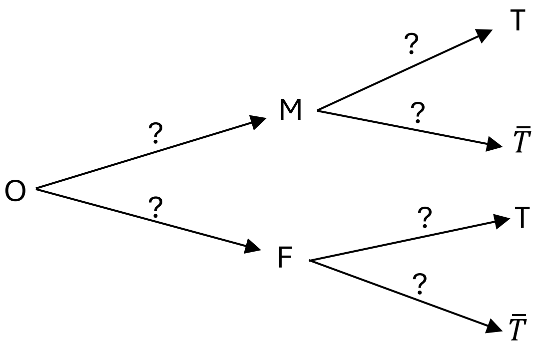


d/ In the probability tree diagram completed in question c/, show the paths from O to T, and state how to calculate the probability that the selected student is taller than 1.80 m.

e/ Show that $P(T) = P(T/M).P(M) + P(T/F).P(F)$.

f/ Write a formula to calculate the probability that the selected student is female, knowing that the student is taller than 1.80 m.

g/ Show that $P(F/T) = \frac{P(T/F).P(F)}{P(T/F).P(F)+P(T/M).P(M)}$.

h/ Calculate $P(F/T)$.

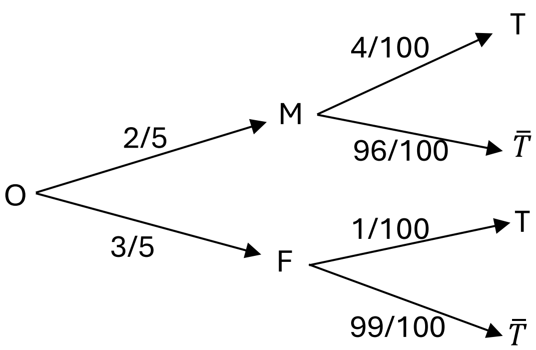
The correct answers we expect from students include:

a/ The requirement of the problem is to calculate the probability that a randomly selected student is female given that the student is taller than 1.8 m.

b/ According to the given data, we get

$$P(M) = 2/5; P(F) = 3/5; P(T/M) = 4/100; P(T/F) = 1/100.$$

c/



d/ On the probability tree diagram, there are two paths from O to T: $O - M - T$ and $O - F - T$. To calculate the probability that the selected student is taller than 1.80 m, $P(T)$, we multiply the probability values on the branches in each of the two paths $O - M - T$ and $O - F - T$, and then add the two results together.

e/ According to the calculation method of $P(T)$ mentioned in question d/, we get the formula

$$P(T) = P(M).P(T/M) + P(F).P(T/F) \text{ (*)}.$$

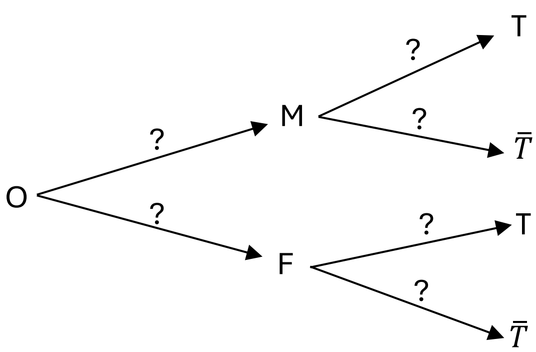
f/ $P(F/T) = \frac{P(F\cap T)}{P(T)}$.

g/ Since $P(F \cap T) = P(T/F).P(F)$, and from the relation (*), we have:

$$P(F/T) = \frac{P(F\cap T)}{P(T)} = \frac{P(T/F).P(F)}{P(T/F).P(F)+P(T/M).P(M)}.$$

h/ $P(F/T) = \frac{\frac{1}{100}.\frac{3}{5}}{\frac{1}{100}.\frac{3}{5}+\frac{4}{100}.\frac{2}{5}} = 3/11.$

Table 1. Questions for students in the teaching process

Situations	Questions	Objective
1	a/ Calculate the probability that the patient will be treated with drug M. b/ Calculate the probability that a patient has been cured. c/ Calculate the probability that a patient is cured and is treated with drug M.	Preparing to build a specific model of the problem 1 (model of).
	d/ Calculate the probability that the patient will be treated with drug M, knowing that the patient is cured. Explain how to calculate it. We denote the probability to be calculated as $\mathrm{P(M/C)}$. e/ Calculate the probability that the patient will be cured, given that the patient was treated with drug N. Explain how to calculate it. We denote this probability as $\mathrm{P(C/N)}$.	Building a specific model of the problem 1 (model of).
2	a/ Calculate $\mathrm{P(A)}$, $\mathrm{P(B)}$; b/ Calculate $\mathrm{P(A \cap B)}$;	Preparing to build a general model of the problem 1 (model for).
	c/ Calculate the probability of event A, knowing that event B has occurred before, $\mathrm{P(A/B)}$, and prove that $\mathrm{P(A/B) = P(A \cap B)/P(B)}$;	Building a general model of the problems 1 (model for) and conditional probability formulation.
	d/ Calculate the probability of event B, knowing that event A has occurred before, $\mathrm{P(B/A)}$, and and prove that $\mathrm{P(B/A) = P(A \cap B)/P(A)}$;	Building a general model of the problems 1 (model for) and conditional probability formulation.
	e/ Compare $\mathrm{P(A/B)}$ and $\mathrm{P(B/A)}$. Explain why?	Prove that the conditional probability formula is not commutative, and emphasizes the order of occurrence of events.
3	Calculate the probability that the health insurance buyer works in a foreign company, knowing that the person is a woman.	Apply the formula to calculate the conditional probability of an event.
4	a/ What is the problem's requirement? b/ Calculate the probabilities $\mathrm{P(M)}$, $\mathrm{P(F)}$, $\mathrm{P(T/M)}$, $\mathrm{P(T/F)}$. c/ In the following tree diagram, called a probability tree, fill in all the probability values in the place of the question marks:  	Preparing to build a specific model of the problem 4 (model of).
	d/ In the probability tree diagram completed in question c/, show the paths from O to T, and state	Building the first specific model of the problem 4 (model of).

how to calculate the probability that the selected student is taller than 1.80 m.	
e/ Show that: $P(T) = P(T/M).P(M) + P(T/F).P(F)$.	Building the first general model of the problem 4 (model for) and the total probability theorem.
f/ Write a formula to calculate the probability that the selected student is female, knowing that the student is taller than 1.80 m.	Building the second specific model of the problem 4 (model of).
g/ Show that: $P(F/T) = \frac{P(T/F).P(F)}{P(T/F).P(F)+P(T/M).P(M)}$.	Building the second general model of the problem 4 (model for) and the Bayes' theorem.
h/ Calculate $P(F/T)$.	Apply Bayes' theorem to calculate the conditional probability of an event.