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Epistemic actions in proving two-triangle problems by considering mathematical reading and writing ability

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Abstract

Mathematical abstraction is essential in constructing mathematical concepts, particularly in proof. The RBC+C epistemic actions—recognizing, building-with, constructing, and consolidating—are key cognitive processes in proof construction. However, the impact of mathematical reading and writing abilities on these processes remains unexplored. This study investigates how students' mathematical reading and writing abilities affect their epistemic actions when proving the congruence of two triangles. This qualitative research adopts a case study design involving three undergraduate students who have completed a geometry course. The participants were selected based on their reading and writing proficiency levels: high, moderate, and low. Data were collected through reading and writing assessments, proof-solving tasks, and semi-structured interviews. The analysis follows the RBC+C framework to identify patterns in students' cognitive process during proof construction. Findings reveal that students with high mathematical reading and writing abilities demonstrate a more structured proof strategy, effectively recognizing key properties, building logical connections, and constructing valid arguments. High-proficiency students also exhibit flexibility in using both geometric and algebraic approaches in proving. In contrast, students with lower reading and writing abilities struggle with symbolic representation, logical coherence, and notation consistency, leading to incomplete or incorrect proofs. Moreover, consolidation of mathematical ideas, such as reusing known theorems and revisiting proof steps, occurs more frequently in high-achieving students, enabling deeper conceptual understanding. This study highlights the critical role of mathematical literacy in the proof process. It suggests that strengthening reading and writing instruction in mathematics education can enhance students' ability to construct rigorous proofs. The findings contribute to the development of instructional strategies that integrate mathematical literacy into proof-based learning, ultimately fostering students' reasoning and problem-solving skills in mathematics.

Keywords: Abstraction, Epistemic Action, Geometry, Mathematical Reading Ability, Mathematical Writing Ability

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For most mathematicians, abstraction refers to mathematical objects or formal structures such as vector spaces. However, within the field of mathematics education, abstraction is understood as a cognitive process through which students construct, reorganize, and internalize mathematical knowledge (Hershkowitz et al., 2001, 2020). It is seen not only as a pathway toward understanding mathematical structures but also as one of the highest human intellectual achievements (Dreyfus & Kidron, 2014). As such, mathematical abstraction plays a central role in the learning process, particularly in facilitating problem-solving and concept formation (Hong & Kim, 2016; Sahrudin et al., 2022).

Researchers have extensively examined abstraction mechanisms in mathematics learning. These include generalization, decontextualization, and vertical mathematization. Abstraction is the processes that transform concrete experiences into abstract structures (Dreyfus et al., 2015; Ferrari, 2003). Skemp (Yang, 2016) and Tall (1991) highlight abstraction as a mental reorganization of prior knowledge, while the Nested RBC Model developed by Hershkowitz et al. (2001) describe abstraction through a sequence of epistemic actions: Recognizing (R), Building-with (B), Constructing (C), and later, Consolidating (C) (Dreyfus & Tsamir, 2004). These cognitive processes represent how learners dynamically interact with and build upon their prior mathematical understandings.

The Abstraction in Context (AiC) framework provides a theoretical-methodological lens to investigate how abstraction occurs in specific social and instructional contexts (Dreyfus & Kidron, 2014). It allows researchers to analyze micro-level cognitive dynamics by observing students' epistemic actions. These actions are often visible in verbal and physical behaviors as learners engage with mathematical tasks (Hershkowitz et al., 2001; Sahrudin et al., 2022). AiC has been used to explore abstraction in topics ranging from cube nets (Sahrudin et al., 2022) and computational thinking (Çakiroğlu & Çevik, 2022) to definite integrals (Park & Lee, 2022) and 3D visualization (Fitriani et al., 2018). However, studies employing AiC in the context of geometry, particularly in proof construction, remain limited. Thus, it is leaving room for further investigation.

Geometry plays a crucial role in mathematics education due to its capacity to support spatial reasoning, critical thinking, and visualization of abstract concepts (Battista, 1990; Jones, 2002; Sunzuma & Maharaj, 2019). Nonetheless, it is also among the most challenging domains for both learners and teachers (Levenberg & Shaham, 2014). When it comes to constructing formal proofs in geometry, abstraction becomes particularly complex, as it requires both structural understanding and logical coherence. This underscores the importance of tools such as AiC in examining students' thinking during geometric proof processes.

In the Indonesian higher education system, students enrolled in mathematics education programs are typically introduced to formal proof tasks during their geometry coursework, usually in the first or second year of study. While proof construction is a standard part of the curriculum, the integration of mathematical literacy, specifically reading and writing mathematical texts, as explicit learning objectives remains limited (Retnawati et al., 2018). As a result, students are often expected to develop proof competencies without systematic support in reading comprehension or written mathematical communication. Understanding this curricular structure is crucial for interpreting how students engage in abstraction and proof-related tasks, particularly when viewed through the lens of epistemic actions such as recognizing, building-with, constructing, and consolidating.

Despite the centrality of abstraction in proof construction, one underexplored aspect is the role of mathematical literacy, specifically students' reading and writing abilities. Mathematical reading refers to the skill of interpreting symbols, extracting meaning from definitions, and understanding mathematical relationships within texts (Österholm, 2006; Wallace & Clark, 2005). It involves engaging with vocabulary, symbolic representations, and mathematical syntax that are often denser than other academic texts (Gullatt, 1986; Harris & VanDevender, 1990). Effective mathematical reading requires metacognitive regulation and problem-solving awareness (Smith et al., 1992; Woolley & Woolley, 2011).

Complementing reading, mathematical writing is an essential process through which learners articulate ideas, clarify reasoning, and represent abstract concepts in coherent form (Freitag, 1997; Kesorn et al., 2020). Writing allows students to consolidate understanding, formulate conjectures, and present formal arguments (Grossman et al., 1993; Shibli, 1992). As such, it serves not only as a communicative medium but also as a pedagogical tool for conceptual development (Kesorn et al., 2020; Sipka, 1990).

Several scholars argue that integrating writing into mathematics instruction enhances students' ability to reflect on problem-solving, develop logical arguments, and strengthen overall comprehension (Bicer et al., 2013; Teuscher et al., 2015). Yet, while the individual impacts of reading or writing on mathematics learning are well documented, the intersection of these skills with abstraction, particularly in the process of constructing geometric proofs, has not been extensively addressed.

Although existing research has applied the AiC and RBC+C frameworks to investigate abstraction in various mathematical contexts, few studies have explored how students' mathematical reading and writing abilities shape their epistemic actions during proof construction in geometry. Furthermore, prior literature has largely examined literacy and abstraction in isolation rather than as interdependent processes. This study seeks to bridge this gap by analyzing how students' literacy levels influence their engagement in the epistemic actions of recognizing, building-with, constructing, and consolidating mathematical ideas. Specifically, it focuses on proof tasks involving triangle congruence, a foundational yet cognitively demanding area of geometry. Therefore, this study aims to explore how undergraduate students with varying levels of mathematical reading and writing abilities engage in epistemic actions (RBC+C) during the process of constructing geometric proofs involving triangle congruence.

This study offers several significant contributions to the existing body of research. Theoretically, it extends the Abstraction in Context (AiC) framework by embedding mathematical literacy, specifically students' reading and writing abilities, into the structure of epistemic actions. By doing so, the study proposes a more integrated and holistic model of abstraction that accounts not only for students' cognitive operations but also their representational and communicative practices during proof construction. Methodologically, the study employs a qualitative case study design to capture the nuanced cognitive processes involved in abstraction, particularly in the context of proving triangle congruence. This approach allows for a detailed examination of how literacy skills interact with the nested RBC+C model in real problem-solving situations. Practically, the findings inform mathematics educators and curriculum designers about the importance of incorporating reading and writing instruction into proof-based learning environments. By demonstrating that students' mathematical literacy influences their ability to engage in abstract reasoning and construct rigorous arguments, this study provides a foundation for designing literacy-integrated instructional strategies that can support deeper conceptual understanding and improved mathematical communication.

METHODS

This study employed a qualitative case study approach to explore how undergraduate students with different levels of mathematical reading and writing abilities engage in epistemic actions (RBC+C) during the construction of geometric proofs. The research was situated within the framework of Abstraction in Context (AiC), which allows for micro-analytic observations of abstraction as it emerges through interaction with mathematical tasks.

Research Participants

Participants were selected from a cohort of 42 undergraduate students who had completed a geometry course. Initially, all 42 students were administered two assessment instruments: a Mathematical Reading Ability Test (T1) and a Mathematical Writing Ability Test (T2). Based on the results of these instruments, three students were purposively selected to represent three literacy profiles: high, moderate, and low.

This stratified sampling strategy was used to allow comparative analysis and to reflect the variability of students' mathematical literacy in relation to their abstraction processes.

The criteria for selection were based on performance thresholds: scores above 20 on both T1 and T2 were categorized as high, scores between 11–20 as moderate, and scores 10 or below as low. The selected students were coded as S1 (high), S2 (moderate), and S3 (low).

Research Instruments and Data Collection

All instruments (T1, T2, and T3) were developed by the research team and underwent an expert validation process to ensure alignment with the constructs of mathematical reading, writing, and proof-related abstraction. The validation involved two subject matter experts: the first was a full professor with recognized expertise in mathematical abstraction, and the second was a senior mathematics education lecturer with over ten years of teaching experience at the undergraduate level. Both experts reviewed the instruments for content accuracy, construct clarity, and task appropriateness based on the study's conceptual framework. Their feedback led to several refinements, including clarifying item wording, removing ambiguous terminology, and ensuring that symbolic representations conformed to established mathematical conventions.

1. T1 (Mathematical Reading Ability Test)
   This test consisted of 25 items designed to measure students' ability to interpret mathematical symbols, extract relevant information, and transform written mathematical information into other forms (e.g., diagrams or verbal explanations). An example item asked students to represent a given geometric statement visually and label corresponding angles and sides.
2. T2 (Mathematical Writing Ability Test)
   Also consisting of 25 items, this test assessed students' ability to articulate mathematical ideas using proper notation, logical sequencing, and symbolic representation. Items included prompts such as "Draw two congruent triangles and write down the corresponding angles."
3. T3 (Triangle Congruence Proof Task)
   A single, open-ended problem was used to elicit epistemic actions in geometric proof construction. The task required students to prove that two triangles were congruent using a geometric diagram and deductive reasoning. The problem given to the research subjects was:
   "*Given an ΔABC with right angle at B. From point B, an altitude is to the hypotenuse AC, intersecting it at D. Prove that* $|\overline{BD}|^2 = |\overline{AD}| \times |\overline{CD}|$"
   This problem was designed to engage students in recognizing geometric relationships, constructing formal arguments, and applying congruence principles - thereby enabling the identification of their epistemic actions as outlined in the RBC+C framework. The task was followed by semi-structured interviews and video-recorded observations to gain further insights into the students' cognitive processes.

Based on their feedback, revisions were made to simplify the language in several T1 and T2 items, particularly those involving diagram interpretation and verbal restatements of symbolic expressions. In T3, minor adjustments were made to ensure that the proof prompt elicited all stages of the RBC+C process without overly directing student responses.

Data Analysis

All data sets were systematically organized and categorized according to the structure of the data analysis process, including written responses, observation notes, and interview transcripts. Each mathematical problem-solving task was treated as a unit of analysis, allowing the researchers to trace students' reasoning and abstraction processes at a micro level. The interview transcripts were transcribed verbatim and analyzed using a multi-stage qualitative coding approach grounded in the RBC+C framework.

The first stage involved **open coding**, where researchers identified segments of data that reflected epistemic actions, such as recognizing known properties, building on prior knowledge, constructing new reasoning chains, or consolidating ideas. In the second stage, **axial coding** was used to relate these codes to specific phases of the RBC+C model, thus revealing patterns in students' abstraction processes. The coding scheme was adapted from Kim et al. (2020), with modifications to suit the context of geometric proof construction.

In the final stage, a **thematic synthesis** was conducted to describe the characteristics of each participant's abstraction process across the RBC+C categories. This included mapping out the sequence and interconnection of epistemic actions based on students' verbal explanations, written work, and behaviors observed during the tasks. The overall structure of the analysis followed a process outlined by Magiera & Zawojewski (2011), which involves: (1) coding interview and observational data, and (2) interpreting those codes within the theoretical lens of RBC+C to track the emergence and development of abstraction.

RESULTS AND DISCUSSION

Students' Mathematical Reading and Writing Ability

Table 1 explains the number of students who could read and write in the high, moderate, and low categories.

Table 1. The number of students who could read and write in the high, medium, and low categories

Category		Reading Ability			Total
		High	Moderate	Low	
Writing Ability	High	8	6	3	17
	Moderate	6	5	4	15
	Low	2	2	6	10
Total		16	13	13	42

In accordance with Table 1, there were eight students as the prospective subjects with high reading and writing abilities, five students with moderate reading and writing abilities, and six students with low reading and writing abilities. Furthermore, the subjects were randomly selected to obtain research subjects, which were then coded S1, S2, and S3.

Epistemic Actions in Proof Construction of S1

The results of the interview were based on the proof process of the following questions. "*Given an* ΔABC *with right angle at B. From point B, an altitude is to the hypotenuse AC, intersecting it at D. Prove that* $|\overline{BD}|^2 = |\overline{AD}| \times |\overline{CD}|$". The data obtained were described as follows: subject S1 demonstrated their reading and writing abilities by constructing the given triangle, writing down the known information and the statement to be proven, and reusing angle notation, angle measurements, line segments, segment lengths, congruent angles, similar angles, and similar triangles as shown in Figure 1.

The proving process carried out by S1 was as follows.

1. $m\angle ABC = 90°$ with premise arguments,
2. $\overline{BD} \perp \overline{AC}$ with premise arguments.
3. $m\angle ADB = m\angle BDC = 90°$ with premise arguments, and construct $\angle ADB \cong \angle BDC$ with new premise arguments.
4. For example, $\angle BAC = \theta$ with new premise arguments.
5. $m\angle BAC + m\angle ABC + m\angle ACB = 180°$ with the arguments that the sum of the angles in a triangle was 180°.
6. $m\angle ACB = 90° - \theta$ with the arguments in steps 1, 3, and 4.
7. $m\angle ADB + m\angle ABD + m\angle BAD = 180°$ with the arguments that the sum of the angles in a triangle was 180°.
8. $m\angle ADB = m\angle CDB = 90°$
9. $m\angle BAD = \theta$ with step 4 argumentation
10. $m\angle ABD = 90° - \theta$ with the arguments in steps 4 and 6.
11. $m\angle BCD = 90° - \theta$ with the same angle argument, step 10
12. $\angle ABD \cong \angle BCD$ with the arguments in steps 10 and 11
13. $\Delta ABD \sim \Delta BCD$ with arguments step 3, 12
14. $\frac{BD}{CD} = \frac{AD}{BD}$ with the argumentation in step 13
15. $|\overline{BD}|^2 = |\overline{AD}| \times |\overline{CD}|$ with arguments in step 14.

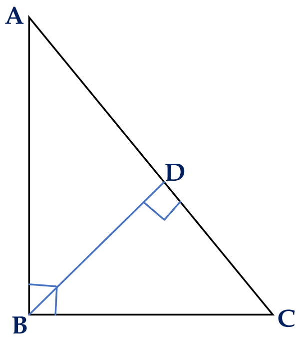

Figure 1. A figure made by S1 subject to answer questions

The consolidation performed by S1 involved the frequent sharing and reuse of mathematical ideas, particularly the concepts of angle congruence, line segment congruence, and repetition as shown in Diagram 1. The subject recognized that the sum of the interior angles of a triangle is 180°. Repetition was observed in $\triangle ABC$, where the subject constructed $\angle BAC = \theta$ and formulated the equation $m\angle BAC + m\angle ABC + m\angle ACB = 180°$ leading to $m\angle ACB = 90° - \theta$.

Similarly, in $\triangle ABD$, S1 applied the same reasoning, stating that $m\angle ADB + m\angle ABD + m\angle BAD = 180°$ and recognizing that $m\angle BAD = \theta$. Consequently, the subject established that $m\angle ABD = 90° - \theta$, ensuring congruence between the two triangles. Through this proof, the subject demonstrated the reuse of reasoning, concluding that $\triangle ABD$ and $\triangle BCD$ are similar due to the correspondence of their angles. Thus, the subject exhibited an awareness of the fundamental property that the sum of a triangle's interior angles is always 180°, reinforcing their understanding through repeated verification.

Considering the Recognition (R), Building-with (B), and Constructing (C) abilities of subject S1 in proving that $|\overline{BD}|^2 = |\overline{AD}| \times |\overline{CD}|$ presented in Diagram 2, as well as evidence from interviews, this study obtained the following findings:

The subject recognized (R1) that $\triangle ABC$ is right-angled at B and constructed (B1) $m\angle ABC = 90°$. The subject also recognized (R2) that $\triangle BDC$ is right-angled at D and constructed (B2) $m\angle BDC = 90°$. Similarly, the subject recognized (R3) that $\triangle ADB$ is right-angled at D and constructed (B3) $m\angle ADB = 90°$.

Furthermore, the subject recognized (R4 = B2) that $m\angle BDC = 90°∘$ and (R5 = B3) that $m\angle ADB = 90°$, leading to the construction (B4) of $m\angle ADB = m\angle BDC = 90°$. The subject further recognized (R6 = B4) that $m\angle ADB = m\angle BDC = 90°$ and constructed (C1) $\angle ADB \cong \angle BDC$.

Next, the subject formulated a new premise (C2) that $\angle BAC = \theta$. The subject recognized (R7) that the sum of the angles in a triangle is $180°$, expressed as $m\angle BAC + m\angle ABC + m\angle ACB = 180°$. Additionally, the subject recognized (R8 = C2) that $\angle BAC = \theta$ and (R9=B1) that $m\angle ABC = 90°$, constructing (B5) $m\angle ACB = 90° - \theta$.

The subject then recognized (R10) that $m\angle ADB + m\angle ABD + m\angle BAD = 180°$, accepted (R8 = C2) that $m\angle ADB = 90°$, and constructed (B6) $\angle BAD = \theta$, (B7) $m\angle ABD = 90° - \theta$, and (B8) $m\angle BCD = 90° - \theta$. Furthermore, the subject constructed (C3) $\angle ABD \cong \angle BCD$.

Finally, the subject recognized (B5 = R10) that $m\angle ADB = m\angle BDC = 90°$ and (C3 = R13) that $\angle ABD \cong \angle BCD$, and concluded (B6) that if two corresponding angles in two triangles are congruent, then the triangles are similar. The subject ultimately proved (C4) that $\triangle ABD \sim \triangle BCD$, and derived the proportional relationship (C5) $\frac{BD}{CD} = \frac{AD}{BD}$, leading to the final proof that $|\overline{BD}|^2 = |\overline{AD}| \times |\overline{CD}|$.

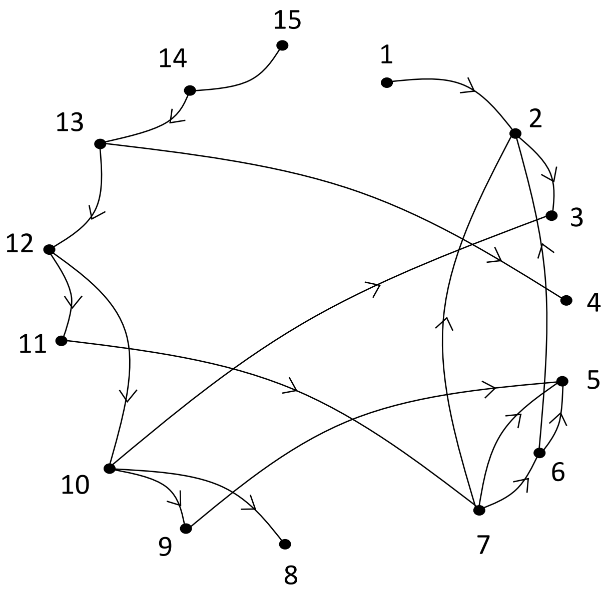

Diagram 1. The S1 subject's process of proving

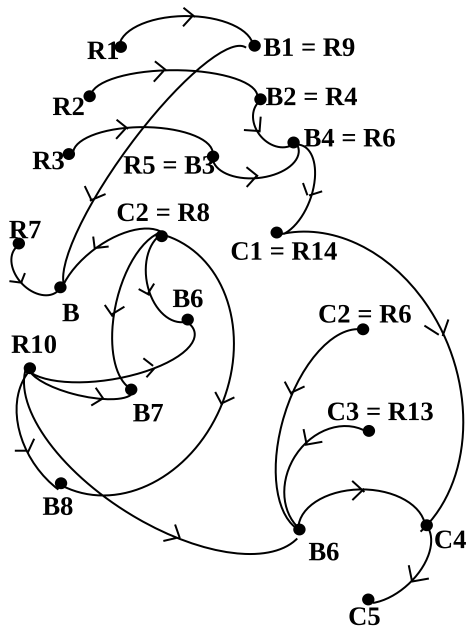

Diagram 2. RBC of S1 subject

S1 was able to use the Pythagorean theorem to verify a problem that had already been proven using the congruence of two triangles. The subject identified the three right triangles that were formed, namely $\triangle ABC$, $\triangle BDC$, and $\triangle ADB$, and thus recognized that the Pythagorean theorem applied to them. The subject demonstrated recognition, assembly, and construction of the Pythagorean theorem in

$\triangle ABC$, leading to the equation: $AB^2 + BC^2 = AC^2$ (R1, B1, C1). Similarly, the subject applied the theorem to $\triangle BDC$ and derived: $BD^2 + DC^2 = BC^2$ (R2, B2, C2). For $\triangle ADB$, the subject obtained: $BD^2 + AD^2 = AB^2$ (R3, B3, C3). Through algebraic manipulation, the following transformations were obtained:

$$AB^2 + BC^2 = AC^2$$
$$(BD^2 + AD^2) + (BD^2 + DC^2) = AC^2$$
$$BD^2 + AD^2 + BD^2 + DC^2 = AC^2$$
$$2BD^2 + AD^2 + DC^2 = AC^2$$
$$2BD^2 + AD^2 + DC^2 = (AD + DC)^2$$
$$2BD^2 + AD^2 + DC^2 = AD^2 + 2AD \times DC + DC^2$$
$$2BD^2 = 2AD \times DC$$
$$BD^2 = AD \times DC$$

Thus, it was proven that: $BD^2 = AD \times DC$.

S1 needed to be more consistent in using line segment length notation, such as distinguishing between AD and $|\overline{AD}|$ (*see the proving process of S1 at step 14 and 15*) also $\angle BAC = \theta$ and $m\angle BAD = \theta$ (*see step 4 and 9*). The subject explained that different reference books used varying notations for line segment lengths. The consolidation carried out by S1 in this proof demonstrated that mathematical ideas were often shared and reused, particularly through the repeated application of the Pythagorean theorem. The theorem was applied in $\triangle ABC$ to derive $AB^2 + BC^2 = AC^2$, in $\triangle BDC$ to obtain $BD^2 + DC^2 = BC^2$, and in $\triangle ADB$ to establish $BD^2 + AD^2 = AB^2$. However, S1 did not reassess the proof's purpose (repurposing) using a geometric approach, such as congruence, but instead relied on algebraic manipulation.

Epistemic Actions in Proof Construction of S2

Interviews based on the evidentiary process revealed that subject S2 demonstrated the ability to read and write mathematical proofs by constructing the relevant triangles, noting given information and what needed to be proven, using perpendicular line segment notation, accurately measuring angles, and applying algebraic operations to segment lengths. However, S2 inconsistently applied notations, as seen in expressions like: $\overline{BD} \times \overline{BD} = \overline{AD} \times \overline{DC}$. Additionally, the subject struggled with the correct use of symbolic notation for equal and congruent angles illustrated in Figure 2.

The proving process carried out by S2 was as follows.

1. ΔABC is a right triangle and $\overline{BD} \perp \overline{AC}$ with premise arguments.
2. $m\angle CDB = m\angle BDA$ with arguments $\overline{BD} \perp \overline{AC}$
3. $m\angle BCD = 180° - (m\angle DAB + 90°)$with the argument the sum of the angles in a triangle $= 180°$
4. $m\angle ABD = 180° - (m\angle DAB + 90°)$ with the sum of the angles in a triangle argument $180°$.
5. $m\angle BCD = m\angle ABD$ with the argument result of $m\angle ABD = 180° - (m\angle DAB + 90°)$ and $m\angle BCD = 180° - (m\angle DAB + 90°)$
6. $\Delta CDB \sim \Delta BDA$ with the argument that the measure of the angles adjacent to the two triangles were equal.
7. $\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{BD}}{\overline{DC}}$ (R, B, C) with consequential arguments of $\Delta CDB \sim \Delta BDA$ and with algebraic operations arguments $\overline{BD} \times \overline{BD} = \overline{AD} \times \overline{DC}$ then $\overline{BD}^2 = \overline{AD} \times \overline{DC}$

8. Draw the conclusion, “If the right angle is at B. From the point drawn an altitude intersects the hypotenuse at D then $\overline{BD}^2 = \overline{AD} \times \overline{DC}$."

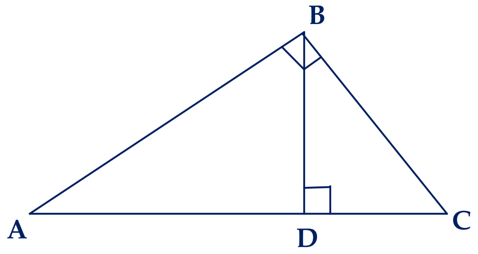

Figure 2. A Figure Made by S2 Subject to Answer Questions

The process of proving undertaken by subject S2 is presented in Diagram 3.

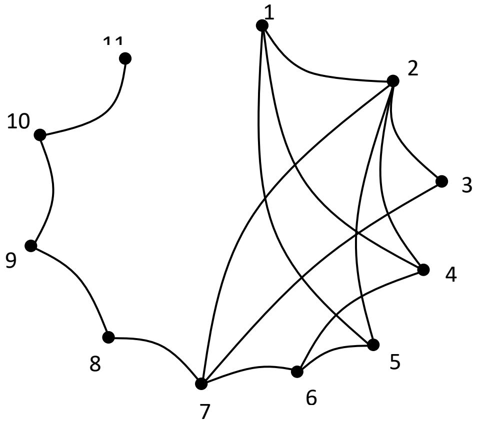

Diagram 3. The process of proving of S2 subject

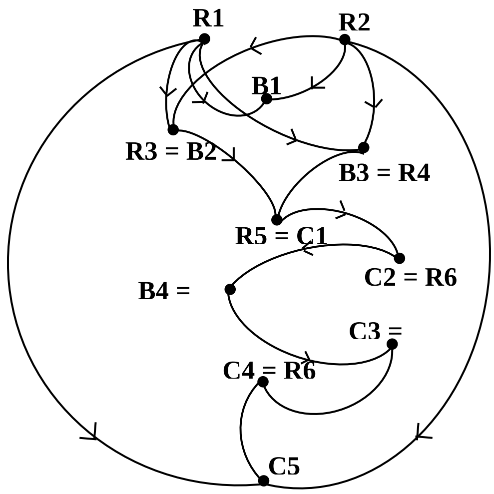

Diagram 4. RBC of S2 subject

The consolidation process carried out by the subject involved recognizing and applying mathematical concepts, particularly the identification of two similar angles. Additionally, the subject demonstrated repetition in using the fundamental property that the sum of the angles in a triangle is 180°. In ΔABC, the subject established that $\angle BAC = \theta$, then recognized and formulated the equation $m\angle BAC + m\angle ABC + m\angle ACB = 180°$, further deducing that $m\angle ACB = 90° - \theta$. The same reasoning was applied to ΔABD, where the subject formulated $m\angle ADB + m\angle ABD + m\angle BAD = 180°$, identified that $m\angle BAD = \theta$, and recognized that $m\angle ABD = 90° - \theta$. Through this proving process, S2 engaged in repurposing, which led to the recognition that ΔABD and ΔBCD are similar due to their corresponding angle relationships. The subject also revisited the fundamental understanding that the sum of the angles in a triangle is always 180°.

S2’s Recognition (R), Building-with (B), and Constructing (C) abilities in proving $(BD)^2 = (AD) \times (DC)$ were supported by evidence-based interviews presented in Diagram 4. The subject identified that ΔABC is a right triangle with $\angle B = 90°$ (R1) and recognized that $(BD) \perp (AC)$ (R2), leading to the conclusion (R1, R2, B1) that $m\angle BDC = m\angle BDA = 90°$. Furthermore, S2 formulated (R1, R2, B2) the equation $m\angle BCD = 180° - (m\angle DAB + 90°)$ and similarly formulated (R1, R2, B3) $m\angle ABD = 180° - (m\angle DAB + 90°)$. The subject then constructed (R3 = B2, R4 = B3, C1) the relationship $m\angle BCD = m\angle ABD$, followed by a logical progression through additional constructions

(R5 = C1, C2), (R6 = C2, B3), (R7 = B3, C3), and (R8 = C3, C4). Finally, S2 arrived at the conclusion (R1, R2, R6 = C4, C5) that "If ΔABC is a right triangle with a right angle at B, and an altitude is drawn from B to intersect the hypotenuse at D, then $\overline{BD}^2 = \overline{AD} \times \overline{DC}$". Diagram 4 illustrates S2's recognition abilities in this proof process.

Epistemic Actions in Proof Construction of S3

Based on the interview data obtained from the evidentiary process, S3 demonstrated the ability to read and write mathematical notation effectively. S3 was able to construct the given triangle, write down the known and required elements, and accurately use notations for congruent angles, perpendicular lines, congruent triangles, and segment lengths presented in Figure 3. However, an inconsistency was found in the representation of right angles in the diagram, as the right-angled triangle was not depicted in a standard orientation, which affected the clarity of the proof structure.

The proving process carried out by S3 was as follows.

1. $\Delta\ ABC$ had a right angle at B, premise argument
2. $\overline{BD} \perp \overline{AC}$, premise.
3. $m\angle CDB = m\angle BDA = 90°$, argumentation due to step 2
4. $m\angle ABC = m\angle BDA = 90°$, argumentation due to steps 1 and 2
5. $\angle ABC \cong \angle CDB \cong \angle ADB$, argumentation due to steps 3 and 4
6. $\angle BAC \cong \angle DAB$, the arguments of the two angles coincide
7. $\angle BCA \cong \angle DCB$, arguments the two angles coincide
8. $\triangle\ BAC \sim\triangle\ DAB$ argumentation step 5, step 7 and similarity of two triangles
9. $\triangle\ DAB \sim\triangle\ DBC$, argumentation of step 6, step 7 and the similarity of the two triangles.
10. $\frac{AD}{BD} = \frac{BD}{DC}$ as a result of step 9.
11. $AD.DC = BD.BD$, due to step 10.
12. $AD.DC = BD^2$ algebraic manipulation of step 11.
13. Drawing the following conclusion, "If $\Delta\ ABC$has a right angle at B, and from point B is drawn an altitude intersecting the hypotenuse at D, then $\overline{BD}^2 = \overline{AD} \times \overline{DC}$", as a result of steps 1, 2, and 12.

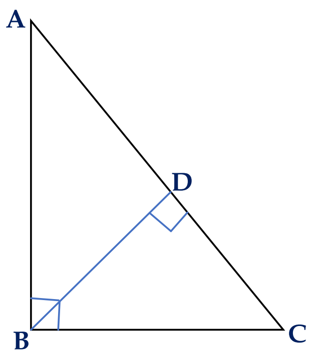

Figure 3. A Figure Made by S3 Subject to Answer Questions

Diagram 5 illustrates the sequence of proving steps undertaken by S3.

The consolidation process revealed that S3 frequently used mathematical concepts related to equal angles and triangle similarity while also demonstrating repetition in reasoning about the congruence of angles and triangle similarity. Regarding the Recognition (R), Building-with (B), and Constructing (C) abilities, S3 recognized (R1) that ΔABC was a right triangle with $\angle B = 90°$ and that BD was perpendicular to AC (R2) presented in Diagram 6. She also identified and structured key angle relationships (R2, B1), such as $m\angle CDB = m\angle BDA = 90°$.

In the building-with phase, she established (R3=B1, C1) that $\angle CDB \cong \angle ADB$ and (R2, B3, C2) $\angle ABC \cong \angle CDB$. The subject concatenated (R3 = C1, R4 = C2, C3) to establish that $\angle ABC \cong \angle CDB \cong \angle ADB$. She also recognized (R5) that ΔBDA had a right angle at D. Additionally, she concatenated (R1, R5, B4) to conclude that $\angle BAC \cong \angle DAB$. The subject further recognized (R6) that ΔBDC had a right angle at D. Then, she structured (R1, R6, B5) the relationship $\angle BCA \cong \angle DCB$. Next, the subject constructed (B4 = R7, C3) to establish the similarity relation $\Delta BAC \sim \Delta DAB$ and (B5 = R8, C4) to establish $\Delta DAB \sim \Delta DBC$. Furthermore, she constructed (R9 = C4, C5) the proportion $\frac{AD}{BD} = \frac{BD}{DC}$ and (R10 = C5, C6) the equation $AD.DC = BD.BD$, leading to (R11 = C6, C7) the final algebraic expression $AD.DC = BD^2$. Finally, the subject formulated the conclusion (R1, R2, R12): "If ΔABC has a right angle at B, and an altitude is drawn from point B to intersect the hypotenuse at point D, then $\overline{BD}^2 = \overline{AD} \times \overline{DC}$".

The conclusion drawn by S3 was a direct result of recognizing, structuring, and constructing mathematical relationships through systematic reasoning. This process highlighted S3's ability to integrate prior knowledge with logical proof techniques. Diagram 6 further illustrates S3's recognition abilities throughout the proof process, providing a visual representation of the conceptual steps involved.

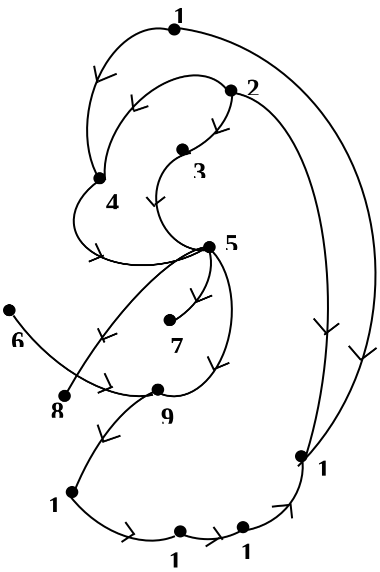

Diagram 5. The Process of Proving of S3

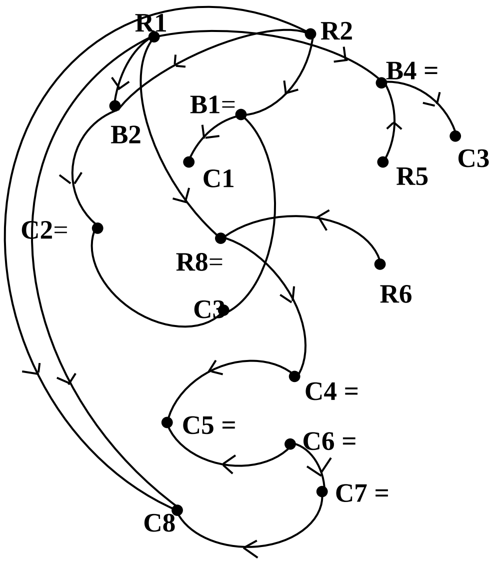

Diagram 6. RBC of S3 Subject

Discussion

The process of writing mathematical proofs provides undergraduate students with an opportunity to think critically and reflect on their experiences when proving a problem based on the congruence of two triangles and the application of the Pythagorean theorem. Writing proofs not only enhances students' ability to follow procedures but also helps bridge the gap between procedural fluency and conceptual understanding. This aligns with the findings of King et al. (2016), who stated that writing in mathematics fosters critical thinking.

However, subjects S2 and S3 were unable to explore alternative methods to solve the problem beyond using the congruence of two triangles, even though they selected different pairs of triangles.

Writing proficiency enables students to articulate mathematical reasoning, demonstrate conceptual understanding, and develop problem-representation skills. This is consistent with previous research (Baxter et al., 2005), which suggests that writing significantly aids in understanding mathematical reasoning, conceptual comprehension, and problem representation.

In the proof process, students recognized that if a right angle exists at a given point, drawing an altitude from that point to the hypotenuse establishes the congruence of two triangles. This finding is in line with research by Jones et al. (2013), which states that students can determine pairs of congruent triangles to prove a geometric theorem. The ability to read and construct proofs enhances problem-solving skills (Bicer et al., 2013) and has a positive impact on academic achievement (Cooper, 2012). Moreover, proof-writing skills help students develop conceptual, procedural, and mathematical communication abilities (Teuscher et al., 2015). These findings reaffirm that proof construction is not merely a technical task but one that integrates cognitive and communicative competence.

The process of abstraction follows a sequence of epistemic actions, including recognizing, building-with, constructing, and consolidating (Dreyfus et al., 2015). The three subjects required time to determine their proof strategy, ultimately choosing to rely on triangle congruence. Recognition occurred as subjects made analogies to previously encountered mathematical structures, such as the sum of angles in a triangle being 180°. In the building-with phase, subjects combined existing elements to achieve specific goals, such as problem-solving or constructing arguments (Dreyfus et al., 2002; Jirotková & Littler, 2005). This process led them to construct the proof that $|\overline{BD}|^2 = |\overline{AD}| \times |\overline{CD}|$, demonstrating their ability to reorganize and restructure knowledge (Dreyfus et al., 2015). For example, recognizing that two pairs of congruent angles imply triangle congruence allows for corresponding sides to be proportionate.

During the consolidation phase, subjects demonstrated a clear and rapid understanding of the relationships among the three triangles—ΔABC (right-angled at B), ΔABD (right-angled at D), and ΔCDB (right-angled at D) (Dreyfus et al., 2015). They realized that the abstraction established in one triangle facilitated their comprehension of similar structures in other triangles, supporting further abstraction (Breive, 2022).

Although the RBC+C abstraction model allowed us to observe abstraction micro-analytically, the findings also revealed that students' epistemic actions were not uniform across literacy levels. For example, high-proficiency students demonstrated the ability to reorganize ideas and recognize patterns through multiple representations, including diagrams, symbolic notation, and verbal justification. In contrast, low-proficiency students often focused on isolated aspects of the problem and struggled to maintain logical coherence between steps.

These differences in abstraction quality are likely influenced by students' mathematical reading and writing abilities, which were the focal point of this study. In the Indonesian higher education context, proof is introduced relatively early, typically during geometry courses in the first or second year. However, mathematical literacy is rarely treated as an explicit learning objective in the curriculum, specifically the ability to read and write mathematical arguments (Retnawati et al., 2018). Students are often expected to construct proofs based on examples or procedural models without sufficient scaffolding in interpreting symbolic language or articulating reasoning. This may explain why low-literacy students in this study struggled with consolidation and symbolic consistency, despite demonstrating basic recognition of geometric relationships.

The contrast between students with high and low literacy levels supports findings from international literature (Bicer et al., 2013; Österholm, 2006), which suggests that reading and writing in mathematics are critical not only for understanding text but also for navigating abstract concepts and engaging in deep mathematical thinking. Furthermore, the fact that high-proficiency students reused and repurposed theorems across multiple steps indicates a stronger ability to internalize and manipulate abstract constructs—a hallmark of successful abstraction in the AiC framework (Dreyfus & Kidron, 2014; Gilboa et al., 2019).

The RBC+C abstraction model describes the process through which the subjects abstracted knowledge while solving the problem. The analysis of RBC+C suggests that students with strong language, writing skills, and mathematical reasoning exhibited a structured approach to proof construction. These students successfully utilized all epistemic actions within the RBC+C model and achieved a formal level of proof for triangle congruence.

From a theoretical perspective, the analysis reveals a nested abstraction process. Students recognized previously established structures and assembled them to meet the requirements of the proof. The interview design was intended to establish a network of relationships that substantiates the reasoning process and allows for the extension of existing knowledge into new mathematical structures. The subjects' activities—recognizing, assembling, and constructing relationships between quadrilaterals—were not strictly hierarchical but rather interconnected, forming a chain-like progression of thought.

Their cognitive and communicative abilities, both written and verbal, played a crucial role in proof construction and interview discussions. In some instances, high-achieving students demonstrated creative thinking by proving the theorem not only through geometric methods but also via algebraic approaches. High-achieving subjects consolidated mathematical concepts by repeatedly applying, revisiting, and refining their ideas, whereas lower-achieving students exhibited a more fragmented approach, frequently reusing but not deeply engaging with mathematical concepts.

Furthermore, the analysis suggests that abstraction processes differ among students with high, medium, and low abilities. High-achieving students demonstrated a refined abstraction process, systematically recognizing and assembling previously established structures to meet verification and interview tasks. The interview process guided students toward proving geometric theorems while also encouraging them to extend beyond known knowledge to construct new mathematical structures. In this process, students engaged in recognizing, assembling, and constructing knowledge in a structured manner, where the complexity of the abstraction process corresponded to their proficiency in mathematical reading and writing.

CONCLUSION

The Recognition, Building-with, Construction, and Consolidation (RBC+C) abilities of subjects with high mathematical reading and writing proficiency are presented in detail, aligned with the proof steps and further refined through interviews to develop a nested RBC network. This network consists of 16 recognitions (4 from building-with and 4 from construction), 8 building-with, and 6 construction actions. However, the subject did not consistently apply line segment length notation, likely due to differences in notation across reference books. Nevertheless, the subject demonstrates consolidation through frequent reuse of mathematical concepts, particularly the repeated application of the Pythagorean Theorem across three right triangles within the given problem. Although the subject did not reconfirm the proof using geometric congruence, their use of algebraic reasoning reflected strong abstraction and flexible problem-solving strategies.

Similarly, the RBC+C abilities of subjects with moderate mathematical reading and writing proficiency are analyzed in detail according to proof steps, refined through interviews, and mapped into a nested RBC network. This network comprises 8 recognitions (3 from building-with and 4 from construction), 4 building-with, and 6 construction actions. The subject effectively utilizes reading and writing skills to construct the relevant triangles, document known and to-be-proven elements, and apply precise notations for congruent angles, perpendicular line segments, congruent triangles, and segment lengths. Inaccuracies emerged in the use of right-angle indicators and symbolic representations, which may have affected the coherence of the proof structure. However, reinforcement of understanding was evident through the repeated application of the triangle angle sum property (180°), demonstrating partial consolidation of ideas.

For subjects with low mathematical reading and writing proficiency, the RBC+C abilities are also examined in relation to proof steps, supported by interviews, and represented through a nested RBC network. This network consists of 12 recognitions (3 from building-with and 4 from construction), 5 building-with, and 8 construction actions. Although they demonstrated some level of consolidation by frequently referencing congruent angle relationships, these students faced greater challenges in maintaining logical flow and using consistent mathematical notation throughout the proof. Their abstraction processes appeared more fragmented, with limited coherence between recognition and construction stages.

This study has several limitations, including a small sample size of three undergraduate students, which limits the generalizability of the findings. The narrow focus on triangle congruence as a proof domain may also constrain applicability to other mathematical topics. Furthermore, the study did not employ longitudinal tracking, making it difficult to assess the development of RBC+C abilities over time. Variations in participants' prior knowledge could also have influenced their performance and abstraction patterns.

Despite these limitations, the study offers valuable insights into how students' mathematical reading and writing abilities influence their abstraction processes in proof construction, directly addressing the research objective. The findings highlight that students with stronger literacy skills exhibit more structured and interconnected epistemic actions—recognizing, building-with, constructing, and consolidating—while those with lower proficiency demonstrate inconsistencies in reasoning and difficulty in maintaining logical coherence. This affirms the central aim of the study: to explore the relationship between mathematical literacy and students' engagement in epistemic actions during geometric proof.

The results reinforce the importance of integrating mathematical reading and writing into proof-based instruction. Developing students' literacy skills can enhance not only their communication abilities but also their conceptual reasoning and abstraction. Future research is encouraged to investigate this relationship in broader mathematical contexts and across different educational levels. Studies involving instructional interventions that strengthen mathematical literacy may yield strategies for improving students' proof competencies and overall mathematical understanding.

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Author Contribution	: RS: Conceptualization, Formal Analysis, Investigation, Methodology, Project Administration; Visualization, and Writing - Original Draft. MTB: Conceptualization, Formal Analysis, Investigation, and Methodology. AFJ: Visualization, Writing - Original Draft, and Writing-review and editing.
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