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Classification of inductive thinking in mathematical problem solving

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Abstract

Inductive thinking is a method of thinking that involves recognizing patterns, understanding relationships, and deconstructing general rules. This method of thinking develops through a variety of factors that support complex problem solving. Using mathematical problems that describe the inductive thinking process within the context of number problems helps investigate students' inductive thinking process. This study, employing a qualitative descriptive research approach, seeks to develop a novel classification framework for students' inductive thinking in the context of mathematical problem solving. The study was conducted in a structured manner on 21 students enrolled in the Department of Mathematics during their fifth semester at a university in Indonesia using number sequence as the problem material. The collection of data was executed through the administration of tests and the observations of problem-solving behaviors. The analysis was conducted using constant comparative procedures (CCP). The instruments used in this study included mathematical problems and recording devices. The findings of this study are presented in the form of three different classifications of inductive thinking: the use of variables, the use of visual, and the use of formulae. The study offers significant theoretical insights for future research and practical implications for the implementation of inductive thinking in improving mathematical problem-solving.

Keywords: Classification, Inductive Thinking, Mathematical Problems, Problem-Solving

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Mathematics is a vital part of daily life, affecting various aspects of personal and professional development. Mathematics is also essential for problem solving, critical thinking, and effective communication (Khan & Salman, 2020). Fields such as natural sciences, engineering, economics, and the arts all make use of mathematics (Schiemer, 2019). Mathematics is a fundamental field of study with applications in various fields including weather forecasting, banking, technology, and research (Khan & Salman, 2020; Rani et al., 2023). The importance of mathematics is evident in its role in scientific advancement, as advancements in mathematics often precede significant discoveries in the field of science (Rani et al., 2023). Despite the perceived difficulties, mathematics is indispensable for progress in the modern world (Barete & Taja-on, 2024). By understanding the real-life use of mathematics, students can better appreciate the relevance and importance of mathematics in their education and future careers (Tan, 2023; Vos et al., 2024). The Realistic Mathematics Education (RME) approach is generally considered an effective method for achieving this objective (Sutarni et al., 2024).

Mathematics, in addition to its practical role in everyday life, involves a cognitive process that helps students understand and solve complex problems with a more structured approach. Thinking is an important component of solving mathematical problems (Kholid, Sa'dijah et al., 2022; Sa'dijah et al., 2020). Mathematical thinking is defined as a creative process involving making predictions, induction, interpretation, description, abstraction, and reasoning (Adawiyah et al., 2017; Kasmer & Kim, 2011). This process helps students understand complex structures and solve problems effectively. Mason's cognitive framework subdivides the stages of thinking into three phases: input, impact, and evaluation. These phases can be applied to the solution of trigonometric equations (Adawiyah et al., 2017). The concept of mathematical thinking, as it is currently understood, originated from the book "Thinking Mathematically" in 1982, a seminal work by Mason, Burton, and Stacey. This book introduced the concept of mathematical thinking as a dual process, comprising two pairs of complementary activities: specialization and generalization, and conjecture and convincing (Delima et al., 2018). Nevertheless, the conceptualization of mathematical thinking has evolved over time. In the domain of mathematics education, the focus is on the outcome and on the process (Kapur, 2014). The process helps students understand complex structures and solve problems effectively (Sachdeva & Eggen, 2021; Supandi et al., 2019). One mathematical thinking that can be used in problem solving is inductive thinking, which is a process of deriving conclusions from specific observations to establish general principles (Vo & Csapó, 2022).

Inductive thinking, which falls into the category of mathematical thinking, plays an important role in strengthening students' conceptual understanding and problem-solving skills (Sanjaya et al., 2018), as well as in cognitive development and educational success. The Taba Inductive Thinking Model has demonstrated effectiveness in improving academic achievement in science and cultivating creative thinking skills, including originality, fluency, and flexibility, especially among high school students (Kauts et al., 2020). In the context of mathematics education, the implementation of inductive-deductive approach helps to foster reasoning skills and conceptual comprehension. Specifically, inductive thinking helps to discern general patterns, while deductive thinking improves mathematical proofing and reasoning skills (Rahmah, 2017). Inductive thinking involves the recognition of patterns, the understanding of relationships, and the deconstruction of general rules, which develop through various factors that support complex problem solving, as evidenced by the Raven Progressive Matrix test (Perret, 2015).

Several studies have yielded equivocal results regarding the effectiveness of inductive-deductive approaches in mathematical problem solving. While one study did not find significant improvements in mathematical comprehension or problem-solving skills using an inductive-deductive approach (Rahmah, 2017), the other found better problem-solving skills and confidence among students taught with this approach than conventional learning (Wright, 1977). A study identified three main areas of inductive thinking: data collection, pattern discovery, and hypothesis generation. Pattern discovery is of particular importance in successful problem solving (Haverty et al., 2000). An analysis of students' mathematical thinking types revealed that deductive and inductive thinking is used in problem-solving tasks. Some students demonstrated proficiency in inductive thinking while others struggled to fully apply it (Miswanto et al., 2019). Therefore, while the effectiveness of the inductive-deductive approach in enhancing mathematical problem-solving remains inconclusive, the critical role of elements such as pattern discovery in inductive thinking suggests that the success of this method largely depends on how well these components are integrated into the learning process.

Mathematical problem solving involves complex cognitive processes and reasoning abilities. Research has shown that students tend to rely on inductive thinking more often than deductive thinking when confronted with mathematical problem-solving tasks (Miswanto et al., 2019; Sanjaya et al., 2018). This inclination toward inductive thinking is attributed to its capacity to facilitate the transition from specific examples to more general concepts (Miswanto et al., 2019). It is also considered to be advantageous for students who demonstrate an aptitude for working with concrete examples to understand abstract concepts. When applied appropriately, inductive thinking has the potential to lead to more efficient mathematical problem solving and improve intelligence. A study suggests that training in inductive reasoning can enhance fluid intelligence and academic performance (Klauer et al., 2002). Additionally, the process of conceptual thinking in mathematical problem solving involves important stages, including the description of problems, the association of concepts, the determination of key concepts, and the formulation of solutions (Hamda, 2018).

The importance of fostering inductive thinking through structured strategies and conceptual thinking lies in their capacity to establish a foundation for profound mathematical understanding and enhanced problem-solving effectiveness. The process of inductively solving mathematical problems involves collecting data, discovering patterns, and creating hypotheses (Haverty et al., 2000). A strategy commonly used in this context is known as "Pursuit", wherein students engage in the creation of new quantities, the detection of patterns, and the subsequent expression of these patterns in terms of variables (Haverty et al., 2000). To improve problem-solving skills, a conceptual thinking approach has been proposed. This approach involves a series of steps: problem description, concept association, key concept definition, and solution formulation (Hamda, 2018). In addition, there have been suggestions of didactic strategies that focus on developing students' inductive thinking skills to improve their problem-solving abilities in mathematics.

A comprehensive review of extant studies related to inductive thinking reveals three predominant focuses. The focuses are as follows: (1) the application of inductive thinking methods in mathematics learning; (2) the stages (indicators) of inductive thinking in solving mathematical problems; and (3) a comparison between inductive thinking and other approaches.

A body of research has been conducted on the application of inductive thinking methods in mathematics learning, and the following findings have been reported. The reflective learning model has been demonstrated to improve students' mathematical inductive thinking, surpassing the effectiveness of conventional learning (Kurniawati et al., 2021). The integration of mathematical inductive thinking in teaching has been shown to facilitate students' active construction of knowledge, thereby improving their thinking and problem-solving skills (Li et al., 2022). However, the use of inductive-deductive approaches in mathematics education has not been observed to result in significant improvements in comprehension or problem-solving skills among junior high school students (Rahmah, 2017).

Several studies have been conducted on the stages (indicators) of inductive thinking in mathematics learning. These include a study by Moguel et al. (2020) that reported six stages of inductive thinking in teachers who solve generalization problems and emphasized the importance of associating regularity with mathematical structure. Another study was conducted by Helviyana et al. (2020). The study identified stages of inductive thinking in students' mathematical thinking during an inquiry learning model, which are: perception of an inquiry learning model, perception of generality, expression of generality, and manipulation of generality. In the study by Haverty et al. (2000), three fundamental areas of inductive activity were identified: data collection, pattern discovery, and hypothesis generation. The role of pattern discovery was found to be significant.

A number of studies that compared inductive thinking with other approaches yielded the following findings: (1) Inductive approaches are more effective than deductive approaches in teaching EFL grammar to high school students (Benitez-Correa et al., 2019); (2) Inductive teaching methods are generally more effective than traditional deductive methods in achieving a variety of learning outcomes (Prince & Felder, 2006); and (3) The deductive approach is more effective than the inductive approach in improving students' conceptual understanding, while the inductive approach is more effective for theory

(Wardani et al., 2020). Following a review of studies related to inductive thinking, the topics identified for further research were categorized into three classifications: (1) the application of inductive thinking to solve mathematical problems; (2) the integration of collaborative learning to improve inductive thinking; and (3) the development of STEAM-based learning media to promote inductive thinking.

The present study focuses on the classification of students' inductive thinking in the context of solving mathematical problems. The results of the study are expected to provide an overview of the classification of students' inductive thinking based on indicators of inductive thinking in the context of mathematical problem solving. These results can be used as a guide to assess the extent of students' inductive thinking in solving mathematical problems. Moreover, the present study can serve as a foundational framework for further research endeavours that focused on the improvement of students' inductive thinking, thereby underscoring its significance.

The objective of this study is to examine the manner in which students employ inductive thinking to solve mathematical problems. The goal is to categorize students' inductive thinking when confronted with mathematical problems. As Rahmah (2017) stated, the application of inductive thinking can facilitate the development of mathematical problem-solving skills.

METHODS

Research Design

Given the purpose of the present study, which is to describe the classification of inductive thinking in mathematical problem solving, and in consideration of the qualitative nature of the data, a qualitative method and a descriptive approach were employed. Qualitative research is a systematic approach to understanding human experiences, behaviours, and beliefs that cannot be easily measured (Awasthy, 2019; Mistry, 2012). Descriptive qualitative research is an approach that aims to systematically describe current conditions by providing detailed explanations and portrayals of the phenomenon under study, without testing hypotheses or seeking causal relationships (Creswell, 2012). This approach is particularly valuable for gaining an in-depth understanding of current conditions, perceptions, or practices, and often serves as a foundation for subsequent stages of research.

Participants

The subjects of this study were 21 fifth-semester students from the Department of Mathematics at a university in Indonesia. These students were selected using the purposive sampling method because the results of their problem-solving supported the research objectives (Kholid, Swastika et al., 2022). Participants were selected based on their willingness to volunteer in this study and their ability to work on mathematical problem solving by implementing think-aloud and to use inductive thinking to solve problems. Students who did not engage in inductive thinking, students who engaged in inductive thinking but could not implement think-aloud, and students who were not willing to participate as research subjects were not selected as research participants. Of the 21 students, only 10 employed inductive thinking systematically in their problem-solving process. Through comprehensive analysis, three distinct classifications of inductive thinking were identified, with their distribution detailed in Table 1. In the Results and Discussion section, one subject representative from each classification is presented. These subjects were selected based on the strength and clarity with which they exhibited the defining features of inductive thinking within their respective group.

Table 1. Distribution of participants

Classification Inductive Thinking	Number
Classification 1	3
Classification 2	5
Classification 3	2
Sum	**10**

Instruments

The main instruments employed in qualitative research are researchers, who plan and execute data collection, analyse results, formulate conclusions, and compose reports (Kholid, Sa'dijah et al., 2022). The present study incorporated additional instruments, including mathematical problems and audio-visual recording devices. Prior to its utilization for data collection, the test instrument underwent a validation process. The validation process was carried out by a qualitative research expert in mathematics education and an expert in inductive mathematics problem solving. The suggestions provided by the validators included adjustments to the dictions to enhance the test's manageability for students, and improvements to the image quality to render it more straightforward and engaging. The participants were not provided with any specific instructions to ensure that the data obtained were natural. Figure 1 shows an example of a question used to classify students' inductive thinking.

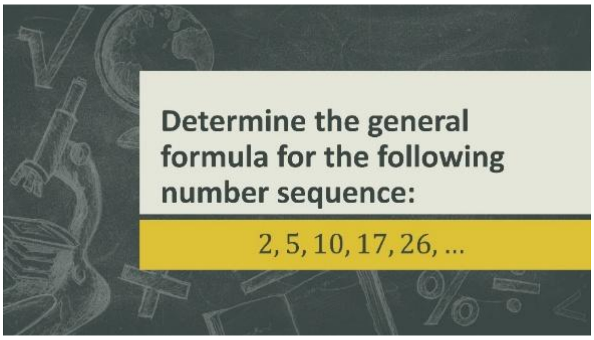


Figure 1. The written test question

Data Collection Procedure

The data collection procedure began with the formulation of a test containing mathematical problems designed to elicit students' use of inductive thinking. The selection of questions was informed by indicators of inductive thinking, such as pattern identification, generalization, and hypothesis formulation. Subsequently, the subjects of the study were instructed to solve the problem using the think-aloud method. This entailed the subjects verbally reporting their thought process during the problem-solving process, thereby allowing for the acquisition of first-hand data concerning students' problem-solving cognitive strategies.

The entire think-aloud process was recorded using audiovisual devices to allow for more thorough observation and ensure the accuracy of the subsequent data analysis. The students' answer sheets were collected to examine their final results and finishing steps, which were then compared to the thought processes revealed from the think-aloud sessions.

A data analysis was conducted by examining answer sheets and audio-visual recordings to identify inductive thinking classification patterns and to evaluate the strategies used by students. The results of the analysis were used to categorize the students' inductive thinking and to determine the stages that often arise and the level of use of inductive strategies in solving mathematical problems. The findings of the study were subsequently compiled into a report, which included a detailed classification of students' inductive thinking abilities and recommendations for teaching strategies designated to improve these abilities.

Data Analysis

The data were collected and subsequently categorized, and indicators of inductive thinking were employed to facilitate the classification process. Hewitt-Taylor (2001) and Jobes et al. (1997) described this procedure as a constant comparative procedure (CCP). In qualitative research, CCP is an inductive data analysis method used to generalize and classify data into specific categories to develop new theories.

In this study, CCP was applied by processing original data derived from think-aloud session transcripts and student answer sheets. The data were collected from various subjects through two data collection techniques. During the analysis process, the indicators of inductive thinking that appeared in each data set were compared to the categories that had been created. New data were continually compared with existing categories to identify patterns, similarities, and differences. This process yielded evidence that lends support to the classification of students' inductive thinking and helped eliminate redundant data.

The present study aims to classify students' inductive thinking in the context of mathematical problem-solving. The results of the CCP analysis enabled the development of a classification of inductive thinking in mathematical problem solving. This classification includes a variety of indicators such as pattern identification, generalization, and hypothesis formulation. Figure 2 shows the CCP used in this study.

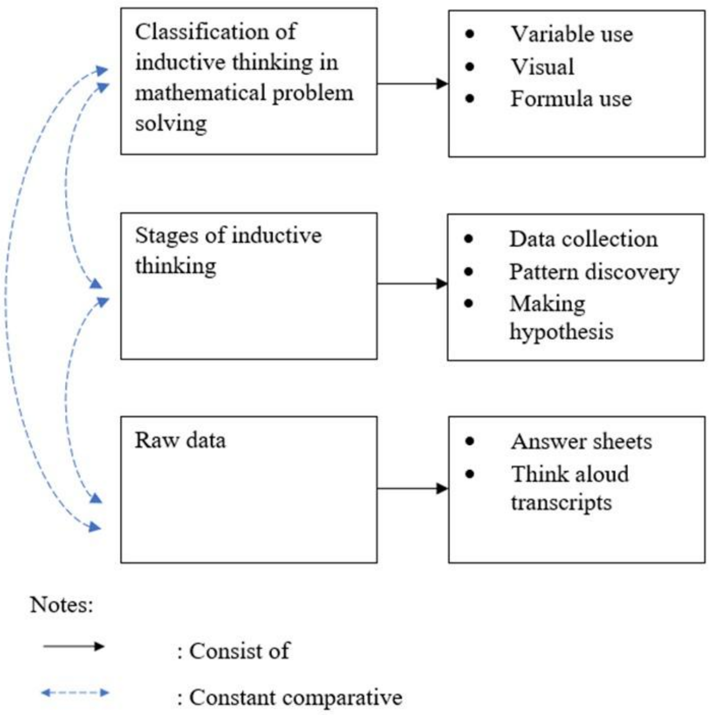


Figure 2. Implementation of CCP in data analysis

RESULTS AND DISCUSSION

The data analysis yielded three classifications of students' inductive thinking in the context of solving mathematical problems. Subject 1 exemplified a student who employed inductive thinking by using variables. Subject 2 exemplified a student who used visual inductive thinking. Finally, Subject 3 exemplified a student who utilized inductive thinking through the application of formulae.

Classification I: Variable Use

The first step taken by Subject 1 was to analyze the differences of each expression in order. Upon determining that the number sequence was a multi-level sequence of numbers, Subject 1 elected to use second-degree polynomial regression to solve the problem, thereby ascertaining the general formula of the number sequence. Subject 1 applied the first, second, and third terms to the general formula of second-degree polynomials, i.e., $U_n = an^2 + bn + c$. From $U_1 = 2$, it can be deduced that $a + b + c = 2$. Subsequent to this, with $U_2 = 5$, it can be established that $4a + 2b + c = 5$. Furthermore, from $U_3 = 10$, it can be concluded that $9a + 3b + c = 10$. Consequently, Subject 1 derived the system of equations (1) $a + b + c = 2$, (2) $4a + 2b + c = 5$, and (3) $9a + 3b + c = 10$. In this step, Subject 1 gathered the information needed to solve the problem, as shown in Figure 3.

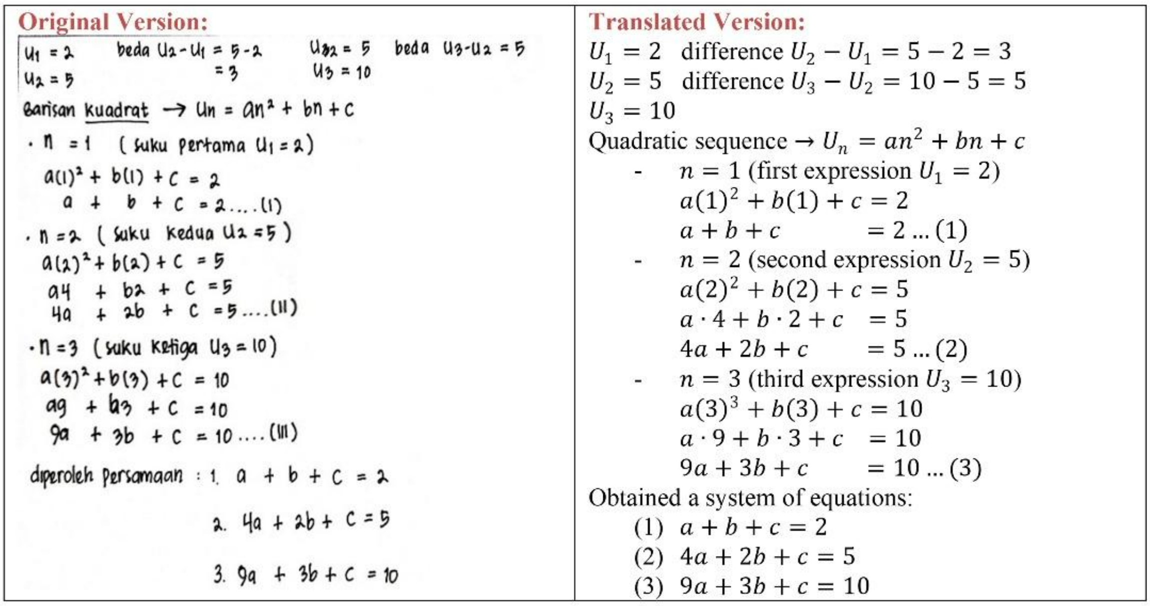
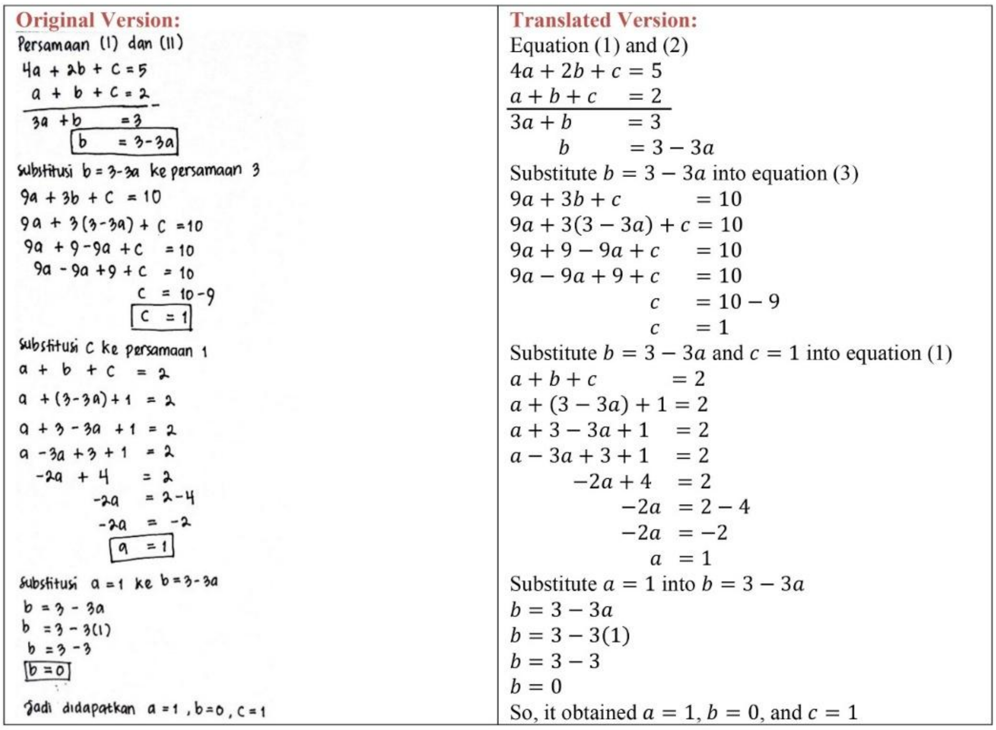


Figure 3. Answer sheet of Subject 1

In the next step, Subject 1 eliminated equations (1) and (2) so that $b = 3 - 3a$. Furthermore, Subject 1 substituted $b = 3 - 3a$ into equation (3), resulting in $c = 1$. After obtaining the value, Subject 1 substituted $c = 1$ and $b = 3 - 3a$ into equation (1) and obtained $a = 1$. With $a = 1$, Subject 1 obtained b through substitution of $a = 1$ to $b = 3 - 3a$. This was followed by Subject 1 obtaining $b = 0$. Therefore, the final outcome for Subject 1 was $a = 1$, $b = 0$, and $c = 1$, as presented in Figure 4.



Original Version:	Translated Version:
Persamaan (I) dan (II)	Equation (1) and (2)
4a + 2b + c = 5	$4a + 2b + c = 5$
a + b + c = 2 −	$a + b + c = 2$
3a + b = 3	$3a + b = 3$
b = 3−3a	$b = 3 - 3a$
substitusi b = 3-3a ke persamaan 3	Substitute $b = 3 - 3a$ into equation (3)
9a + 3b + c = 10	$9a + 3b + c = 10$
9a + 3(3-3a) + c = 10	$9a + 3(3 - 3a) + c = 10$
9a + 9 - 9a + c = 10	$9a + 9 - 9a + c = 10$
9a - 9a + 9 + c = 10	$9a - 9a + 9 + c = 10$
c = 10-9	$c = 10 - 9$
c = 1	$c = 1$
substitusi c ke persamaan 1	Substitute $b = 3 - 3a$ and $c = 1$ into equation (1)
a + b + c = 2	$a + b + c = 2$
a + (3-3a) + 1 = 2	$a + (3 - 3a) + 1 = 2$
a + 3 - 3a + 1 = 2	$a + 3 - 3a + 1 = 2$
a - 3a + 3 + 1 = 2	$a - 3a + 3 + 1 = 2$
-2a + 4 = 2	$-2a + 4 = 2$
-2a = 2-4	$-2a = 2 - 4$
-2a = -2	$-2a = -2$
a = 1	$a = 1$
substitusi a = 1 ke b = 3-3a	Substitute $a = 1$ into $b = 3 - 3a$
b = 3 - 3a	$b = 3 - 3a$
b = 3 - 3(1)	$b = 3 - 3(1)$
b = 3 - 3	$b = 3 - 3$
b = 0	$b = 0$
Jadi didapatkan a = 1, b = 0, c = 1	So, it obtained $a = 1$, $b = 0$, and $c = 1$

Figure 4. Answer sheet of Subject 1

After obtaining the values of *a*, *b*, and *c*, Subject 1 proceeded to determine the general formula of the sequence of numbers by substituting $a = 1$, $b = 0$, and $c = 1$ into the general formula of the second-degree polynomial, $U_n = an^2 + bn + c$. This substitution resulted in the following general formula being derived from the number sequence $2, 5, 10, 17, 26, ...$: $U_n = n^2 + 1$. At this stage, Subject 1 drew conclusions using the information that had been obtained. This indicated that Subject 1 used an inductive thinking approach, whereby the subject collected a certain set of specific information and subsequently used it to draw possible conclusions. Then, subject 1 verified the accuracy of the formula by substituting $n = 1, 2, 3, ...$ This yielded $U_1 = 2$, $U_2 = 5$, and $U_3 = 10$, thereby confirming the accuracy of the general formula $U_n = n^2 + 1$ for the sequence of numbers $2, 5, 10, 17, 26, ...$, as illustrated in Figure 5.

It was observed that Subject 1 used a variable to identify relationships between terms and determine the general formula for the sequence of numbers. Subject 1 derived a conclusion or general formula from the number sequence through information or data obtained using variables, namely the use of second-degree polynomial regression. The data obtained were in the form of a system of equations that were substituted and eliminated. The aforementioned steps demonstrate that Subject 1 used an inductive thinking method, predominantly involving variables in mathematical problems.

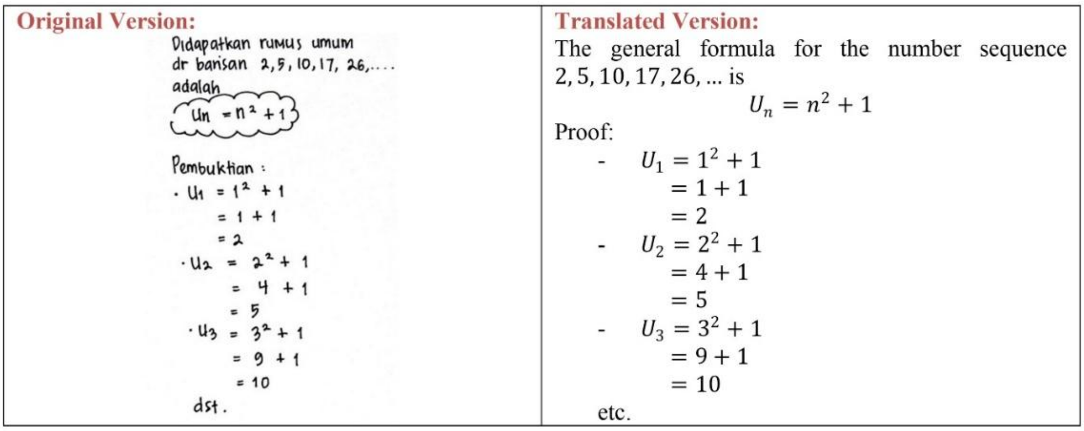


Figure 5. Answer sheet of Subject 1

Classification II: Visual Use

In order to solve the presented problem, Subject 2 employed a visual representation of the problem to obtain pertinent information. The sequence of numbers $2, 5, 10, 17, 26, ...$ was represented as a series of dots arranged in a pattern. This method facilitated the acquisition of information regarding each expression by Subject 2. It was observed by Subject 2 that the sequence of numbers can form a particular pattern in the formation of the dots. Then, Subject 2 identified the pattern and obtained that $U_1 = 1^2 + 1 = 2$, $U_2 = 2^2 + 1 = 5$, $U_3 = 3^3 + 1 = 10$, $U_4 = 4^2 + 1 = 17$, and $U_5 = 5^2 + 1 = 26$. From this identification, Subject 2 arrived at the conclusion that the sequence of numbers $2, 5, 10, 17, 26, ...$ should be expressed as $U_n = n^2 + 1$, as shown in Figure 6.

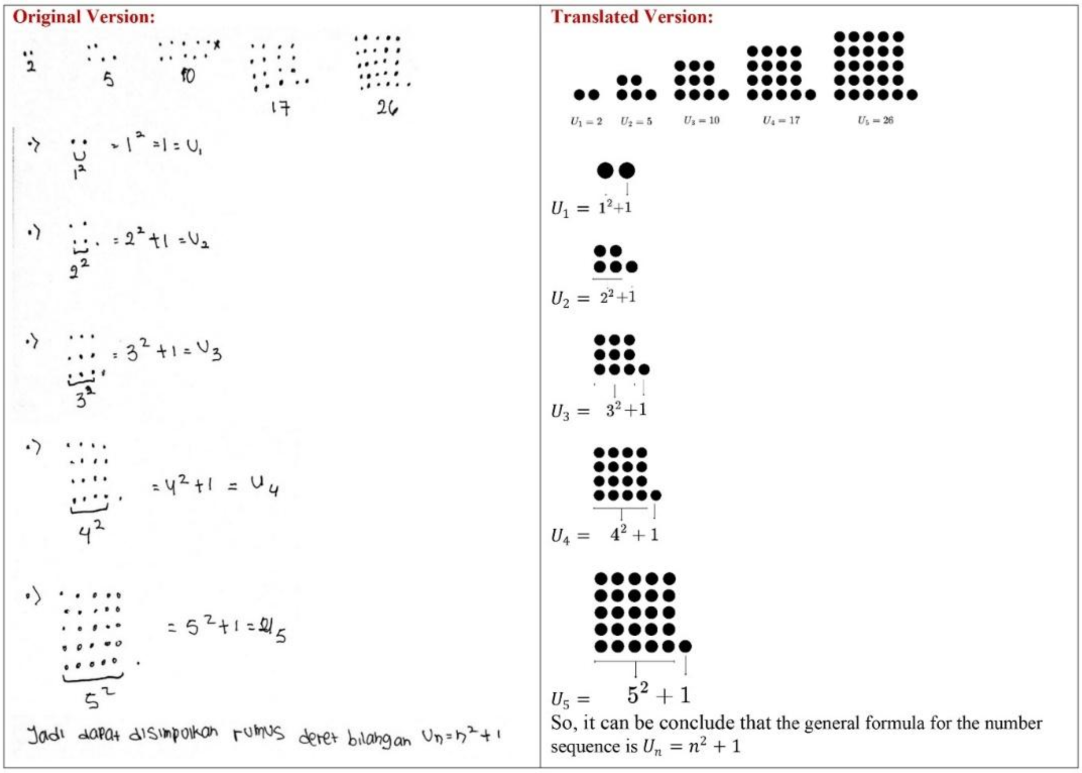
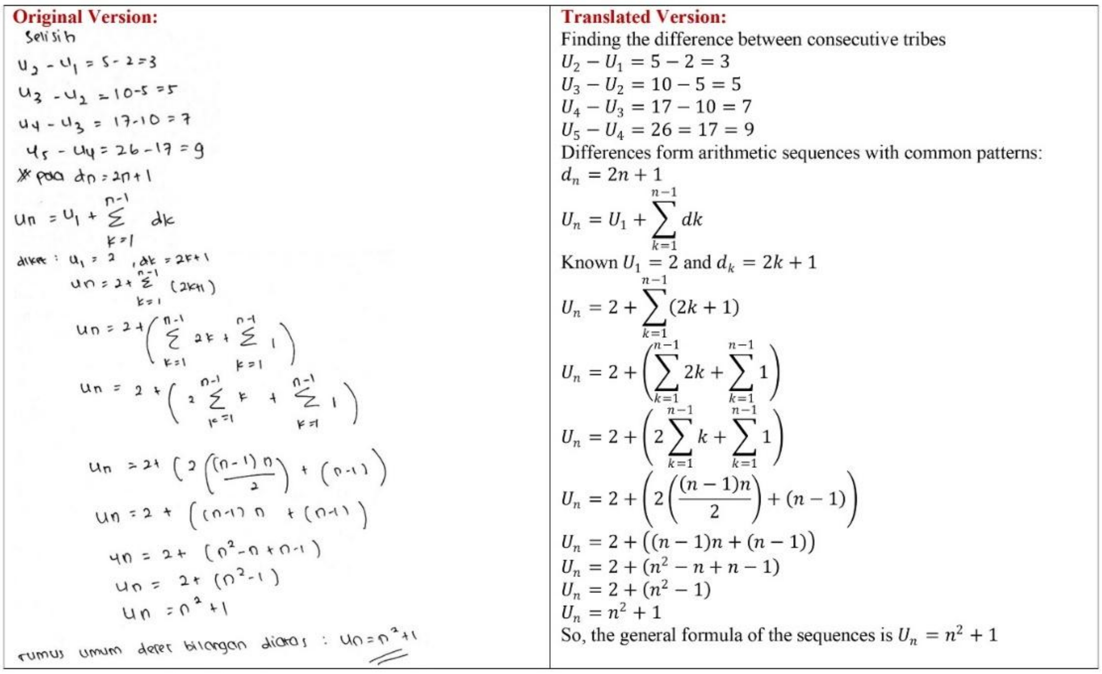


Figure 6. Answer sheet of Subject 2

The problem-completion steps taken by Subject 2 demonstrate that the subject employed inductive thinking, which is the process of formulating conclusions based on specific information observed. The subject's attempt to obtain information through visual representation and subsequent conclusions regarding the specific information obtained were demonstrated. Subject 2 used visual representations in the form of dots to facilitate problem solving and stimulate inductive thinking during the problem-solving process. Subject 2 derived great benefit from the implementation of visual representation, a strategy that facilitated the identification of a solution to the problem presented.

Classification III: Formula Use

The first step in solving the problem presented by Subject 3 was to analyze the differences between consecutive terms. This step was an effort made by Subject 3 to obtain information to solve the problem. It was found by Subject 3 that the difference between the expressions forms an arithmetic sequence: $3, 5, 7, 9,$ .... The sequence has a general pattern, with $d_n = 2n + 1$. With this information, the next step taken by subject 3 was to deduce the general formula for the sequence of numbers $2, 5, 10, 17, 26,$ .... This was achieved by summing the 1st quarter of the number line with the number of rows of difference from 1 to $n - 1$, or $U_n = U_1 + \sum_{k=1}^{n-1} dk$. In this process, Subject 3 demonstrated an application of an inductive thinking method by formulating conclusions from the information obtained to solve the problem. Upon determining that $U_1 = 2$ and $d_k = 2k + 1$, through the simplification of the formula, Subject 3 arrived at the general formula for the number sequence, $U_n = n^2 + 1$, as shown in the Figure 7.


Original Version:

Selisih
$u_2 - u_1 = 5 - 2 = 3$
$u_3 - u_2 = 10 - 5 = 5$
$u_4 - u_3 = 17 - 10 = 7$
$u_5 - u_4 = 26 - 17 = 9$
pola $d_n = 2n+1$
$u_n = u_1 + \sum_{k=1}^{n-1} dk$
diket : $u_1 = 2$, $dk = 2k+1$
$u_n = 2 + \sum_{k=1}^{n-1} (2k+1)$
$u_n = 2 + \left(\sum_{k=1}^{n-1} 2k + \sum_{k=1}^{n-1} 1\right)$
$u_n = 2 + \left(2\sum_{k=1}^{n-1} k + \sum_{k=1}^{n-1} 1\right)$
$u_n = 2 + \left(2\left(\frac{(n-1)n}{2}\right) + (n-1)\right)$
$u_n = 2 + ((n-1)n + (n-1))$
$u_n = 2 + (n^2 - n + n - 1)$
$u_n = 2 + (n^2 - 1)$
$u_n = n^2 + 1$
rumus umum deret bilangan diatas : $u_n = n^2 + 1$

Translated Version:

Finding the difference between consecutive tribes
$U_2 - U_1 = 5 - 2 = 3$
$U_3 - U_2 = 10 - 5 = 5$
$U_4 - U_3 = 17 - 10 = 7$
$U_5 - U_4 = 26 = 17 = 9$
Differences form arithmetic sequences with common patterns:
$d_n = 2n + 1$
$U_n = U_1 + \sum_{k=1}^{n-1} dk$
Known $U_1 = 2$ and $d_k = 2k + 1$
$U_n = 2 + \sum_{k=1}^{n-1} (2k + 1)$
$U_n = 2 + \left(\sum_{k=1}^{n-1} 2k + \sum_{k=1}^{n-1} 1\right)$
$U_n = 2 + \left(2\sum_{k=1}^{n-1} k + \sum_{k=1}^{n-1} 1\right)$
$U_n = 2 + \left(2\left(\frac{(n-1)n}{2}\right) + (n-1)\right)$
$U_n = 2 + ((n-1)n + (n-1))$
$U_n = 2 + (n^2 - n + n - 1)$
$U_n = 2 + (n^2 - 1)$
$U_n = n^2 + 1$
So, the general formula of the sequences is $U_n = n^2 + 1$

Figure 7. Answer sheet of Subject 3

The problem was solved by Subject 3 through the collection of pertinent information by implementing the formula $d_n = 2n + 1$ as a general pattern of divergence for arithmetic sequences. Subsequently, Subject 3 used the information obtained to formulate a potential conclusion aimed at solving the problem through the application of a simplified formula. Therefore, Subject 3 was regarded as demonstrating an inductive thinking process, whereby general conclusions were derived from specific data or information through the implementation of mathematical formulae.

Discussion

According to the results of the analysis, students in the Department of Mathematics in their fifth semester exhibited a tendency to use all stages of inductive thinking, although not all stages were fully actualized. Of the 21 students, only 10 applied inductive thinking to solve problems (Belcastro, 1966; Rizzuto, 1970). In the context of mathematical problem solving, students often have difficulty understanding the intended purpose of the problem, which impedes their ability to effectively utilize their prior experiences and conceptual ideas (Kholid et al., 2020; 2021). Despite the assertion that inductive thinking is more effective than other approaches, the reality is that students in the Department of Mathematics have a low level of inductive thinking. Some students have been observed making incorrect generalizations or hastily formulating generalizations without sufficient observation (Tóth et al., 2021). This finding suggests a lack of skills in developing and validating patterns more systematically. Students often encounter patterns that are either irrelevant or incorrect, especially when mathematical problems have some potential patterns. This underscores the need for teaching strategies to help students recognize patterns more precisely.

A more innovative learning approach is necessary to improve students' inductive thinking. Technology-based approaches, such as interactive software, have demonstrated effectiveness in assisting students in accurately identifying patterns and systematically validating generalizations (Kholid, Putri et al., 2022). Other innovative approaches, such as the portfolio-based think-pair-share (TPS) strategy, have also been shown to improve students' inductive thinking and mathematics learning outcomes by fostering a collaborative and active learning environment (Kholid et al., 2019). The integration of metacognitive strategies, encompassing planning, monitoring, and evaluation, has been demonstrated to enhance students' comprehension of problems and cultivate systematic and consistent solution strategies (Masduki et al., 2020). This approach aligns with the importance of integrating teaching methods that promote the development of critical and analytical thinking skills in the context of mathematics learning (Sa'dijah et al., 2020). The present study found that students employed inductive thinking methods to solve mathematical problems. These methods include identifying patterns or information, generalizing or drawing conclusions from patterns found, and proving formula (Haverty et al., 2000).

Students are expected to identify patterns and formulate solutions to problems. The present study has yielded that allow for the classification of inductive thinking in mathematical problem solving into three classifications. The first classification, designated as the use of variables, involves the use of variables to facilitate the collection of information, thereby enabling problem solving. In the context of mathematical problem solving, the use of variables is important, as it has been demonstrated to improve understanding and efficiency. These variables function as placeholders, facilitating the understanding and manipulation of algebraic expressions (Moss et al., 2019). The use of variables in lieu of numbers during problem-solving processes has been shown to influence the efficiency of a given strategy, especially among students with advanced algebraic competencies (Chan et al. 2022). Substitution, the primary method involving variables, is effective when simplifying expressions and exploiting symmetry or special properties in equations (Lingefjärd, 2023). However, the effectiveness of this approach depends on the student's algebraic proficiency and the complexity of the problem at hand (Chan et al., 2022). In the context of mathematics education, contents play an important role in helping students express unknown numbers, define functions, and engage in mathematical discourse more effectively (Moss et al., 2019).

The second classification (visual use) involves the process of problem solving through the collection of information through visual representations. Visualization strategies have been shown to improve students' performance in solving mathematical word problems (Clements, 2014). Illustration and visualization interventions improve students' ability to solve problems, select the correct operation, and identify equations (Benson et al., 2023; Kohen et al., 2022). Visual use positively correlates with higher math problem-solving performance (Hegarty & Kozhevnikov, 1999). Using visual strategies when solving math problems can facilitate the demonstration of student understanding (Zulu & Mudaly, 2023). The use of visualization strategies helps students develop problem-solving skills by allowing them to interpret and understand mathematical concepts with greater ease (Kholid, Rofi'ah et al., 2022). Visualization can improve problem-solving utility and performance, although individual and contextual factors can influence its effectiveness (Carden et al., 2015). Visual thinking skills are crucial, as they help in comprehending complex problems, breaking them down into simpler components, and recognizing connections to related issues (Sholihah & Maryono, 2020; Sosa & Aguilar, 2021).

In the third classification (use of formulae), students typically use formulae to obtain the information necessary to solve problems. Mathematical formulae are often considered challenging by students, frequently leading to mathematical anxiety (Ngu et al., 2017; Ramirez et al., 2018; Trigueros et al., 2020), However, they can also be utilized as a method for solving mathematical problems through optimization and simplification. Hosseini et al. (2014) presented a new approach, namely, the use of formulae to solve arithmetic word problems through a process of sentence analysis, variable identification, and equation generation. The use of existing mathematical formulae, their simplification, and the application of optimization techniques have been shown to be beneficial for solving problems efficiently across various domains (Iwane & Anai, 2017). In the context of mathematics education, the implementation of these strategies can improve students' problem-solving abilities and prepare them for more complex mathematical challenges (Anggo et al., 2021). Finally, Ishartono et al. (2022) further posit that students' self-regulated learning is a contributing factor to their problem-solving abilities.

CONCLUSION

This study has identified and elaborated three distinct classifications of inductive thinking strategies employed by students in solving mathematical problems: variable-based, visual-based, and formula-based strategies. These classifications differ in the cognitive pathways through which individuals extract, process, and synthesize information to arrive at a solution. In the variable-based strategy, students utilize symbolic representations (variables) to encode given information, which they then manipulate to infer general patterns or reach conclusions—this approach emerged as the most prevalent among participants. In the visual-based strategy, learners construct or interpret diagrams and other forms of visual representation to support data organization and pattern recognition, thereby enhancing inductive reasoning processes. The formula-based strategy involves applying known mathematical formulae to derive new relationships or simplify expressions to resolve the problem at hand. These findings underscore the multifaceted nature of inductive reasoning in mathematical problem-solving and suggest that students engage in varied yet interconnected modes of reasoning that can be systematically categorized.

Despite its contributions, the study is limited by the relatively small and homogeneous sample, which restricts the generalizability of its findings. A more diverse participant pool with varying educational backgrounds and levels of mathematical proficiency could potentially yield a broader and more nuanced taxonomy of inductive thinking strategies. Nevertheless, the study offers valuable theoretical insights for both educational research and instructional practice. For practitioners, understanding the distinct inductive reasoning strategies may inform more responsive pedagogical approaches that support students' cognitive development in mathematics. For future research, it is recommended to investigate the progression and fragmentation of inductive thinking across different educational stages and problem types. Furthermore, the design and implementation of instructional materials tailored to each identified strategy may enhance the effectiveness of mathematics education and support differentiated instruction aligned with students' cognitive profiles.

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REFERENCES

Adawiyah, R., Muin, A., & Khairunnisa, K. (2017). Mathematical inductive-creative reasoning, a theoretical study. *Proceedings of the 2016 International Conference on Mathematics and Science Education*. https://doi.org/10.2991/icmsed-16.2017.53

Anggo, M., Masi, L., & Haryani, M. (2021). The use of metacognitive strategies in solving mathematical problems. *Journal of Physics: Conference Series*, *1752*(1), 012078. https://doi.org/10.1088/1742-6596/1752/1/012078

Awasthy, R. (2019). Nature of qualitative research. In *Methodological Issues in Management Research: Advances, Challenges, and the Way Ahead* (pp. 145–161). Emerald Group Publishing Ltd. https://doi.org/10.1108/978-1-78973-973-220191010

Barete, M., & Taja-on, E. (2024). Students' perception in learning the course mathematics in the modern world: A qualitative study. *East Asian Journal of Multidisciplinary Research*, *3*(7), 2721-2738. https://doi.org/10.55927/eajmr.v3i7.10071

Belcastro, F. P. (1966). Relative effectiveness of the inductive and deductive methods of programming algebra. *The Journal of Experimental Education*, *34*(3), 77–82. https://doi.org/10.1080/00220973.1966.11010941

Benitez-Correa, C., Gonzalez-Torres, P., Ochoa-Cueva, C., & Vargas-Saritama, A. (2019). A comparison between deductive and inductive approaches for teaching EFL grammar to high school students. *International Journal of Instruction*, *12*(1), 225–236. https://doi.org/10.29333/iji.2019.12115a

Benson, I., Marriott, N., & McCandliss, B. D. (2023). Interventions to improve equational reasoning: Replication and extension of the Cuisenaire-Gattegno curriculum effect. *Frontiers in Psychology*, *14*, 1116555. https://doi.org/10.3389/fpsyg.2023.1116555

Carden, J., & Cline, T. (2015). Problem-solving in mathematics: The significance of visualisation and related working memory. *Educational Psychology in Practice*, *31*(3), 235–246. https://doi.org/10.1080/02667363.2015.1051660

Chan, J. Y. C., Ottmar, E. R., Smith, H., & Closser, A. H. (2022). Variables versus numbers: Effects of symbols and algebraic knowledge on students' problem-solving strategies. *Contemporary Educational Psychology*, *71*, 102114. https://doi.org/10.1016/j.cedpsych.2022.102114

Clements, M. A. (2014). Fifty years of thinking about visualization and visualizing in mathematics education: A historical overview. In *Fifty Years of Thinking About Visualization and Visualizing* (pp. 177–192). https://doi.org/10.1007/978-94-007-7473-5_11

Creswell, J. W. (2012). *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research* (4th ed.). Pearson Education.

Delima, N., Rahmah, M. A., & Akbar, A. (2018). The analysis of students' mathematical thinking based on their mathematics self-concept. *Journal of Physics: Conference Series*, *1108*(1), 012104. https://doi.org/10.1088/1742-6596/1108/1/012104

Hamda, H. (2018). Mathematical problem-solving strategy based on conceptual thinking. *Journal of Physics: Conference Series*, *1028*(1), 012163. https://doi.org/10.1088/1742-6596/1028/1/012163

Haverty, L. A., Koedinger, K. R., Klahr, D., & Alibali, M. W. (2000). Solving inductive reasoning problems in mathematics: Not-so-trivial pursuit. *Cognitive Science*, *24*(2), 249–298. https://doi.org/10.1207/s15516709cog2402_3

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual–spatial representations and mathematical problem solving. *Journal of Educational Psychology*, *91*(4), 684–689. https://doi.org/10.1037/0022-0663.91.4.684

Helviyana, G., Susanti, E., Indaryanti, Sari, N., & Simarmata, R. H. (2020). Students' mathematical reasoning in inquiry learning model. *Journal of Physics: Conference Series*, *1480*(1), 012058. https://doi.org/10.1088/1742-6596/1480/1/012058

Hewitt-Taylor, J. (2001). Use of constant comparative analysis in qualitative research. *Nursing Standard*, *15*(42), 39–42. https://doi.org/10.7748/ns2001.07.15.42.39.c3052

Hosseini, M. J., Hajishirzi, H., Etzioni, O., & Kushman, N. (2014). Learning to solve arithmetic word problems with verb categorization. *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, 523–533. https://doi.org/10.3115/v1/D14-1058

Ishartono, N., Nurcahyo, A., Waluyo, M., Prayitno, H. J., & Hanifah, M. (2022). Integrating GeoGebra into the flipped learning approach to improve students' self-regulated learning during the covid-19 pandemic. *Journal on Mathematics Education*, *13*(1), 69–86. https://doi.org/10.22342/jme.v13i1.pp69-86

Iwane, H., & Anai, H. (2017). Formula simplification for real quantifier elimination using geometric invariance. *Proceedings of the International Symposium on Symbolic and Algebraic Computation, ISSAC*, *Part F129312*, 213–220. https://doi.org/10.1145/3087604.3087627

Jobes, P. C., Aldea, A., Cernat, C., Icolisan, I.-M., Iordache, G., Lazeru, S., Stoica, C., Tibil, G., & Udangiu, E. (1997). Using the constant comparative method in the foreign classroom: Shopping as a social problem in Romania. *Teaching Sociology*, *25*(4), 292–302. https://doi.org/10.2307/1319297

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, *38*(5), 1008–1022. https://doi.org/10.1111/cogs.12107

Kasmer, L., & Kim, O. (2011). Using prediction to promote mathematical understanding and reasoning. *School Science and Mathematics*, *111*(1), 20–33. https://doi.org/10.1111/j.1949-8594.2010.00056.x

Kauts, D. S., & Kauts, A. (2020). Effect of Taba's inductive thinking model on achievement in science and creative thinking in relation to intelligence of students at secondary school stage. *MIER Journal of Educational Studies, Trends & Practices*, *10*(2), 151–166. https://doi.org/10.52634/mier/2020/v10/i2/1347

Khan, S. B. S., & Salman, R. (2020). Influence of mathematics in our daily lives. *Arts & Humanities Open Access Journal*, *4*(2), 50–52. https://doi.org/10.15406/ahoaj.2020.04.00152

Kholid, M. N., Agustin, R. L., & Pradana, L. N. (2019). Effect of TPS strategy with portfolio assessment and learning interest on mathematical learning achievement. *International Journal of Scientific & Technology Research*, *8*(09), 616–620.

Kholid, M. N., Hamida, P. S., Pradana, L. N., & Maharani, S. (2020). Students' critical thinking depends on their cognitive style. *International Journal of Scientific & Technology Research*, *9*(1), 1045–1049.

Kholid, M. N., Imawati, A., Swastika, A., Maharani, S., & Pradana, L. N. (2021). How are students' conceptual understanding for solving mathematical problem? *Journal of Physics: Conference Series*, *1776*(1), 012018. https://doi.org/10.1088/1742-6596/1776/1/012018

Kholid, M. N., Putri, Y. P., Swastika, A., Maharani, S., & Ikram, M. (2022). What are the pupils' challenges in implementing reflective thinking for problem-solving? *The International Conference on Mathematics and Learning Research (ICOMER) 2021*, 020012. https://doi.org/10.1063/5.0099600

Kholid, M. N., Rofi'ah, F., Ishartono, N., Waluyo, M., Maharani, S., Swastika, A., Faiziyah, N., & Sari, C. K. (2022). What are students' difficulties in implementing mathematical literacy skills for solving PISA-like problem? *Journal of Higher Education Theory and Practice*, *22*(2), 180–199. https://doi.org/10.33423/jhetp.v22i2.5057

Kholid, M. N., Sa'dijah, C., Hidayanto, E., & Permadi, H. (2022). Students' reflective thinking pattern changes and characteristics of problem solving. *Reflective Practice*, *23*(3), 319–341. https://doi.org/10.1080/14623943.2021.2025353

Kholid, M. N., Swastika, A., Ishartono, N., Nurcahyo, A., Tin Lam, T., Maharani, S., Ikram, M., Murniasih, T. R., Majid, M., Wijaya, A. P., & Pratiwi, E. (2022). Hierarchy of students' reflective thinking levels in mathematical problem solving. *Acta Scientiae*, *24*(6), 24–59. https://doi.org/10.17648/acta.scientiae.6883

Klauer, K. J., Willmes, K., & Phye, G. D. (2002). Inducing inductive reasoning: Does it transfer to fluid intelligence? *Contemporary Educational Psychology*, *27*(1), 1–25. https://doi.org/10.1006/ceps.2001.1079

Kohen, Z., Amram, M., Dagan, M., & Miranda, T. (2022). Self-efficacy and problem-solving skills in mathematics: The effect of instruction-based dynamic versus static visualization. *Interactive Learning Environments*, *30*(4), 759–778. https://doi.org/10.1080/10494820.2019.1683588

Kurniawati, L., Miftah, R., & Indriani, R. (2021). Improving students' mathematical inductive reasoning ability through reflective learning model. *Journal of Physics: Conference Series*, *1836*(1), 012071. https://doi.org/10.1088/1742-6596/1836/1/012071

Li, Y., Xu, W., Xu, Y., & Liu, B. (2022). Mathematics learning and teaching directed to process construction: Reflection on the application of mathematical induction. *Annals of Mathematical Modeling*, *2*(2), 82–89. https://doi.org/10.33292/amm.v2i2.23

Lingefjärd, T. (2023). A cognitive aspect on substitution in problem-solving in mathematics. *Journal of Mathematics and Science Teacher*, *3*(1), em034. https://doi.org/10.29333/mathsciteacher/13085

Masduki, Kholid, M. N., & Khotimah, R. P. (2020). Exploring students' problem-solving ability and response towards metacognitive strategy in mathematics learning. *Universal Journal of Educational Research*, *8*(8), 3698–3703. https://doi.org/10.13189/ujer.2020.080849

Mistry, K. B. (2012). Research & statistic: Qualitative research methods. *Pediatrics in Review*, *33*(11), 521–523. https://doi.org/10.1542/pir.33-11-521

Miswanto, A., Susanti, E., Hapizah, H., Meryansumayeka, M., & Nurzalena, A. (2019). Analysis of mathematical thinking types reasoning students in completing the problem-solving question. *Journal of Physics: Conference Series*, *1318*(1), 012101. https://doi.org/10.1088/1742-6596/1318/1/012101

Moguel, L. E. S., Landa, E. A., & Cabañas-Sánchez, G. (2020). Fases del razonamiento inductivo que presentan profesores de matematicás al resolver un problema de generalización. *PNA Revista En Didactica de La Matematica*, *14*(2), 118–141. https://doi.org/10.30827/PNA.V14I2.9118

Moss, D. L., Boyce, S., & Lamberg, T. (2019). Representations and conceptions of variables in students' early understandings of functions. *International Electronic Journal of Mathematics Education*, *15*(2), em0564. https://doi.org/10.29333/iejme/6257

Ngu, B. H., & Phan, H. P. (2017). Will learning to solve one-step equations pose a challenge to 8th grade students? *International Journal of Mathematical Education in Science and Technology*, *48*(6), 876–894. https://doi.org/10.1080/0020739X.2017.1293856

Perret, P. (2015). Children's inductive reasoning: Developmental and educational perspectives. *Journal of Cognitive Education and Psychology*, *14*(3), 389–408. https://doi.org/10.1891/1945-8959.14.3.389

Prince, M. J., & Felder, R. M. (2006). Inductive teaching and learning methods: Definitions, comparisons, and research bases. *Journal of Engineering Education*, *95*(2), 123–138. https://doi.org/10.1002/j.2168-9830.2006.tb00884.x

Rahmah, M. A. (2017). Inductive-deductive approach to improve mathematical problem solving for junior high school. *Journal of Physics: Conference Series*, *812*(1), 012089. https://doi.org/10.1088/1742-6596/812/1/012089

Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, *53*(3), 145–164. https://doi.org/10.1080/00461520.2018.1447384

Rani, G., Kumar, P., Devi, R., Kumar, R., Kumar, S., & Kumar, M. (2023). Mathematics as a part of the real life. *International Journal of Advanced Research in Science, Communication and Technology*, *3*(5), 409–418. https://doi.org/10.48175/ijarsct-11665

Rizzuto, M. F. (1970). Experimental comparison of inductive and deductive methods of teaching concepts of language structure. *Journal of Educational Research*, *63*(6), 269–273. https://doi.org/10.1080/00220671.1970.10884008

Sachdeva, S., & Eggen, P.-O. (2021). Learners' critical thinking about learning mathematics. *International Electronic Journal of Mathematics Education*, *16*(3), em0644. https://doi.org/10.29333/iejme/11003

Sa'dijah, C., Kholid, M. N., Hidayanto, E., & Permadi, H. (2020). Reflective thinking characteristics: A study in the proficient mathematics prospective teachers. *Infinity Journal*, *9*(2), 159-172. https://doi.org/10.22460/infinity.v9i2.p159-172

Sanjaya, A., Johar, R., Ikhsan, M., & Khairi, L. (2018). Students' thinking process in solving mathematical problems based on the levels of mathematical ability. *Journal of Physics: Conference Series*, *1088*(1), 012116. https://doi.org/10.1088/1742-6596/1088/1/012116

Schiemer, G. (2019). Mathematik in den Wissenschaften. In *Einheit und Vielfalt in den Wissenschaften* (pp. 38–68). De Gruyter. https://doi.org/10.1515/9783110614831-003

Sholihah, U., & Maryono, M. (2020). Students' visual thinking ability in solving the integral problem. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *5*(2), 175–186. https://doi.org/10.23917/jramathedu.v5i2.10286

Sosa, J. J. C., & Aguilar, F. K. M. (2021). Chess, visual memory and geometric transformations. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *6*(4), 299–315. https://doi.org/10.23917/jramathedu.v6i4.14269

Supandi, S., Suyitno, H., Sukestiyarno, Y. L., & Dwijanto, D. (2019). Adaptation and creativity in mathematics learning. *Journal of Physics: Conference Series*, *1321*(2), 022132. https://doi.org/10.1088/1742-6596/1321/2/022132

Sutarni, S., Sutama, Prayitno, H. J., Sutopo, A., & Laksmiwati, P. A. (2024). The development of realistic mathematics education-based student worksheets to enhance higher-order thinking skills and mathematical ability. *Infinity Journal*, *13*(2), 285–300. https://doi.org/10.22460/infinity.v13i2.p285-300

Tan, L. G. (2023). Real-life valuation and re-configuring instructional approach of learning mathematics among college non-math-oriented students. *Journal of Namibian Studies: History Politics Culture*, *33*, 1745–1765. https://doi.org/10.59670/jns.v33i.3165

Tóth, P., Horváth, K., & Kéri, K. (2021). Development level of engineering students' inductive thinking. *Acta Polytechnica Hungarica*, *18*(5), 107–129. https://doi.org/10.12700/APH.18.5.2021.5.8

Trigueros, R., Aguilar-Parra, J. M., Mercader, I., Fernández-Campoy, J. M., & Carrión, J. (2020). Set the controls for the heart of the maths: The protective factor of resilience in the face of mathematical anxiety. *Mathematics*, *8*(10), 1–11. https://doi.org/10.3390/math8101660

Vo, D. Van, & Csapó, B. (2022). Measuring inductive reasoning in school contexts: A review of instruments and predictors. *International Journal of Innovation and Learning*, *31*(4), 506. https://doi.org/10.1504/IJIL.2022.10046982

Vos, P., Wiik, A., & Hernandez-Martinez, P. (2024). "Imagine, maths is used anywhere, and we don't get to know this"—upper secondary students and the relevance of advanced mathematics. *Frontiers in Education, 9*, 1338205. https://doi.org/10.3389/feduc.2024.1338205

Wardani, S., Kusuma, I. W., Liu, S. T., & Harjito. (2020). Comparison of learning in inductive and deductive approach to increase student's conceptual understanding based on international standard curriculum. *Jurnal Pendidikan IPA Indonesia*, *9*(1), 70–78. https://doi.org/10.15294/jpii.v9i1.21155

Wright, D. P. (1977). Interactions between instructional methods and styles of concept learning. *Journal of Educational Research*, *70*(3), 150–156. https://doi.org/10.1080/00220671.1977.10884973

Zulu, M. W., & Mudaly, V. (2023). Unveiling problem-solving strategies of pre-service mathematics teachers: A visual and discursive exploration. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(7), em2299. https://doi.org/10.29333/ejmste/13344