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Investigation of the impact online single and multiple representation scaffolding on mathematical concept mastery and mathematical problem-solving skill

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Abstract

Representation plays a central role in mathematical problem solving, serving as a cognitive bridge between abstract concepts and concrete understanding. However, while prior studies have examined the effects of scaffolding in mathematics learning, limited attention has been given to the comparative impact of single versus multiple online representations, particularly in relation to students' cognitive processes such as eye movement behavior. This study addresses this gap by investigating the effectiveness of online single and multiple representation scaffolding in enhancing students' mathematical concept mastery, problem-solving performance, and eye movement patterns during problem-solving tasks. A quasi-experimental design was employed involving 300 high school students, randomly assigned to either a multiple representation scaffolding group (n = 150) or a single representation scaffolding group (n = 150). Data were analyzed using one-way MANCOVA, ANCOVA, MANOVA, and ANOVA tests. The results revealed that students who received multiple representation scaffolding outperformed their peers in mastering mathematical concepts, solving complex problems—including advanced-level tasks—and demonstrating more efficient visual processing, indicated by shorter fixation durations and rereading times. Furthermore, these students exhibited more adaptive strategies across varying question types (basic, combination, and advanced). The findings highlight the pedagogical advantage of using multiple representation scaffolding in online mathematics instruction, suggesting that it offers more comprehensive cognitive support and promotes deeper conceptual understanding. This study contributes to the growing body of research on digital scaffolding by evidencing the cognitive and performance-related benefits of multimodal representation and underscores its potential to inform the design of technology-enhanced mathematics learning environments.

Keywords: Mathematical Conception, Mathematical Problem Solving, Multiple Representation Scaffolding, Online Scaffolding, Single Representation Scaffolding

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The ability to solve mathematical problems is a central objective in mathematics education, as it contributes significantly to the development of students' conceptual understanding. Problem-solving practices form the core of mathematical competencies, facilitating the construction of new mathematical knowledge and enabling students to address problems in diverse contexts (Capone et al., 2021; Doruk & Doruk, 2022). As a hallmark of mathematical activity, problem solving not only fosters higher-order thinking but also serves as a vehicle for conceptual growth (Faulkner et al., 2023; Suwarto et al., 2023).

Recent empirical findings emphasize the benefits of integrating online dynamic geometry environments in mathematics instruction. Such environments have been shown to enhance students' problem-solving abilities and conceptual understanding more effectively than traditional classroom methods (Hinton & Flores, 2019; Smith & Mancy, 2018). In particular, geometry—characterized by the interplay between figure-based (spatial and perceptual) and conceptual (abstract and formal) features—offers fertile ground for exploring visual reasoning and conceptual development (Tinungki et al., 2024).

In the process of solving geometry problems, students will use the knowledge they already have. Diagrams can be used to make it easier for students to understand information for problem solving. Other elements that can be used to convey information in mathematics are lines and spatial. Students who have skills in these areas will be better able to use visual representations as additional important information in the process of solving mathematical problems (Smith & Mancy, 2018; Toikka et al., 2024). So, in this study, researchers use important features in geometry which include diagrammatic shapes and concepts that can create interactions between shapes and concepts in the learning process.

Representation is also interpreted as the center of mathematical problem solving. Some researchers argue that representation has the function of organizing, structuring, and conveying mathematical concepts (Tesfaw et al., 2024; Tinungki et al., 2024). In addition, representation can also be used to select, apply, and interpret problems so that it is easier for students to demonstrate and interpret mathematical phenomena or problems. In this study, representation is a configuration that represents a concept. Representation is also determined by media that depicts external reality or real subjects (Capone, 2022; Toikka et al., 2024). So, the term representation means the process, product, and action to understand mathematical concepts through various forms.

A growing body of research has demonstrated that the use of representations can enhance the quality of mathematics instruction and foster students' mathematical problem-solving abilities (Hidajat, 2024; Leitner & Gabel, 2024). However, relying solely on a single form of representation may constrain the presentation and comprehension of mathematical information. To address this limitation, scholars have advocated for the integration of multiple representations, arguing that such an approach is more effective for conveying complex mathematical ideas. The use of multiple representations can support students in developing deeper conceptual understanding, facilitate more effective integration of information, and promote insight into the relationships among mathematical concepts (Chinofunga et al., 2025; Zhang et al., 2024).

Multiple representations are often operationalized within multimedia learning environments that combine various representational formats, including text, graphics, diagrams, tables, and audio, in an integrated and flexible manner. Research into the cognitive mechanisms underlying the interaction between external and internal representations has gained increasing attention in recent years (Paivio, 1986). Dual coding theory, which posits the existence of two distinct but interconnected cognitive subsystems—one for processing verbal or linguistic information and the other for processing nonverbal or visual information—offers a theoretical foundation for understanding how learners process mathematical representations (Herold-Blasius, 2024; Martinez et al., 2021). The coordination of these two subsystems is believed to enhance learning outcomes through the complementary integration of verbal and visual inputs. This perspective aligns with Mayer's cognitive theory of multimedia learning (Mayer & Moreno, 2003), which emphasizes the dual-channel nature of human information processing—specifically, the separate processing of visual and auditory information. Despite the advantages of multimedia and multiple representations, some studies have also highlighted that students may encounter difficulties in interpreting and transitioning between representational forms, particularly when required to move beyond static visual representations to construct meaning and solve problems.

The integration of textual information with spatially related visuals has been shown to more effectively support students in mathematical problem solving. Empirical findings suggest that students demonstrate higher academic performance when instructional materials are presented in audiovisual formats, as opposed to visual-only formats. These results lend support to Ainsworth's (2006) framework, which posits that multiple external representations can operate synergistically to enhance the construction of conceptual understanding. The interaction between modalities can foster the development of coherent internal propositional representations and mental models. Such models are constructed and refined through iterative processes of model generation and inspection, enabling learners to integrate and comprehend information from text, images, and diagrams (Fitzsimons & Ní Fhloinn, 2023; Sidenvall et al., 2022).

Building on these theoretical perspectives and empirical findings, it can be inferred that the use of multiple representations facilitates the formation of complementary internal models that support deeper understanding. However, there is a notable gap in the existing literature: few, if any, empirical studies have systematically compared the effectiveness of single versus dual representations in enhancing mathematical problem-solving skills. This study aims to address that gap by investigating the differential impact of these representational formats.

In addition to cognitive theories of multimedia learning, this study also draws on Vygotsky's (1978) sociocultural theory, particularly the concept of the Zone of Proximal Development (ZPD). Within this framework, instructional scaffolding plays a critical role in facilitating learners' progression from their actual level of development to their potential level. Scaffolding—defined as the tailored support provided by more knowledgeable others—enables students to construct new knowledge and engage more fully in the learning process (Haataja et al., 2025; Smit et al., 2023). The application of online scaffolding that aligns with the decomposition of problem-solving tasks has been found to significantly improve students' mathematical performance. Specifically, research has shown that the use of scaffolding media is more effective in supporting students' understanding of geometric concepts compared to conditions without such support (Brandsæter & Berge, 2025; Pereda Loriente et al., 2025). Furthermore, in this study, online scaffolding will be designed to support students' movement from actual to potential levels of problem-solving ability, particularly in the context of geometric problem solving. The scaffolding will serve as a step-by-step guide, aimed at fostering the development of students' internal mental models. Each stage of scaffolding builds upon the previous one, thereby enabling students to incrementally advance toward more complex levels of understanding and performance.

Eye movement analysis is a widely used method for investigating learners' cognitive processes. According to the mind-eye hypothesis, there is a strong correspondence between gaze direction and cognitive activity, suggesting that where individuals look reflects what they are thinking about (Nückles, 2021; Rohde Poole, 2022). Moreover, fixation duration serves as an important indicator of cognitive processing depth, with longer fixations generally associated with more comprehensive data processing.

Empirical studies have demonstrated that multimedia environments can influence cognitive engagement through processes such as mitosis and meliosis—terms used to describe phases of conceptual differentiation and integration during learning. For instance, students exposed to multimedia learning materials combining visual and auditory elements (e.g., narrative explanations) tend to allocate more attentional resources to visual components. This is reflected in greater total fixation duration, inspection time, and average fixation compared to students receiving animation accompanied by written text (Geraniou et al., 2024; Kohen, 2025). These visual-narrative interventions are also associated with improved academic outcomes in science learning.

Additional research has shown that design features such as color coding can further enhance cognitive processing by affecting the average fixation duration during information encoding (Peretz et al., 2023; Siller et al., 2023). Students with higher academic achievement often exhibit greater cognitive effort, demonstrated by increased integration of visual elements and more frequent rereading of scientific texts (Rezat et al., 2022; Vogel et al., 2022). Furthermore, when learners are presented with visual cues that explicitly direct attention to functional relationships across spatial and temporal dimensions, they exhibit improved fixation patterns and achieve higher performance outcomes. These findings suggest that attentional guidance through visual design can enhance both visual processing and conceptual understanding in multimedia learning contexts.

Eye movement data have been shown to be reliable predictors of students' performance in computer-based assessments, particularly in science and mathematics domains. For example, studies have demonstrated that average fixation duration is a significant predictor of the likelihood of a correct response, with rereading time also contributing to performance prediction (Borchers et al., 2025; Iwuanyanwu & Ogunniyi, 2020). In the context of multimedia learning, eye-tracking research has revealed that the use of static versus dynamic 3D representations influences the construction of students' mental models. Learners exposed to dynamic 3D visualizations tend to develop more complex and sophisticated representations of molecular hybridization compared to those using static 3D models (Bliss et al., 2019; Rezat et al., 2022).

Building on these findings, the present study employs eye-tracking techniques to explore how students process single and multiple representational scaffolds in an online learning environment aimed at improving mathematical problem-solving skills. This research seeks to extend the existing literature in several keyways. First, it investigates the use of representational scaffolding in online formats, contrasting single with multiple representations. Second, it examines the combined impact of these scaffolds on three interrelated competencies: conceptual understanding of mathematics, problem-solving ability, and cognitive processing as indicated by eye movement metrics during problem-solving tasks.

Despite increasing interest in representation-based instructional support, empirical studies exploring how single and multiple forms of scaffolding affect students' mathematical understanding and performance—particularly using real-time eye-tracking data—remain limited. Thus, this study is guided by the following research questions:

1. How do single and multiple online scaffolds affect students' conceptual understanding of mathematics?
2. How do single and multiple representational online scaffolds influence students' mathematical problem-solving performance?
3. How do single and multiple representational online scaffolds influence students' eye movement patterns, including total fixations, inspection time, average fixation duration, and rereading behavior when solving mathematical problems?

METHODS

Design and Participants

The research method used in this study was a quasi-experimental involving 300 high school students' class 10-12 who were randomly selected and divided into two groups. The first group was students who received an online single representational scaffolding intervention, and the second group was an online double representational scaffolding intervention. Each student consisted of 150 students. The subjects focused on online geometry problem solving designed with two-unit theorems and the application of a triangle's inner and exterior angle sum. The class was conducted in 8 periods with each period of 50 minutes.

Instruments

Mathematical Concept Mastery Assessment

The purpose of the mathematical concept test was to assess students' understanding of geometric concepts related to triangles, specifically focusing on two core topics: the sum of interior angles (Figure 1) and the sum of exterior angles of a triangle (Figure 2). The test was developed collaboratively by a panel of five experts, comprising two experienced secondary school mathematics teachers and three subject-matter specialists in mathematics education.

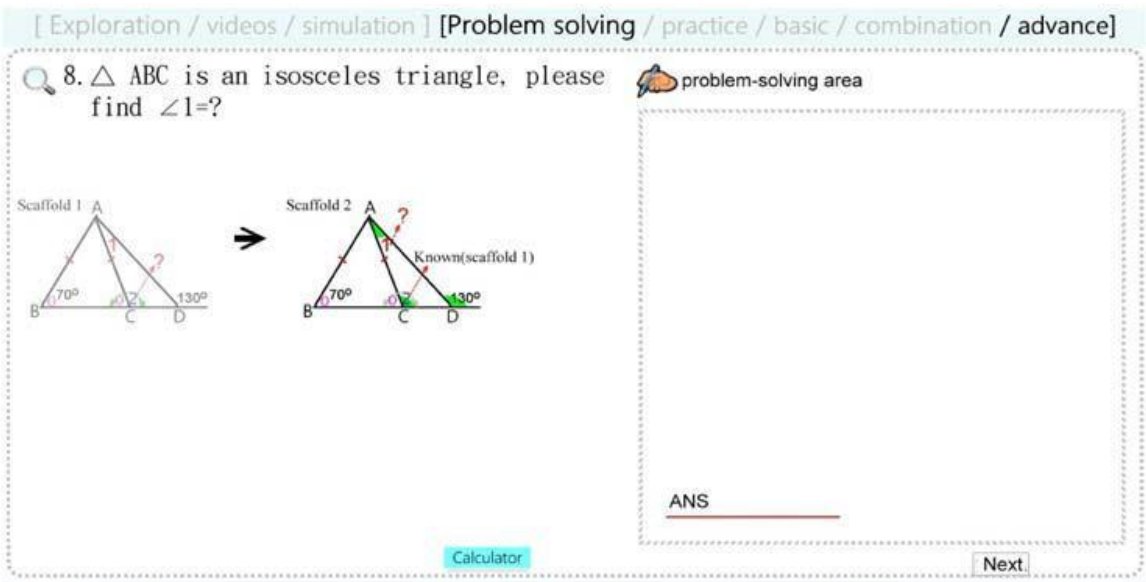

Figure 1. Screenshot of online single representational scaffolding

The instrument consisted of multiple-choice items, with 32 questions per unit, yielding a maximum total score of 30 points. The same panel of experts also evaluated the content validity of the test to ensure alignment with the intended learning objectives. Reliability analysis indicated high internal consistency, with a Cronbach's alpha coefficient of 0.88 for both the pretest and posttest.

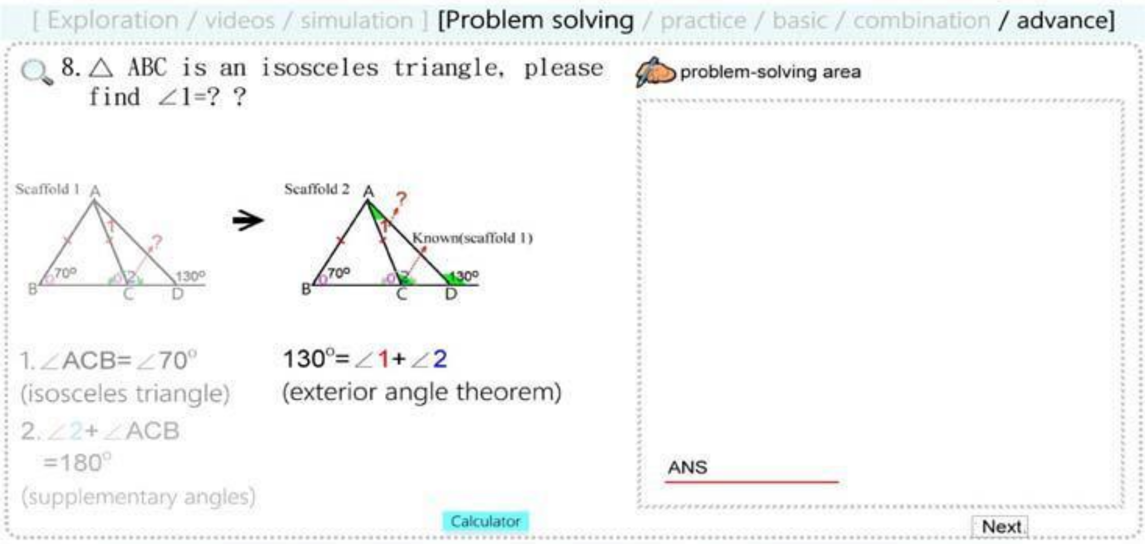

Figure 2. Screenshot of online dual representational scaffolding

Problem Solving Test based on Mathematical Concepts

The problem-solving test was designed to evaluate students' ability to solve problems requiring mastery of geometric concepts related to the interior and exterior angles of triangles. The instrument comprised 20 items, equally distributed across two conceptual domains: the sum of interior angles and the sum of exterior angles of triangles. Each item was scored dichotomously, with one point awarded for a correct response and zero points for an incorrect response, resulting in a maximum possible score of 20.

To ensure the validity of the instrument, Pearson's product-moment correlation analysis was conducted for each item, yielding an average correlation coefficient of 0.89, indicating strong item validity. Instrument reliability was assessed using the split-half method, which evaluates the internal consistency of the test by examining the linear correlation between two halves of the instrument. The reliability analysis produced a coefficient of 0.99, demonstrating a very high level of consistency. Based on these findings, the problem-solving instrument was deemed valid and reliable for use in this study.

Process of Online Mathematics Problem-Solving

The instrument used to assess students' online mathematical problem-solving processes focused on tasks involving the sum of interior and exterior angles of triangles. The test included three categories of problem-solving questions: five basic-level questions, eight combination-level questions, and eight advanced-level questions, all designed to capture varying levels of cognitive demand. Each item was scored on a four-point rubric: 3 points were awarded for a correct answer with an appropriate procedure, 2 points for an incorrect answer with a correct procedure, 1 point for a correct answer with an incorrect procedure, and 0 points for both an incorrect answer and incorrect procedure. The maximum possible score on the instrument was 63 points.

To evaluate the validity of the instrument, Pearson's product-moment correlation analysis was conducted, yielding an average item correlation coefficient of 0.87. Reliability was assessed using the split-half method, which produced a coefficient of 0.99, indicating excellent internal consistency. These results suggest that the instrument is both valid and reliable for assessing students' mathematical problem-solving processes in an online learning environment.

Intervention Design for Online Mathematics Problem Solving using Single and Multiple Representation Scaffolds

This study developed an online intervention aimed at enhancing students' problem-solving abilities in geometry, specifically focusing on the concepts of interior and exterior angles of triangles. The intervention was designed using two types of representational scaffolds: single representation (visual only) and multiple representations (combined visual and textual). The content was structured into two instructional units. The first unit focused on theoretical understanding, conceptual knowledge, and problem-solving strategies related to the sum of the interior angles of a triangle. The second unit addressed similar aspects concerning the sum of the exterior angles of a triangle.

The intervention was delivered via tablet devices equipped with digital pens, enabling students to write formulas, draw geometric shapes, and perform calculations interactively. The entire problem-solving process was displayed on a computer screen, which also recorded students' written responses and calculation behaviors in real time. The instructional design of the intervention was collaboratively developed by a team consisting of one science educator and two secondary school mathematics teachers, ensuring alignment with curriculum standards and classroom practices.

Furthermore, the intervention was implemented in two phases. In the first phase, students were introduced to the foundational concepts and theorems of interior and exterior angles through videos and interactive simulations to build conceptual understanding. In the second phase, students engaged in problem-solving tasks organized into three levels of complexity: basic, combination, and advanced problems. Participants were assigned to different experimental conditions based on the type of scaffold provided. Students in the single representation condition received visual-only support in the form of diagrams displaying geometric elements such as signs, angles, and symbols. In contrast, students in the multiple representation condition received both visual and textual scaffolds, including diagrams, calculation prompts, and explanatory text designed to guide problem-solving at each stage. This design aimed to examine how different types of representational scaffolds influence students' cognitive engagement and mathematical performance in an online learning environment.

Procedure

Before receiving the intervention, students completed a series of assessments, including a pretest, posttest, and retention test, to evaluate their understanding of mathematical concepts and problem-solving abilities. These assessments focused on online geometry content related to triangles, specifically targeting students' ability to apply their knowledge of interior and exterior angles. During problem-solving sessions, students engaged in tasks that involved drawing diagrams, writing formulas, and performing calculations by hand on a tablet device. All written and drawn activities were recorded and later analyzed by trained mathematics teachers. The scoring process demonstrated a high degree of consistency, with an inter-rater reliability coefficient of 0.99 across all samples.

To further investigate students' cognitive processing during problem solving, an Eye Tribe eye tracker was employed to monitor and record students' eye movements as they worked through mathematical tasks. A visual representation of the Eye Tribe setup is provided in Figure 3. Eye movement data were analyzed using key metrics, including total number of fixations, inspection time, average fixation duration, and rereading time. Rereading time was defined as the time spent revisiting a previously fixated location, and average rereading time represented the mean duration of these revisits. The Eye Tribe device captured data at a sampling rate of 30 Hz. Finally, the intervention was delivered individually on computer-based platforms over ten instructional sessions. Each session involved the use of an Eye

Tribe tracker to continuously monitor and collect data on students' learning behaviors and visual attention patterns throughout the problem-solving process. This design enabled a comprehensive analysis of both performance and cognitive engagement during the intervention.

Figure 3. An Eye Tribe tracker

Data Analysis

Before further analysis is carried out, preliminary tests for normality and homogeneity were performed to ensure that the data met the assumptions required for parametric statistical procedures. The results confirmed that the data were normally distributed and homogeneous, allowing for further analysis to proceed. The selection of statistical tests in this study was guided by the research questions and the nature of the dependent variables, which included conceptual mastery, problem-solving performance, and eye movement behavior.

A MANCOVA was used to examine the overall impact of the intervention—comparing single versus multiple online representational scaffolds—on students' mastery of triangle geometry concepts and their related problem-solving abilities, while controlling for covariates. To explore these outcomes in greater detail, an ANCOVA was conducted as a follow-up to determine whether there were statistically significant differences between the two intervention groups specifically in terms of conceptual understanding of triangle geometry. Furthermore, a MANOVA was employed to assess the effects of the intervention on students' performance in specific content areas, namely the addition of interior and exterior angles of triangles in online learning tasks. Additionally, an ANOVA was used to evaluate the impact of the intervention on students' problem-solving performance across three levels of task complexity: basic, combination, and advanced problems. Finally, another one-way ANOVA was conducted to investigate the effect of the intervention on students' eye movement behavior, as measured by metrics such as total fixations, inspection time, average fixation duration, and rereading time.

RESULTS AND DSCUSSION

Mathematical Conception Test

Based on the results of the descriptive statistical analysis of the mathematical conception test, it was found that the group of students who received online dual representation intervention showed better scores than students who received online single scaffold intervention on the posttest with a score of

(27.82 vs. 26.61) and the results of the retention test showed a score of (28.52 vs. 28.03). In addition, based on the results of the t-test, it was found that students with dual and single representational scaffolds showed almost the same significant increase in the mathematical conception test from the pretest to the posttest phase with a score of (T single scaffold = 9.70, $p < 0.001$; T scaffold of multiple = 12.19, $p < 0.001$). Improvements were also seen in the results of the retention test with a score of (T scaffold of single = 3.02; T multiple scaffold = 2.10, $p < 0.05$, $p < 0.001$). Furthermore, to determine the impact of two types of single and double representational scaffolding on the performance of the mathematical concept test, a MANCOVA test was conducted. The posttest score and retention score in the analysis were used as independent variables and the pretest score was used as a covariate. Based on the results of the one-factor MANCOVA test, it was found that double representational scaffolding had important effect on performance in mastering mathematical concepts on the posttest and retention with a value (Wilk's Λ value= 0.942, $p < 0.05$) presented in Table 1. Based on the results of the MANCOVA test and the post hoc test, it was found that students with double representational scaffolding showed a more significant increase than the performance of mathematical conceptions who received online single representational scaffolding intervention with a value ($F = 5.87$, $p < 0.05$).

Table 1. Results of the one-way MANCOVA test of the triangle geometry conception test and the problem-solving ability test based on the concept

Tests multivariate	Value of Wilk's Λ	df	F Value	Sig	Partial eta
Test of the concept of triangle geometric					
Value of intercept	0.163	2	263.31***	0.000	0.861
Pretest	0.540	2	40.52***	0.000	0.481
Group of instructional	0.942	2	4.31*	0.046	0.070
Test of the problem solving about triangle geometry					
Value of Intercept	0.070	2	594.95***	0.000	0.941
Pretest	0.640	2	25.72***	0.000	0.372
Instructional Group	0.940	2	2.89	0.060	0.065

*p < 0.05, **p < 0.01, ***p < 0.001, SS: Single Scaffold, MS: Multiple Scaffold

Problem-Solving Ability Test based on Mathematical Concepts

The results of descriptive statistical analysis on mastery of mathematical concepts showed that students who received the double representational scaffolding group had superior scores than the group of students who received single scaffolding in the posttest phase with scores (17.10 vs. 16.40) and retention tests (17.82 vs. 17.20). The results of the t-test on problem-solving ability based on mathematical concepts showed a significant increase in both groups from the pretest to the posttest phase with a value (single scaffolding; = 9.21, $p < 0.001$, multiple scaffolding 13.30, $p < 0.001$ and retention test (single scaffolding 2.61, $p < 0.05$; and multiple scaffolding 2.89, $p < 0.01$). Furthermore, the results of the one-factor MANCOVA test were conducted to determine the impact of scaffolding on problem-solving ability based on mathematical concepts. The posttest and retention values on this problem-solving ability were used as independent variables, the pretest value was used as a covariate. Based on the results of the analysis, it was found that the type of representational scaffolding that varied had a significant impact on problem-solving ability based on mathematical concepts with a value (Wilk's Λ = 0.940, $p < 0.060$).

Advanced ANCOVA analysis and post hoc test were conducted to determine differences in detail. Based on the results of the analysis, it was found that the group that received dual representational scaffolding intervention showed significantly better performance than students who received single scaffolding intervention in the posttest phase with a value (F = 5.92, p <0.05). The results of the analysis are presented in Table 2. The process of solving online mathematical problems is carried out using various types of questions, ranging from basic, combination, and advanced questions.

Table 2. Results of the one-way ANCOVA test of the mathematical conception test and the problem-solving ability test

	SS group			MS group				Sig	Test of post-hoc
	N	Mean	S.D.	N	Mean	S.D.	F		
Test of the concept of triangle geometric									
Post-test	150	26.61	6.61	150	27.82	4.15	6.89*	0.017	MS > SS
Retention-test	150	28.03	5.63	150	28.50	3.78	1.16	0.290	
Post-test	150	16.41	4.67	150	17.10	3.14	6.91*	0.019	MS > SS
Retention-test	150	17.20	4.21	150	17.82	1.80	4.78	0.062	

*p < 0.05, **p < 0.01, ***p < 0.001., SS: Single Scaffold, MS: Multiple Scaffold

Students' abilities in solving online mathematical problems with the topic of angles in triangles are grouped into several levels including levels 0, 1, 2, and 3 for each question. The results of the analysis showed that the group of students who received dual representational scaffolding intervention showed a upper percentage of ability to solve level 3 questions than level 0 questions. This finding was reversed in the group of students who received intervention with single representational scaffolding which was dominated by level 0 questions. So, this dual representational scaffolding is able to facilitate students to solve problems up to the difficulties level of level 3 contrasted to the group of students who received single representational scaffolding intervention.

Next, a MANOVA test was managed to investigate the impact of different interventions on online problem-solving skills in mathematics on each type of basic to advanced questions. The results of the MANOVA test are presented in Table 3. Based on the results of the MANOVA test, it was found that both types of single and double scaffolds had a positive impact on online problem-solving ability with a value (Wilk's Λ = 0.820, F = 6.63, p <0.001). Furthermore, based on the results of the one-factor ANOVA test and the post-hoc test, it was found that this double representational scaffold gives more significant influence on problem-solving ability in the advanced question aspect than the single representational scaffold with a value (F = 16.30, p <0.001).

Table 3. Results of the one-way MANOVA test of online problem-solving ability on triangular geometry material

Tests of multivariate	Value of Wilk's Λ	df	F	Sig value	Value of partial eta
Online questions about angles in triangles					
Intercept	0.053	3	660.72**	0.000	0.960
Group of instructional	0.820	3	6.63**	0.001	0.192
Online questions about exterior angles of triangles					
Intercept	0.062	3	462.14***	0.000	0.952
Group of Instructional	0.760	3	9.32***	0.000	0.352

*p < 0.05, **p < 0.01, ***p < 0.001, SS: Single Scaffold, MS: Multiple Scaffold

The percentage of mathematical problem-solving levels in the double representational scaffolding group was the highest at level 3 and the lowest at level 0. This percentage is higher than the percentage of problem-solving levels in the single scaffolding group which is dominated by levels 1 and 2. This double representational scaffolding is able to facilitate students to solve problems to the highest level, while single scaffolding is only able to facilitate up to level 2.

The results of the one-factor MANOVA analysis are presented in Table 3 which stated that both types of scaffolding (single and double representational) have a significant impact on students' online problem-solving abilities with a value (Wilk's Λ Value = 0.760, F = 0.32, p < .001). The results of the ANCOVA analysis and post hoc test showed that the group of students who received double representation scaffolding showed a more significant increase in mathematical solve problem abilities than the group of students with single representation scaffolding in the aspect of advanced questions with a value (F = 16.30, p < .001). The results of the analysis are presented in Table 4.

Table 4. Results of one-way ANOVA test of online problem-solving ability on triangular geometry material.

	Single scaffold group			Multiple scaffold group			F	Sig value	Test of post-hoc
	N	M	SD	N	M	SD			
Basic	150	13.30	4.52	150	14.62	4.10	3.42	0.084	
Combination	150	22.91	7.90	150	25.05	6.72	3.92	0.096	
Advance	150	27.60	11.61	150	34.10	4.78	16.50***	0.000	MS > SS
Basic	150	12.42	5.40	150	13.08	4.92	0.78	0.391	
Combination	150	16.82	10.12	150	20.89	7.34	7.80*	0.012	MS > SS
Advance	150	17.30	8.60	150	23.45	3.42	28.89***	0.000	MS > SS

SS: Single Scaffold, MS: Multiple Scaffold

Eye Movement Behavior in Online Math Problem Solving

Eye movement behavior includes (total of fixations, review time, average fixation duration, and average rereading time) in solving online math problems on each type of question starting from essential, mixture, along with more advanced questions regarding the subject of interior and exterior angles of triangles. ANOVA test was used to assess the impact of the two interventions on students' eye movement behavior. The results of the ANOVA analysis are presented in Table 5. From the results of the analysis, the single representation student group used slightly more inspection time than the dual representation group. The most significant difference in time usage was found only in combination questions with the topic of exterior angles with a value (F = 7.52, p < 0.013).

Furthermore, in terms of the total of fixations, the single representation student group used slightly more fixations than the dual representation student group in the topic of interior angles. A significant difference in the use of the number of fixations only occurred in basic questions on the topic of interior angles with a value (F Value = 5.10, p < 0.027). The students using single representation scaffolding used significantly more time on average than the dual representation with value (F Value= 4.70, p value < 0.05; F Value = 4.97, p < 0.05; F = 5.30, p < 0.05). This also occurred in the average rereading time of the single representation group using more time than the dual group with value on the topic of deep corners in all types of questions (F Value = 4.23, p < 0.05; F Value = 4.73, p < 0.05; F Value = 5.82, p < 0.05).

It was also found that the scaffolding of single group used a greater average fixation duration than the dual scaffolding group with value (F = 6.52, p < 0.05) and was also found in the time used to reread with value (F = 5.91, p < 0.05) on the combination question with the topic of exterior angles. The findings of the one-way ANOVA analysis on students' eye movements during the problem-solving process are presented in Table 5.

Table 5. Results of the one-way ANOVA test on eye movement behavior

	Single scaffold group			Multiple scaffold group				Value	Test of post-hoc
	N	M	SD	N	M	SD	F	sig	
Fixation Total									
Total of interior angle of triangle									
Essential	150	24.31	14.05	150	31.52	15.89	6.08*	0.030	MS > SS
Combination	150	43.92	20.27	150	47.61	17.51	0.90	0.362	
Advance	150	40.32	20.81	150	44.52	16.42	1.32	0.382	
Sum of exterior angle of triangle									
Essential	42	30.72	15.15	150	32.87	16.89	0.50	0.502	
Combination	150	42.31	16.40	150	35.05	15.72	4.56	0.072	
Advance	150	45.71	21.91	150	40.08	19.50	1.82	0.189	
Inspection time total									
Total of interior angle of triangle									
Essential	150	5721	4587	150	6862	4382	1.72	0.312	
Combination	150	12,812	6810	150	12,148	5723	0.19	0.783	
Advance	150	11,253	6230	150	9984	5034	0.06	0.931	
Total of exterior angle of triangle									
Essential	150	8760	4952	150	8740	4762	0.00	0.987	
Combination	150	12,089	6486	150	9089	5424	6.52*	0.014	SS > MS
Advance	150	13,491	8720	150	8578	6845	4.52	0.070	
Average fixation duration									
Sum of angles in a triangle									
Essential	150	242.70	56.78	150	214.50	47.72	4.70*	0.042	SS > MS
Combination	150	273.72	67.30	150	230.95	67.32	4.97*	0.030	SS > MS
Advance	150	264.62	72.50	150	232.89	59.82	5.30*	0.025	SS > MS
Total of exterior angle of triangle									
Essential	150	250.62	70.72	150	230.42	56.89	3.09	0.162	
Combination	150	273.78	82.82	150	231.38	60.73	5.72*	0.035	SS > MS
Advance	150	274.09	102.34	150	242.81	71.62	2.71	0.107	
Average reread time									
Sum of angles in a triangle									
Essential	150	263.61	62.51	150	235.22	53.18	4.23*	0.051	SS > MS
Combination	150	284.21	68.71	150	253.50	65.67	4.73*	0.042	SS > MS
Advance	150	273.34	82.62	150	230.62	68.09	5.82*	0.020	SS > MS
Sum of exterior angle of triangle									
Essential	150	270.17	90.42	150	241.09	61.23	4.72	0.073	
Combination	150	284.30	88.62	150	252.92	62.89	6.89*	0.019	SS > MS
Advance	150	283.08	115.2	150	256.14	71.09	4.60	0.070	

*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$. SS: Single Scaffold, MS: Multiple Scaffold

As shown in Table 5, students who received single representation scaffolding exhibited a significantly longer overall inspection time compared to those who received multiple (dual) representation scaffolding. The most pronounced differences between the groups emerged in tasks related to the exterior angles of triangles. Tables 5 and 6 summarize the detailed findings of the eye-tracking analysis. In the multiple representation condition, students allocated between 63% and 75% of their total inspection time to visual components (images), while the remaining 25% to 39% was directed toward textual information.

Table 6 further highlights that across all three levels of question complexity—basic, combination, and advanced—the single scaffolding group consistently spent more time inspecting images than the multiple scaffolding group. Specifically, students in the single representation condition dedicated between 130% and 220% more time to image inspection compared to their counterparts in the multiple representation group. This substantial difference suggests that the absence of supporting textual scaffolds may lead students to rely more heavily on visual representations, resulting in longer processing times when interpreting diagrams and visual cues.

Table 6. Results of the analysis of the amount of inspection time for images and texts in both scaffolding groups

	Single scaffold group	**Multiple scaffold group**				**SRS group/MS group image**
	Image	**Image**	**Text**	**Image + text**	**Image/picture + text**	
Total of inspection time of interior angles of triangle						
Basic	5723	4392	2572	6842	64%	142%
Combination	12,807	8340	2812	12,140	75%	153%
Advance	11,257	6960	3116	9982	70%	149%
Exterior angles of a triangle						
Basic	7860	5512	2343	7740	72%	143%
Combination	12,089	5023	3192	9089	63%	234%
Advance	13,491	6280	3420	9578	66%	202%

The findings of this study provide strong empirical evidence for the significant impact of online scaffolding on enhancing students' mathematical problem-solving abilities. The results underscore the critical role of scaffolding in supporting students' learning, both in terms of conceptual understanding and procedural problem solving. Irrespective of the type of scaffolding implemented, the interventions proved effective in improving students' mathematical performance (Huang et al., 2024; Kohen, 2019). However, students who received dual representational scaffolding—integrating both text and image—demonstrated superior outcomes compared to those who received single representation scaffolding. These students exhibited greater mastery of mathematical concepts and higher performance in problem-solving tasks.

While both forms of scaffolding led to measurable improvements, the comparative advantage of dual scaffolding was statistically and pedagogically significant, particularly in supporting deeper understanding and the successful resolution of more complex problems. This suggests that the integration of multiple representations provides cognitive support that enhances students' ability to interpret, connect, and apply mathematical concepts. These findings align with previous research indicating that mathematics instruction becomes more effective when it incorporates varied and well-designed scaffolding strategies (Calor et al., 2022; Rodríguez-Nieto et al., 2022).

Furthermore, the results support Vygotsky's sociocultural theory, particularly the concept of the ZPD, by demonstrating that appropriately designed scaffolding can help students progress from their current level of understanding to more advanced stages of competence (Capone et al., 2021; Suwarto et al., 2023). The study highlights the importance of scaffolded online learning environments that not only address immediate problem-solving needs but also foster long-term conceptual development. In particular, the use of dual scaffolding appears to offer enhanced support in bridging the gap between students' actual and potential levels of mathematical performance.

The research findings also demonstrate that multiple representation scaffolding is more effective than single representation scaffolding in enhancing students' understanding of geometric concepts. This result is consistent with previous research showing that online instructional scaffolding supports students in mastering geometric concepts (Faulkner et al., 2023; Partanen et al., 2020), and more broadly, that representational support can facilitate mathematics learning (Hinton & Flores, 2019). The current findings thus extend and reinforce prior studies by providing further empirical evidence that online scaffolding—particularly when involving multiple representations—is both effective and efficient in supporting students' conceptual development in mathematics.

More specifically, students who received multiple representation scaffolding outperformed those who received single representation scaffolding in the posttest phase, particularly on measures of conceptual understanding. This aligns with research suggesting that varied external representations (e.g., visual and verbal formats) offer richer and more complete information, thereby fostering deeper comprehension (Smith & Mancy, 2018; Tinungki et al., 2024). The present study also supports findings that suggest diverse representations can serve as effective navigational tools for students, guiding them through problem-solving processes and helping them to make meaningful connections across mathematical concepts (Capone, 2022; Toikka et al., 2024). Furthermore, these results affirm theoretical claims that integrating multiple forms of representation enhances students' ability to construct, organize, and relate conceptual knowledge more effectively (Hidajat, 2024; Leitner & Gabel, 2024).

Importantly, the benefits of online scaffolding—both single and multiple—were not only evident in immediate posttest performance but were also sustained over time, as demonstrated by the retention test results. This suggests that online scaffolding interventions have the potential to support long-term conceptual understanding in mathematics. However, students who received multiple representation scaffolding showed particularly notable improvements in their ability to solve high-level problems, while those in the single representation group were generally limited to moderate-level problem solving across all item types. These findings are supported by earlier research indicating that multiple representations enhance students' mathematical problem-solving abilities (Chinofunga et al., 2025; Zhang et al., 2024). They also align with theoretical perspectives that highlight the close interrelationship between geometric reasoning, visual representations, and conceptual understanding (Herold-Blasius, 2024; Sidenvall et al., 2022). The integration of images and abstract concepts appears to facilitate students' problem-solving processes and may serve as a powerful tool in geometry instruction.

Another key finding of this study concerns students' eye movement behavior during the online problem-solving intervention. Students in the dual representation scaffolding group exhibited shorter average fixation durations and rereading times compared to those in the single representation group. Despite this reduced attentional time, they consistently demonstrated higher problem-solving performance across all levels of question complexity—basic, combination, and advanced. This pattern suggests more efficient cognitive processing when both visual and textual scaffolds are available. According to prior research, longer fixation durations often reflect greater cognitive effort or processing difficulties, particularly when learners struggle to extract key information from a task (Fitzsimons & Ní Fhloinn, 2023; Smit et al., 2023). Longer fixations are frequently associated with difficulties identifying keywords or contextual cues essential for conceptual understanding. This is supported by findings in science education, where more complex problems in physics have been shown to elicit longer and more intense fixations (Brandsæter & Berge, 2025; Haataja et al., 2025).

In the current study, students in the single representation group—who received only image-based scaffolding—required longer inspection times and exhibited prolonged fixations yet achieved lower problem-solving scores. This indicates that visual scaffolds alone may be insufficient for facilitating efficient cognitive integration and conceptual comprehension. The absence of textual support likely contributed to students' difficulties in interpreting visual representations, which in turn hampered their problem-solving effectiveness (Avcı & Deniz, 2022; Calor et al., 2022). Notably, the most substantial differences in performance between the two groups were observed in the combination-type questions, particularly those involving exterior angles of triangles.

Further analysis revealed that students in the dual representation group distributed their visual attention more strategically spending approximately two-thirds of their inspection time on images and one-third on text (Nückles, 2021; Pereda Loriente et al., 2025). Despite spending less time inspecting images than their single-representation peers, these students performed better, underscoring the importance of integrated verbal-visual scaffolding. This supports the theoretical proposition that the interaction between text and images facilitates the development of internal propositional representations and coherent mental models (Rohde Poole, 2022; Sui et al., 2024). The combination of visual and textual input helps students integrate and organize information more effectively, enabling them to navigate problem-solving tasks with greater accuracy and efficiency.

Additionally, the dual representation group's more efficient use of inspection time—spending less overall time while achieving higher levels of performance—suggests that multimodal scaffolding enhances not only learning outcomes but also the cognitive economy of the problem-solving process (Geraniou et al., 2024; Kohen, 2025). These findings further confirm that varied external representations, when presented in a complementary manner, can facilitate the construction of well-structured mental models that support higher-order mathematical reasoning. Ultimately, this study highlights the significant advantages of dual representational scaffolding in promoting more effective eye movement behavior, deeper conceptual understanding, and stronger problem-solving performance compared to single-image scaffolding.

CONCLUSION

Dual online representation scaffolding—integrating both textual and visual elements—has a significantly greater impact on students' understanding of mathematical concepts, problem-solving proficiency, and eye movement behavior compared to single representation scaffolding. Students supported by dual representations demonstrated superior performance across all levels of mathematical tasks, including basic, combination, and advanced problem types. The effectiveness of this scaffolding strategy lies in the synergistic interaction between textual and visual representations, which together facilitate the construction of coherent mental models and enhance cognitive processing during problem-solving. In contrast, single representation scaffolding—whether visual or textual—was less effective, as it limited students' ability to integrate information and develop a comprehensive understanding of the problem context. These findings underscore the pedagogical value of employing dual representations in digital mathematics instruction, particularly for promoting higher-order thinking and advanced problem-solving capabilities.

Despite these promising results, the study is subject to several limitations. The participant sample was limited to high school students, and the instructional content focused narrowly on a single geometry topic, namely triangle angle properties. Furthermore, the research design relied exclusively on quantitative data and examined a limited set of variables, specifically conceptual understanding, problem-solving ability, and eye movement metrics. These constraints may limit the generalizability of the findings. Future research should consider expanding the scope by including participants from diverse educational levels and backgrounds, exploring additional mathematical domains, and incorporating a broader range of cognitive and affective variables. Moreover, the integration of qualitative methods—such as student interviews, think-aloud protocols, or reflective accounts—could provide deeper insights into learners' cognitive strategies and experiences during the problem-solving process. Such multidimensional investigations would contribute to a more comprehensive understanding of how varied scaffolding designs support mathematical thinking and could inform the development of adaptive, representation-rich learning environments in mathematics education.

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