





The development of web-based learning environment to enhance students' numeracy and reasoning

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Abstract

Despite the growing emphasis on numeracy as a critical outcome of mathematics education, many instructional approaches fail to connect numeracy learning with students' reasoning development in meaningful ways. Existing research has not sufficiently explored the integration of technology-supported environments for fostering numeracy through theoretically grounded task design. Addressing this gap, the present study introduces a novel web-based learning environment grounded in numeracy theory and task design principles aimed at enhancing students' numeracy competence and mathematical reasoning. The development and implementation process involved iterative trials with 25 fifth-grade students: a one-to-one trial ($n = 2$), a small-group trial ($n = 5$), and a field trial ($n = 18$). Data were collected from students' written responses on the web-based platform and their oral explanations. Findings demonstrate that the developed environment meets three key criteria: validity, as it aligns with relevant theoretical and empirical foundations; practicality, based on its usability and feasibility for students; and effectiveness, as evidenced by improved reasoning skills. These results highlight the potential of well-designed web-based learning environments to meaningfully support the development of numeracy competence while simultaneously fostering mathematical reasoning in primary education.

Keywords: Design Research, Fraction, Mathematical Reasoning, Numeracy, Web-Based Learning Environment

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The Programme for the International Assessment of Adult Competencies (PIAAC) defines numeracy as the ability to access, use, interpret, and communicate mathematical information in order to engage with and respond to the mathematical demands of diverse adult contexts (OECD, 2021). This definition emphasizes the functional and critical application of mathematical knowledge, information, and concepts as they are encountered in various forms and settings.

From a broader sociocultural perspective, numeracy is increasingly recognized as a social practice, embedded within and shaped by cultural, social, personal, and emotional dimensions (Díez-Palomar et al., 2023). Accordingly, numeracy encompasses more than mere arithmetic proficiency; it involves the meaningful application of mathematics to solve problems situated in real-world contexts.

Díez-Palomar et al. (2023) further argue that numeracy is essential for full and equitable participation in society. It spans a range of mathematical domains, content areas, and processes—particularly mathematical reasoning—given its close relationship to sense-making, application, and informed decision-making (O'Donoghue, 2002). In this regard, numeracy contributes significantly to the

development of students' critical thinking skills, providing a cognitive foundation for both reasoning and problem-solving (Sellars, 2017).

Numeracy is widely regarded as a fundamental objective of mathematics education, with the goal of cultivating learners' capacity to address quantitative challenges in everyday life (Hoogland, 2016; Niss & Jablonka, 2014; O'Donoghue, 2002). To achieve this, it is imperative that both numeracy and mathematics are taught effectively—either within the mathematics classroom by subject-specialist teachers (Tout, 2020), or through an integrated, cross-curricular approach (Bennison, 2016; Goos et al., 2018). The importance of numeracy lies in its long-term relevance, as it shapes what students retain and apply beyond formal schooling (Gal et al., 2020).

Although the Indonesian mathematics curriculum explicitly emphasizes the development of numeracy and literacy (Pusat Asesmen dan Pembelajaran, 2020), national data indicate a declining trend in students' average performance in mathematical literacy (see Figure 1). This suggests that challenges may lie not only in curriculum design but also in its implementation, pointing to a need for more effective pedagogical practices and systemic support in promoting numeracy.

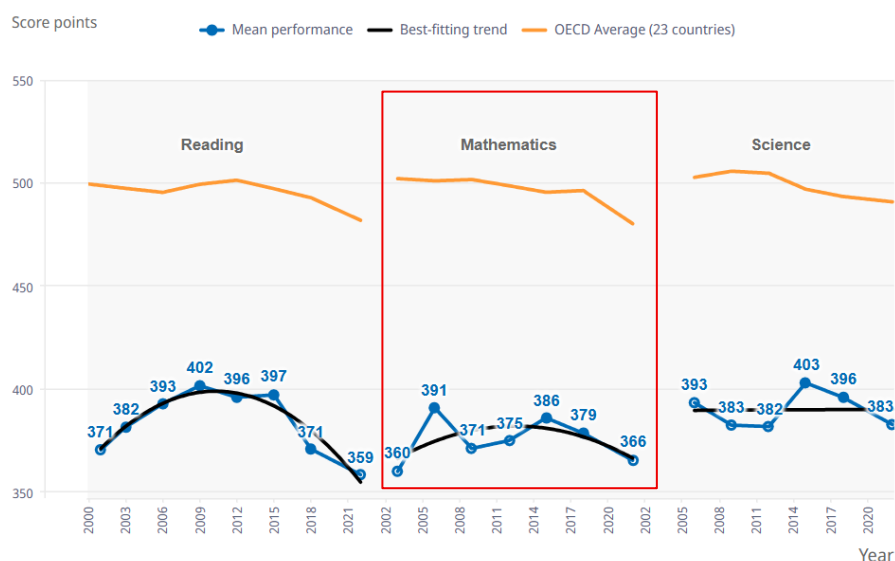


Figure 1. Indonesian PISA 2022 result

Preliminary findings from our study indicate that mathematics teachers in Indonesia—particularly those in Palembang—face significant challenges in providing meaningful and supportive learning experiences that effectively develop students' numeracy skills (Adelia et al., 2024a). This aligns with the recommendation by Xiao et al. (2019), who emphasize the importance of creating opportunities for students to apply their numeracy skills in authentic contexts and through problem-solving activities that reflect core mathematical processes. Our study further reveals that one of the key barriers teachers face in implementing such practices is the burden of extensive administrative responsibilities (Adelia et al., 2024a), which limits their capacity to design and deliver engaging numeracy-focused instruction.

To address these challenges, there is a pressing need for accessible, flexible, and pedagogically sound learning resources that support teachers in facilitating numeracy development. Digital tools offer a promising solution, as they have been shown to support rich and varied numeracy practices (Geiger et al., 2015). Moreover, the increasing integration of technology in education has been positively associated with improvements in students' learning outcomes across subjects (Hillmayr et al., 2020). In particular,

digital learning environments are recognized for their potential to create realistic, contextually relevant scenarios that help students apply mathematical concepts to real-life situations—an essential component of numeracy competence (Cirneanu & Moldoveanu, 2024).

In response to these needs, the present study aims to develop a web-based learning environment that can be utilized by teachers to support elementary students' engagement in numeracy learning. The focus is specifically on enhancing students' mathematical reasoning, one of the key processes underpinning numeracy. The study is guided by the following research question: What are the characteristics of a web-based learning environment that strengthens elementary students' numeracy competence for reasoning in a way that is valid, practical, and demonstrates potential effects?

METHODS

Research Procedure

To address the research question, we conduct a design research study following the development study stages. This study is structured into three main stages, as described by Nieveen et al. (2006) and Plomp (2010). The first stage is the preliminary research stage, in which we analyze the problem and context while developing a conceptual framework grounded in a literature review. The conceptual framework for this study focusses on numeracy learning, the digital learning environment, and fraction learning. The second stage is the prototyping stage, where we are establishing design principles and refining prototypes of the intervention. The first prototype of the developed learning environment consisted of two components: instructional videos and the numeracy test items. Both components will go through formative evaluation, expert review and one-to-one tryout, and then will be revised based on the formative evaluation results. The final stage is assessment phase, where we conduct the formative evaluation to assess the practicality and the efficiency of the final prototype of the learning environment.

Analytical Procedure for Development

The prototyping process of the learning environment will be concluded with a formative evaluation (Zulkardi, 2002). Formative evaluation is an important component of each prototyping approach, since it analyses the potentials of the intervention and its key characteristics and uncovers the shortcomings of an object during its development process. Therefore, empirical data are essential to evaluate the quality of the intervention and design principles (Nieveen, 2010).

The formative evaluation was conducted to evaluate the validity, practicality, and the potential effect of the learning environment (Nieveen, 2010). She suggested three criteria for high-quality interventions. First, the components of the intervention must be grounded in cutting-edge knowledge and consistently interconnected. The intervention is deemed valid if it fulfills these requirements. The subsequent consideration is the intervention's practicality in being "user-friendly". The last is, if the result yields the expected outcomes, then the intervention is considered effective.

To evaluate the validity, practicality, and the effectiveness of the learning environment Nieveen (2010) had proposed an example of how various formative evaluation methods can be used. In this study, expert review, one-to-one trial, small group trial, and field trial are utilized to analyse the quality of the learning environment. Table 1 summarizes the formative evaluation process of this study.

Expert review is utilized to assess the validity of the learning environment. The practicality of the learning environment is assessed through one-to-one trial and small group trial. The effectiveness of the web-based learning environment was evaluated through field trial using Guskey's (2000) five

development levels evaluation. The first level, participants' reaction, assessed students' satisfaction with the web-based learning environment through post-activity surveys, focusing on their perceptions and experiences. The second level, participants' learning, examined students' learning processes by documenting their engagement during the field trial and analyzing their responses submitted on the website. The third level, organizational support and change, evaluated the school's commitment to supporting and sustaining the implementation of the web-based learning environment, which was assessed through interviews with the headmaster and teachers. The fourth level, participants' use of new knowledge and skills, explored how students applied their numeracy knowledge for reasoning, measured through students' responses to numeracy test items. Finally, the fifth level, students' learning outcomes, determined whether students achieved the intended learning objectives, as evidenced by their performance on written numeracy tests. Through these five levels, the study comprehensively assessed the effectiveness of the developed web-based learning environment in supporting students' numeracy and reasoning.

Table 1. Formative evaluation of the prototypes

	First prototype		Final prototype
	Experts	Users	Users
Validity	ea (video, test)		
Practicality		oto (video, test)	sg
Effectiveness			ft

Methods of formative evaluation:

ea = expert appraisal; oto = one-to-one trial; sg = small group trial; ft = field trial

Participant

This study employed several methods during the formative evaluation stage, as summarized in [Table 1](#). Consequently, different groups of participants were involved. In the one-to-one trial, two fifth-grade students participated: one high-performing student and one middle-performing student. They worked with the first prototype of the numeracy test item and the instructional video.

For the evaluation of the final prototype, five fifth-grade students from one classroom participated in the small group trial, while eighteen fifth-grade students from a different classroom participated in the field trial. Each group included students with diverse performance levels, high, middle, and low. The selection of students and classrooms was based on the teacher's recommendations, as teachers are most familiar with their students' abilities.

RESULTS AND DISCUSSION

First Stage: Preliminary Research

This study is grounded in two interrelated domains: numeracy learning and digital learning environment. By intertwining these perspectives, we aim to investigate how the integration of numeracy-oriented task design and digital technology can support students' reasoning, particularly in the context of fraction learning.

Numeracy Learning

In the numeracy learning domain, our work is anchored in the task design that emphasizes numeracy-oriented learning experiences. Goos and her colleagues (2018) defined numeracy as a fundamental



ability required to solve real-life situations problem that incorporate mathematical elements. They further formulated the 21st century numeracy model that consists of five dimensions:

1. Context: numeracy is highly related to everyday situations; therefore, it requires consideration of context to be effective. Even fundamental aspect of numeracy learning is the teachers' comprehension of the practical applications of mathematics in real situations (Adelia et al., 2024b).
2. Mathematical knowledge: an understanding of mathematical knowledge is needed to reason mathematically, solve problems, and interpret situations in real-life contexts.
3. Disposition: one of the characteristics of a numerate person is the dispositional elements such as eagerness and persistence when facing challenges in working with real-life problems.
4. Tool: real-world problems generally contrast with textbook problems; therefore, it requires different problem-solving tools.
5. Critical orientation: this dimension supports the individual to be preventive about the increasing utilization of mathematical information in social, political, and national or international issues.

Digital Learning Environment

In the domain of digital learning environments, our focus is on task design that leverages the affordances of digital technologies to foster mathematical reasoning. Digital tools offer students opportunities to enhance their conceptual understanding of fractions (Zhang et al., 2020). To achieve this, Drijvers (2015) emphasized three essential factors: the design of the digital tool, the role of the teacher while using the tool, and the educational context in which the tool is embedded.

Prior studies in mathematics education highlights that multiple representation used in digital learning activities—such as constant splitting and pizza game—help students understand the meaning of fraction (Martin et al., 2015), equivalent fraction, and complementary fractions (Gaggi et al., 2018). Additionally, a systematic use of technology enables students to explore and identify new mathematical results (Santos-Trigo et al., 2015).

By incorporating digital technologies, we aim to develop and investigate how such tools facilitate students' reasoning in numeracy learning, particularly in the domain of fractions. Given that fraction concepts are often challenging for elementary students due to their abstract nature (Siegler et al., 2013). We argue that digital environments can provide interactive tasks and visual representations that support students in making sense of fraction conceptual understanding.

Second Stage: Prototyping

The design process of the learning environment was guided by the prototyping approach (Zulkardi, 2002). He continued that, by employing this approach, the learning environment carried on through two stages within the prototyping cycle and ended with a formative evaluation. The main components of the learning environment are instructional video and numeracy test item.

Instructional Video

As one of the main components, the instructional video provides theory-grounded fraction learning. The designed instructional video sequence included four videos on Fraction as Part of a Whole, Fraction Equivalence, Unlike Denominator Fraction Addition, and Fraction Division. These four videos are embedded in the web-based learning environment "*BangunRuang*" (www.bangunruang.net).

Based on the literature, the construct of fractions as a part of a whole is a crucial understanding that students need to comprehend (Charalambous & Pitta-Pantazi, 2007). This construction enables

pupils to articulate a situation where a whole is partitioned into equal parts, with a fraction denoting how many of these parts are being considered (Charalambous & Pitta-Pantazi, 2007). They further emphasized that students must be proficient in partitioning a continuous area and discrete set into equal parts, as well as determining whether the whole has been partitioned in equal parts.

For fraction equivalence, students must grasp the notion of proportionality—recognizing that different fractions can have the same value (Adelia et al., 2022; Pedersen & Bjerre, 2021). These studies highlight the importance of engaging students in a situation where the denominator varies while still representing an equivalent quantity.

In fraction addition, understanding the magnitude of fractions with unlike denominators plays a fundamental role in students' conceptual development (Amuah & Davis, 2023). Tsai and Li (2017) stated that the ability to unitize quantities in different ways and the flexibility in handling units are essential for developing equivalence competence, which is also important for fraction addition with unlike denominator (Laughlin, 2022).

Some studies differentiate fraction division into two interpretations: measurement and partitive (Adu-Gyamfi et al., 2019; Son & Senk, 2010). This study focuses only on partitive division. Prior study has identified key design principles to support students' partitive fraction division understanding: 1) developing pupils' sense of fair-sharing through whole number partitive division, 2) strengthening the concept of unit in fraction and partitioning, 3) choosing appropriate context and graphical representations, and 4) sequencing the fraction used (Wahyu et al., 2020).

These four fraction concepts formed the foundation for the instructional videos. We ensured that the instructional videos addressed a real-world problem, outlined the solution with the involved process, provided learner support and emphasized the presentation of the video. We adopted the framework of Wirth and Greefrath (2024) to design the instructional video.

The instructional videos subsequently proceed to formative evaluation of expert appraisal. The experts involved in this study were a researcher in educational technology and a mathematics educator. In general, the experts agree that the instructional videos present a problem that is situated in real-world context, contributes to students being able to recognize the procedure, call for knowledge-generating activity and engage students in subsequent problems, also the presentation of the video is well-designed. Overall, the experts concluded that the instructional videos are valid for the intended educational purposes. However, certain points require further discussion. The first concern is how pupils connect mathematics to everyday life. To address this, we emphasize that the fraction unit used in the video is practical and frequently encountered by students in their daily lives. The next concern involves pupils' ability to make predictions and validate their solutions. We addressed this by conducting a pilot test to determine whether the problem presented in the video helps students make predictions and validate their solutions. Another concern relates to feedback: does the video help students articulate their solution, explain the underlying principles to themselves, or anticipate the next step? We address this by allowing the teacher to provide opportunities for students to reflect on their learning and confirm their understanding.

To assess the practicality of the instructional video, we conducted a one-to-one trial. During the trial, the participating students engaged with the instructional video and answered the problems presented at the end of each video. Nieveen and van den Akker (1999) stated that practicality is determined by whether the instructional videos are usable and engaging in real-world settings. While the students faced minor difficulties, such as occasional challenges in following specific parts of the video, they appeared to enjoy the experience overall. They remained focused throughout the session and were

able to answer most of the questions correctly. To illustrate, we provide a detailed account of the participants, Andy and Sarah, as shown in Figure 2.

Andy engaged actively with the instructional videos, demonstrating attentiveness and interest in the content. He answered the questions correctly, indicating an understanding of the material presented. For instance, in fraction as parts of a whole video, we presented an incomplete situation problem adapted from Pramudiani et al. (2022). The problem asked students to determine what fraction one part of a pizza represents when one part is missing. Andy's answer was $\frac{1}{8}$, and he confidently explained that the missing part still counts toward the whole. This reasoning aligns with Čadež and Kolar (2018) concept of comprehending fraction as a part-whole subconstruct. In contrast, Sarah answered incorrectly for the same question. She responded $\frac{1}{7}$, explaining that there were only seven visible pizza slices, disregarding the missing slice. This highlights her misconception that fractions should be based solely on visible parts. Sarah required support to understand that fractions must account for the entire whole, including parts that are not immediately visible.

In another video, the problem required finding a fraction equivalent of $\frac{1}{2}$ using the unit fraction of $\frac{1}{8}$. The video context, adapted from Adelia et al. (2022), involved measuring cups. Andy and Sarah correctly determined that four measuring cups of $\frac{1}{8}$ are needed to fill the $\frac{1}{2}$ -cup. Based on these findings, it can be concluded that Andy and Sarah perceived the instructional videos to be rather practical and maintained a positive attitude toward their use in his learning process. Furthermore, the incorrect answers by Sarah led us to consider the need for additional assistance when working with the video.

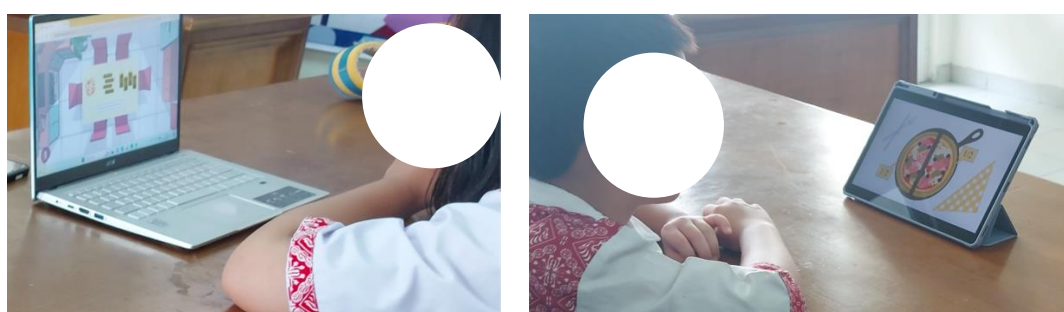




Figure 2. Sara and Andy work on instructional video

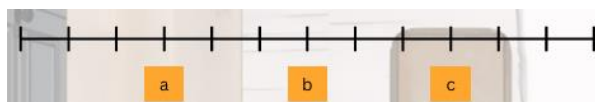
Numeracy Test Item

In the development of numeracy problems, Goos et al. (2018) identified two distinct processes. *Structuring* involves using the numeracy dimensions to assess how the selected context effectively involves students, identify pertinent mathematical knowledge, consider how the task enhances student dispositions, introduce to suitable tools, and includes elements to encourage students to adopt a critical orientation. *Fitting to circumstances*, focuses on creating, selecting, and adapting tasks to accommodate or take advantage of the specific characteristics of the school.

In line with the first stage of numeracy problem development proposed by Goos et al. (2018), we aim to design numeracy tasks that arise from everyday situations involving fraction. These tasks are crafted to integrate contextual understanding and mathematical knowledge, utilize appropriate mathematical tools, support students' dispositions toward mathematics, and evoke students' critical orientation presented in Table 2.

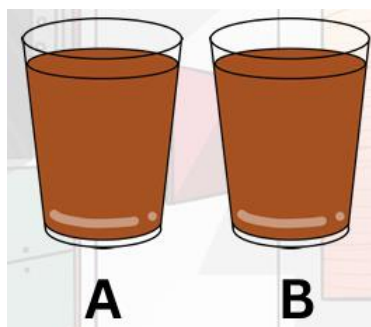
Table 2. The numeracy test items

Problem	Numeracy dimension
Subtopic: Fractions as part-whole relationship	
 <p>a. Why do you think the price written is for $\frac{1}{7}$ cow?</p> <p>b. How do you find the price for 1 cow?</p>	<p>Mathematical Knowledge:</p> <ul style="list-style-type: none"> • Multiplication with decimal number • Multiplication of large number <p>Context:</p> <p>Qurban (Sacrificial Animal)</p> <p>Dispositions:</p> <ul style="list-style-type: none"> • Curiosity • Problem-solving attitude <p>Critical orientation:</p> <p>Reflecting on how mathematical representations are used in public communication.</p> <p>Tool:</p> <p>Procedural tool: multiplying procedure</p>
<p>A regular column in the local newspaper invites readers to ask a famous chef for a recipe.</p> <p>Q: I want to make a small sponge cake in a 15 cm pan. How do I measure the quantity of ingredients needed?</p> <p>A: Just adjust the recipe; for example, if the recipe is for a 30 cm pan, then divide the recipe in half.</p> <p>Do you think the Chef's answer is the best advice? Give your answer and why.</p>	<p>Mathematical Knowledge:</p> <p>Fraction proportional sizes</p> <p>Context:</p> <p>Baking</p> <p>Dispositions:</p> <ul style="list-style-type: none"> • Critical thinking attitude • Curiosity <p>Critical orientation:</p> <p>Decision making on choosing whether to follow the advice</p> <p>Tool:</p> <p>Fraction illustration</p>
Subtopic: Ordering fractions	
<p>A platform is a place where you could wait for your train in a train station.</p> <p>Below is a ticket from London to Hogwarts, the ticket show that we must wait in Platform $9\frac{3}{4}$.</p>  <p>Place the $9\frac{3}{4}$ in this number line.</p>	<p>Mathematical Knowledge:</p> <p>Fraction magnitude</p> <p>Context:</p> <p>Pop culture</p> <p>Dispositions:</p> <ul style="list-style-type: none"> • Flexibility • Curiosity <p>Critical orientation:</p> <p>Interpreting visual representation critically</p> <p>Tool:</p> <p>Number line</p>



Subtopic: Fraction equivalence

These two glasses contain a same tea type and volume. If cup A has $\frac{3}{4}$ spoon of sugar added, and cup B has $\frac{2}{3}$ spoon of sugar added. Will the sweetness of the two teas be the same? Give your reason.



Mathematical Knowledge:

Comparing fractions

Context:

Measurement

Dispositions:

- Persistence
- Curiosity

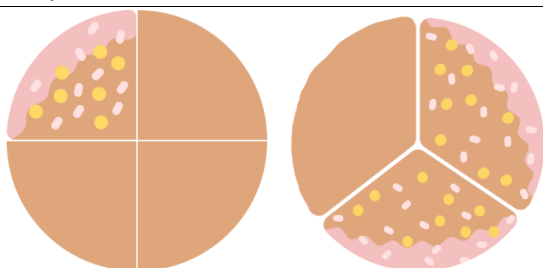
Critical orientation:

Reflecting on how fractions visualized spatially

Tool:

Procedural tool: referencing known benchmarks

Subtopic: Fraction addition with unlike denominators



- If the topped cake parts are combined, will it be one whole cake, less than one whole cake, or more than one whole cake?
- What is the fraction of the combination of the topped parts?

Mathematical Knowledge:

- Part-whole relationship
- Fraction equivalence
- Fraction addition

Context:

Fair sharing activity

Dispositions:

- Persistence
- Curiosity

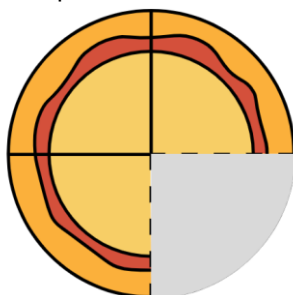
Critical orientation:

Evaluating representations

Tool:

Procedural tool: finding a common denominator or reasoning through visual models

Subtopic: Fraction division



Mathematical Knowledge:

Part-whole relationship

Context:

Fair sharing activity

Dispositions:

- Persistence
- Curiosity

- | | |
|---|---|
| a. What is the fraction of that pizza leftover? | Critical orientation: |
| b. What do you suggest if the leftover pizza is shared by 6 people? | Avoiding surface-level visual assumptions |
| c. What fraction of a pizza did each person receive? | Tool: |
| d. How do you explain the result of $\frac{3}{4} : 6$ | Visual model |
-

Same as instructional video, the numeracy test item validity was also assessed through expert appraisal, by the expert of mathematics educator. The assessment indicator followed five dimensions of the 21st century numeracy model: context, mathematical knowledge, disposition, tools, and critical orientation (Goos et al., 2018). In general, the expert agrees that the content and construct of the numeracy test items are valid and align with the numeracy dimensions of 21st century numeracy model. The expert highlights that the test items excel at connecting mathematics to real-world contexts, which is a critical goal of numeracy learning. However, he noted that some items could be further refined to better suit students in different regions or countries. For instance, item 1 may be particularly beneficial for students from areas with a strong livestock farm culture. Regarding language, the expert affirms that the phrasing is appropriate and communicative for elementary school students. Furthermore, the expert specifically commends item 3 and 5 for encouraging students to construct mathematical arguments and interpret solutions when addressing everyday problems, demonstrating their potential to enhance students' mathematical reasoning (Jeannotte & Kieran, 2017).

The numeracy test item was further evaluated for practicality through one-to-one trials, covering topics: fraction definition, ordering fraction, fraction equivalence, unlike denominator fraction addition, and fraction division as shown Figure 3. Results indicate that guidance within items impacts students' success. For instance, both Andy and Sarah provided correct answers to the partitive fraction division problem, $\frac{3}{4} : 6$. The guided structure of this item helped students by directing them to first notice the fraction $\frac{3}{4}$, then to equal partition it six people, and finally to interpret the mathematical solution to the problem. In contrast, when the problem lacked explicit guidance, as in item 3 (focused on fraction equivalence by comparing $\frac{3}{4}$ and $\frac{2}{3}$), performance varied. Andy identified that $\frac{3}{4}$ is not equivalent to $\frac{2}{3}$ by calculating the least common denominator, while Sarah incorrectly claimed that $\frac{3}{4}$ is equivalent to $\frac{2}{3}$, revealing a misunderstanding of fraction equivalence. These findings suggest that the practicality of test items depends on their structure and level of guidance, which can influence students' ability to engage with and solve fraction problems.



Figure 3. Sara and Andy work on numeracy test item

Third Stage: Assessment

The final prototype is the fully developed web-based learning environment, accessible at www.ruangnumerasi.myportfolio.com. The development process began after completing the initial prototype. The final prototype design commenced with creating a storyboard for the website, guided by the framework of online mathematics environment proposed by Barlovits et al. (2022). According to this framework, designing an effective online mathematics environment requires careful consideration of several key aspects. First, technical equipment and digital competencies, ensuring device and internet access availability and educators with varying levels of digital proficiency. It also incorporates curricular resources, to create appropriate content for mathematics lessons. Additionally, adequate formative assessment is essential, so that the assessment is congruent with the intended processes and outcomes of the lesson. Finally, the design should foster personal interaction, facilitating meaningful connections between students and educators to build a supportive and collaborative learning community. By addressing these considerations, the web-based learning environment ensures a pedagogically sound and technologically robust platform for its users.

Practicality

To further investigate the enacted web-based learning environment, a follow-up small group trial was conducted with 15 fifth-grade students that represented a variety of mathematics abilities. The specific goal of this trial was to identify to what extent the practicality of the web-based learning environment when engaged with the students. Focusing on its usability and accessibility. The trial revealed several key insights regarding the practical feasibility of implementing the web-based learning environment in a classroom setting.

One of the main challenges observed was the need for stable internet access. For the web-based platform to function effectively, all students needed to be able to access the website simultaneously (see Figure 4). Besides that, most students were able to navigate web-based activities without significant difficulties. The teacher also played a crucial role in monitoring and assisting students during the trial. The verbal explanations are still needed for students to fully uncover their reasoning. This suggests that while the platform is a valuable supplementary tool, it works best when integrated with teacher-led discussions.

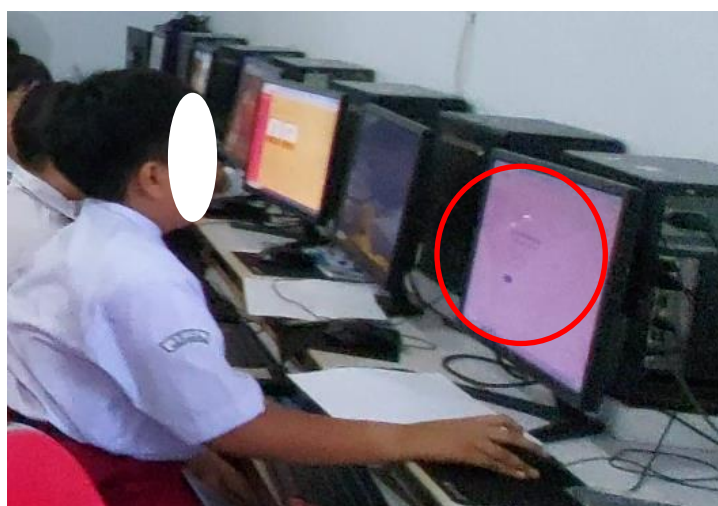


Figure 4. The internet problem faced by a student

Effectiveness: Potential Effect of the Learning Environment

Participants' reaction. Students' satisfaction was measured using a questionnaire administered to conclude the field testing (see Figure 5). The questionnaire results revealed that 85% of students show positive attitude toward the web-based learning environment. This indicates a high level of alignment and satisfaction among students with the web-based learning environment.

Nama: [REDACTED]

a. Lingkari kata-kata di bawah ini (satu atau lebih) yang mendeskripsikan perasaanmu saat berinteraksi dengan website BangunRuang? Kamu dapat menambahkan kata-kata lain di kolom yang disediakan.

① Senang	⑤ Hebat
② Pintar	⑥ Bersemangat
3. Tidak bisa terlibat	11. Depresi
4. Khawatir	12. Puas
5. Tidak senang	13. Merasa bodoh
③ Bingung	④ Tertarik
7. Antusias	15. Frustasi
8. Bosan	

b. Lingkari kata-kata di bawah ini (satu atau lebih) yang mendeskripsikan permasalahan serta video yang diberikan di website BangunRuang? Kamu dapat menambahkan kata-kata lain di kolom yang disediakan.

① Menyenangkan	
2. Sangat mudah	
③ Menarik	
④ Menantang	
5. Mengecewakan	
6. Membosankan	
7. Berbeda	
⑤ Sulit	
9. Seperti soal yang biasa saya kerjakan	

Circle one or more words below that describe your feelings when interacting with the website. You can add other words in the provided column.

Happy <input checked="" type="checkbox"/>	Great <input checked="" type="checkbox"/>
Smart <input checked="" type="checkbox"/>	Excited <input checked="" type="checkbox"/>
Unable to participate	Depressed
Worried	Satisfied
Unhappy	Feeling stupid
Confused	Interested <input checked="" type="checkbox"/>
Enthusiastic	Frustrated
Bored	

Circle one or more words below that describe the problems and videos provided on the website. You can add other words in the provided column.

Fun <input checked="" type="checkbox"/>	Boring
Very easy	Different
Interesting <input checked="" type="checkbox"/>	Difficult <input checked="" type="checkbox"/>
Challenging <input checked="" type="checkbox"/>	Just like usual <input checked="" type="checkbox"/>
Disappointing	Trivial

Figure 5. Students' reaction questionnaire

Participants' learning. The observation revealed that students actively participated in the activities, demonstrated curiosity, and frequently interacted with the tasks provided. Most students were able to follow the instructions independently, while some required guidance to navigate the platform and understand the task. The analysis of students submitted responses indicated that the majority of students could correctly apply the concepts introduced in the web-based learning environment. Students' answers reflected their understanding of fraction concepts, such as recognizing fractions in contextual problems and providing justifications for their solutions.

Organizational support and change. The web-based learning environment was endorsed by supportive backing from the school principal, providing formal approval for conducting the study and encouraging mathematics teachers to integrate the platform into their classroom practices. Also, teachers expressed positive perceptions of the web-based learning environment, acknowledging its relevance in supporting students' numeracy learning and its alignment with the school's educational goals. While some teachers noted initial challenges related to technological access and students' digital literacy, they demonstrated a willingness to adapt and incorporate the platform into their lessons. The school management also showed openness to future collaborations and considered the potential for

wider implementation of the web-based learning environment.

Participants' use of new knowledge and skills. The focus of this level was on finding indications of the effects with respect to students' mathematical reasoning when solving numeracy tasks about fractions. In this study, we use Lithner's (2008) framework of reasoning. According to him, there are two main types of reasoning: imitative and creative mathematical reasoning. The reasoning that has a mathematical foundation is creative mathematical reasoning.

We separate students' answers into imitative reasoning or creative mathematical reasoning. In general, from 108 answers, sixty-eight answers are classified as imitative reasoning and forty answer classified as creative mathematical reasoning ($n = 18$).

Fraction Definition

The question is: "Al Irsyad Mosque accepts and distributes sacrificial animals: the price for $\frac{1}{7}$ of a cow is IDR 2.9 million. Why do you think the displayed price is for $\frac{1}{7}$ of a cow?". Luna's answer provided an example of the creative mathematical reasoning answer.

Luna was able to connect the question to the animal sacrifice (*Qurban*), where one cow can be sacrificed on behalf of seven people. Through this understanding, she recognized that the listed price corresponds to $\frac{1}{7}$ of the total cost of one whole cow, as each person contributes their fair share. Furthermore, Luna explained that the total price of one whole cow is IDR 20.3 because the price per person ($\frac{1}{7}$ of a cow) is IDR 2.9 million.

Luna's answer demonstrates creative mathematical reasoning (Lithner, 2008) because she did not merely perform a calculation but also connected the question to a meaningful real-world context. She supports her conclusion by explaining how price distribution aligns with fraction, ensuring fairness in cost allocation. Most importantly, her reasoning is anchored in proportional reasoning, as she effectively links price distribution with fraction operations.

Fraction Equivalence

The question is: "Two glasses contain tea of the same type and volume, if glass A is given $\frac{3}{4}$ of a spoon of sugar and glass B is given $\frac{2}{3}$ of a spoon of sugar, will the sweetness of both teas be the same? Provide your reasoning". Ron's answer provided an example of the imitative reasoning answer.

Ron's answer demonstrated imitative reasoning, even though it was correct. He stated that glass A is sweeter than glass B because glass A contains three spoons of sugar, while glass B contains two spoons. He was likely referring to fraction using the part-whole subconstruct, where $\frac{3}{4}$ represents three parts (of something) out of four parts in total. According to Sumpter (2018), this type of response indicates that the foundation of the student's reasoning is mathematical, but it is not central to the task.

Fraction Addition with Unlike Denominator

The question is: "If the cake slices with toppings are combined, will they form exactly one whole cake, less than one whole cake, or more than one whole cake?" (see Figure 6). Harry's answer provided an example of the imitative reasoning answer.

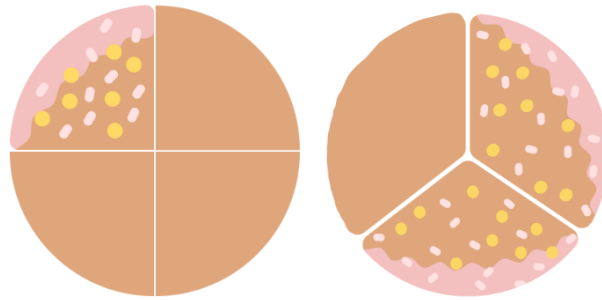


Figure 6. Fraction addition problem

Harry knew that he needed to add the fractions, which is the main mathematical problem in the question, but instead of adding $\frac{1}{4}$ with $\frac{2}{3}$, he added $\frac{1}{4}$ and $\frac{1}{3}$. Then he recalled an incorrect procedure. Instead of finding a common denominator for fourths and thirds, he mistakenly added the denominators, arriving at an answer of $\frac{1}{7}$. Harry's reasoning was not based on the mathematical properties of fraction addition with unlike denominator. According to him, he used this approach because the problem felt familiar (Lithner, 2006). Unfortunately, he made no attempt to verify his answer, such as by illustrating the addition.

Fraction Division

The question is: "What is your suggestion if the remaining pizza, as shown in the image, needs to be divided equally among six people? What fraction of the pizza will each person receive?" (see Figure 7). Ginny's answer provided an example of creative mathematical reasoning.

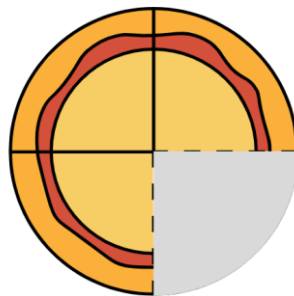


Figure 7. Fraction division problem

"The remaining $\frac{3}{4}$ of the pizza can be divided into six equal portions. If we imagine the pizza as a whole circle divided into four parts, and then each remaining part is further split into smaller pieces for six people, each person gets $\frac{1}{8}$ of the total pizza. This shows how $\frac{3}{4}$ divided by six equals $\frac{1}{8}$." -Ginny

Ginny performed creative mathematical reasoning because she adapts her knowledge of fair sharing to the specific scenario. Her explanation is also logical and addressed the problem, and rooted in the principles of fractions, division, and proportionality (Lithner, 2008). Ginny noticed that $\frac{1}{4}$ of the pizza is missing, so $\frac{3}{4}$ of the pizza remains. She visualized by dividing the remaining pizza into six equal parts, so that each person gets $\frac{1}{8}$. This reasoning is based on dividing fractions and proportional thinking.

To gain a broader perspective on students' responses, we conducted a quantitative analysis of their scores on numeracy test items. We scored their answer based on the processes of mathematical

reasoning as defined by Lithner (2008). Since there are four processes, each process a student demonstrated was awarded one point, making the maximum possible score for each question four points. For $n = 18$, the mean was 9.5 and the standard deviation of 5.6. The range of the dataset was 16, which suggests a relatively large spread of scores. The coefficient of variation, which measures relative dispersion, is 59%, suggesting a moderate to high variability in students' mathematical reasoning scores. This means that while some students demonstrated strong reasoning skills, others struggled. The presence of a wide range suggests that the learning environment was effective for some students but may need refinement to support lower-performing students more effectively. The results indicate that the learning environment provided opportunities for students to engage in mathematical reasoning, but the variability in scores suggests differing levels of success across students. A further breakdown of responses could identify which aspects of the environment were most effective and which need improvement.

Students' learning outcome. The results indicated that students showed understanding in applying fraction concepts in contextualized problems. They were able to recognize fractions as part-whole relationships, accurately identify fractions from visual representations, and perform fraction addition and comparison involving unlike denominators. Furthermore, students demonstrated the ability to explain their reasoning by connecting mathematical procedures to the problem context, reflecting a deeper conceptual understanding beyond mere procedural skills. Compared to their initial performance, students became more consistent in using appropriate strategies, such as finding common denominators and justifying their answers with relevant contextual explanations.

Based on the development and analysis conducted in this study, the characteristics of a web-based learning environment that effectively strengthens elementary students' numeracy competence for reasoning, while ensuring validity, practicality, and potential effects, can be categorized as: First, alignment with theoretical and empirical foundations. The developed learning environment integrates five key dimensions of numeracy concepts in fraction learning. The experts agreed that its components effectively present problems situated in real-world contexts, enable students to recognize mathematical procedures, encourage knowledge generation, and engage students in subsequent problems. Furthermore, the environment is encouraging students to construct mathematical arguments and interpreting solutions when addressing everyday problems. For instance, in item 1, students demonstrate their ability to make sense of fractional representations in real-world settings. This is evident in their responses to a numeracy problem involving Qurban, where they recognize that one cow can be sacrificed on behalf of seven people. As a result, the price displayed represents $\frac{1}{7}$ of a cow rather than the full price of one cow. Students successfully determined the price of a whole cow by multiplying the given price for one whole cow by seven. This reasoning aligns with prior studies, which emphasize that numeracy demands a solid mathematical understanding in solving real-world problem (Adelia et al., 2024a; Goos et al., 2018; OECD, 2023).

The second one is the feasibility and usability for students. For a web-based learning environment to be practical, its design must be user-friendly and technically accessible (Drijvers, 2015; Zulkardi, 2002). Additionally, teachers must be proficient in using the web-based learning environment, including its instructional videos and numeracy test items, to effectively support students (Alvarez et al., 2009; Drijvers, 2015). Lastly, the learning environment's context should align with the learning objectives and students' prior knowledge to ensure meaningful engagement and knowledge construction (Drijvers, 2015).

The final one is the potential effects on promoting students reasoning. The evaluation of the web-

based learning environment through Guskey's five levels provided comprehensive insights into its effectiveness in supporting students' numeracy and reasoning, particularly in the domain of fractions. Students responded positively to the learning experience, as reflected in their engagement and satisfaction during the field trial. Observations and analysis of their responses indicated active participation and meaningful learning processes. The school demonstrated strong organizational support, with both leadership and teachers showing commitment to integrating the platform into classroom practice. Furthermore, the web-based learning environment has the potential to evoke students' reasoning skills by prompting students to justify their answer and connect their reasoning to meaningful context (Lithner, 2008). For instance, Ginny clearly provided a solution for dividing $\frac{3}{4}$ by 6. Finally, students' performance on numeracy test item confirmed the achievement of the intended learning objectives, showcasing improvements in their understanding and application of fractions. These findings collectively highlight the web-based learning environment's potential as a valid, practical, and impactful tool for enhancing fraction learning and fostering students' numeracy and reasoning.

This study contributes to the ongoing discussion on integrating technology to enhance students' numeracy and reasoning, especially in the context of fraction learning. Previous studies (e.g., Drijvers (2015) and Zulkardi (2002)) emphasized the role of web-based learning environment in making mathematics learning more accessible and engaging. Our findings confirm these earlier insights, showing that a web-based learning environment designed around real-world contexts and numeracy principles can promote reasoning among elementary students. Furthermore, this study also highlights how students can meaningfully construct arguments and apply mathematical reasoning when tasks are situated in contexts familiar to them.

CONCLUSION

This study identified and analyzed the characteristics of a web-based learning environment designed to strengthen elementary students' numeracy and reasoning in the context of fractions. Beyond confirming validity, practicality, and potential effects, the main findings reveal that integrating five key dimensions of numeracy, situating problems in real contexts, recognizing procedures, encouraging knowledge construction, engaging in sequential tasks, and constructing arguments, can meaningfully promote students' reasoning processes. Students were able to connect mathematical procedures to real-life contexts, such as interpreting the price of Qurban, and justify their solutions, as demonstrated in tasks requiring fraction operations. This shows that thoughtfully designed web-based environments can go beyond procedural fluency to foster deeper conceptual understanding and reasoning.

Despite these contributions, this study also has limitations. The field trial involved a relatively small number of students from a single grade level and context, which may limit the generalizability of the findings. Additionally, the intervention focused specifically on fraction learning, so its effects on other mathematical domains remain unexplored. Technical constraints, such as internet connectivity and device availability, also influenced the learning environment's implementation and may affect broader adoption.

The impact of this study is twofold. Theoretically, it extends previous research by demonstrating how numeracy can inform the design of web-based learning environments to enhance reasoning. Practically, it provides teachers and curriculum developers with concrete examples of tasks and features that can engage students in constructing arguments and solving contextual problems. The findings suggest that integrating numeracy-based design principles into digital tools can help students develop



reasoning skills that are critical for lifelong learning. Future research could explore scaling the learning environment to different contexts, subjects, or student populations, and investigate its long-term impact on students' mathematical thinking.

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Declarations

- Author Contribution : VA: Conceptualization, writing-original draft, formal analysis, editing, and visualization.
RIIP: Writing-review & editing, formal analysis, and methodology.
Z: Writing-review & editing, validation, and supervision.
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REFERENCES

- Adelia, V., Putri, R. I. I., & Zulkardi. (2024a). Mathematics teachers in Palembang teaching practice: Numeracy perspective. *The 5th Sriwijaya University Learning and Education (SULE) International Conference 2023: Reimagining Learning toward Equitable Education*, 020069. <https://doi.org/10.1063/5.0201098>
- Adelia, V., Putri, R. I. I., & Zulkardi. (2024b). A systematic literature review: How do we support students to become numerate? *International Journal of Evaluation and Research in Education (IJERE)*, 13(3), 1816. <https://doi.org/10.11591/ijere.v13i3.26849>
- Adelia, V., Putri, R. I. I., Zulkardi, & Mulyono, B. (2022). Learning trajectory for equivalent fraction learning: An insight. *Journal of Honai Math*, 5(1), 47–60. <https://doi.org/10.30862/jhm.v5i1.233>
- Adu-Gyamfi, K., Schwartz, C. S., Sinicrope, R., & Bossé, M. J. (2019). Making sense of fraction division: Domain and representation knowledge of preservice elementary teachers on a fraction division task. *Mathematics Education Research Journal*, 31(4), 507–528. <https://doi.org/10.1007/s13394-019-00265-2>

- Alvarez, I., Guasch, T., & Espasa, A. (2009). University teacher roles and competencies in online learning environments: A theoretical analysis of teaching and learning practices. *European Journal of Teacher Education*, 32(3), 321–336. <https://doi.org/10.1080/02619760802624104>
- Amuah, E., & Davis, E. K. (2023). Strategies and procedural and conceptual knowledge of addition of unlike denominator fractions: The case of grade 8 children in two districts of the central region of Ghana. *African Journal of Research in Mathematics, Science and Technology Education*, 27(2), 123–136. <https://doi.org/10.1080/18117295.2023.2226546>
- Barlovits, S., Caldeira, A., Fesakis, G., Jablonski, S., Koutsomanoli Filippaki, D., Lázaro, C., Ludwig, M., Mammana, M. F., Moura, A., Oehler, D.-X. K., Recio, T., Taranto, E., & Volika, S. (2022). Adaptive, synchronous, and mobile online education: Developing the ASYMPTOTE learning environment. *Mathematics*, 10(10), 1628. <https://doi.org/10.3390/math10101628>
- Bennison, A. (2016). A sociocultural approach to understanding identity as an embedder-of-numeracy: A case of numeracy and history. *European Educational Research Journal*, 15(4), 491–502. <https://doi.org/10.1177/1474904116643327>
- Čadež, T. H., & Kolar, V. M. (2018). How fifth-grade pupils reason about fractions: A reliance on part-whole subconstructs. *Educational Studies in Mathematics*, 99(3), 335–357. <https://doi.org/10.1007/s10649-018-9838-z>
- Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics*, 64(3), 293–316. <https://doi.org/10.1007/s10649-006-9036-2>
- Cirneanu, A.-L., & Moldoveanu, C.-E. (2024). Use of digital technology in integrated mathematics education. *Applied System Innovation*, 7(4), 66. <https://doi.org/10.3390/asi7040066>
- Díez-Palomar, J., Ramis-Salas, M., Močnik, I., Simonič, M., & Hoogland, K. (2023). Challenges for numeracy awareness in the 21st century: Making visible the invisible. *Frontiers in Education*, 8. <https://doi.org/10.3389/feduc.2023.1295781>
- Drijvers, P. (2015). Digital technology in mathematics education: Why it works (or doesn't). In *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (pp. 135–151). Springer International Publishing. https://doi.org/10.1007/978-3-319-17187-6_8
- Gaggi, O., Ciraulo, F., & Casagrande, M. (2018). Eating Pizza to learn fractions. *Proceedings of the 4th EAI International Conference on Smart Objects and Technologies for Social Good*, 220–225. <https://doi.org/10.1145/3284869.3284921>
- Gal, I., Grotlüschen, A., Tout, D., & Kaiser, G. (2020). Numeracy, adult education, and vulnerable adults: A critical view of a neglected field. *ZDM*, 52(3), 377–394. <https://doi.org/10.1007/s11858-020-01155-9>
- Geiger, V., Goos, M., & Dole, S. (2015). The role of digital technologies in numeracy teaching and learning. *International Journal of Science and Mathematics Education*, 13(5), 1115–1137. <https://doi.org/10.1007/s10763-014-9530-4>
- Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2018). *Numeracy across the curriculum: Research-based strategies for enhancing teaching and learning*. Routledge.
- Guskey, T. R. (2000). *Evaluating professional development* (Vol. 1). Corwin press.



- Hillmayr, D., Ziernwald, L., Reinhold, F., Hofer, S. I., & Reiss, K. M. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis. *Computers & Education*, 153, 103897. <https://doi.org/10.1016/j.compedu.2020.103897>
- Hoogland, K. (2016). *Images of numeracy investigating the effects of visual representations of problem situations in contextual mathematical problem solving* [Technische Universiteit Eindhoven]. <http://dx.doi.org/10.13140/RG.2.2.22839.57767>
- Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, 96(1), 1–16. <https://doi.org/10.1007/s10649-017-9761-8>
- Laughlin, A. J. (2022). Lesson Study and the new teacher: Adding fractions with unlike denominators, for grade 4 (8- and 9-year-old) students. In A. Takahasi, T. McDougal, S. Friedkin, & T. Watanabe (Eds.), *Educators' Learning from Lesson Study* (1st ed.). Routledge. <https://doi.org/10.4324/9781003230915>
- Lithner, J. (2006). *A framework for analysing creative and imitative mathematical reasoning*.
- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, 67(3), 255–276. <https://doi.org/10.1007/s10649-007-9104-2>
- Martin, T., Petrick Smith, C., Forsgren, N., Aghababayan, A., Janisiewicz, P., & Baker, S. (2015). Learning fractions by splitting: Using learning analytics to illuminate the development of mathematical understanding. *Journal of the Learning Sciences*, 24(4), 593–637. <https://doi.org/10.1080/10508406.2015.1078244>
- Nieveen, N. (2010). Formative evaluation in educational design research. In T. Plomp & N. Nieveen (Eds.), *An Introduction to Educational Design Research* (pp. 89–101). SLO.
- Nieveen, N., McKenney, S., & Van Den Akker, J. (2006). Educational design research: The value of variety. In J. Van Den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational Design Research* (pp. 151–158). Routledge.
- Nieveen, N., & van den Akker, J. (1999). Exploring the potential of a computer tool for instructional developers. *Educational Technology Research and Development*, 47(3), 77–98. <https://doi.org/10.1007/BF02299635>
- Niss, M., & Jablonka, E. (2014). Mathematical literacy. In *Encyclopedia of Mathematics Education* (pp. 391–396). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_100
- O'Donoghue, J. (2002). Numeracy and mathematics. *Irish Mathematical Society Bulletin*, 0048, 47–56. <https://doi.org/10.33232/BIMS.0048.47.56>
- OECD. (2021). *The assessment frameworks for cycle 2 of the programme for the international assessment of adult competencies*. OECD. <https://doi.org/10.1787/4bc2342d-en>
- OECD. (2023). *PISA 2022 assessment and analytical framework*. OECD. <https://doi.org/10.1787/dfe0bf9c-en>
- Pedersen, P. L., & Bjerre, M. (2021). Two conceptions of fraction equivalence. *Educational Studies in Mathematics*, 107(1), 135–157. <https://doi.org/10.1007/s10649-021-10030-7>
- Plomp, T. (2010). Educational design research: An Introduction. In T. Plomp & N. Nieveen (Eds.), *An Introduction to Educational Design Research* (pp. 9–35). SLO. www.slo.nl

- Pramudiani, P., Herman, T., Turmudi, Dolk, M., & Doorman, M. (2022). How does a missing part become important for primary school students in understanding fractions? *Journal on Mathematics Education*, 13(4), 565–586. <https://doi.org/10.22342/jme.v13i4.pp565-586>
- Pusat Asesmen dan Pembelajaran. (2020). *AKM dan implikasinya pada pembelajaran [AKM and its implications for learning]*.
- Santos-Trigo, M., Reyes-Martínez, I., & Aguilar-Magallón, D. (2015). *The use of digital technology in extending mathematical problem solving reasoning* (pp. 298–309). https://doi.org/10.1007/978-3-319-22629-3_24
- Sellars, M. (2017). Numeracy across the curriculum: A pathway to critical thinking. *International Journal Innovation Creativity and Change*, 3(2), 75–83.
- Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences*, 17(1), 13–19. <https://doi.org/10.1016/j.tics.2012.11.004>
- Son, J.-W., & Senk, S. L. (2010). How reform curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics*, 74(2), 117–142. <https://doi.org/10.1007/s10649-010-9229-6>
- Sumpter, L. (2018). Grade 9 students' reasoning about division of fractions: What are their arguments anchored in? In *students' and teachers' values, attitudes, feelings and beliefs in mathematics classrooms* (pp. 135–143). Springer International Publishing. https://doi.org/10.1007/978-3-319-70244-5_13
- Tout, D. (2020). *Issues in the teaching of mathematics: Critical connections between numeracy and mathematics*. https://research.acer.edu.au/learning_processes/29
- Tsai, T.-L., & Li, H.-C. (2017). Towards a framework for developing students' fraction proficiency. *International Journal of Mathematical Education in Science and Technology*, 48(2), 244–255. <https://doi.org/10.1080/0020739X.2016.1238520>
- Wahyu, K., Kuzu, T. E., Subarinah, S., Ratnasari, D., & Mahfudy, S. (2020). Partitive fraction division: Revealing and promoting primary students' understanding. *Journal on Mathematics Education*, 11(2), 237–258. <https://doi.org/10.22342/jme.11.2.11062.237-258>
- Wirth, L., & Greefrath, G. (2024). Working with an instructional video on mathematical modeling: Upper-secondary students' perceived advantages and challenges. *ZDM–Mathematics Education*, 56(4), 573–587. <https://doi.org/10.1007/s11858-024-01546-2>
- Xiao, F., Barnard-Brak, L., Lan, W., & Burley, H. (2019). Examining problem-solving skills in technology-rich environments as related to numeracy and literacy. *International Journal of Lifelong Education*, 38(3), 327–338. <https://doi.org/10.1080/02601370.2019.1598507>
- Zhang, L., Shang, J., Pelton, T., & Pelton, L. F. (2020). Supporting primary students' learning of fraction conceptual knowledge through digital games. *Journal of Computer Assisted Learning*, 36(4), 540–548. <https://doi.org/10.1111/jcal.12422>
- Zulkardi. (2002). Developing a learning environment on realistic mathematics education for Indonesian student teachers. Doctoral dissertation. University of Twente.