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Bridging mathematics and communication: Implementing realistic mathematics education principles for skill development

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Abstract

Effective mathematical communication is essential for developing conceptual understanding and applying mathematics in real-life contexts. However, empirical studies consistently report that students' mathematical communication skills remain insufficiently developed, highlighting a critical gap in instructional practices that effectively foster these competencies. The Realistic Mathematics Education (RME) approach, grounded in six core principles—reality, levels, intertwinement, activity, interactivity, and guidance—has demonstrated promise in promoting meaningful and contextualized learning. Yet, there is limited empirical research that systematically investigates how these principles specifically support the enhancement of students' mathematical communication. This study addresses that gap by examining how the application of RME principles contributes to the development of communication skills in junior high school mathematics classrooms. Utilizing a qualitative research design, data were gathered from nine schools across three districts through focus group discussions, lesson plan analysis, and classroom observations. Participants included RME experts, education policymakers, school principals, and mathematics teachers. Thematic analysis was conducted to identify how the six RME principles are operationalized in practice and how they align with indicators of students' mathematical communication. The findings reveal that each principle plays a distinct yet interconnected role in supporting communication—particularly through meaningful contexts, dialogic interactions, and structured teacher guidance. The study offers empirical support for the broader adoption of RME and highlights its potential to increase teacher motivation and instructional quality. It further underscores the importance of sustained professional development—such as mentoring and lesson study—in enhancing teachers' capacity to implement RME effectively. Future research should focus on developing RME-based instructional materials and digital tools, including GeoGebra-integrated worksheets and context-rich problem sets, to amplify its impact within mathematics education in Indonesia.

Keywords: Communication Skills, Mathematics Learning, Realistic Mathematics Education Principles, Skill Development, Thematic Analysis

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Effective communication is recognized as one of the essential soft skills required in the 21st century. It can be cultivated through various educational activities and the integration of digital tools (Astuti et al., 2019; Jose, 2021). Broadly defined, communication is a process through which individuals achieve mutual understanding and influence one another by exchanging information. This information encompasses a wide range of content—including ideas, emotions, and skills—and may be conveyed through words, numbers, symbols, images, and other modalities, whether intentionally or unintentionally (Berelson & Steiner, 1964; Rogers & Kincaid, 1981; Shannon, 1997). Beyond everyday contexts, communication plays a pivotal role in academic disciplines, including mathematics.

Mathematical communication refers to the expression of mathematical ideas, both orally and in writing, using various forms such as images, symbols, graphs, tables, equations, and specialized terminology. It also involves the presentation, justification, and discussion of problem solutions (Gatsmir & Palupi, 2023). This form of communication is critical in facilitating the explanation of mathematical concepts and supports other core processes such as reasoning, problem-solving, and proof. Moreover, it assists learners in articulating and refining their mathematical thinking, thus deepening their conceptual understanding (Ekaludini & Darhim, 2020; Mulbar et al., 2019; Puspa et al., 2019; Wandari & Anggara, 2021). From a theoretical standpoint, communication functions as a vehicle for reasoning and proof, enabling learners to construct logical arguments and discern relationships between mathematical objects (Bagchi & Wells, 1998). Shannon and Weaver's theory further underscores the role of semantic communication in interpreting meaning within mathematical discourse (Klüver, 2012). Communication is also integral to successful collaboration in educational settings, particularly within cooperative learning environments (Wang et al., 2020).

Cooperative communication, which takes place in interactive and team-based learning settings, fosters mathematical dialogue among students as they discuss ideas, explain reasoning, and work collaboratively to solve problems. This interaction not only enhances students' mathematical communication—both oral and written—but also contributes to the development of social competencies and increases engagement through pedagogical strategies such as peer learning and group tasks (Budiono et al., 2020; Kholidah & Qohar, 2021; Marasabessy & Qohar, 2023).

Despite these benefits, several studies have identified substantial deficiencies in Indonesian students' mathematical communication skills (Darto, 2021). National assessments consistently show low levels of proficiency, despite various initiatives, such as the development of assessment tools that address cognitive, affective, and psychomotor aspects (Musdi et al., 2023; Nurfadhillah et al., 2018). These challenges are widespread, with evidence from multiple regions across Indonesia, including provinces on Java and Sumatra (Aziz et al., 2024; Jailani et al., 2020; Sanjaya et al., 2018).

On the international stage, Indonesian students also underperform. According to the 2018 PISA results, the average mathematics literacy score decreased from 386 to 379, while the OECD average stood at 459 (Habibi & Prahmana, 2022). The 2023 PISA results reflected a further decline to 366 (Balán, 2025). This deterioration is indicative of weakening mathematical communication skills, given the centrality of communication in mathematical literacy (Mitari & Zulkardi, 2019).

Among the key obstacles is students' limited ability to articulate mathematical ideas, attributed to inadequate conceptual understanding, the complexity of mathematical language, and insufficient language development (Yarman et al., 2021). Some students particularly struggle with written explanations and higher-order problem solving (Ekaludini & Darhim, 2020; Wandari & Anggara, 2021). These challenges highlight the need for more engaging and contextually relevant instructional strategies (Fauziyah & Jupri, 2020; Panggabean & Turmudi, 2022).

Pedagogical approaches such as collaborative learning, problem-based learning, and interactive discussions have been shown to foster communication skills. Such methods provide opportunities for students to articulate ideas, pose questions, and explain reasoning. Structured discussions—whether in whole-class or group settings—encourage active participation and improve clarity of expression

(Dallimore et al., 2008). Collaborative learning in particular supports peer interaction, which enhances both communication and reasoning (Guajardo et al., 2025; Poornesh et al., 2021). Similarly, Contextual Teaching and Learning (CTL) promotes relevance and engagement, which in turn supports the development of communication competencies (Medriano & Bautista, 2020).

Various instructional strategies—including cooperative learning, the integration of digital tools, and constructivist multimedia—have been shown to improve communication skills and student performance, as evidenced by gains in both pre- and post-test scores. Group work and collaborative environments provide authentic contexts for students to express their thoughts and engage in mathematical reasoning, thus reinforcing communication abilities (Palinussa et al., 2021; Saleh et al., 2017).

Among these strategies, the Realistic Mathematics Education (RME) approach has received considerable attention and empirical support. Despite variation in research designs, implementation strategies, and educational contexts, numerous studies have consistently demonstrated RME's positive impact on students' mathematical communication skills. Quantitative research using quasi-experimental designs has reported significant improvements in communication among elementary and secondary students taught with RME-based materials (Armiati et al., 2019; Noperta et al., 2020). Students exposed to RME generally outperform peers in conventional classrooms, as reflected in higher communication scores (Siregar et al., 2019). RME emphasizes student participation and interaction, creating opportunities for meaningful mathematical dialogue (Bayu et al., 2023; Purba & Surya, 2020). Moreover, combining RME with video-assisted instruction has been effective in promoting students' ability to express mathematical ideas (Kristiani et al., 2024).

Qualitative findings further support these conclusions, particularly with regard to students' ability to express mathematical ideas in response to real-life problems. This benefit extends across diverse learner profiles, including those who engage through visual, auditory, reading/writing, and kinesthetic modes (Ismail et al., 2020; Noperta et al., 2020; Palinussa et al., 2021). Classroom Action Research employing the Problem-Based Learning (PBL) model—which also emphasizes real-world problem solving—similarly demonstrates improvement in mathematical communication skills (Diponegoro et al., 2024; Panggabean & Turmudi, 2022). When combined with collaborative learning, RME has also been shown to enhance students' ability to articulate mathematical ideas and draw conclusions (Kristiani et al., 2024).

Despite the demonstrated effectiveness of RME, its implementation remains limited. Although RME incorporates core principles such as reality, levels, intertwinement, activity, interactivity, and guidance—which collectively foster communication skills—teachers often apply RME only at the beginning of lessons rather than throughout the instructional sequence (Reinke & Casto, 2022). This limited use diminishes its potential impact on student learning. Teachers frequently encounter challenges in designing tasks that are simultaneously mathematically rigorous and contextually relevant. Moreover, encouraging active participation remains a difficulty, particularly for those unfamiliar with interactive pedagogies (Rianasari & Guzon, 2024).

Additional barriers include teachers' perceptions that RME is unsuitable for lower-achieving students, insufficient professional development, and limited access to instructional materials and resources (Russo et al., 2020; Mariana et al., 2021). Cultural and contextual differences also influence the implementation of RME (Revina & Leung, 2019), while institutional factors, such as a lack of autonomy and inadequate support, further hinder its use.

To date, most studies have focused on evaluating the outcomes of RME, particularly in terms of student communication skills. However, limited attention has been given to the process of implementing

RME's core principles to support communication development. Addressing this gap, the present study seeks to contribute new insights by examining how RME principles can be applied systematically and effectively to enhance students' mathematical communication. In doing so, it aims not only to inform theoretical discourse and educational policy, but also to support teachers in adopting instructional practices that promote meaningful mathematical engagement.

METHODS

This study employed a qualitative research approach to investigate how the principles of Realistic Mathematics Education (RME) support the development of students' mathematical communication skills. The research design included three primary stages: (1) preparation, (2) data collection, and (3) data analysis.

Research Design

The initial phase of the study involved designing the research framework, which included articulating the research background, objectives, relevant literature, methodology, and research timeline. A detailed procedural plan was developed to guide data collection and analysis. Subsequently, data collection instruments were designed, comprising structured guidelines for focus group discussions (FGDs), classroom observation protocols, and document analysis frameworks—specifically, for analyzing lesson plans.

Data Collection Procedures

Data were collected through FGDs, classroom observations, and document analysis. The study received ethical approval from the Social and Humanities Ethics Committee of the National Research and Innovation Agency of the Republic of Indonesia. The data presented in this article represent one component of a broader qualitative investigation and were organized around thematic clusters to enable focused analysis (Forero et al., 2020).

Research Settings and Sampling

Three research sites were purposively selected based on the implementation of RME at the junior secondary school level: Palembang City (South Sumatra), West Bandung Regency (West Java), and Trenggalek Regency (East Java). Within each region, three junior high schools were selected, resulting in a total of nine participating schools. The selection of junior high schools was informed by Indonesia's consistently low performance in the Programme for International Student Assessment (PISA) from 2001 to 2018, which revealed deficits in students' mathematical reasoning and communication (Avvisati et al., 2019; Mitari & Zulkardi, 2019). This concern is particularly relevant in light of the 2022 PISA framework, which emphasizes reasoning competencies aligned with the objectives of RME (Zulkardi, 2022). Implementing RME in this context holds promise for addressing these persistent challenges (Kamaliyah et al., 2013).

Participants

The participants consisted of a diverse group of stakeholders, including RME experts, education office representatives at the district or municipal level, school principals, and mathematics teachers. All participants contributed to the FGDs, drawing from their respective professional roles and areas of expertise. RME experts provided insights into the current state of mathematics education post-pandemic and shared recommendations for nationwide implementation of RME. They also offered solutions to common challenges faced by teachers and school administrators. Education officials discussed local policy frameworks and expressed institutional support for the use of RME in schools. Teachers and principals reflected on their experiences with RME, identifying both enabling and constraining factors in its implementation.

Classroom Observation

In addition to the FGDs, classroom observations were conducted to gather empirical evidence of mathematical communication during instruction. Prior to these observations, researchers reviewed the lesson plans developed by participating teachers. Each observation session spanned a minimum of two instructional periods and focused on verbal and non-verbal interactions among teachers and students, with particular attention to peer discussions during collaborative tasks. Observational data were used to document the enactment of communication practices, the interactional dynamics of group work, and the use of questioning and scaffolding techniques. This method enabled the identification of communication patterns and challenges that may not be apparent through interviews or written assessments, thereby providing a more comprehensive understanding of how communication unfolds in real instructional settings.

Instruments

The FGD guide consisted of structured questions exploring participants' perspectives on RME, their assessments of implementation feasibility, perceived benefits, and the factors supporting or hindering adoption. Semi-structured interviews were also conducted with school principals to probe institutional influences on RME implementation. Following the classroom observations, participating teachers were interviewed to clarify instructional decisions, with a focus on strategies employed to create engaging and communicative learning environments.

Analytical Framework

The data were analyzed using thematic analysis, following the six-phase framework outlined by Sandhiya and Bhuvaneswari (2024): (1) familiarization with the data, (2) generation of initial codes, (3) identification of themes, (4) review of themes, (5) definition and naming of themes, and (6) production of the report. The analysis was divided into two complementary strands; both aimed at evaluating the role of RME in supporting students' mathematical communication.

The first strand involved content-based analysis focused on the RME principles of reality, levels, and intertwinement. This strand examined how these principles support mathematical communication, particularly in relation to the teacher's mastery of mathematical content and their ability to contextualize learning. Furthermore, the second strand entailed individual analysis of the remaining RME principles: activity, interactivity, and guidance. Each was explored for its specific contribution to communication development:

1. Activity: Examined student engagement through their responsiveness to teacher prompts, participation in class discussions, and demonstration of active listening.
2. Interactivity: Focused on collaborative learning processes, especially instances where students articulated group responses and negotiated meaning through dialogue.
3. Guidance: Assessed the quality and effectiveness of teacher scaffolding, including the use of prompts, feedback, and support strategies during instructional interactions.

Together, these analytical categories allowed for a nuanced interpretation of how RME principles manifest in classroom practice and contribute to the development of students' mathematical communication skills.

RESULTS AND DISCUSSION

The presentation of findings is structured according to the sequence of data collection, beginning with the results from the focus group discussions (FGDs), followed by findings from classroom observations. The discussion is organized around the six core principles of Realistic Mathematics Education (RME). The principles of reality, levels, and intertwinement are analyzed collectively under the dimension of learning content, while the principles of activity, interactivity, and guided reinvention are discussed individually based on their pedagogical manifestations.

Findings from the FGDs revealed several challenges that hinder teachers from implementing RME consistently. First, teachers reported difficulties in accessing comprehensive, contextualized instructional materials that reflect real-life situations across all three junior high school grade levels (Grades VII, VIII, and IX). The absence of a standardized curriculum aligned with RME principles poses a barrier to continuity in implementation. One teacher remarked:

> *"It is difficult for us to find realistic, contextual, and comprehensive learning materials for Grades VII, VIII, and IX. We also do not always teach the same grade level, as it can change from year to year. So even if we want to implement realistic mathematics, it is hard to do so consistently."*

Second, teachers expressed concerns over the limited availability of teaching aids and resources necessary for contextual instruction. Third, a lack of deep conceptual understanding of RME principles among teachers further constrained effective implementation.

Furthermore, a review of an eighth-grade teacher's lesson plan on linear equations revealed alignment with the five phases of inquiry-based instruction: observing, questioning, collecting information, reasoning, and communicating. During the questioning phase, students were encouraged to formulate their own questions, with the teacher providing scaffolded prompts when necessary. The communicating phase was particularly focused on developing mathematical modeling skills through guided questions, such as:

> *"What is the relationship between the manager's salary and total sales in this problem?"*

These structured prompts aimed to enhance students' mathematical communication by encouraging them to articulate relationships and represent problems symbolically.

Embedding the Reality Principle in Instruction

Classroom observations demonstrated various applications of the reality principle. In Trenggalek Regency, where a flood had recently occurred, teachers designed tasks that involved calculating the amount of aid distributed to flood victims using linear equations. In West Bandung Regency and Palembang City, teachers used familiar contexts such as mirror reflections to introduce geometric transformations, and mountainous roads to teach gradients and slopes. These examples connected abstract mathematical content to tangible, everyday experiences.

In seventh-grade classrooms in Palembang City, students engaged with practical problems involving the calculation of product prices and travel times, illustrating the use of one-variable linear equations. Student engagement was observed to increase particularly when teachers posed contextualized questions and invited individual responses. The majority of student answers were accurate, suggesting that the use of real-life scenarios facilitated comprehension.

Intertwinement of Content and Social Context

In several lessons, teachers integrated real-world events to strengthen conceptual understanding. For instance, recent flood events were used to contextualize tasks involving gradients and linear modeling. Students calculated aid requirements based on the number of flood victims, establishing connections between mathematics and environmental/social issues. These activities not only illustrated the reality and intertwinement principles but also highlighted the relevance of mathematics in civic contexts.

In West Bandung, local topography (e.g., hilly terrain) was used to frame discussions around slope and distance, reinforcing connections between mathematics and students lived experiences. The intertwinement principle was evident in how the algebraic domain (equations and variables) was linked with the geometric domain (Cartesian coordinates) and further extended to social themes such as disaster relief.

Classroom observations revealed that the principle of activity was consistently reflected in student behavior across lesson phases. Students actively responded to teacher questions, participated in collaborative assignments, and demonstrated attentive listening during peer presentations. Engagement was particularly noticeable during group problem-solving tasks and in the application of mathematics to real-life scenarios.

The principle of interactivity was prominently reflected in group-based learning activities, where students collaboratively solved mathematical problems and presented their solutions to the class. This instructional strategy promoted student discourse, encouraged the exchange of ideas, and fostered the co-construction of mathematical understanding. As illustrated in Figure 1, one teacher assigned a set of six linear equation problems to eighth-grade students during the first semester.

Across all observed classrooms, group work was conducted in small groups of four to six students. While the ideal instructional model encouraged regular rotation of group members to facilitate exposure to diverse viewpoints and enhance collaborative learning skills, many teachers permitted fixed groupings to accommodate students' preference for working with familiar peers. This decision, though a compromise, supported classroom harmony and sustained engagement.

The task illustrated in Figure 1 represents a typical outcome of student collaboration. The problems were designed to apply mathematical concepts to realistic contexts, aligning with the reality principle of RME. Specifically, one item (Problem 3) required students to model a post-disaster aid distribution scenario using linear equations. This task exemplifies the intertwinement principle by integrating multiple mathematical domains—namely, algebra (equations and variables) and geometry (Cartesian coordinates)—within a broader social context involving disaster relief.

The complete set of six problems assigned in the group task included:

1. Plot the points A(1,3), B(2,6), and C(3,9). What type of line is formed when these points are connected?
2. Calculate the gradient of the line connecting points A(1,3) and B(2,6).

3. A flood recently affected the region. Each victim receives 5 kg of rice as aid. Represent this situation using a linear equation: let the Y-axis denote the total rice aid and the X-axis denote the number of victims (for 1 to 6 individuals).
4. What is the gradient of the equation $y = 5x + 5$?
5. What is the gradient of the equation $2y = 8x + 16$?
6. Calculate the gradient of the line connecting points A(2, −3) and B(1, 3).

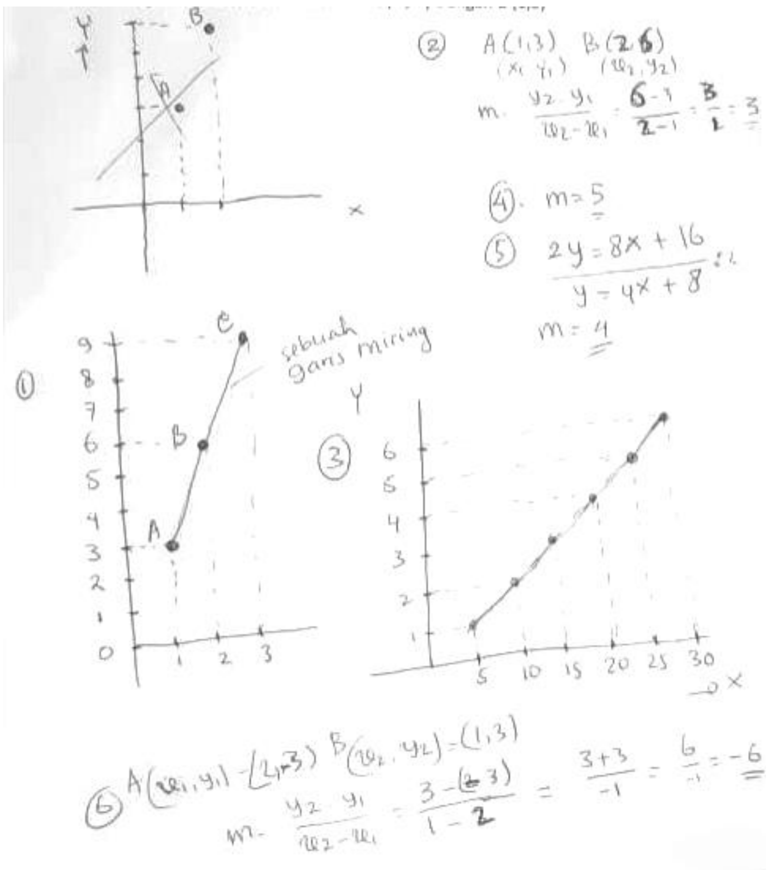

Figure 1. Group learning answer of provided task on linear equations in a sample junior high school

An example of student work on Problem 3 is shown in Figure 1, where students plotted the number of disaster victims against the total aid received. However, the plotted points lacked full coordinate labeling, omitting explicit reference to the abscissa and ordinate. Ideally, the labeled points would be A(1,5), B(2,10), ..., F(6,30). These values demonstrate a linear relationship where each additional victim corresponds to an increase of 5 kg of rice. The task effectively engages students in constructing and interpreting linear models relevant to real-world humanitarian efforts. Given that the number of disaster victims is typically known and publicly communicated, this task illustrates the authentic applicability of mathematics to societal needs.

Integration of Technology in RME-Based Instruction

Teachers employed a range of technological tools to support mathematical instruction. Most commonly, projectors were used to display digital lesson content prepared on laptops. Some educators integrated GeoGebra, a free and open-source dynamic mathematics software that combines geometry, algebra, statistics, and calculus in a unified visual environment. Its interactive features enabled dynamic visualization and manipulation of mathematical objects, enhancing students' conceptual understanding.

Students also utilized mobile phones for academic purposes. Specifically, they captured images of their completed worksheets and uploaded them to teacher-designated online platforms. This practice not only streamlined the submission process but also promoted digital literacy and accountability. Collectively, the integration of technology enhanced the visibility of student thinking, facilitated multimodal communication, and supported the goals of RME by enriching the learning environment.

The Principles of Reality, Levels, and Intertwinement and Their Relationship to Communication

In RME, the principle of reality emphasizes the use of meaningful, real-world contexts to facilitate students' understanding of mathematical concepts. By situating learning within situations familiar to students, mathematics becomes relevant and applicable beyond the classroom (Bray & Tangney, 2016; Heuvel-Panhuizen & Drijvers, 2020; Tamur et al., 2020). This contextualized approach supports the development of essential 21st-century competencies, including critical thinking, problem-solving (Szabo et al., 2020), collaboration, creativity (Anwar et al., 2024), numeracy, and logical reasoning (Pickering et al., 2025), as well as fostering self-confidence and learner autonomy (Caridade & Rasteiro, 2024). Engaging with real-life contexts allows students to connect abstract mathematical ideas with everyday experiences, thereby enhancing their ability to communicate mathematical reasoning and apply knowledge to practical problems (Sawatzki & Sullivan, 2018; Uy, 2022). Empirical research supports the claim that RME-based instructional materials—such as e-books and context-rich lesson designs—can significantly improve students' mathematical communication skills by offering opportunities to explore and articulate contextual problems (Ahmad et al., 2023).

The level principle in RME refers to the structured progression of learning from informal, intuitive understanding to formal, abstract mathematical reasoning. This pedagogical approach scaffolds students' communication and problem-solving skills by ensuring that cognitive development occurs gradually and meaningfully (Sa'id et al., 2021). The principle is congruent with the Indonesian national curriculum, which draws upon Piaget's theory of cognitive development and Bruner's spiral curriculum. These theoretical foundations emphasize the importance of sequencing concepts from simple to complex to promote deep understanding and long-term retention. As students' comprehension deepens through this structured progression, they become better equipped to communicate complex mathematical ideas with clarity and confidence (Sa'id et al., 2021).

The principle of intertwinement refers to the integration of different mathematical domains—such as arithmetic, algebra, geometry, measurement, data analysis, and probability—within and across learning experiences (Jupri, 2017). Beyond intra-mathematical integration, this principle extends to the connection between mathematics and other disciplines, including environmental science, culture, and societal issues (Arnellis et al., 2020; Bingölbali & Coşkun, 2016; Day et al., 2024). Such interdisciplinary interconnections enrich students' understanding of mathematical concepts and support the development of mathematical communication. As students engage with learning materials that connect mathematical ideas across domains and contexts, they become more capable of expressing, interpreting, and representing these ideas through multiple modalities, including language, symbols, diagrams, and tables (Fitriani & Rohman, 2021; Puspa et al., 2019; Zulhelmi & Anwar, 2021).

To effectively guide students in developing mathematical understanding, teachers must possess strong communication skills, particularly in presenting mathematical content accurately and clearly (Strömberg et al., 2022). According to Shannon and Weaver's model of communication, subject-matter expertise is a prerequisite for effective knowledge transmission (Fiske, 1990). However, many Indonesian teachers encounter difficulties in mastering core mathematical concepts, which undermines their instructional clarity and effectiveness. For example, Indonesian mathematics teachers often struggle with foundational concepts in numeracy and mathematical definitions. These challenges are partly attributed to limited capacity in designing contextual learning activities that encourage critical thinking (Adelia et al., 2024). At the elementary level, misconceptions persist, such as misinterpretations of repeating decimals, which can result in the transmission of incorrect knowledge to students (Rahaju et al., 2019; Yopp et al., 2011). Furthermore, teachers frequently encounter challenges in teaching advanced topics like three-dimensional geometry, which demand conceptual understanding and the use of appropriate visual models and instructional media (Asrafil et al., 2020).

The preparation of prospective teachers also remains a concern. While many pre-service teachers demonstrate theoretical knowledge of RME, they often lack the ability to distinguish RME from other pedagogical approaches or to formulate contextual problems that align with its principles (Yilmaz, 2019). These deficiencies suggest a need for targeted interventions to strengthen both content knowledge and pedagogical competence among current and future mathematics educators.

One such intervention is Lesson Study, a collaborative model of professional development in which teachers jointly plan, observe, and reflect on lessons. Originating from Japan, Lesson Study has been adopted in various countries as a strategy for enhancing instructional quality and student learning outcomes (Lundbäck & Egerhag, 2020; Runesson, 2013). By promoting structured reflection and peer learning, Lesson Study supports the continuous development of teaching practices aligned with RME principles. The Indonesian government has implemented a range of initiatives to strengthen mathematics teacher competencies, including international partnerships (e.g., IMSTEP-JICA, SISTTEM), national programs (e.g., PMRI workshops, teacher professional education), local collaborations between universities and regional education offices, and integration of STEM-oriented approaches (Ekawati & Kohar, 2016; Laksmiwati et al., 2020; Marsigit, 2015; Saito et al., 2006; Situmorang et al., 2018; Wijaya et al., 2018; Winarso & Udin, 2024). These efforts have been supplemented by continuous training models such as Lesson Study and other in-service programs.

Despite these various reforms, significant challenges remain. Variability in the quality of programs, inequitable access to professional development resources, and inconsistent institutional support continue to impede the overall effectiveness of these initiatives. To ensure that Indonesian students are adequately prepared with the communication and problem-solving skills required for success in 21st-century learning environments, strategic, equitable, and sustained interventions are essential.

The Principle of Activity

The activity principle in Realistic Mathematics Education (RME) posits that students learn most effectively when they are actively engaged in constructing knowledge, particularly through problem-solving processes (Heuvel-Panhuizen & Drijvers, 2020). Active engagement includes verbal and written communication, both of which are instrumental in developing mathematical understanding and communication skills (Budiono et al., 2020; Puspa et al., 2019). For instance, when a teacher asked, "What are the characteristics of an equation?", a student responded: "The result of the calculation before the equal sign is equal to the result of the calculation after the equal sign." This type of verbal expression reflects students' attempts to articulate mathematical reasoning and demonstrates an early stage of communication development.

Similarly, when students engage in written mathematical tasks—individually or collaboratively—they are required to express their reasoning in structured form, negotiating meaning through personal interpretation or group consensus. These solution-oriented activities are essential in fostering both conceptual understanding and communication competence. Previous research has established a significant positive relationship between students' problem-solving abilities and their mathematical communication skills. Strong communication supports the formulation and refinement of mathematical models, which are necessary not only for academic problem-solving but also for addressing real-world challenges (Erozkan, 2013; Kim & Jee, 2018).

In the classroom context, students' participation extends beyond producing written or spoken responses. They also engage in active listening, which constitutes a critical component of the activity principle. Active listening involves attentively focusing on the speaker's message, resisting distractions, and demonstrating cognitive and affective engagement with the content being delivered (Figl & Bauer, 2008; Spataro & Bloch, 2018). It encompasses both verbal techniques—such as paraphrasing, questioning, and confirming understanding—and non-verbal cues such as maintaining eye contact, nodding, and adopting a responsive posture. This form of listening fosters greater comprehension, minimizes misunderstandings, and helps to build a positive and empathetic classroom climate (Çiftçi, 2007; Figl & Bauer, 2008).

Moreover, active listening plays a crucial role in promoting communication, empathy, and collaborative problem-solving. When students listen respectfully and attentively, they are more likely to appreciate diverse perspectives, offer constructive feedback, and convey mathematical ideas with clarity and precision (Bletscher & Lee, 2021; Flavia & Enachi-Vasluianu, 2016; Sari et al., 2020). Observations from the current study indicate that students in all participating classrooms demonstrated active listening. This was evidenced by their ability to accurately respond to teacher questions—ranging from brief factual answers to more elaborated explanations—after listening to instructional prompts.

The development of active listening skills involves several strategies, including paraphrasing, asking clarifying questions, offering confirmations, and engaging non-verbally with the speaker (Bakhshandeh, 2023; Kerzner, 2023). However, to practice these strategies effectively, students must overcome internal and external barriers such as personal biases, emotional distractions, and environmental noise. These skills require continuous development through intentional practice and reflection (Kerzner, 2023; Sproull, 2018). A common classroom challenge that undermines active listening is off-task peer conversation (Flowers & O'Neill, 2005). Notably, classroom observations in this study did not reveal such behavior, suggesting a high level of attentiveness among students. Finally, students' attentiveness was likely supported by several key factors:

1. Effective classroom management, where the teacher successfully created a structured, respectful, and comfortable learning environment that encouraged students to pay attention.
2. Engaging teaching strategies, such as dynamic vocal delivery with strategic emphasis on important points, consistent eye contact, frequent movement around the classroom, and expressive body language and facial gestures, which collectively helped maintain student interest.
3. Contextually relevant learning materials, which made abstract concepts meaningful. For example, in a Grade IX class on transformations, students were tasked with creating a floor plan using the Cartesian coordinate system via GeoGebra—an activity that linked mathematical content to real-world applications and thereby enhanced student engagement.

The Principle of Interactivity

Unlike the principles of reality, levels, and intertwinement—which pertain to the content domain—the principle of interactivity is categorized under the domain of activity. This principle emphasizes interactions between students as well as between students and the teacher, with communication forming a central component. Interactivity plays a pivotal role in developing students' communication skills by fostering communicative competence (Agago et al., 2021; Krishnasamy, 2016), enhancing confidence and motivation (Braga & Da Silva, 2010; Hullman et al., 2010; Light & McNaughton, 2014), and promoting meaningful classroom dialogue (Chen, 2013; Henderson & Barker, 2018; Stockwell & Ito, 2022; Zhang & Wu, 2025).

Interactive learning practices—such as group work, collaborative problem-solving, and dialogic instruction—create opportunities for students to engage in both verbal and nonverbal communication. Although participation levels may vary among group members, structured collaboration can promote greater communication confidence (Agago et al., 2021; Krishnasamy, 2016; Omar et al., 2020). Moreover, interactive environments encourage students to co-construct knowledge, reflect on peer contributions, and negotiate meaning through shared problem-solving processes.

Numerous studies have highlighted the effectiveness of interactive strategies in fostering students' oral communication skills (Türkben, 2019; Winanti & Hartati, 2019). Dialogic engagement enhances students' ability to process mathematical ideas, articulate reasoning, and engage critically with their peers (Stefano et al., 2022; Sundari, 2018). The integration of technology further enriches interactivity by supporting real-time collaboration, dynamic visualizations, and peer feedback (Dumitrache & Gheorghe, 2021; Kaewunruen, 2019; Marchiori & McLean, 2022).

During classroom observations, group work was followed by student presentations, where three students from different groups were selected to present their solutions on the whiteboard. Each presenter explained their reasoning while addressing questions from classmates. This activity functioned not only as formative assessment but also as practice in public speaking, offering authentic opportunities for students to develop communication skills relevant to academic and professional contexts (Siddiqui, 2024; Timm & Bienvenu, 2011). Furthermore, presenting to peers supports the development of communicative clarity, reduces performance anxiety, and builds confidence (García-López et al., 2013; Kamaruddin, 2020; Picanço Marchand et al., 2024). It also cultivates critical thinking, promotes constructive peer feedback, and allows students to apply communication strategies in authentic settings (Penrose, 2024; Prosenjak & Lučev, 2020). These benefits reflect the implementation of the interactivity principle through lesson components such as "questioning" and "communicating."

In summary, the principle of interactivity fosters reciprocal communication and collaboration through structured dialogue and discussion. It enhances cognitive engagement and plays a critical role in developing deep mathematical understanding and communication competence.

The Principle of Guidance Reinvention

The principle of guided reinvention supports students in constructing formal mathematical knowledge through a structured yet exploratory learning trajectory (Gravemeijer, 1994). In this approach, students are encouraged to rediscover key mathematical concepts independently, while the teacher provides strategic, implicit guidance to scaffold their cognitive processes (Bayu et al., 2023; Passos et al., 2019). This guided pathway aligns with progressive formalization, wherein students evolve from informal intuitive reasoning to semi-formal model-based understanding and eventually to formal mathematical abstraction (Arnellis et al., 2020).

Empirical research underscores the efficacy of guided instruction in enhancing mathematical communication. Compared to conventional teaching methods, guided reinvention leads to significantly higher communication proficiency among students (Kurani & Syarifuddin, 2020; Suratno et al., 2019). For instance, Junaila and Yerizon (2021) reported an increase in students' mathematical communication scores from 60 to 70.5 after instruction grounded in this approach.

Guided reinvention supports the development of mathematical communication skills through multiple mechanisms:

1. Promoting Active Interaction and Discussion – Teachers encourage students to engage in problem-solving, share ideas, and justify their reasoning, thereby developing articulation skills (Shreyar et al., 2010; Solomon et al., 2021).
2. Redefining the Teacher's Role as Facilitator – Rather than delivering knowledge directly, the teacher adopts a facilitative stance, empowering students to take ownership of their learning and express ideas more freely (Shreyar et al., 2010; Solomon et al., 2021).
3. Embedding Real-World Contexts – The use of contextualized tasks grounds mathematical exploration in familiar experiences, which enhances both comprehension and the ability to communicate findings (Bayu et al., 2023).
4. Encouraging Collaborative Learning – Group discussions and collaborative tasks provide a platform for peer explanation and negotiation, reinforcing mathematical ideas while improving communication fluency (Solomon et al., 2021; Suratno et al., 2019).

Overall, the principle of guided reinvention creates a learning environment where students progressively formalize mathematical concepts through inquiry, reflection, and guided support. This process not only deepens conceptual understanding but also cultivates essential communication competencies required in both academic and everyday mathematical practices.

CONCLUSION

This study provides evidence that the six foundational principles of Realistic Mathematics Education (RME)—namely, reality, levels, intertwinement, activity, interactivity, and guided reinvention—hold considerable promise in advancing students' mathematical communication skills. The principles situated within the content domain (reality, levels, and intertwinement) facilitated students' conceptual understanding and enabled them to articulate mathematical ideas with greater clarity and coherence, including using paraphrasing. Meanwhile, the principles grounded in the domain of activity—activity, interactivity, and guided reinvention—fostered communication through dialogic exchanges, group-based problem-solving, classroom presentations, and teacher-facilitated rediscovery of mathematical concepts. These findings underscore the integral role of RME in supporting the development of both mathematical thinking and its communicative expression. Consequently, there is a critical need for ongoing teacher professional development to support the effective enactment of RME-based instruction. Such development should be pursued both individually and institutionally through initiatives that promote reflective practice, collaboration, and pedagogical innovation.

Notwithstanding these contributions, the study is not without limitations. The research was conducted within a bounded set of instructional contexts, which may not fully represent the diversity of teaching practices or student populations in broader educational settings. The reliance on qualitative data, primarily from classroom observations, introduces the potential for interpretive bias, despite efforts to maintain analytical rigor. Furthermore, the study did not investigate longitudinal outcomes or measure changes in students' communication skills over extended periods. Nor did it differentiate findings across students' varying levels of mathematical proficiency or sociocultural backgrounds, both of which may shape the effectiveness of RME-based approaches. These limitations suggest the need for further research employing more varied contexts, mixed-methods designs, and longitudinal data to better understand the sustained impact and contextual adaptability of RME principles.

Future research should build upon these findings by exploring the integration of digital technologies within RME frameworks to enhance mathematical communication. Investigations should examine teachers' readiness for technology integration, the pedagogical challenges involved, and the influence of digital tools—such as dynamic geometry software, culturally contextualized multimedia, and interactive learning platforms—on students' mathematical discourse. Furthermore, research might address how RME principles operate in online, blended, or remote learning environments, particularly when aligned with culturally responsive teaching practices. Collaborative models such as lesson study may be especially effective for supporting teacher learning and the iterative refinement of RME-based instruction. Ultimately, continued inquiry in this area can contribute to more inclusive, engaging, and communicatively rich mathematics classrooms, while informing curriculum design, instructional policy, and professional learning systems that prepare students to engage in meaningful mathematical reasoning and dialogue in both academic and real-world contexts.

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Declarations

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