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A PRAXEOLOGICAL ANALYSIS OF PRE-SERVICE ELEMENTARY TEACHER-DESIGNED MATHEMATICS COMICS

Zetra Hainul Putra[1], Dahnilsyah[1], Ayman Aljarrah[2]

[1]University of Riau, Jalan H.R. Soebrantas Km. 12,5, Pekanbaru, Indonesia
[2]Acadia University, Wolfville, Nova Scotia, Canada
Email: zetra.hainul.putra@lecturer.unri.ac.id

Abstract

Mathematical and didactic knowledge presented in mathematics textbooks and other resources, like mathematics comics (MCs), needs to be evaluated from a lens of appropriate theoretical framework in mathematics education before it can be used as a medium for teaching and learning mathematics. Therefore, this study investigates mathematical and didactic competencies that were reflected in MCs designed by pre-service elementary teachers. The framework for analysing mathematical knowledge embedded in these MCs is based on the Anthropological Theory of the Didactic, specifically a praxeology. This study utilized a content analysis technique within a qualitative approach. Thirteen MCs were analysed using a praxeological analysis; the type of task and techniques (praxis block) as well as the possible technology and theory (logos block). The findings demonstrate that the mathematical praxeologies embedded in MCs belong to five mathematical domains, namely numbers and operations; number theory; fractions, decimals, and percentages; ratio and proportion; as well as measurement. Additionally, the analysis revealed that seven of these MCs were related to a single domain, while the others belong to two or three mathematical domains. Concerning the mathematical praxeologies, most of MCs focus on presenting the practical blocks, the type of task and the techniques, while only a few could provide the theoretical lens to justify the practical blocks.

Keywords: A Type of Task, Logos, Mathematics Comics, Praxeological Analysis, Techniques

Abstrak

Pengetahuan matematika dan didaktik yang disajikan dalam buku teks matematika dan sumber lain, seperti komik matematika (KM), perlu dievaluasi dari kacamata kerangka teori yang sesuai dalam pendidikan matematika sebelum digunakan sebagai media dalam proses belajar mengajar matematika. Karena itu, studi ini menyelidiki kompetensi matematika dan didaktika yang tercermin dalam komik yang dirancang oleh calon guru sekolah dasar. Kerangka dalam menganalisis pengetahuan matematika yang tertanam dalam KM tersebut didasarkan pada Teori Antropologi Didaktik, khususnya praksiologi. Studi ini menggunakan teknik analisis konten dengan pendekatan kualitatif. Tiga belas KM dianalisis menggunakan analisis praksiologi; jenis soal dan teknik penyelesaian (blok praksis), serta kemungkinan teknologi dan teori (blok logos). Hasil studi ini menunjukkan bahwa praksiologi matematis yang tertuang dalam KM termasuk dalam lima domain matematika, yaitu bilangan dan operasinya; teori bilangan; pecahan, desimal, dan persentase; rasio dan proporsi; serta pengukuran. Selain itu, hasil analisis mengungkapkan bahwa tujuh dari KM tersebut terkait dengan satu domain, sementara lainnya termasuk dalam dua atau tiga domain matematika. Sehubungan dengan praksiologi matematis, sebagian besar KM berfokus pada presentasi blok praktis, tipe soal dan teknik, dan hanya beberapa yang dapat memberikan lensa teoritis untuk membenarkan blok praktis tersebut.

Kata kunci: Tipe Soal, Logos, Komik Matematika, Analisis Praxeologi, Teknik

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Currently, students encounter more challenges in their lives due to the ever-changing digital era, therefore, they need more skills necessary for success in their future careers. Critical thinking, creativity, communication, and social skills are some of the twenty-first-century competencies (21 ccs) that students need to deal with the modern societal challenges (As'ari et al., 2019; Novita & Putra, 2016; Suh & Seshaiyer, 2013). These skills are highly recommended to be incorporated into all subject areas, including teaching, and learning mathematics in several parts of Indonesia and around the globe. However, rote learning is still emphasized rather than providing students with rich activities that promote creativity and critical thinking.

To cope with the challenges of teaching mathematics in the 21st century, teachers need to create and develop learning activities to support students' creativity and critical thinking. The use of comics or picture books in supporting students' mathematical understanding has been popular for several decades (Toh et al., 2017; van den Heuvel-Panhuizen et al., 2016; van den Heuvel-Panhuizen & Elia, 2011; van den Heuvel-Panhuizen & van den Boogaard, 2008), especially in kindergarten. However, many comics or picture books used in mathematics teaching are not directly related to the subject. For instance, van den Heuvel-Panhuizen et al. (2016) used picture books from trade books in which mathematical topics were not the authors' intention. Therefore, teachers need to be critical and selective in choosing suitable picture books or comics to be exploited in teaching or may need to design the picture books or comics for mathematical purposes.

This study aims to analyse mathematics content in comics. In contrast to previous studies, it focuses on analysing MCs designed by pre-service elementary teachers. In a recent study (Putra, Witri, & Syahrilfuddin, 2020), which was the first study of teacher-designed MCs, the focus was to analyse the contexts applied by the authors to present the mathematics content. Furthermore, this study investigated other mathematical contents that appeared in MCs and how they communicate mathematical ideas to readers. Also, through the comprehensive analysis of mathematical ideas represented in MCs, this study elucidated how pre-service elementary teachers communicate mathematical knowledge to support students' learning through MCs.

Practical suggestions were provided to teachers on designing and selecting MCs for mathematics instruction. It also provided mathematics education researchers with a new theoretical lens to conduct content analysis of MCs or other resources. Moreover, the analysis of MCs offered in this study may stimulate new paradigms that are valuable for future debate and may lead to a more in-depth analysis of MCs.

Mathematics Comics

Comics or picture books comprise texts and pictures in which the images play an essential role in communicating and promoting understanding (van den Heuvel-Panhuizen & Elia, 2012). The presentation of mathematical ideas through comics or picture books helps children to grasp mathematical concepts in an exciting and engaging manner. Children are likely fond of reading books and more engaged in learning mathematical concepts when pictures are present. According to Ardiansyah and Setyadi (2014), MCs are mixture of drawings and texts that act as content translators. In this study, MCs are characterised as comics mixed with texts, illustrations, and mathematical principles, which are interrelated to students' experiences in their everyday lives.

Previous studies found that MCs or mathematics picture books improved students' motivation and mathematical skills (Indaryati & Jailani, 2015; Kurniati, 2017; Negara, 2014; van den Heuvel-Panhuizen & Elia, 2011; van den Heuvel-Panhuizen & van den Boogaard, 2008). For example, Van den Heuvel-Panhuizen and Elia (2011), which examined nursery school children in the Netherlands, observed a significant impact of reading mathematics picture books on children's calculation ability. This is consistent with Indaryati and Jailani (2015) which found an increase in grade five students' learning outcomes and motivation after being exposed to MC-based learning.

MCs or mathematics picture books often provide opportunities to create and handle informative discussions while selecting the correct text (van den Heuvel-Panhuizen & van den Boogaard, 2008). Van den Heuvel-Panhuizen and van den Boogaard (2008) proposed specific criteria to select and design MCs or mathematics picture books that can promote students' mathematics knowledge. These criteria include valuable and convincing tales about students' daily life and experiences, as well as mathematical terms that are less complicated or filled with mathematical jargon. The comics seemed to improve students' logical thinking during the lesson. Therefore, they can be considered as good criteria for the design and development of MCs.

Theoretical Background for the Analysis of MCs

Several approaches have been developed to study and analyse mathematics textbooks, apart from MCs. Wijaya, van den Heuvel-Panhuizen, and Doorman (2015) developed an analysis framework to investigate the characteristics of tasks in textbooks from four perspectives, namely the type of context used, the purpose of context-based tasks, the type of information provided, and the type of cognitive demands of the tasks. Furthermore, Pepin and Haggarty (2001) proposed an approach to analyse mathematics textbooks by suggesting four areas of analysis, namely the knowledge represented in books and beliefs about the nature of mathematics as reflected in texts, the pedagogical intention of textbooks addressing various ways students are guided to learn the content, the sociological contexts of books to evaluate the adaptation of texts referring to the different students' performance levels, and the cultural traditions portrayed in books depicting the national curricular goals.

Mathematical knowledge represented in textbooks was the primary focus of recent studies (Abdullah & Shin, 2019; Hidayah & Forgasz, 2020; Miyakawa, 2017; Quiroz & Rodríguez, 2015; Wijayanti & Winsløw, 2017). Wijayanti and Winsløw (2017) developed a new method for textbook analysis based on praxeological reference models which focused on specific content. This method suggests that the mathematical knowledge represented in textbooks can be analysed in tasks and techniques. Subsequently, they can be interpreted and supplemented by a discussion and analysis of technological or theoretical discourse. Miyakawa (2017) analysed mathematical contents from three aspects of proof taught in geometry, namely statement, evidence, and theory. The study focused on the technological and theoretical discourse in other evidence to be taught in French and Japanese lower secondary school mathematics. Also, Quiroz and Rodríguez (2015) focused on the praxeological analysis of mathematical modelling in primary education textbooks. They stated that praxeological elements can be exploited as a model to describe the mathematical modelling cycle that appeared in the text.

In the present study, textbook analysis methods developed by Wijayanti and Winsløw (2017) were adopted, while praxeology as a model was used to describe the mathematical contents that appeared in the MCs. The following section briefly describes the theoretical framework used in analysing MCs.

The Theoretical Framework of the Analysis of MCs

The theoretical framework that underpins this study is the *Anthropological Theory of the Didactic* (ATD) (Barquero & Bosch, 2015; Chevallard, 2006; Putra, 2020; Winsløw et al., 2014). The theory provides a detailed tool to model human actions through the notion of *praxeology* which links human practice (know-how) and theory (knowledge). According to Chevallard (2006):

> … a praxeology is, in some ways, the basic unit into which one can analyse human action at large. (The concept of praxeology is essential to praxeology as a science – in the dictionaries' definition of the word.) What exactly is a praxeology? We can rely on etymology to guide us here – one can analyse any human doing into two main, interrelated components: *praxis*, i.e. the practical part, on the one hand, and *logos*, on the other hand. *"Logos"* is a Greek word that has been used steadily to refer to human thinking and reasoning – particularly about the cosmos from pre-Socratic times. [...], and how do [*praxis* and *logos*] affect one another? The answer draws on one fundamental principle of ATD – the anthropological theory of the didactic –, according to which no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast. *Praxis* thus entails *logos* which in turn back up *praxis*. For *praxis* needs support – just because, in the long run, no human doing goes unquestioned.. (p. 23)

The above quotes clearly describe human actions (including doing mathematics) which consist of two interrelated components, namely *praxis* and *logos*. The praxis or practical block can be classified into two inseparable units, *a type of task* and *techniques*. The type of task in ATD is broadly understood, but could be explicitly stated (Rasmussen, 2016). A technique can serve as a way to solve the task. An example of a mathematical task is how to add two whole numbers, while the technique for solving can be a standard procedure. The logos or theoretical block also consists of two interrelated components, which are *technology* and *theory*. A technology implies a justification or explanation for the techniques. One may argue that the positions of the number determine their value. In this case, place value is the standard theory to explain the given technology.

To analyze the pre-service elementary teacher-designed MCs, praxeology was used as a basic theoretical model to describe mathematical knowledge shared in MCs. The practical blocks were also examined as they could be directly identified and presented in MCs, while the theoretical block could be explicitly interpreted. Thus, this study is driven by the following questions:

RQ1. What mathematical praxeologies appear in pre-service elementary teacher-designed MCs?

RQ2. How can the mathematical praxeologies be elaborated between praxis and logos to support students' learning mathematics in MCs?

METHODS

Research Approach and MCs for the Analysis

This study applied a content analysis technique within a qualitative approach. According to Cohan, Manion, and Morrison (2007), this technique is a theoretical tool to analyse the broad spectrum issue where information quality is the foundation for inference. Also, appropriate categories and units of analysis can be meticulously identified through this approach.

In the present study, the content to be analysed is presented in 13 MCs with titles and the number of pages for each MCs in Table 1. The MCs were designed and written by thirteen second-year pre-service elementary teachers from a public university in Riau, Indonesia. The MCs were final projects from a mathematics education course for upper grades of the elementary school offered in the second semester of 2019. Each MC was written by pre-service elementary teachers. In the instructional process, each group could present and discuss their MC with other students and the instructor (the first author).

Table 1. The MCs designed and written by the pre-service elementary teachers

MC Code	Title	Number of pages (Not including the cover)
MC1	Our Beautiful Bracelet	9
MC2	Hobbies Bring us Together	7
MC3	Come on, Let's Share!	5
MC4	Helping my Mom	6
MC5	Fishermen Village	5
MC6	Let's Find Out!	6
MC7	Sarong for the Father	5
MC8	This is our Clog	8
MC9	Siak Palace	13
MC10	Aris' New Books	5
MC11	Mathematics Professor	13
MC12	Harvesting Watermelon	7
MC13	What is the Price of the Shoes?	5

The Procedure of MC Analysis

Following Wijayanti and Winsløw (2017), the thirteen MCs were analyzed using a praxeology from the *Anthropological Theory of the Didactic* (Chevallard, 2006). The praxeology serves as a tool to evaluate the mathematical contents presented in MCs. The research team created and used a praxeological reference model from which the design was constructed based on the content to be analysed, allowing the analysis to be completely explicit (Putra, 2019; Putra & Winsløw, 2018; Wijayanti & Winsløw, 2017).

Data Analysis

The analysis of MCs was meant to investigate the mathematical praxeologies that appeared in MCs. For this objective, an analysis framework (Table 2) was developed to address the mathematical praxeologies, type of tasks, techniques, and logos. The technologies and theories (logos part) were not described separately due to the challenge of differentiating from the MCs. Besides, the part of the logo could be inferred from the explanation of techniques presented to solve a mathematical task.

Table 2. Analysis framework for MCs

Categories	Explanation
Domains	Mathematics domains presented in MCs were categorised into numbers and operations, number theory, fractions, decimals and percentages, ratio and proportion, as well as measurement.
Types of Tasks	Types of tasks presented in MC were interpreted from the authors' described tasks. An MC can provide a single and several types of tasks. For instance, MC12 (Harvesting watermelon) provided two tasks, namely subtraction of fractions and finding the discount.
Techniques	Mathematical techniques presented in MCs were differentiated from two broader categories, namely non-standard/non-procedural techniques, such as using tens, fives, and units to solve a task, subtracting of the whole number, as well as standard/procedural technique, such as using algorithm to solve that task.
Logos	Logos presented in MCs were interpreted from what the authors directly wrote in MCs, which includes techniques learned at school. Alternatively, they were explicitly presented in their mathematical explanation, such as place value used for adding two-digit numbers.

The first author conducted a praxeological analysis of the thirteen MCs and classified them into five mathematics domains (Table 2). The classifications were based on the mathematical knowledge shared in each MC. Some of which were classified into a single mathematical domain, while others involved 2 or more. Subsequently, the analysis extended to the mathematical praxeologies that appeared in those MCs. The research team evaluated the type of mathematical tasks presented in MCs and the techniques for solving them. Also, the logos were considered to explain the practical blocks. For questionable information, the first author discussed with the other two research teams and even requested some information from the MCs' authors.

RESULTS AND DISCUSSION

Thirteen MCs were grouped based on the mathematical domains. As previously noted, some of them were parts of a single domain, with others belonging to two or more. The mathematical domains include numbers and operations; number theory; fractions, decimals, percentages; ratio and proportion as well as measurement. The mathematical praxeologies were subsequently analysed from these domains.

The Praxeologies Incorporated into the Domain of Numbers and Operations

Table 2 illustrates a summary of mathematical praxeologies related to numbers and operations. Two MCs (MC5 and MC10) were classified under this domain, and each provided two types of mathematical tasks. As demonstrated in Table 3, only three types of tasks involved a single operation. The number facts are the standard technique presented in these MCs, influenced by the numbers involved in the task. Although the MCs did not provide technology nor the theory to justify those techniques, some possible theoretical blocks can be interpreted to explain the practical block, such as place value and distributive property of multiplication.

Table 3. The mathematical praxeologies of numbers and operations

Types of tasks	Techniques	Logos
Subtraction of whole numbers	Subtracting by ten, five, and units	Place value
Multi-digit addition	Number facts	-
Multi-digit multiplication	Decomposing and subsequently multiplying by tens and units	Distributive property of multiplication and place value
Multi-digit multiplication and subtraction	Number facts	-

To illustrate how mathematical praxeologies appear in MCs, an example was taken from MC5. This comic proposed two tasks, namely subtraction of whole numbers and multi-digit multiplication. Figure 1 shows the task type of multi-digit multiplication. The mathematical tasks were presented as fishers selling their fish in a traditional market. A buyer asks about the price and demands to buy 18 fishes. Subsequently, the fisher explains that the price for a fish is IDR 3,000 and further analyse the price for 18 by decomposing 18 into 10 and 8 and multiply each number by 3 instead of 3,000 to get 30+24=54. Finally, the fisher illustrates that three 0s should be added behind 54 to get IDR 54,000. The distributive property of multiplication could be the theoretical discourse underpinning the technique of decomposing and multiplying by tens and units. Also, place value implicitly appeared at the end of the fisher’s explanation, and it was included as theoretical discourse that can be used to justify the technique.

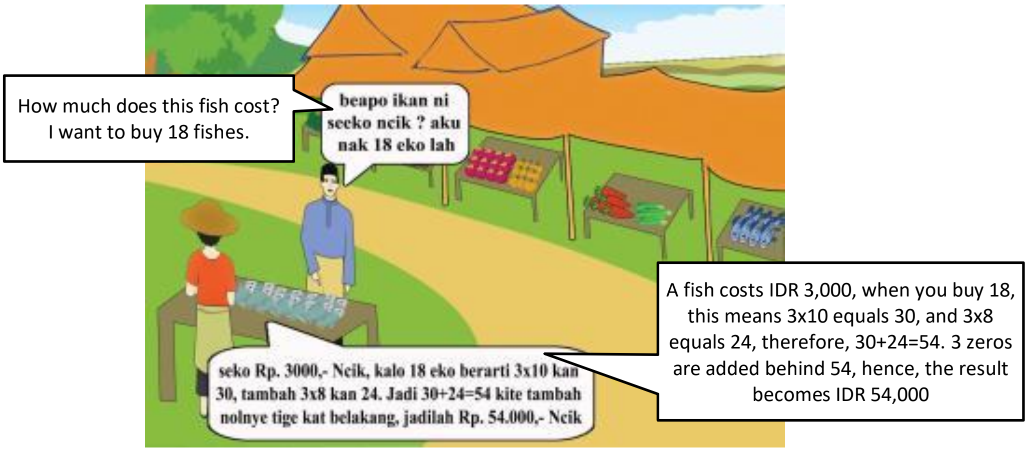

Figure 1. An illustration of a mathematical task and technique from MC5

The Praxeologies Incorporated into the Domain of Number Theory

Table 4 shows a summary of mathematical praxeologies related to number theory. Three MCs (MC1, MC2 and MC11) were grouped in this domain and each presented a type of mathematical task. According to Table 4, there are two types of tasks, which are finding the least common multiple and greatest common divisor. Using prime factorisations is the standard technique provided on both task types, while reasoning is built on what students learned at school, which automatically becomes a technological discourse to justify this common technique.

Table 4. The mathematical praxeologies of number theory

Types of tasks	Techniques	Logos
Least common multiple of two integers	Using prime factorisations.	-
Greatest common divisor of three integers	Using prime factorisations	These are the techniques learned at school.
	Table method	-

An example can be taken from MC1 to illustrate how mathematical praxeology has exploited this domain. MC1 proposed a task of finding the greatest common divisor of three integers. The mathematical tasks were presented in the context of two girls making bracelets with three different colourful beads, namely 90 red, 60 blue, and 36 purple. They want to make bracelets and share the beads from each group into the same number: how many bracelets can they make? Two techniques were proposed to find the number of bracelets. The first is based on the table method. According to Figure 2, the process was initiated by finding the smallest prime number that could divide the three numbers until no prime number could divide them. Here, 2 is a prime number that can divide 90, 60, and 36 beads to give 45, 30, and 18 (see Figure 2). The following prime number is 3 which gives 15, 10, and 6. Since there are no more standard prime numbers that can divide these numbers, the process is terminated as it results in 6 numbers of bracelets.

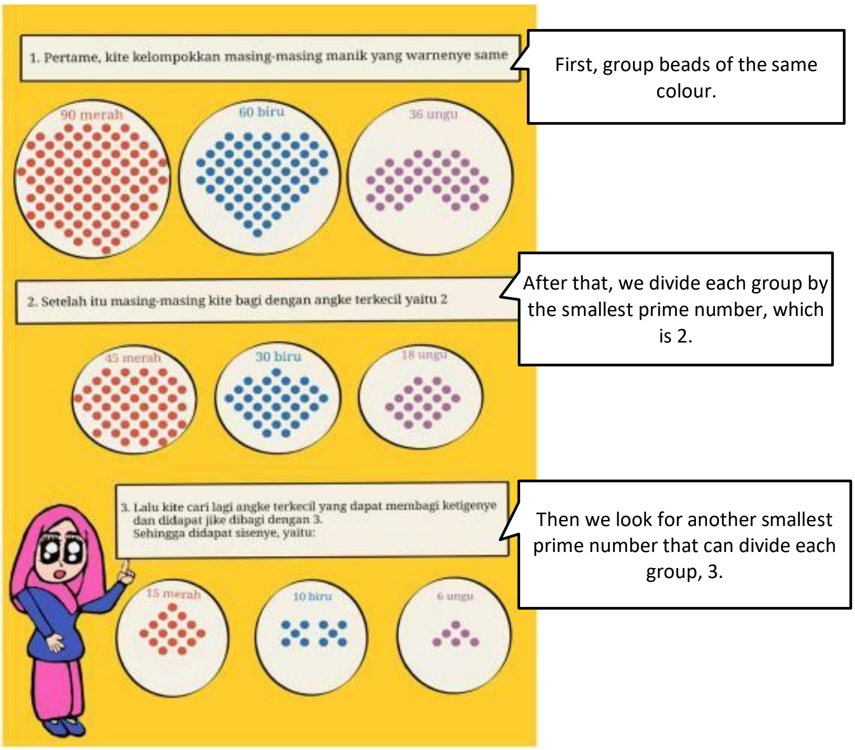

Figure 2. An illustration of a mathematical technique based on a table method

The second technique is based on prime factorisations, and the technological discourse behind this technique is related to what the students learned at school (see Figure 3). As illustrated in Figure 3, a girl remembers how to figure out the maximum number of bracelets that can be made with the same numbers and colours using the most effective common divisors as learned at school.

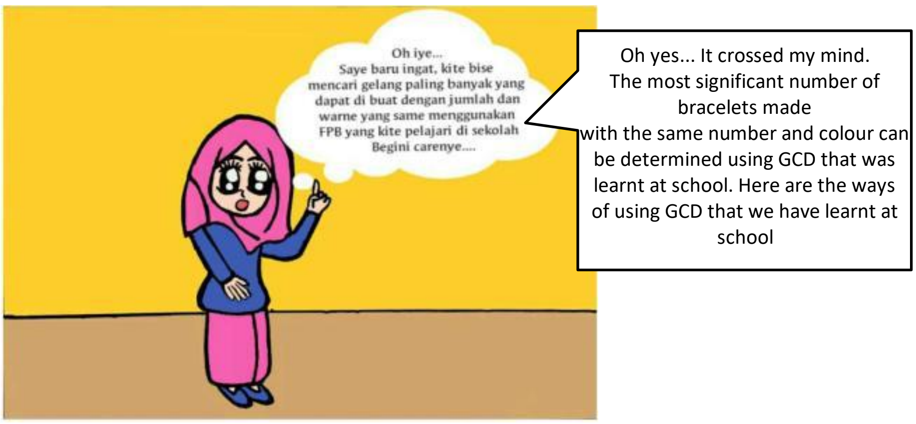

Figure 3. An illustration of a technological discourse for the prime factorisation technique

The Praxeologies Incorporated into the Domain of Fractions, Decimals, and Percentages

Five MCs (MC3, MC6, MC7, MC12, and MC13) were classified under this domain, four

(MC3, MC6, MC7, MC13) of which presented one type of mathematical task, while MC12 consists of two tasks, namely subtraction of fractions and finding the discount (see Table 5). Finding the discount can be considered a common task presented in this domain because three MCs accommodated this task. Meanwhile, the type of task about comparing fractions proposes more techniques than the others. There are also some technologies (logos part) to justify these techniques and how the teacher at school teaches students by exploiting these MCs by presenting the formula or using standard procedures to solve the tasks.

Table 5. The mathematical praxeologies of fractions, decimals, and percentages

Types of tasks	Techniques	Logos
Subtraction of fractions	Mental calculation	
Comparing fractions	Change both fractions into fractions with a common denominator,	The teacher at school teaches these techniques
	Change fractions into decimals, and	-
	Change fractions into whole numbers based on their units and using the scales.	Heavier can be proven using the scales
Multiplication of fractions	Mental calculation	-
Finding the discount	Proportional reasoning	-
	Change the percentage into a fraction with the denominator of 100, followed by the multiplication of a fraction.	Using formula

MC6 was the only MC that proposed a task with three different techniques. The mathematical tasks were presented in an account of a boy struggling to compare the weight of ¼ kg of red chili peppers and ½ kg of onions because the volume of the former is greater than the latter.

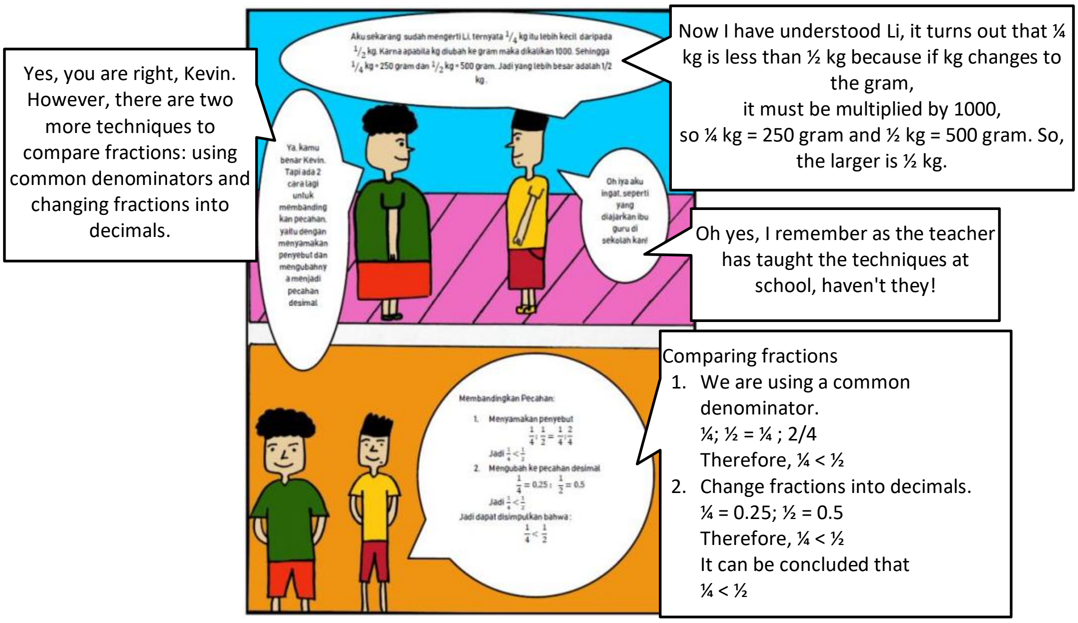

Figure 4. An illustration of three different techniques for the task of comparing fractions

Figure 4 presented the three techniques employed to solve the task of comparing fractions. The first technique is to change fractions into whole numbers based on their units. This technique is supported by using the scale. It is a practical, non-standard technique to be understood by students in an informal situation. The two standard techniques, using common denominator and changing fraction into decimals, are the teacher's techniques at school, and accordingly, this reasoning becomes their technological discourse.

The Praxeologies Incorporated into the Domain of Ratio and Proportion

Six MCs (MC3, MC4, MC7, MC10, MC12, and MC13) were classified under this domain. Three of these MCs contained one type of mathematical task, and the others presented two types (see Table 6). The most common task presented in these MCs is to find the value of *n* objects when given the value of an object. The task of finding the discount intersects with the praxeologies, which are part of fractions, decimals, and percentages domains. The proportional reasoning is the most common technique presented in these MCs. This could be influenced by the tasks presented in MCs' regular contexts, and the numbers involved could be understood using the known fact. Although MCs do not explicitly state the part of the logo, the mathematical thinking process in some MCs can serve as reference, and a linear mapping can be interpreted as one of the technological discourses to justify the proportional reasoning technique.

Table 6. The mathematical praxeologies of ratio and proportion

Type of tasks	Techniques	Logos
Given the value of an object, find a value for *n* objects	Proportional reasoning	Linear mapping
	Number fact	-
	Cross multiplication	-
Finding the ratio	Comparing two units	Ratio
Finding the discount	Proportional reasoning	-
	Change the percentage into a fraction with the denominator of 100, followed by the multiplication of the fraction.	Using formula

Figure 5 illustrates a praxeology which is part of the domain of ratio and proportion from MC7. This MC presented the mathematical task in a narrative recount where a boy saves money to buy a sarong for the father. The boy plans to save IDR 2,000 a day, hence, it is necessary to determine the number of days required to save IDR 40,000. This task is classified as giving the value of an object to find that of the *n* objects. The conversation presented in Figure 5 shows cross multiplication and proportional reasoning are mathematical techniques to solve such a task. From Figure 5, the mother explicitly states the cross-multiplication technique at the beginning of the conversation as follows, "…the technique is to multiply the days by the amount of money saved each day….". Based on this technique, the boy, Doni, develops his thinking through proportional reasoning. This could be seen from the statement, "In 5 days, I can save IDR 10,000, therefore, in 10 days I can save IDR 20,000 because 10 days times IDR 2,000 equal IDR 20,000". The boy can compare the amounts of money saved using multiplicative thinking. Although MC7 does not explicitly state any theoretical discourse to justify those techniques, one could interpret that linear mapping played an essential role in the conversation in Figure 5.

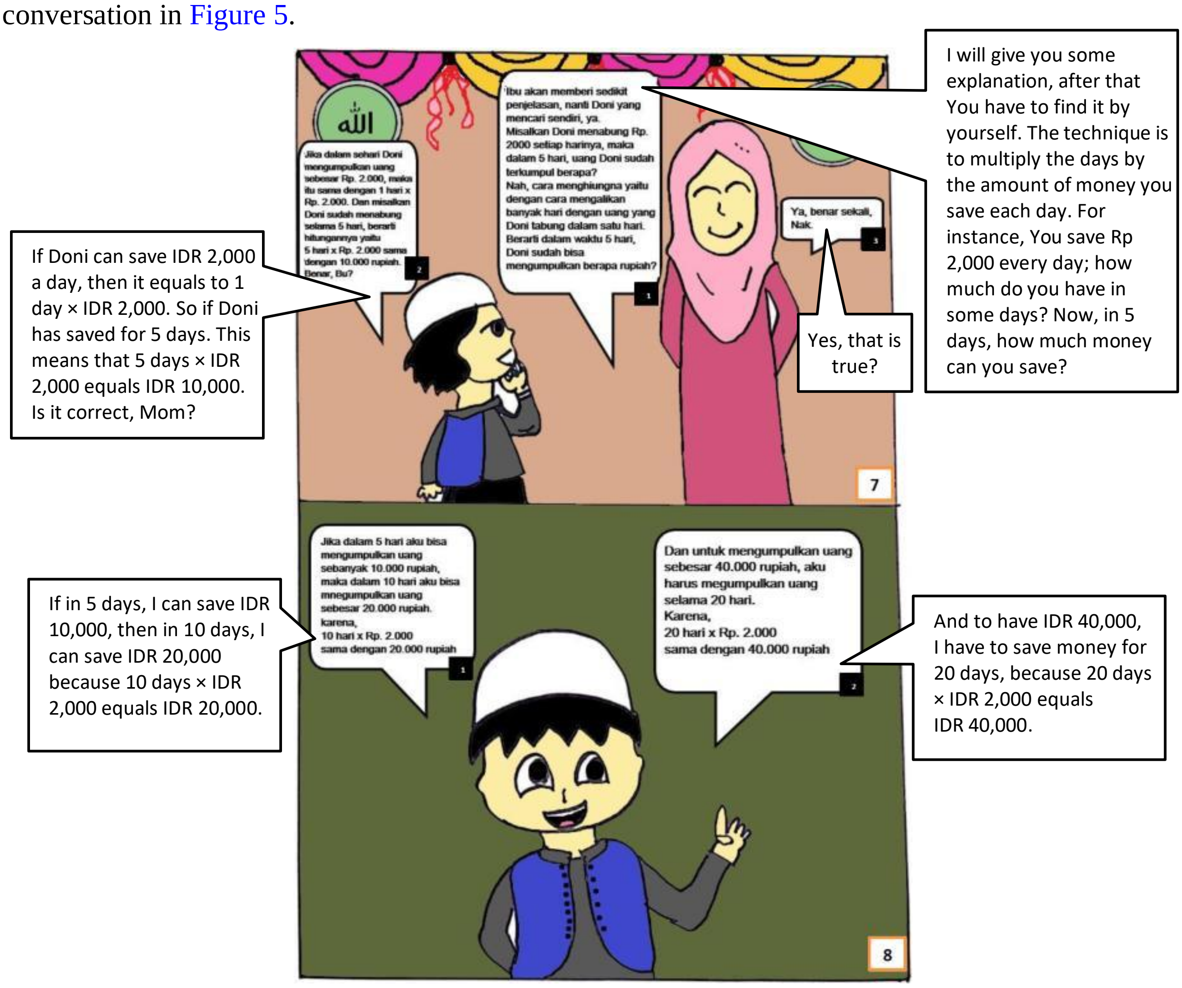

Figure 5. An illustration of techniques for the task of ratio and proportional reasoning

The Praxeologies Incorporated into the Domain of Measurement

Four MCs (MC3, MC4, MC8 and MC9) were grouped in this domain. Each MC presented one type of mathematical task. Table 7 shows a summary of mathematical praxeologies related to measurement, and it also shows there were three types of tasks. MC3 and MC4 presented the task of measuring weight, and they intersected with the praxeologies incorporated into the domain of fractions, decimals, and percentages.

In contrast, the task of measuring length intersected with the praxeology and the domain of number operations. Two techniques were associated with a formal or standard mathematical procedure, such as the conversion to units. For instance, the time measurement task proposed a mathematical situation of finding the equivalent time in years for 16 centuries. The technique is based on the known fact that a century is a hundred years, therefore, multiplying 16 by 100 gives 1,600. The other two tasks focused on concrete measurement activities. For example, during measurement of length, students must use a ruler to measure some objects to make a pair of clogs (see Figure 6).

Figure 6. A pair of clogs in Indonesia is called *Bakiak*

The mathematical task is presented in a problem-solving situation. In line with the part of the logo, MC3 and MC4 implicitly stated possible discourses to justify those techniques. For instance, it is stated in MC4 that the scale is an instrumental technique to show and compare the weights of two objects, and this idea can be used as a technological discourse to justify a technique of comparing two measuring numbers, such as $\frac{1}{2}$ kg and $\frac{1}{4}$ kg.

Table 7. The mathematical praxeologies of measurement

Type of task	Techniques	Logos
Measuring time	Converting the time units	-
Measuring length	Using a ruler to measure the length of objects	Problem-solving
	Decomposing tens and units and dividing using number sense	Mixed number operation taught by the teacher
Measuring weight	Using a scale to measure the weight of objects	The heavier one can be proven by using the scales
	Converting the weight units	The teacher at school teaches the techniques.

To illustrate how the praxeology appeared in these MCs, an example was taken from MC8. This MC proposes a task in a set of non-standard mathematical tasks. This mathematical task is about three students who want to make a pair of clogs (see Figure 7), so they are required to measure the length of wood and decide where they must put the wood's rubbers. Figure 7 shows the different techniques used to solve the task. First, they use a ruler to measure the rubber's width, and this technique is known as using a standard measurement unit to measure the length. Since the task involves several mathematical praxeologies, including number operations, the kids in Figure 7 discussed arithmetic operations, such as multiplication and division of natural numbers.

Interestingly, the kids used a technological discourse to justify those techniques related to arithmetic operations. They used the term of mixed number operation, as this is the terminology their teacher uses at school. Therefore, it can be concluded that the praxeologies students learn at school are primarily used and presented in most MCs.

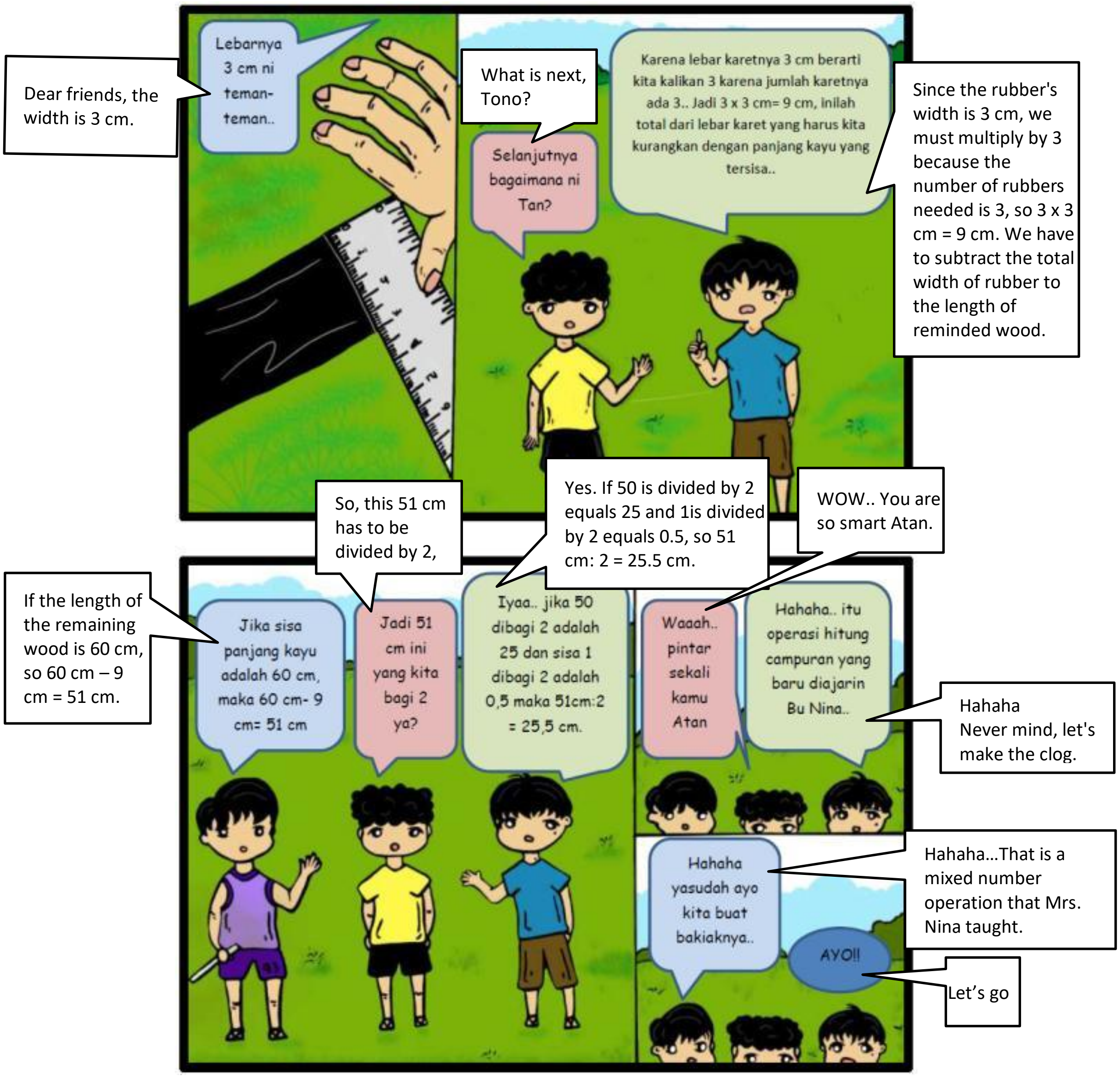

Figure 7. An illustration of techniques for the task of measurement

Discussion and Answering Research Questions

This study aims to investigate and analyse the mathematical praxeologies in teacher-designed MCs. The analysis mainly focused on the practical block and also elaborated on the theoretical block to some extent. The results summarised and explicitly answered questions concerning the two research questions.

The mathematical praxeologies that emerged in the thirteen teacher-designed MCs were associated with five mathematical domains, namely numbers and operations; number theory; fractions, decimals, percentages; ratio and proportion; as well as measurement. Seven MCs were classified as a single domain, while the others belonged to two or three domains. Furthermore, six were associated with praxeologies in ratio and proportion, while five were in fractions, decimals, and percentages domains. Interestingly, four of these MCs proposed mathematical praxeologies based on these two domains (i.e., ratio and proportion as well as fractions, decimals, and percentages). For instance, MC7, Sarong for the Father, provided two types of tasks: given the value of an object and find the value for *n* objects, and find the discount. The first task was included in ratio and proportion, while the second belongs to both domains. Concerning the practical block, some MCs accommodated more than a single type of task followed by some techniques to solve them. All the tasks are presented in real-life contexts, such as daily life and social activities. This criterion was suggested by Heuvel-Panhuizen and Boogaard (2008) as a stimulus for students' mathematical thinking when they learn using mathematics picture books or MCs. Additionally, some teacher-designed MCs are rich in mathematical concepts regarding the techniques proposed and the domains involved. For example, the task of finding the greatest common divisor of three integers (MC1) suggested two different techniques, initiated by presenting a non-standard procedure, followed by the standard technique using factorisation.

Real-life contexts in MCs provide opportunity-to-learn for students in solving context-based tasks. Students' experiences in various contexts provide a meaningful foundation for the mathematical principles they have to learn (Cooper & Harries, 2002). Also, the contexts from everyday life can be utilised as a didactical tool to help students learn mathematics (Wijaya et al., 2015). Consequently, MCs designed by pre-service elementary teachers could be used as mathematics instructional learning in elementary school.

Most of MCs focused on the practical rather than the theoretical block. Several others presented a type of task followed by a technique to solve it. When MCs propose an alternative technique to solve a task, mostly a standard procedure or practice, a technological discourse is provided for justification. The technology depended more on the direct instruction of teaching the standard algorithm that attempts to teach mathematics. This result is consistent with Wijayanti and Winsløw (2017), which discovered that Indonesian school textbooks were more concerned with the tasks and techniques, and most Indonesian teachers use textbooks closely in the classroom (Wijayanti & Winsløw, 2017). Therefore, pre-service teachers who developed mathematical praxeologies on designed MCs were greatly influenced by mathematical knowledge they gained as students at elementary school. Besides, lack of mathematics instructional learning in teacher training and education, which support pre-service elementary teachers' mathematical reasoning, hinders them from developing MCs that are more concerned with theoretical blocks. As supported by Putra's study (2019), Indonesian pre-service elementary teachers' concern is on teaching a mathematical technique to solve the tasks rather than supporting students' understanding through explanations of the given technique.

Concerning the approach of analysing the mathematical knowledge presented in MCs, this study applied praxeological analysis from the anthropological theory of the didactic (Chevallard, 2006). A more detailed nature of mathematics presented in MCs was captured through this theoretical lens. The praxeological elements can be exploited as a model to describe the mathematical practices and theories that appeared in MCs. This is in line with Quiroz and Rodríguez (2015) which stated that praxeology is a fruitful model to describe the mathematical modelling process.

CONCLUSION

MCs designed by pre-service teachers had the potential to engage elementary school students in their learning through entertaining and enjoyable ways. MCs provide the mathematical praxeologies in context-based tasks, and they are different from what appear in school mathematics textbooks. Wijaya et al. (2015) mentioned that Indonesian mathematics textbooks mainly provide tasks without context or those that do not require students' mathematisation and modelling activities. Therefore, it is controversial that MCs can be an alternative for teachers to support and stimulate students' mathematical thinking because they provide rich mathematics contexts. Also, the teachers need to carefully select an MC and develop or manage their educative discussions. Some of these MCs need further development, especially in the theoretical block, to stimulate creativity and critical thinking. Further studies could explore the best way to support pre-service and in-service mathematics teachers in designing, adopting, and adapting MCs to promote the students' higher thinking skills or abilities.

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