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THE NEUTRALIZATION ON AN EMPTY NUMBER LINE MODEL FOR INTEGER ADDITIONS AND SUBTRACTIONS: IS IT HELPFUL?

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Abstract

The number line and the neutralization model have been used very extensively in teaching integer additions and subtractions for decades. Despite their advantages, issues concerning subtractions on these models are still debatable. Therefore, the neutralization on an empty number line (NNL) model is proposed in this research to help students better understand the meaning of integer subtractions as well as additions. This report is a part of a design research study conducted in a classroom of 28 elementary school students at the fifth grade. Data were gathered by collecting students' written work, conducting interviews and observations during the teaching experiment. This paper focuses on students' perceptions of the NNL model and what factors that might contribute to students' achievements in understanding integer additions and subtractions. The analysis revealed that most students overemphasized on the use of the NNL model as a procedural method instead of as a model for thinking. Moreover, students' mathematical beliefs and conceptions on the use of the column strategy and the absence of a discussion on the need of using the model are found to be some factors that could cause students' misunderstandings. However, with a thorough planning, the NNL model has a potential to help students developing a meaning of integer additions and subtractions.

Keywords: Addition, Subtraction, Negative, Neutralization on an Empty Number Line (NNL) Model

Abstrak

Model garis bilangan dan model netralisasi telah digunakan secara luas dalam pembelajaran konsep penjumlahan dan pengurangan bilangan bulat. Terlepas dari kelebihan pada masing-masing model, permasalahan pada operasi pengurangan yang melibatkan bilangan bulat negatif masih menjadi perdebatan yang hangat. Oleh karena itu, model netralisasi pada garis bilangan kosong (NNL) digunakan dalam penelitian ini untuk membantu siswa lebih memahami makna operasi pengurangan bilangan bulat serta penjumlahan. Makalah ini merupakan bagian dari penelitian *design research* yang dilakukan di kelas V suatu Sekolah Dasar dengan 28 siswa. Data diperoleh dari hasil jawaban siswa, observasi dan interview selama pembelajaran di kelas berlangsung. Makalah ini fokus pada persepsi siswa tentang model NNL dan faktor-faktor apa yang mungkin berkontribusi terhadap kesuksesan siswa dalam memahami penjumlahan dan pengurangan bilangan bulat. Analisis mengungkapkan bahwa sebagian besar siswa terlalu menekankan pada penggunaan model NNL sebagai metode prosedural daripada sebagai model untuk berpikir. Selain itu, keyakinan dan konsepsi matematika siswa tentang penggunaan strategi susun ke bawah dan tidak adanya diskusi tentang kebutuhan menggunakan model tersebut ditemukan menjadi beberapa faktor yang dapat menyebabkan kesalahpahaman siswa. Namun, dengan perencanaan yang matang, model NNL memiliki potensi untuk membantu siswa mengembangkan makna penjumlahan dan pengurangan bilangan bulat..

Kata kunci: Penjumlahan, Pengurangan, Negatif, Model Netralisasi Pada Garis Bilangan (NNL)

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Studies indicated that the use of models and contexts could promote students' thinking in performing addition and subtraction of integers (Liebeck, 1990; Stephan & Akyuz, 2012; Bofferding, 2014; Teppo & van den Heuvel-Panhuizen, 2014; Sahat, Tengah, & Prahmana, 2018). Basically, there are two models for teaching addition and subtraction of integers, i.e. the neutralization model and the number line model (Van de Walle, 2004). In dealing with integer addition and subtraction involving negative numbers, the neutralization model or the cancellation model applies the principle of cancellation, where the sum of every number and its opposite is always zero. Whereas the number line model uses the idea of direction to represent the operation of addition and subtraction.

The neutralization model is usually introduced by using counters of two different colors representing positive and negative integers. If blue counter represents positive integer and red counter represents negative integer, then each pair of blue and red counter is equal to zero, and thus implies "$a + (-a) = 0$". When working with this model, integer addition and subtraction can be interpreted as "adding" and "taking away" respectively. As an example, to model the addition of "6 + (-2)", students first put six blue counters and then 'add' two red counters. Since two red counters cancel out two blue counters, then four blue counters remain as the sum. However, the problem happens when students are dealing with subtraction problem such as "6 – (-2)". First, students put six blue counters and then they must 'take away' two red counters. In this case, the red counter does not exist, so they have to 'add' two pairs of red and blue counters or zero pairs so they can 'take away' two red counters and 8 blue counters remain as the result (see Figure 1). Hence, students must understand that when pairs of opposite colors of counters are added to a quantity, the value of the original counters remains unchanged (Van de Walle, 2004).

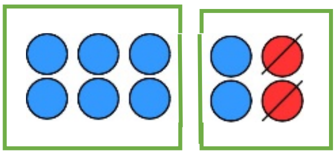


Figure 1. The neutralization model for solving "6 – (-2)"

On the other hand, the number line model is depicted as a horizontal line in which positive integers are located to the right of zero and negative integers are located to the left of zero. NCTM (2000) recommends students to use the number line model to explore negative numbers as extensions of positive numbers through familiar applications. The operation of addition and subtraction can be interpreted as "walking in the same direction" and "walking in the opposite direction". For example, to solve "6 + (-2)", students can start at zero facing right and move forward six steps (+6), then move two steps backwards representing negative two. Although the use of number line model is helpful for supporting integer addition, but students tend to have problems when dealing with subtraction with negative numbers because the procedure is more complicated. For example, to solve "6 – (-2)" using the number line model, students can start at zero facing right and then move forward six steps (+6), afterwards students have to face in the opposite direction (turn around) before they move two steps backward (-2) on the number line to represent the subtraction operation (see Figure 2).

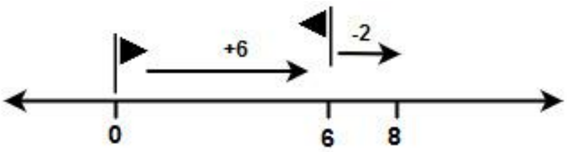


Figure 2. The number line model for solving "6 – (-2)"

Freudenthal (1973) showed that the number line model could be helpful for supporting integer addition, and Stephan & Akyuz (2012) indicated that the number line model together with a financial context (assets and debts) could support students' development of integer addition and subtraction, both procedurally and conceptually. However, Küchemann (1981) pointed out that the number line model should be avoided in teaching subtraction of integers, thus a discrete model or the neutralization model was offered as a better solution. Liebeck (1990) also emphasized that the existing concept of addition and subtraction are related to "adding" and "taking away" objects. When students use the number line model in subtracting a negative number, for example "6 – (-2)" as in Figure 2, there is no relevance between the intuitive concepts of subtraction as "taking away" and the "turn around" movement on the number line that represent the subtraction operation. On the other hand, the neutralization model could also be confusing for students, since the subtraction operation involves both addition and subtraction – that is, adding the zero pair first and then taking away the number indicated in the problem (Bofferding, 2014).

Some studies on the use of the combination of the neutralization and the number line model were identified. Steiner (2009) in his dissertation used the novel model with a context of money to spend (as a positive integer) and debt (as a negative integer). By placing red bills (debts) to the left side of zero and white bills (money to spend) to the right side of zero on a number line, the operation of addition and subtraction of integers were carried out using the cancellation principle. Similarly, Shutler (2017) utilized the banking model as a development of hills and dales model. Positive and negative integers represented as stacks of black circles and wells of white circles respectively. However, the situation of assets and debts is not suitable for Indonesian context, since the smallest amount of money in Indonesia is a hundred rupiah.

Despite the fact that some studies have been conducted in Indonesia on developing students' understanding of integer additions and subtractions (Aris, et al. 2019; Prahmana, 2017; Shanty, 2016; Muslimin, et al. 2012), research on developing a combination model of the neutralization and the number line model has not been done. Therefore, this study offers the neutralization on the number line (NNL) model to promote students' understanding in performing addition and subtraction of integers. The NNL model applies the procedure of neutralization model where the representations of positive and negative integers are located on an empty number line. Sari, et al. (2019) suggested that the NNL model could give meaning to students that subtracting a negative means adding a positive and subtracting a positive from a negative means adding two negatives, and it also allows us to work with big numbers.

Lesh & Doerr (2000) defined model as a system consisting of elements; relationships among elements; operations that describe how the elements interact; and patterns or rules, such as symmetry, commutativity, and transitivity that apply to the preceding relationships and operations. In the NNL model, there are three main elements, (i) a mound (a curve above the empty number line) to represent a positive integer, (ii) a hollow (a curve below the empty number line) to represent negative integers, and (iii) an empty number line to record and track students' strategies in performing the operation. The relationship between the mound and hollow is that every pair of mound and hollow counts as zero, meaning that “1 + (-1) = 0”. The operations on this model constitute addition as ‘adding’ a number of mounds or hollows and subtraction as ‘taking away’ a number of mounds or hollows as indicated in the problem. Furthermore, the rule that applies to this model is that when pairs of a mound and a hollow are added to the existed mounds or hollows, the value of the initial mounds or hollows remains unchanged. Figure 3 illustrates how the NNL model is used in a subtraction problem.

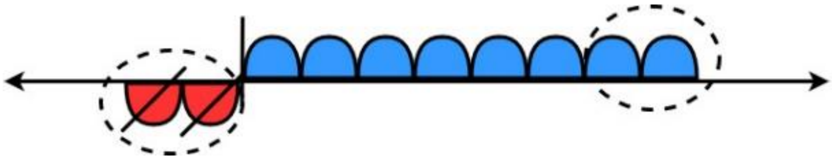


Figure 3. The NNL model for solving “6 – (-2)”

In the present research, we used a theory of RME (Realistic Mathematics Education) which was developed in the 1970s by Hans Freudenthal who perceived mathematics as a *human activity*. Opposite to traditional mathematics education, RME emphasizes mathematics education as a process of doing mathematics in reality that leads to a result, mathematics as a product (Gravemeijer & Terwel, 2000). The main features of RME are the use of contexts and models, students’ own constructions and productions, interactivity, and intertwinement (Treffers, 1987). The use of contexts and models are the two fundamental elements in RME to support students’ progressive mathematizing. Whitacre et al. (2012) suggested that reasoning about opposite magnitudes could serve as a basis for integer reasoning. Therefore, the scoring context which reveals two opposite magnitudes that are positive and negative scores has a potential to develop students’ reasoning with integers. Thus, the context of scoring was chosen in the present research as a situation that students can discuss within it and as a basis for the development of the NNL model.

Hence, aiming at developing the local instructional theory on integer addition and subtraction as well as improving practice, this paper seeks to address the following questions, first, how do students perceive the NNL model in solving integer additions and subtractions? Second, what factors that might contribute to students‘ achievement in solving integer additions and subtractions?

METHOD

This report is a part of the retrospective analysis conducted in a design research study. Design research is also known as developmental research (Freudenthal, 1991), design-based research (Cobb, et al. 2016), educational design research (McKenney & Reeves, 2014), or classroom-based design research (Stephan & Cobb, 2013). There are three phases in a design research study, first, the design and preparation, second, the implementation of the design, and third, the retrospective analysis (Gravemeijer & Cobb, 2006; Cobb, et al. 2003).

In the first phase, a design of an instructional sequence was developed based on the local instructional theory about integer addition and subtraction. The use of models and contexts in developing the concept of integer addition and subtraction was explored. A hypothetical learning trajectory (Simon, 1995) was elaborated to describe both a sequence of learning activities and conjectures of how students engage in the activities. In the second phase, the hypothetical learning trajectory (HLT) was tried out in a classroom of 28 students in SD Nurul Islam, Jakarta. Data such as video recordings of classroom activities, photos of students' activities, students' written works, classroom observations and interviews were collected. Observations took place on the classroom level, group level, and individual level, while interviews were made occasionally during the lesson or purposively after the lesson.

After the lessons took place, the data from different sources were gathered, selected, and analyzed by comparing the actual learning process and the hypothetical learning trajectory. Students' written works were chosen, examined and analyzed in accordance with other sources of data to improve the triangulation. The retrospective analysis reported in this paper will focus on how students' perceptions of the NNL model in solving integer additions and subtractions, do they find it helpful or not. Moreover, we will also find out what factors that might contribute to students' achievement in solving integer additions and subtractions, related to the classroom practices in the classroom.

RESULT AND DISCUSSION

This section summarizes the findings of the teaching experiment in conjunction with the hypothetical learning trajectory (HLT), followed by the retrospective analysis focusing on students‘ perception of the NNL model.

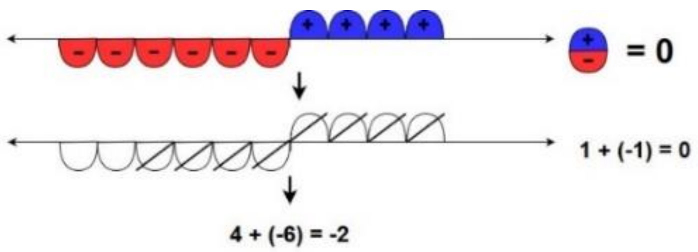
The Comparison between the HLT and the Implementation of HLT

Based on Simon (1995), a hypothetical learning trajectory (HLT) is made up of three components: learning goals that defines the direction, learning activities, and hypothetical learning processes–a prediction of how students' thinking and understanding will evolve in the context of the learning activities. Similar to the present design research, Simon (2018) conducted a Learning Through Activity research program aiming at the construction of a HLT for a specific topic and the elaboration of theory and instructional design to promote students' conceptual understanding. In this design research, HLT serves as a guideline in the teaching experiment and also as a framework in the retrospective analysis where the actual learning process is compared to the HLT (Bakker, 2004). In the present research, a sequence of activities was designed aiming at developing students understanding of negative numbers and constructing a meaning of integer addition and subtraction by means of the NNL model and the scoring context.

Table 1 shows how the actual learning process took place in a classroom compared to the designed HLT. The designed sequence of activities in this research is basically divided into three parts. First, understanding negative numbers by exploring various contexts. Second, developing the context of scoring to introduce integer addition and the NNL model. Third, exploring integer subtraction on the NNL model. It was expected that by experiencing the scoring context, students will develop a meaning of addition and subtraction as ‘adding‘ and ‘taking away‘ positive or negative scores. However, it was also hypothesized that a problem might occur when students are dealing with subtraction, that is, when students have to take away scores or numbers that does not exist before. Thus, a term ‘neutral pairs‘ or ‘zero pairs‘ might be introduced to help students solve the problem.

Table 1. The comparison between the HLT and the implementation of HLT

The HLT	The Implementation of HLT
Understanding Negative Numbers Goals: Students develop a meaning of negative integer. Activities: A classroom discussion about the existence of negative numbers as the extension of positive numbers on the number line by using the context of temperature below zero. A further discussion about negative numbers as the opposite of positive numbers by using the context of height and depth, assets and debts, positive and negative scores. Hypotheses: Various contexts could help students build the meaning of negative integers. The number line model emerges naturally from the temperature context. Hence, students can compare positive and negative integers by using the number line. The number line can also be used to show that any two integers having the same distance from zero in opposite directions are called opposites. Other contexts might be introduced to students to enrich their understanding of negative numbers.	The temperature context can introduce the concept of negative numbers very well. The number line emerges from the picture of a thermometer with negative and positive temperatures on it. However, the students did not have any experience related to negative numbers. Thus, other contexts were introduced to students, including the scoring context. There was less attention in developing the idea of opposite numbers.
The Scoring Context and Integer Addition Goals: Students can perform integer additions involving negative numbers. Activities: The scoring context is introduced to students by playing a game. One positive score is signified as a mound, and one negative score is signified as a hollow. Students are asked to record their scores and find the total score after playing each game. A discussion	Students were very excited when playing the game. They can record their scores correctly and find the total score by the neutralization principle. For those who already have knowledge in performing integer addition involving negative numbers,

about the idea of one positive cancels out one negative, i.e. " 1+(-1)=0" precedes the game. Hypotheses: It was conjectured that the game will promote students' participation in the classroom. The idea of canceling out every pair of positive and negative scores will help them to find the total score. There might be also students who will use subtraction instead of the neutralization principle. Figure 4 shows how the NNL model emerges from the activity.  **Figure 4.** The emergence of the NNL model for integer addition	they can perform the addition as a subtraction: $\boldsymbol{a+(-b)=a-b}$. If the number of negative scores is bigger than the positive scores, then they find the positive difference between the two numbers and put a negative sign in front of the result. However, some students failed in determining the sum of two negative numbers, because they did not go back to the scoring context in adding the two negative numbers.
The Scoring Context and Integer Subtraction Goals: Students can perform integer subtractions involving negative numbers. Activities: The rule of the game in scoring context is modified: If they win they could take away three negative scores, and if they loose they have to take away two positive scores. Hypotheses: The rule was modified to provoke students' understanding that taking away a negative number means adding the opposite, that is $\boldsymbol{a-(-b)=a+b}$ and taking away a positive number means adding the opposite, $\boldsymbol{-a-b\ =(-a)+(-b)}$. To develop those idea, the problem that they might encounter is taking away a score that does not exist before, for example, to solve " -3 – 2", they have to take away two positive scores or two mounds from three negative scores or three hollows. If there is none of them come up with a solution, the teacher may introduce the idea of adding 'neutral pairs' or 'zero pairs' that counts as zero and will not change the initial score. Figure 5 shows how the NNL model can be used in representing the idea:	 There were some evidences that most of the students had difficulties in performing integer subtractions, so the teacher had to introduce the idea of neutral pairs and showed the students how to do the subtractions. It was observed that the difficulty in grasping the idea of subtraction involving negative numbers made them frustrated. The scoring context was no longer predominant in the classroom discussion since they over emphasized the idea of neutral pairs. It happened that the students keep adding the neutral pair though in some

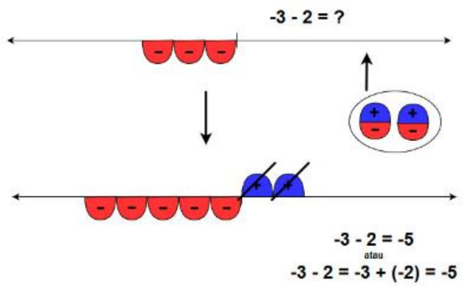


Figure 5. The emergence of the NNL model for integer subtraction

cases it was not always needed in solving a problem.

Some evidences also showed that their previous knowledge in applying the algorithm strategy were quite dominant.

The Retrospective Analysis on the NNL Model

Table 2 describes an interpretative framework that underpins our analyses of students' mathematical learning both as individuals and as a community in a classroom mathematical practice. This framework combines two different perspectives in learning, the psychological or individual perspective and the social perspective. In the view of constructivism theory, the learning process evolves as a result of the contribution of an individual students' reasoning to the classroom community and reflectively the influence of the classroom community to the development of students learning as an individual.

Table 2. An interpretative framework for analyzing communal and individual mathematical activity and learning (Cobb et al, 2001)

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Socio-mathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical interpretations and reasoning

Cobb et al. (2001) explained that while the psychological perspective emphasizes on students' various ways of reasoning during their participations in mathematical practices, the social perspective brings out the development of mathematical practices as a result of the classroom discourse. More specifically, the classroom social norms include explaining and justifying solutions, indicating agreement or disagreement, and trying to understand others' reasoning. The socio-mathematical norms are more specific to mathematical activity in which the classroom community come to an agreement of what counts as an efficient mathematical solution and an acceptable mathematical reasoning. Furthermore, classroom mathematical practices focus on particular mathematical ideas.

In this research, the classroom mathematical practices that were expected to evolve in the classroom discussion are about the ideas of (i) zero pairs or neutral pairs, in other words, the sum of any integer and its opposite is zero; (ii) adding two numbers means putting together the two numbers and then finding what is left over after cancelling out the zero pairs; (iii) subtracting means taking away a certain quantity from another quantity; and (iv) if zero pairs are added to a quantity then the value of the original quantity remains unchanged

Therefore, to answer our questions about students' perception of the NNL model in the teaching experiment, we will look at students' perception of the NNL model as an individual and how this contribute to the classroom mathematical practices. Similarly, we will also analyze how the three aspects of the social perspective contribute to the development of students' perception of the NNL model in performing integer addition and subtraction.

To begin with, we will expound how the scoring activity was developed as a context for the emergence of the NNL model. A game was introduced to the students and they played the game in pairs. Every student had to record their score along the game and found the sum at the end of the game. If they win, they earn a positive score, and if they lose, they earn a negative score. The teacher demonstrated the game in front of the classroom by using blue cards and red cards representing positive and negative scores respectively, as it is shown in Figure 3. The students were very excited when playing the scoring game, they came up with different representations when finding the sum of positive and negative integers (see Figure 6).

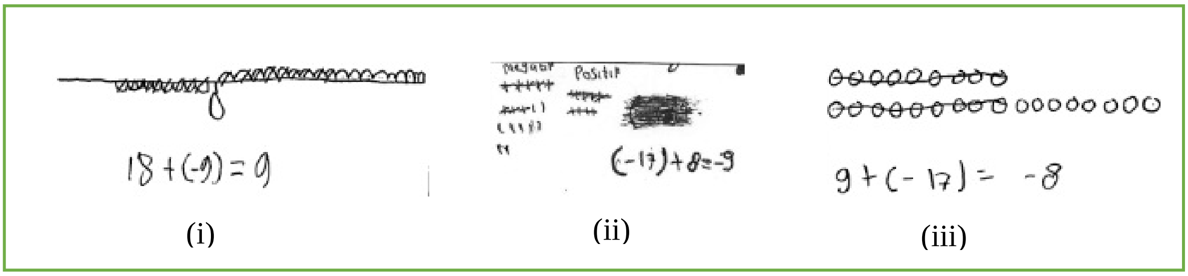


Figure 6. students' representations when solving integer addition

There were three different models identified in solving addition of positive and negative integers. Basically, they were doing the cancellation or neutralization model but with different representations. Based on the observation, some students even already knew the relation of " $\boldsymbol{a + (-b) = a - b}$". The scoring activity can help them understand how to find the sum of a positive number and a negative number. However, there was an absence of a classroom discourse in explaining why the relation "$\boldsymbol{a + (-b) = a - b}$" applies.

The next activity was the scoring activity with a modified rule, that is, if they win then they can take away three negative scores, and if they lose then they must take away two positive scores. By modifying the rules, they were challenged to come up with the idea of 'neutral pair'. However, almost all of the students in the classroom did not come up with the idea of adding a neutral pair when they had to take away a quantity that did not exist before. It might happen because the students did not have enough support to find the idea of adding the neutral pairs to their problem, they need physical objects to work with instead of just drawing mounds and hollows.

Dialog 1. A transcription of a classroom discussion on the idea of a neutral pair

The teacher is introducing the subtraction problem involving negative numbers:

T	*: Now I only have one positive score, and then I win, so I have to take away three negative scores.*
S	*: you can't*
T	*: Ok, how should I find out my score now if I only have one positive score and I have to take away three negative scores? Please discuss this problem with your friend.*

About 15 minutes later… (the whole classroom discussion began and one of the students came forward to explain the solution as shown in Figure 7)

T	*: Now you have one positive score.*
S1	*: I add negative three and positive three*
T	*: Why? You only had one positive in the beginning. Why did you add three negative scores and three positive scores?*
S1	*: To make a neutral pair*
T	*: To make a neutral pair (emphasizing)* *Ok, now let's look at an example, if this is positive one and this is negative one, then what is it?*
S1	*: A neutral pair*
T	*: A neutral pair means…? (asking)*
S1	*: zero*
T	*: zero,,, this is a neutral pair which equals to …? (asking)*
S1	*: zero*
T	*: and for this one, how many neutral pairs are there? (pointing to their solution)*
S1	*: three*
T	*: Three neutral pairs equal to zero … Is that what you mean?* *Now, I want to clarify once again, because your friends might want to know, why did you add three neutral pairs, three positives and three negatives?*
S1	*: neutral pair*

Dialog 1 shows the idea of neutral pair was too dominant in the discussion, but the need of adding the neutral pair in the subtraction problem has not been discussed. Thus, there was a missing discussion about the third and the fourth classroom mathematical practices that were expected to evolve during the discussion. It was also found later that the idea of 'neutral pairs' was very powerful that cause students' misunderstandings in handling problems. Once the teacher emphasized the idea of neutral pairs, they started to build their understanding that a neutral pair is a very important idea in solving such problems. As a result, during a small group discussion, they always think of adding a neutral pair to any kind of problems although it was not needed.

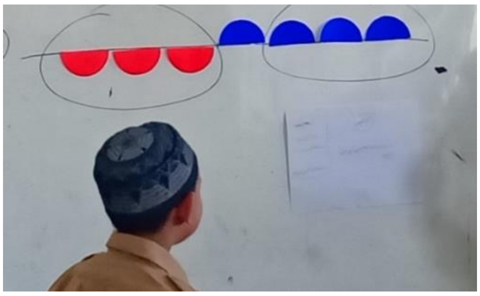
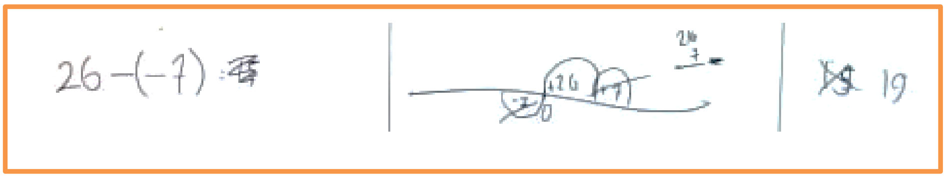


Figure 7. A student is explaining his solution in front of the classroom

Figure 8 (i) shows us one common perception of the NNL model among the students. To solve the problem "46 – 20", the student drew a mound representing 46 together with a neutral pair of positive 20 and negative 20, then she/he took away positive 20 from positive 46. But, there is no need to add a neutral pair of 20 and (-20) to solve this problem, as she/he already had positive 20 that could be subtracted from positive 46. This suggests us that they have beliefs in the classroom that they must add a neutral pair for every subtraction problem. Moreover, it seems that they have to do exactly what the teacher told them to do so, though they could have done it using different strategies. Somehow, this belief could impede their creative and critical thinking in observing and solving a problem situation flexibly.

(i)



(ii)

Figure 8. Students' perception of the NNL model

In another case (Figure 8(ii)), a student was doing a subtraction of minus 7 from positive 26. The student should have crossed only the hollow of negative 7 after adding a pair of 7 and (-7), but in fact she crossed both positive seven and negative seven that did not represent the actual problem. There was also an indication from the picture that the student did a column strategy to find the difference between 26 and 7 which means that she ignored the negative sign of seven, thus she found positive 19 as the result. If the student were going back to the context of scoring, then she might have performed the operation more meaningfully.

Moreover, another student kept repeating the same procedure in modeling the problem using the NNL model. She always adds a neutral pair to every problem, but she did not give meaning to the model. In Figure 9 (ii), the instruction of the problems was to find the difference between -26 and 19. Although she was successful in modeling the problem into the NNL model, but she did not use the representation of two hollows meaning that she had to add the two negative numbers. What she did was subtracted 19 from 26 with an algorithm strategy or column procedure that was not successful. Furthermore, the student's modeling in Figure 9(i) and 9(iii) do not represent the problems whatsoever. It shows how their previous knowledge on the use of the column strategy influenced them in performing integer addition and subtraction. This result is in line with Fuson, et al (1997), that students' concetenated single-digit conception of numbers while proceeding the column strategy could lead to various errors. From the Figure 9, we could see that the student ignored the negative sign of the number and proceed as if the negative sign did not exist.

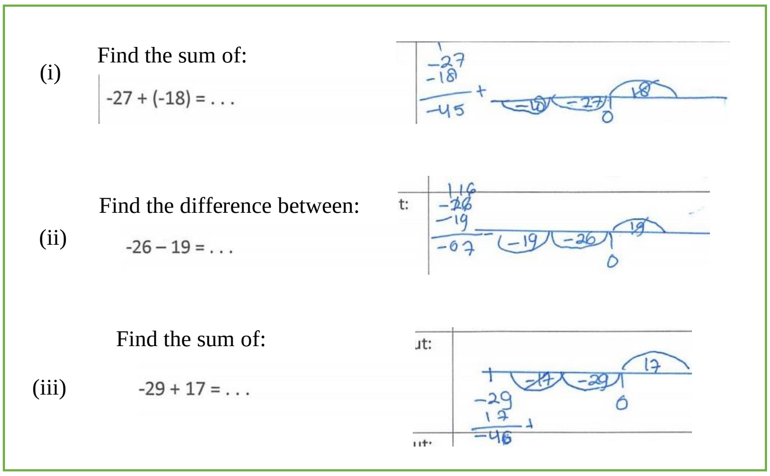


Figure 9. Another student's perception of the NNL model

On the other hand, there were also some evidences that show us the benefit of using the NNL model in performing integer addition involving negative integers. As an example, in Figure 10, the student can correct his mistake when finding the sum of 4 and (-6). By cancelling out the same number of mounds and hollows, the student realized that his first answer was a mistake, then he wrote down the correct answer below the first one. The benefit of using the NNL model together with scoring context is that students can always refer back to the contextual situation when they have diifficulties. They can record their strategy on an empty number line and see how many mounds or hollows must be added or taken away. The NNL model allows students to work with big numbers with mounds and hollows signify positive and negative quantities respectively. While in the neutralization model, both positive and negative numbers are signified with circle with similar shape, they differ only when the circles are filled with different colors or signs.

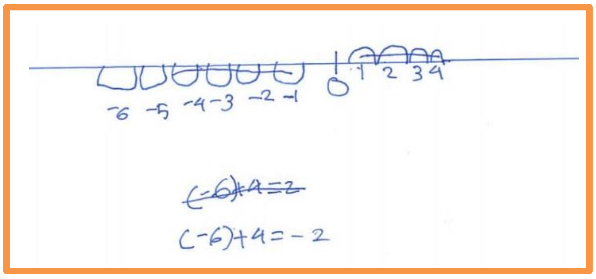


Figure 10. The NNL model in an addition problem

Some factors that might contribute to students' achievement were also the lack of classroom discussion where the three aspects in the social perspective of learning did not establish in the classroom. In most of our classrooms in Indonesia, students do not get used to reasoning and justifying their opinions (Prahmana, Zulkardi, & Hartono, 2012). Most of the students seems to put themselves as a listener instead of as a speaker in sharing their ideas. This might happen because they are afraid of making mistakes. The students need a strong encouragement to change their beliefs and to develop positive classroom cultures such as sharing ideas, understanding each other, and proposing different ideas, as students' contributions and interactivity are important aspects in the theory of RME and other constructivism approaches. Therefore, it needs a serious attention for teachers and researchers to build positive classroom cultures that can support the development of students' learning both as an individu and as a classroom community.

The findings suggest that the designed activity, the instructions given, and the tools provided must be taken into consideration in developing the hypothetical learning trajectory. The problem happened in the subtraction has been conjectured, but the anticipations were not made clearly. If it seems that the students have not developed the need of adding neutral pairs on the NNL model for solving subtraction problems, then the teacher must give an ample space for students to really get involved in the problem and come up with meaningful ideas. As it was stated in Lesh & Doerr (2000) that the teacher plays a critical role in a classroom to create the need for students in sharing their tools and representations, creating and nurturing diverse approaches, also creating meaningful and powerful models through classroom discourse. Moreover, modeling involves the interaction among three types of systems: (a) internal conceptual system, (b) representational systems that function both as externalization of internal conceptual systems and as internalizations of external systems, and (c) external systems that are experienced in nature, or that are artifacts that were constructed by humans. (Lesh & Doerr, 2000). If the interaction among the systems is absent, then the externalization of internal conceptual system will not emerge.

CONCLUSION

The purpose of the present study was to determine students' perceptions of the neutralization on an empty number line 'NNL' model when they are dealing with additions and subtractions of integers. The finding has shown that students found it helpful when they were working with addition problems, although some difficulties were apparent on the subtraction problems. The classroom mathematical practice about the need of adding a neutral pair to a subtraction problem was not developed very well in the classroom. Therefore, some adjustments and revisions must be made related to the hypothetical learning trajectory, particularly on developing the idea of adding a neutral pair in a subtraction problem.

The findings of this study suggest some considerations should be made. First, the need of adding a neutral pair in solving subtraction problems must be clear to students. Although the context has provided a meaningful situation, the absence of classroom discussion on the need of using a neutral pair could be misguided. Second, there must be an ample space for students to develop their thinking, manipulate tools, and collaborate with others to come up with a meaningful representation for them. Third, a teacher should continuously build constructive classroom cultures to improve students' contribution, responsibility, and understanding of their own roles for the development of classroom mathematical practices.

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