Journal on Mathematics Education
Volume 16, No. 3, 2025, pp. 1043-1062

Estimating probabilities through the Mongolian *Shagai* game: A culturally responsive approach to teaching statistics

Itgel Miyejav[1,*], Khajidmaa Otgonbaatar[2]

[1]Department of Mathematics, National University of Mongolia, Ulaanbaatar, Mongolia
[2]Research Department for Education Policy and Planning, Mongolian National Institute for Educational Research, Ulaanbaatar, Mongolia
*Correspondence: itgel@num.edu.mn

Received: 21 May 2025 | Revised: 12 September 2025 | Accepted: 15 September 2025 | Published Online: 18 September 2025

Abstract

Culturally responsive approaches in mathematics education have been widely advocated; however, empirical investigations that embed traditional artifacts into probability and statistics instruction remain limited. This study addresses this gap by employing *Shagai*—a traditional Mongolian four-sided ankle bone—as an ethnomathematical instrument to support the development of statistical reasoning. In total, 10,050 single-throw trials were conducted across three groups: community participants (n = 5,000), pre-service mathematics teachers (n = 5,000), and a researcher-led demonstration (n = 50). Empirical probabilities for the four *Shagai* outcomes—horse, camel, sheep, and goat—were estimated as 0.12, 0.13, 0.39, and 0.36, respectively, with convergence achieved after approximately 8,000 trials, indicating a statistically stable but non-uniform distribution. These results informed the design of a four-hour instructional workshop with nine doctoral students in education. Participants conducted *Shagai*-based experiments, calculated statistical measures, and analyzed data using SPSS. Qualitative reflections were subjected to thematic analysis, which revealed enhanced statistical understanding, interdisciplinary insight, and awareness of cultural integration. A paired-sample t-test confirmed a statistically significant improvement in conceptual understanding $(\mathrm{t}(8) = 4.43, \rho = .002)$. The findings suggest that embedding traditional knowledge systems into statistics education can deepen conceptual comprehension and enrich culturally relevant pedagogy.

Keywords: Culturally Responsive Pedagogy, Ethnomathematics, Higher Education, Probability Education, *Shagai* (Mongolian Ankle Bones)

How to Cite: Miyejav, I., & Otgonbaatar, K. (2025). Estimating probabilities through the Mongolian *Shagai* game: A culturally responsive approach to teaching statistics. *Journal on Mathematics Education, 16*(3), 1043–1062. https://doi.org/10.22342/jme.v16i3.pp1043-1062

Probability is a central component of mathematics education, equipping students with the skills to reason under uncertainty, evaluate risk, and make informed decisions (Jones et al., 2007). Despite its importance, probability is often perceived as abstract and disconnected from everyday experiences, particularly when introduced through conventional tools such as coins, dice, or spinners (Shaughnessy, 1992). This disconnect can hinder engagement and understanding, motivating the adoption of culturally responsive pedagogies that link mathematical concepts to students' cultural backgrounds and lived experiences (Gay, 2010; Rosa & Orey, 2011).

Ethnomathematics provides a theoretical foundation for such approaches. Defined by D'Ambrosio (1985) as the study of mathematical knowledge embedded in cultural practices and artifacts, ethnomathematics challenges the view of mathematics as purely abstract and emphasizes the contexts in which mathematical thinking develops (Knijnik, 2002; Barton, 2008). By engaging with traditional practices, students can encounter mathematics as a dynamic, culturally situated discipline.

One of ethnomathematics' strengths is its capacity to render mathematics meaningful. Rather than presenting rigid rules, it enables students to observe how mathematical principles emerge in cultural activities such as weaving, measurement, or games (Rosa & Orey, 2011), fostering both conceptual understanding and cultural pride, particularly among marginalized learners (Rosa & Orey, 2016).

In multicultural classrooms, ethnomathematics supports the design of lessons aligned with students' cultural backgrounds. For example, probability can be explored using *Shagai*—the ankle bones of sheep or goats traditionally employed in Mongolian games—rather than only through conventional tools. This approach allows students to investigate complex concepts, including randomness and probability, in culturally meaningful contexts (Lipka et al., 2005).

Shagai have been used in Mongolia since at least the 3rd century BCE (Tumurbaatar, 2024). The bones were often modified for usability and aesthetic purposes and can land in four distinct positions—horse (морь), camel (тэмээ), sheep (хонь), and goat (ямаа)—each symbolizing traits such as speed, endurance, abundance, and agility (Chuluun, 1958; Tudev, 2006). These outcomes are both culturally significant and amenable to probabilistic analysis, making *Shagai* an intuitive model for teaching probability as shown in Figure 1.

Figure 1. Four-sided resting positions of the Mongolian *Shagai* (ankle bone), classified as horse, camel, sheep, and goat —as observed when the bone is thrown onto a traditional felt mat (*shirdeg*)

Comparable uses of *Shagai* have been documented in other ancient cultures, including Greece, Rome, and Mesopotamia, where these bones functioned as instruments for games of chance and divination (Culin, 1895; Gardiner, 1930; Simmons, 1997). The inherent asymmetry of *Shagai*, resulting from their irregular shape and uneven weight distribution, produced non-uniform probabilities, making certain outcomes more likely than others. Players in these cultures observed patterns in outcomes and adapted their strategies, accordingly, demonstrating an early, experiential understanding of probabilistic concepts (Culin, 1895). Although bone-based games are found throughout Central Asia—including the

Altai and Siberian regions—the Mongolian variant is distinctive in its combination of symbolic meaning, structured gameplay, and cultural significance (Shemyakin, 2019).

Within Mongolian mathematics education, *Shagai* has traditionally appeared in textbooks and instructional materials to illustrate combinatorial concepts (Dashdorj & Ganbaatar, 1998; Adiyasuren, 2021) and the notion of events (Itgel & Sugir, 2004; Miyejav, 2007). However, these treatments rarely extend to probability and statistics education, reflecting a notable gap in pedagogical resources and research, as well as a lack of systematic curricular integration.

Despite its cultural richness and intuitive probabilistic structure, *Shagai* remains underutilized in formal mathematics instruction (Abdullah, 2017; Elbehary, 2022). Contemporary research highlights the potential of culturally embedded games such as *Shagai* to enhance student engagement and deepen conceptual understanding (Lipka et al., 2005). Systematic empirical investigation of *Shagai* outcomes thus offers a valuable opportunity to teach statistical reasoning in a culturally grounded, data-driven framework.

Previous studies have reported preliminary probability distributions for *Shagai* outcomes based on observational data—for example, horse = 0.1, camel = 0.1, sheep = 0.4, and goat = 0.4 (David, 1955), and horse = 0.12, camel = 0.12, sheep = 0.39, and goat = 0.37 (Maystrov, 1961, as cited in Shemyakin, 2019). However, these early investigations were limited in scope, lacking large-scale systematic experimentation, and their implications for mathematics education have not been explored. Furthermore, precise probabilistic modeling of *Shagai* outcomes has not yet been addressed in the literature.

Beyond its recreational function, *Shagai* has been formally recognized for its cultural significance. In 2014, the competitive knucklebone shooting game—a variant of *Shagai*—was inscribed on UNESCO's Representative List of the Intangible Cultural Heritage of Humanity (UNESCO, 2014), highlighting its potential as a culturally meaningful educational resource.

This study addresses these gaps by conceptualizing *Shagai* not merely as a traditional artifact but as a structured probabilistic system with implications for both cultural relevance and statistical reasoning. Specifically, we examine the probabilities of the four *Shagai* outcomes through empirical experimentation and theoretical modeling. We further investigate how estimating probabilities to a precision of 0.01 can support the development of statistical reasoning within culturally contextualized learning environments.

According to Tudev (2006), more than 145 types of *Shagai* games have been documented, classified into 13 categories based on their structure, objectives, and cultural functions. This diversity reflects the richness of mathematical and logical reasoning embedded in traditional play. Integrating such artifacts into instruction provides learners with opportunities to engage in culturally meaningful problem-solving while developing formal mathematical skills. From a Culturally Responsive Approach (CRA) perspective, incorporating *Shagai* into probability instruction situates learning within familiar cultural practices and legitimizes students' heritage knowledge as a foundation for academic mathematics. In this way, *Shagai* functions simultaneously as a mathematical object and a cultural bridge, enabling students to connect abstract probability concepts with lived experiences, identities, and community traditions (Itgel & Sugir, 2004).

This study seeks to investigate the probabilistic properties of *Shagai*, the traditional Mongolian ankle bone, within the context of culturally grounded mathematics instruction. Specifically, it addresses three interrelated research questions: First, how can the probabilities of the four *Shagai* outcomes—horse, camel, sheep, and goat—be accurately determined using both empirical experimentation and theoretical modeling? Second, to what extent do the empirically derived probabilities align with or diverge from previously reported distributions, and what insights can such comparisons provide regarding the consistency and predictability of *Shagai* outcomes? Third, how does the process of estimating probabilities to a precision of 0.01 within a culturally contextualized learning environment contribute to the development of advanced learners' statistical reasoning and their understanding of culturally embedded mathematical knowledge? By examining these questions, the study integrates traditional cultural artifacts with formal probabilistic analysis, situating statistical inquiry within a meaningful, contextually rich framework.

Aligned with these objectives, the central hypothesis posits that engaging advanced learners in culturally contextualized experiments with *Shagai*, combined with precise probability estimation, will enhance both their statistical reasoning skills and cultural awareness. More specifically, it is anticipated that the systematic collection and analysis of empirical data to a 0.01 level of precision will support learners in constructing rigorous probabilistic models, interpreting outcomes quantitatively, and connecting abstract probability concepts to culturally familiar practices. In turn, this approach is expected to promote data-driven instructional strategies that are simultaneously pedagogically effective and culturally responsive, thereby fostering deeper conceptual understanding and validating learners' heritage knowledge as an integral part of academic mathematics.

METHODS

Research Design

This study employed a design-based empirical research approach, which combines systematic data collection with iterative educational design. In the context of this research, "empirical" refers to the large-scale collection and statistical analysis of *Shagai* outcomes (over 10,000 single throws), ensuring robust probability estimates grounded in observed data. "Design-based" highlights that the empirical findings were not treated as an end in themselves but were deliberately incorporated into the design of an instructional intervention for doctoral students. This cyclical process of generating empirical evidence, applying it to instructional design, and evaluating learner outcomes aligns with the principles of design-based research in education (Cobb et al., 2003). The research unfolded across three phases:

1. Large-scale empirical data collection through controlled single-*Shagai* trials.
2. Statistical modeling and analysis of outcome frequencies.
3. Instructional implementation in a doctoral-level educational setting.

Participants and Data Sources

Data were gathered in two stages, yielding a total of 10,050 single-*Shagai* trials—that is, individual throws of one ankle bone onto a felt mat, with the resting position recorded as horse, camel, sheep, or goat. This standardized unit of observation ensured consistency across participants while capturing the probabilistic behavior of the four possible outcomes. The data for this study were collected during the 2014 academic year across both community and educational contexts.

Stage 1 – Community-Based Trials

50 volunteer participants from the researcher's extended social network each performed 100 independent single-*Shagai* throws under culturally authentic conditions, resulting in 5,000 recorded outcomes. Each trial consisted of tossing a single ankle bone onto a felt mat (*Shirdeg*) and recording the resting position as horse, camel, sheep, or goat. This dataset served as a community-based sample, reflecting traditional play practices outside of formal educational settings (Appendix A).

Stage 2 – Educational Context (Pre-Service Teachers)

A group of 25 fourth-year mathematics education students at the National University of Mongolia, enrolled in the Statistics Methods in Education course, conducted two rounds of 100 throws each, spaced one week apart under researcher supervision. Prior to student participation, the researcher also performed 50 demonstration throws under standard conditions. Together, this produced an additional 5,050 data points. All trials followed a standardized procedure to ensure consistency and cultural authenticity, contributing to a robust and stratified dataset. The procedure involved four key steps:

1. Only sheep or goat ankle bones were used, carefully cleaned of excess tissue. Bones that had been artificially flattened, filed, or modified in any way were excluded.
2. Throws were performed onto a traditional felt mat (*Shirdeg*) or carpet, avoiding surfaces that were either too hard (e.g., wood, concrete) or too soft, as these could bias the landing.
3. Each *Shagai* was released from a moderate height—not excessively high or low—so that the throw resembled customary play conditions.
4. Throws were carried out randomly, without attempting to force or influence a particular outcome.

Data Collection Procedure

Participants performed repeated single-*Shagai* throws onto a traditional felt mat (*Shirdeg*), simulating customary conditions. Each resting position was classified as one of four culturally defined outcomes: horse, camel, sheep, or goat. Results were recorded on tally sheets and later digitized in Microsoft Excel. Duplicates and inconsistencies were reviewed and removed. Duplicates referred to instances where the same throw had been entered twice, while inconsistencies included miscoding errors such as recording more than one outcome in a single cell or placing multiple variables in one column where only a single outcome should have been entered. These cases were carefully reviewed and corrected before analysis. The final dataset was stratified by source: community participants, education students, and researcher-controlled demonstration throws.

Data Analysis Techniques

Quantitative analysis was conducted using Microsoft Excel and SPSS (Versions 20.0 and 28.0). The following procedures were employed.

Empirical probability estimation:

$$\widehat{p_i} = \frac{f_i}{N} \qquad (1)$$

where f_i is the frequency of outcome i, and N is the total number of trials.

Confidence Interval Analysis

Statistical stability was assessed using 95% Wilson score confidence intervals (Brown et al., 2001). To ensure that the estimated probabilities had a precision of approximately ±0.01, we calculated the minimum required sample size using the following approximation for the confidence interval width:

$$\text{Width}_{CI} \approx 2 \cdot z \cdot \sqrt{\frac{p(1-p)}{n}} \qquad (2)$$

where p is the estimated probability of an outcome, n is the sample size (number of throws), and z is the critical value from the standard normal distribution corresponding to the desired confidence level (for 95%, $z \approx 1.96$).

To be conservative, we used the highest observed probability (p = 0.4, corresponding to the sheep outcome), since this maximizes the product $p(1-p)$ and therefore gives the widest interval. Substituting values:

$$n \geq \frac{4 \cdot (1.96)^2 \cdot 0.4 \cdot 0.6}{(0.02)^2}$$

$$n \geq \frac{4 \cdot 3.8416 \cdot 0.24}{0.0004}$$

$$n \geq \frac{3.686}{0.0004} \approx 9{,}215$$

Thus, a minimum of about 9,215 throws is required to estimate probabilities with ±0.01 precision at the 95% confidence level. In our study, we conducted $n = 10{,}050$ throws, which exceeds this requirement and ensures the statistical reliability of the estimated probabilities.

Stabilization Analysis

To examine the convergence of empirical probabilities over time, a stabilization line was constructed. This involved plotting the cumulative empirical probability of each outcome against the cumulative number of throws:

$$\widehat{p_i}^{(t)} = \frac{\sum_{j=1}^{t} x_{ij}}{\sum_{j=1}^{t} n_j} \qquad (3)$$

where x_{ij} is the number of times outcome i occurred in batch j, and n_j is the number of throws in batch j. This time-series approach provides insight into the point at which empirical estimates begin to stabilize, allowing assessment of convergence and data sufficiency.

Shannon Entropy Analysis

To evaluate the overall uncertainty and distributional randomness of the outcomes, Shannon entropy was computed as follows (Jizba & Arimitsu, 2004):

$$H = -\sum_{i=1}^{k} p_i \log_2 p_i \qquad (4)$$

where p_i is the empirical probability of outcome i, and $k = 4$ is the number of possible outcomes. The resulting entropy value was then compared to the maximum entropy for a uniform four-outcome distribution:

$$H_{max} = log_2 4 = 2.0$$

A lower entropy indicates a deviation from uniformity and greater predictability in outcomes, whereas an entropy value approaching 2.0 suggests maximum randomness.

Chi-Square Goodness-of-Fit Analysis

A chi-square (χ^2) goodness-of-fit test was used to assess whether the observed outcome frequencies significantly deviated from a uniform distribution, where the expected probability for each of the four outcomes is $p = 0.25$. The test statistic is calculated as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \qquad (5)$$

where O_i and E_i represent the observed and expected frequencies for outcome i, respectively, and $k = 4$ is the number of possible outcomes. This analysis tested the null hypothesis that the *Shagai* outcomes are uniformly distributed. Additionally, observed frequencies were compared against prior empirical distributions reported by David (1955) and Maystrov (1961, as cited in Shemyakin, 2019), to evaluate historical consistency and possible cultural or physical biases in outcome probabilities.

Instructional Implementation

In 2025, the empirical findings from the *Shagai* probability study were implemented in a 180-minute instructional session for nine doctoral students in education. The workshop was designed to integrate statistical reasoning with culturally responsive pedagogy, under the theme "Connecting Statistical Concepts with Cultural Contexts." The instructional design emphasized experiential learning through traditional artifacts, allowing students to perform data collection, analysis, and reflection within an ethnomathematical framework.

The lesson was organized into nine instructional phases, combining theoretical instruction, hands-on experimentation, and reflective pedagogy as shown in Table 1.

Table 1. Connecting Statistical Concepts with Cultural Contexts

Time (min)	Activity	Content and Pedagogy
0–15	Introduction and Pre-test	Overview of goals; 15-item multiple-choice pre-test
15–30	Cultural-Mathematical Framing	Historical and symbolic context of *Shagai*; discussion of ethnomathematical relevance
30–45	Probability Foundations	Sample space, relative frequency, empirical vs. theoretical probability
45–75	Experimentation (*Shagai* throws)	Each student performs 50–100 throws; records and classifies outcomes
75–105	Group Analysis	Manual computation of frequencies, empirical probability, Wilson CI, Shannon entropy, chi-square test
105–120	Group Sharing and Interpretation	Teams present results; compare statistical outcomes and interpret findings collaboratively
120–150	Discussion: Culturally Responsive Teaching	Guided dialogue: cultural relevance in learning; traditional knowledge in concept formation
150–165	Post-test and Attitudinal Survey	15-item post-test; 5-point Likert survey on cultural responsiveness in mathematics education
165–180	Reflection and Instructional Rubric	Written reflection on learning; completion of the Instructional Impact Rubric for self-assessment

To evaluate instructional impact, three assessment instruments were employed:

1. A 15-item pre/post-test to measure changes in conceptual understanding of probability and inference.
2. A 5-point Likert-scale survey assessing attitudes toward culturally grounded mathematics instruction.
3. An Instructional Impact Rubric allowing learners to self-assess their growth in knowledge, participation, and cultural awareness.

The pre- and post-tests each comprised 15 items, including 10 multiple-choice questions assessing probability and statistics concepts and 5 items evaluating cultural awareness. This design enabled direct comparison of learning gains before and after the instructional intervention. The brief, targeted instructional session proved sufficient to produce measurable improvement, as confirmed by paired-sample t-test results.

The assessment instruments were developed based on established literature in probability education and were adapted to incorporate culturally relevant content related to shagai. To ensure content validity, the instruments were reviewed by three experts in mathematics education and educational measurement, who provided feedback regarding clarity, alignment with learning objectives, and cultural appropriateness. This structured, culturally contextualized instructional sequence facilitated the integration of empirical statistical reasoning with ethnomathematical concepts, consistent with the principles of responsive pedagogy in higher education. Post-workshop analysis indicated a significant increase in student understanding, as evidenced by the paired-sample t-test ($t(8) = 4.43$, $p = .002$), demonstrating the effectiveness of culturally grounded probability instruction in enhancing both conceptual knowledge and cultural engagement.

Thematic Analysis

In addition to quantitative assessment and instructional data, qualitative information was collected through five open-ended reflection questions administered following the workshop. The questions were designed to capture participant perspectives across five domains: (1) attitudinal shift, (2) statistical experience, (3) integrated understanding, (4) instructional applicability, and (5) ethnostatistical conceptualization. Written responses from all nine doctoral participants constituted the primary dataset for qualitative analysis.

Analysis was conducted using Braun and Clarke's (2006) six-phase thematic framework. The process included: (1) familiarization with the data through repeated reading to build initial understanding, (2) generation of initial codes to identify key ideas and recurring patterns, (3) searching for themes by grouping related codes into provisional categories, (4) reviewing themes to ensure coherence and coverage across the dataset, (5) defining and naming themes by finalizing scope and providing concise labels, and (6) producing the report through thematic descriptions supported by illustrative quotations.

The analysis resulted in five overarching themes corresponding to the original reflection prompts, highlighting significant changes in participants' attitudes, conceptual understanding, instructional planning, and philosophical perspectives. Specifically, participants (1) recognized traditional games as valuable educational tools, (2) developed statistical reasoning through hands-on computation, (3) integrated cultural and mathematical knowledge, (4) expressed intent to implement culturally responsive teaching strategies, and (5) reframed ethnostatistics as both a methodological and philosophical lens for mathematical inquiry.

Cultural Contextualization

Within Mongolian nomadic heritage, *Shagai* functions not only as a recreational object but also as a symbolic tool for divination, decision-making, and cultural expression. Each outcome carries culturally ascribed significance (e.g., horse = speed or luck; camel = endurance), positioning *Shagai* as an ideal medium for ethnomathematical instruction. This study illustrates how traditional artifacts can facilitate the development of precise statistical reasoning within culturally meaningful contexts, thereby bridging abstract mathematical concepts with learners' cultural identities.

Ethical Considerations

All participants provided voluntary informed consent prior to participation. No personally identifiable information was collected. The study was conducted in a manner that respected cultural practices and adhered to institutional ethical guidelines for educational research.

RESULTS AND DISCUSSION

Empirical Outcomes of *Shagai* Throws

A total of 10,050 single-*Shagai* throws were conducted across two experimental phases. The observed frequencies for the four resting positions were as follows: horse (1,203), camel (1,296), sheep (3,919), and goat (3,632). Table 2 presents the empirical frequencies, corresponding probabilities, and the 95% Wilson confidence intervals for each outcome.

Table 2. Empirical probabilities and 95% Wilson confidence intervals (N = 10,050)

Outcome	Frequency	Empirical Probability	CI Lower (95%)	CI Upper (95%)	CI Width
Horse	1203	0.11970	0.11336	0.12605	0.01269
Camel	1296	0.12896	0.12240	0.13551	0.01310
Sheep	3919	0.38995	0.38041	0.39949	0.01907
Goat	3632	0.36139	0.35200	0.37079	0.01878

Although the *Shagai* has four possible resting positions, the outcomes do not occur with equal probability because the bone is physically asymmetric in shape and weight distribution. The horse and camel sides are narrower and less stable, while the sheep and goat sides provide broader, flatter resting surfaces, making them more likely to appear. This structural imbalance has been noted in earlier historical analyses (David, 1955; Shemyakin, 2019) and was confirmed in our dataset, where sheep and goat outcomes occurred significantly more often than horse or camel.

Confidence Intervals and Stabilization Analysis

The narrow 95% Wilson score confidence intervals indicate high precision in the estimation of outcome probabilities. To assess the consistency of these estimates over time, cumulative probability plots were generated. As shown in Figure 2, the empirical probabilities stabilize after approximately 8,000 trials, suggesting statistical reliability and sufficient sample size.

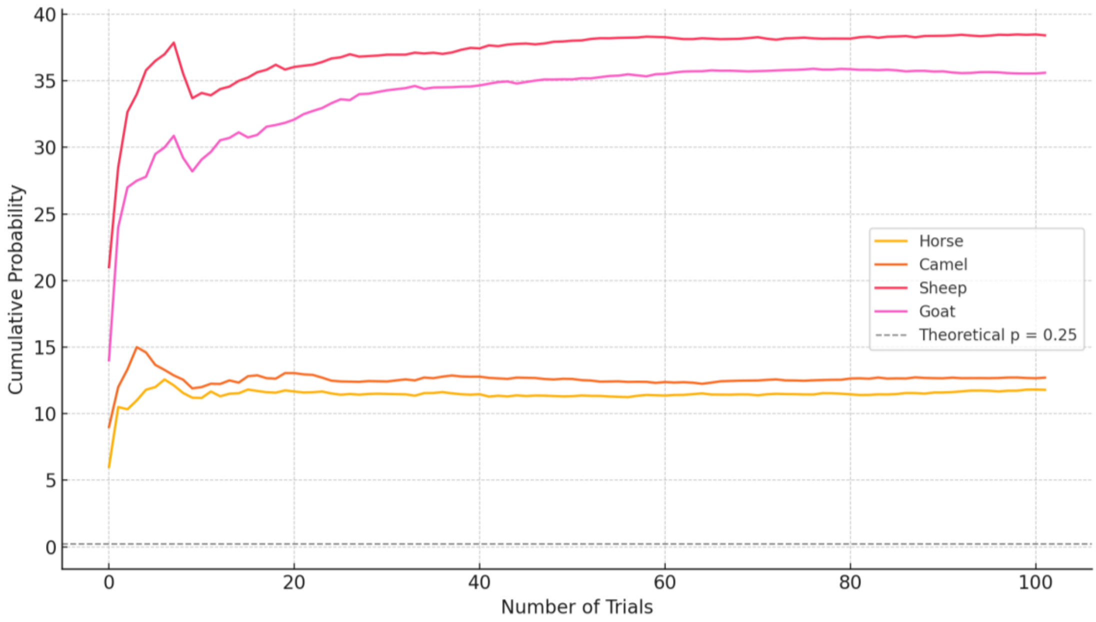


Figure 2. Stabilization of empirical probabilities across over 10,050 trials

Entropy and Distributional Uncertainty

Shannon entropy was computed to evaluate the uncertainty associated with the distribution of *shagai* outcomes:

$$H = -\sum p_i \log_2 p_i = 1.946$$

This value is close to the theoretical maximum entropy of $\log_2 4 = 2.0$, indicating a moderate level of uncertainty. However, the higher frequencies of sheep and goat outcomes, compared to horse and camel, suggest a skewed distribution rather than perfect uniformity.

Chi-Square Goodness-of-Fit Test

To evaluate whether the observed frequencies significantly deviated from a theoretical uniform distribution ($p = 0.25$ for each outcome), a chi-square goodness-of-fit test was conducted:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 185.62, df = 3, p < .001$$

The result indicates a statistically significant departure from uniformity, confirming the non-random nature of the *Shagai*'s four-sided resting positions. These findings are consistent with earlier analyses reported by David (1955) and Maystrov (1961).

Instructional Outcomes for Doctoral Students

The instructional session followed a structured 180-minute lesson plan designed around the integration of statistical reasoning and cultural pedagogy. Participants demonstrated high engagement during hands-on tasks, particularly when interpreting the implications of non-uniform *Shagai* outcomes. The step-by-step structure—spanning cultural framing, data collection, statistical analysis, and reflection—enabled learners to apply theoretical concepts in a meaningful context.

In 2025, a 4-hour instructional workshop was conducted to enhance doctoral students' understanding of culturally grounded statistical reasoning using the *Shagai* dataset. A total of nine doctoral students participated in the session, which integrated hands-on experimentation, statistical analysis, and pedagogical reflection on culturally responsive practices.

Quantitative Results for Pre- and Post Test

To assess the impact of the culturally contextualized statistics lesson, a 15-item multiple-choice test was administered to nine doctoral students both before and after the session. The test consisted of two subscales: (1) statistical understanding (10 items), and (2) cultural awareness (5 items). Descriptive statistics and paired-sample t-tests were conducted to evaluate changes in performance. Results showed a statistically significant improvement in both subscales (Table 3).

Table 3. Pre- and post-test summary

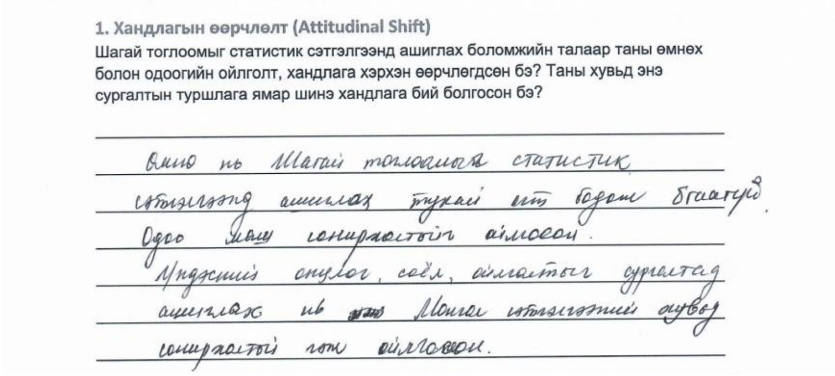
Subscale	Pre-test Mean (SD)	Post-test Mean (SD)	t(8)	p-value
Statistical Knowledge (10 items)	3.22 (1.79)	6.33 (2.12)	4.13	.003
Cultural Awareness (5 items)	3.00 (1.66)	4.44 (0.53)	2.60	.032
Total Score (15 items)	6.22 (2.95)	10.78 (2.11)	4.43	.002

The mean score for statistical understanding increased from 3.22 (SD = 1.79) to 6.33 (SD = 2.12), yielding a significant difference ($t(8) = 4.13$, $p = .003$). Similarly, the mean score for cultural awareness increased from 3.00 (SD = 1.66) to 4.44 (SD = 0.53), also showing a significant gain ($t(8) = 2.60$, $p = .032$). The total score across all 15 items improved from 6.22 (SD = 2.95) to 10.78 (SD = 2.11), with a highly significant difference ($t(8) = 4.43$, $p = .002$), suggesting that the lesson effectively enhanced both cognitive and affective dimensions of learning.

Thematic Analysis

Analysis of participant reflections yielded five overarching themes, each supported by illustrative quotations that highlight the educational impact of culturally contextualized *Shagai*-based instruction.

1. Attitudinal Shift — Participants as shown in Figure 3 reported a notable transformation in their perceptions of traditional games, particularly *Shagai*, within educational contexts. Previously regarded as cultural pastimes, *Shagai* was reconceptualized as a medium for mathematical inquiry and statistical reasoning.



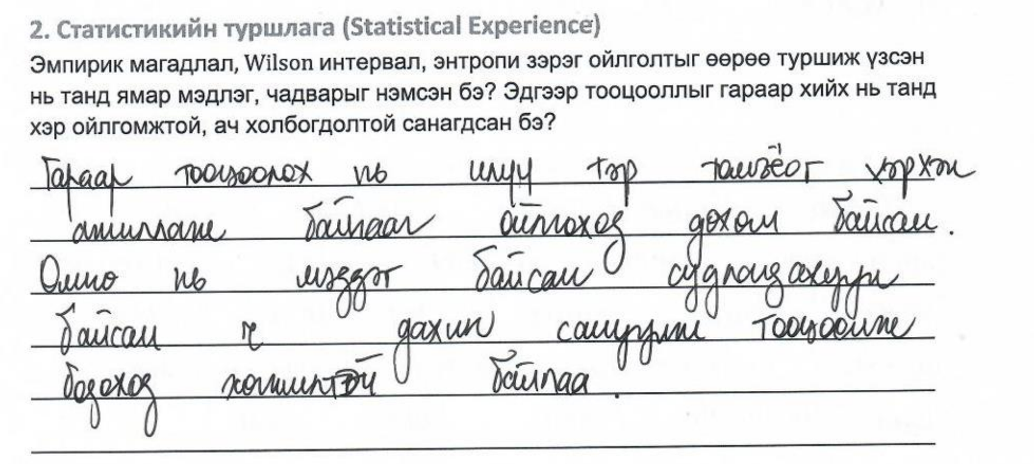
Translated Context:

Previously, I had never thought about using the Shagai game for statistical reasoning. Now, however, I have realized how fascinating it is. I came to understand that using concepts connected to national traditions and culture is particularly engaging from the perspective of Mongolian ways of thinking"

Figure 3. Participant 1 notes

Several participants emphasized an increased appreciation for cultural relevance in learning, noting that the session made statistical concepts more meaningful and personally engaging.

2. Statistical Experience — Engagement in hands-on data collection and probability estimation facilitated a deeper internalization of statistical concepts. Participants indicated that manually calculating empirical probabilities and measures such as entropy fostered a more profound understanding of statistical reasoning than reliance on automated software tools alone presented in Figure 4.

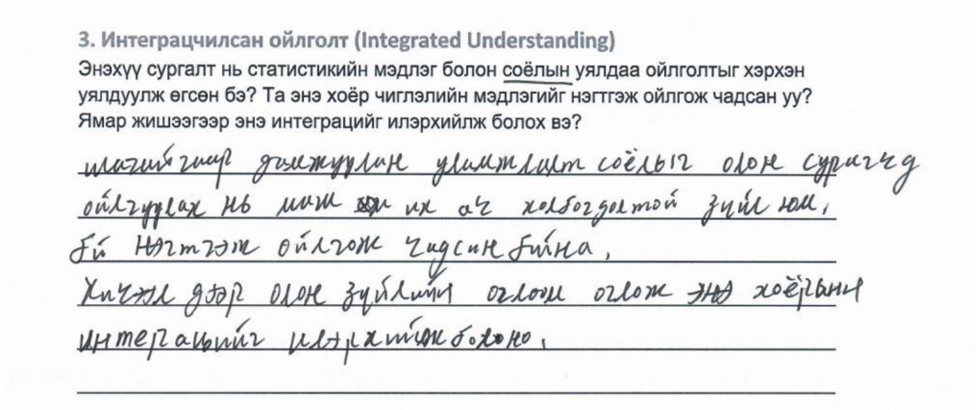

2. Статистикийн туршлага (Statistical Experience)
Эмпирик магадлал, Wilson интервал, энтропи зэрэг ойлголтыг өөрөө туршиж үзсэн нь танд ямар мэдлэг, чадварыг нэмсэн бэ? Эдгээр тооцооллыг гараар хийх нь танд хэр ойлгомжтой, ач холбогдолтой санагдсан бэ?

Гараар тооцоолох нь илүү тэр томъёог хэрхэн ашиглахыг байгаар ойлгоход дөхөм байсан. Өмнө нь мэддэг байсан судлагдахуун байсан ч дахин сануулж тооцоолж болоход хөгжилтэй байлаа.

Translated Context:
Manual calculation made it easier to understand how to apply the formulas. Although I already knew this content before, it was a good reminder and enjoyable to think about it in a different way.

Figure 4. Participant 7 notes

3. Integrated Understanding — Participants demonstrated an emerging ability to synthesize statistical, cultural, and instructional dimensions as shown in Figure 5. They proposed applying *Shagai*-based probability lessons across multiple disciplines, including language, physics, and history. This theme reflects a broader shift toward interdisciplinary thinking and curriculum integration, highlighting the potential for ethnomathematical activities to enrich diverse areas of learning.

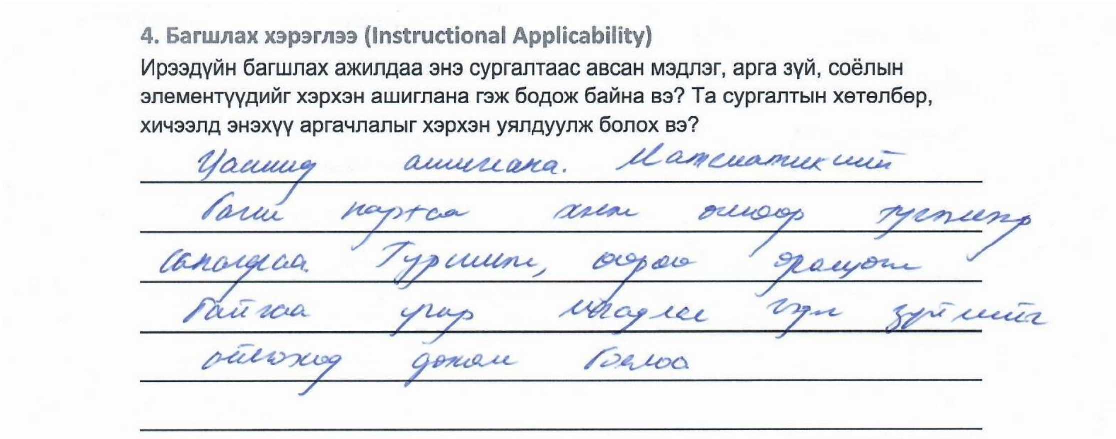

3. Интеграцчилсан ойлголт (Integrated Understanding)
Энэхүү сургалт нь статистикийн мэдлэг болон соёлын уялдаа ойлголтыг хэрхэн уялдуулж өгсөн бэ? Та энэ хоёр чиглэлийн мэдлэгийг нэгтгэж ойлгож чадсан уу? Ямар жишээгээр энэ интеграцийг илэрхийлж болох вэ?

Шагайгаар дамжуулан уламжлалт соёлыг олон сурагчид ойлгуулах нь маш их ач холбогдолтой зүйл юм. Би нэгтгэж ойлгож чадсан байна. Хичээл дээр олон зүйлийн ойлгох ойлгож энэ хоёрын интеграцийг илэрхийлж болно.

Translated Context:
It is very meaningful to help many students understand traditional culture through Shagai. For me, it enabled an integration of mathematics and culture. I realized that many concepts in mathematics lessons can be understood in this integrated way.

Figure 5. Participant 6 notes

4. Instructional Applicability — All participants indicated a strong intention to adapt the workshop experience for future classroom implementation. They identified specific strategies, including the use of pre- and post-assessments, facilitation of learner-centered experimentation, and integration of traditional games into lesson design. Many participants also expressed interest in disseminating these methodologies to peers and colleagues (see Figure 6).


4. Багшлах хэрэглээ (Instructional Applicability)
Ирээдүйн багшлах ажилдаа энэ сургалтаас авсан мэдлэг, арга зүй, соёлын элементүүдийг хэрхэн ашиглана гэж бодож байна вэ? Та сургалтын хөтөлбөр, хичээлд энэхүү аргачлалыг хэрхэн уялдуулж болох вэ?

Цаашид ашиглана. Математикийн багш нартаа хэлж өгмөөр туслмаар санагдсан. Туршиж, өөрөө оролцож байгаа учир магадлал гэх зүйлийг ойлгоход дөхөм болоо

Translated Context:
I will use this in the future. I felt like sharing it with and supporting other mathematics teachers. Because I personally tried it and actively participated, it became much easier for me to understand the concept of probability.

Figure 6. Participant 2 notes

5. Ethnostatistical Conceptualization — Participants articulated a redefinition of ethnostatistics as a comprehensive educational framework. *Shagai* was recognized not only as a convergence of cultural knowledge and statistical practice but also as a tool for cultivating logical thinking and modeling skills. One participant summarized, "*Shagai isn't just a game. This demonstrated that by using traditional games in modern education, mathematics lessons can be taught more effectively and engagingly*."

Discussion

This study investigated the probabilistic characteristics of the traditional Mongolian *Shagai* and its potential as an instructional tool in mathematics education. Large-scale empirical experimentation revealed that *Shagai* outcomes deviate markedly from a uniform distribution, with sheep and goat occurring more frequently than horse or camel. These findings were supported by statistical analyses, including Wilson confidence intervals, entropy calculations, stabilization plots, and a chi-square goodness-of-fit test. The results confirm not only the inherent bias arising from the bone's physical geometry but also its suitability as a culturally grounded, analytically tractable artifact for probability instruction.

From a cultural perspective, the predominance of sheep and goat outcomes aligns with Mongolia's pastoral heritage, in which these animals are the most numerous and economically significant livestock (Batkhuyag, 2024; Tudev, 2006). Sheep symbolize abundance and everyday sustenance, while goats represent agility and resilience within nomadic life (Chuluun, 1958; UNESCO, 2014). Consequently, the observed probabilities reflect both physical properties of the bone (Shemyakin, 2019) and culturally embedded symbolism, underscoring *Shagai*'s dual role as a mathematical object and a culturally responsive pedagogical tool (Rosa & Orey, 2016).

Instructionally, the findings provide compelling evidence that culturally localized tools such as *Shagai* can effectively support the teaching of probability concepts. In the doctoral-level workshop, engagement with culturally relevant experimentation fostered both conceptual understanding and contextual insight. Statistically significant gains in pre- and post-test performance were complemented by reflective responses, indicating that the use of familiar, tangible cultural materials enhanced motivation and engagement. These outcomes align with prior work advocating ethnomathematical approaches as a means to promote equity, engagement, and deeper mathematical thinking through cultural connection (Rosa & Orey, 2011).

The structured progression of the workshop—from cultural contextualization to empirical investigation and interpretive reflection—was integral to facilitating conceptual growth. Participants did not merely compute empirical probabilities; they experienced the complete cycle of statistical reasoning, including data collection, analysis, interpretation, and integration with cultural knowledge. This hands-on approach, grounded in culturally meaningful data, reinforced abstract probability concepts and supported durable learning outcomes, highlighting the pedagogical potential of integrating traditional artifacts into mathematics instruction.

The empirical probabilities determined in this study—approximately 0.12 (horse), 0.13 (camel), 0.39 (sheep), and 0.36 (goat)—have been statistically validated and may serve as reliable reference values in instructional settings. For general classroom applications, these values can be rounded to one decimal place (0.1, 0.1, 0.4, 0.4), closely aligning with the empirical frequencies reported by David (1955). For contexts requiring higher precision, such as secondary or tertiary instruction, the probabilities established in this study are preferable to those of Maystrov (1961) due to the substantially larger dataset and controlled experimental design.

These findings address the study's first two research questions by demonstrating that (a) shagai outcomes are not equally probable and (b) large-scale, standardized trials refine and validate earlier empirical studies. Additionally, the results contribute to the third research question by illustrating that high-precision probability estimation within a culturally contextualized setting supports deeper statistical reasoning. Learners engaged not only in the calculation of frequencies but also in critical reflection on fairness, randomness, and the cultural significance of outcomes, reinforcing prior evidence that culturally grounded instruction enhances conceptual engagement (Rosa & Orey, 2016; Elbehary, 2022). In this context, *Shagai* functions simultaneously as a data-generating object and a culturally responsive pedagogical tool.

By situating probability learning within a familiar cultural framework, this study extends research on ethnomathematics and culturally responsive teaching (Lipka et al., 2005; Abdullah, 2017) and provides empirical support for the integration of traditional games in formal mathematics instruction. The recognition that all four *Shagai* outcomes possess distinct probabilities generated curiosity and engagement among learners, prompting rich classroom discussions on fairness, symmetry, and the interplay between cultural context and probability. The doctoral-level workshop confirmed that presenting *Shagai* as a probabilistic object with non-uniform outcomes is both mathematically valid and pedagogically effective.

Moreover, *Shagai*-based instruction exhibits flexibility across educational levels. At the middle school level, it can introduce empirical and theoretical probability through simple frequency-based analysis. At the secondary level, students may explore compound events and combinatorial reasoning by modeling outcomes from multiple *Shagai*. In higher education, *Shagai* can support connections to binomial and multinomial distributions or serve as a foundation for creative modeling tasks in statistics and game theory. This versatility is particularly valuable in compulsory mathematics education, offering culturally relevant, engaging approaches to abstract topics. By incorporating *Shagai* into classroom practice, probability concepts can be made more accessible and meaningful while simultaneously affirming students' cultural identities.

In addition, the use of traditional casting conditions, such as the felt surface (*Shirdeg*), underscores the importance of authenticity in culturally responsive teaching. This practice aligns with Lipka et al.'s (2005) emphasis on respecting indigenous epistemologies not only as cultural heritage but also as functional and pedagogically meaningful resources. Rather than treating such tools as peripheral or supplementary, this study positions *Shagai* as a central, generative object in mathematics instruction, capable of fostering both conceptual understanding and cultural connection.

The findings demonstrate that ethnomathematical materials provide dual value: as empirical resources for statistical learning and as cultural anchors that support learner identity, engagement, and inclusivity. When embedded within carefully designed instructional sequences, localized tools such as *Shagai* bridge abstract mathematical concepts with students lived experiences, making probability education more relevant, accessible, and equitable.

Thematic analysis revealed that incorporating *Shagai* into statistics instruction positively influenced participants' attitudes, comprehension, and pedagogical intentions. Attitudinal shifts indicated that culturally embedded mathematics fosters personal engagement and epistemological openness, corroborating the work of Lipka et al. (2005) and Barton (2008) on the value of local knowledge systems in formal learning. Participants' statistical experiences suggested that tactile, embodied learning—particularly manual computation—enhanced understanding of abstract concepts. By calculating probabilities and entropy by hand, learners were able to connect theoretical content with intuitive understanding, supporting evidence that hands-on methods promote deeper comprehension (Rosa & Orey, 2011).

Integrated understanding emerged as a particularly salient outcome. Participants connected traditional games to disciplines beyond mathematics, demonstrating advanced capacity for interdisciplinary curriculum thinking. This supports the argument that ethnomathematical tools promote systemic thinking and holistic pedagogical approaches. Regarding instructional applicability, participants reported strong motivation to adapt *Shagai*-based lessons into their own practice. The perceived value of pre- and post-assessments, learner-centered experimentation, and culturally relevant content indicates that such approaches are both scalable and contextually meaningful.

Finally, participants' reconceptualization of ethnostatistics highlights its potential not merely as a method but as a transformative epistemological framework. *Shagai*-based learning was described as bridging ancestral knowledge and modern statistical reasoning, consistent with Rosa and Orey's (2016) framing of ethnomathematics as simultaneously culturally situated and analytically rigorous. These findings reinforce the pedagogical power of integrating culturally embedded artifacts into mathematics education, demonstrating that ethnomathematical tools can enrich conceptual understanding while fostering cultural engagement and identity.

CONCLUSION

This study empirically established the non-uniform probability distribution of the Mongolian *Shagai* through design-based experimentation and rigorous statistical modeling. Drawing on over 10,000 controlled single-throw trials, the analysis employed Wilson score intervals, entropy metrics, stabilization plots, and chi-square tests to confirm both the statistical reliability and cultural specificity of *Shagai* outcomes. Beyond empirical validation, the study examined the pedagogical potential of *Shagai* within a culturally contextualized instructional framework. A 180-minute workshop for doctoral students demonstrated statistically significant gains in conceptual understanding, accompanied by positive attitudinal shifts toward culturally grounded mathematics. These results support the integration of ethnomathematical artifacts into probability education, extending the contributions of Rosa and Orey (2011) and Knijnik (2002).

Importantly, the findings suggest that traditional cultural tools—when intentionally embedded within instructional design—can function as central resources for developing both statistical reasoning and cultural literacy. Rather than treating indigenous materials as supplementary, *Shagai* was positioned as a generative object that fosters cognitive engagement and epistemological inclusion. In this regard, the study contributes to broader efforts to decolonize mathematics education by legitimizing local knowledge systems as viable and productive grounds for mathematical inquiry. *Shagai* emerges not only as a cultural symbol but also as a platform for advancing ethnostatistical thinking within Mongolian educational contexts. On the other hands, the study's limitations include its focus on single-*Shagai* trials and short-term educational interventions. It did not examine longitudinal learning gains, variations in cultural interpretation, or gameplay involving multiple *Shagai* pieces. Furthermore, the instructional outcomes were based on a small cohort of doctoral students, limiting the generalizability of findings across broader educational contexts.

Finally, future research should investigate how the probabilistic structure of *Shagai* can be integrated across educational levels. In middle school, *Shagai* may support instruction on fundamental concepts such as empirical and theoretical probability through frequency-based reasoning. At the secondary level, applications could extend to combinatorics, probability trees, and compound events through multi-*Shagai* modeling. In tertiary education, *Shagai* could inform explorations of binomial and multinomial distributions or serve as a basis for creative modeling tasks in statistics and game theory. Furthermore, future studies also should examine the long-term impact of *Shagai*-based instruction in culturally diverse classrooms and how learners interpret its mathematical relevance in relation to their cultural backgrounds. Comparative investigations of traditional probabilistic games across cultures—such as knucklebones in Central Asia or *Astragaloi* in Ancient Greece—could further illuminate the role of culturally embedded artifacts in developing probabilistic reasoning. Overall, this research underscores the potential of *Shagai* as both a source of empirical data and a culturally meaningful pedagogical tool, capable of bridging abstract mathematical concepts with learners lived experiences, enhancing engagement, and supporting inclusive, culturally responsive mathematics education across multiple grade levels and contexts.

Acknowledgment

We would like to express our heartfelt gratitude to all individuals who contributed to this study, making it possible to explore the cultural and mathematical significance of *Shagai*. Our sincere appreciation goes to the community participants who generously volunteered their time to perform *Shagai* trials, providing valuable data that reflect traditional understandings of this culturally significant game. Your involvement laid a strong foundation for our analysis of *Shagai* as a cultural artifact with probabilistic properties.

We are equally grateful to the undergraduate and doctoral students who participated in the educational phase of this research. Your dedication, careful execution of *Shagai* trials, and willingness to engage under standardized conditions greatly enriched the dataset, enabling meaningful comparisons between community practices and educational settings.

Special thanks are also extended to the educational institutions that facilitated student participation and to the researchers' extended social and professional networks for their support in fostering community engagement. Your cooperation was instrumental in ensuring the cultural authenticity and educational relevance of this study.

Finally, we wish to acknowledge the cultural experts who provided invaluable insights into the traditional practices and symbolic meanings associated with *Shagai*. Your knowledge and guidance ensured that this research remained both culturally respectful and academically rigorous while exploring the mathematical dimensions of this enduring heritage.

Declarations

Author Contribution	:	IM: Conceptualization, Supervision, Validation, Methodology, Project Administration, Investigation, Data Curation, Resources, Writing - Original Draft, Formal Analysis, and Writing - Review and Editing. KHO: Conceptualization, Methodology, Investigation, Data Curation, Resources, Writing - Original Draft, and Writing - Review and Editing.
Funding Statement	:	This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

REFERENCES

Abdullah, A. S. (2017). Ethnomathematics in Perspective of Sundanese Culture. *Journal on Mathematics Education*, *8*(1), 1-16. https://files.eric.ed.gov/fulltext/EJ1173644.pdf

Adiyasuren, V. (2021). *Professional mathematics course 11* [In Mongolian]. Soyombo Printing.

Barton, B. (2008). *The language of mathematics*: *Telling mathematical tales* (Vol. 44). Springer Science and Business Media. https://doi.org/10.1007/978-0-387-72859-9_5

Batkhuyag, B. (2024). *Mongolian traditional game—"Shagai": Features of Mongolian traditional games*. *Journal of Ethnomathematics and Cultural Statistics*, *12*(2), 45–59. https://doi.org/10.17265/2328-2177/2024.02.004

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Brown, L. D., Cai, T. T., & DasGupta, A. (2001). Interval estimation for a binomial proportion. *Statistical Science*, *16*(2), 101–133. https://doi.org/10.1214/ss/1009213286

Chuluun, Ch. (1958). *Games with Shagai*. State Publishing.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, *32*(1), 9–13. https://doi.org/10.3102/0013189X032001009

Culin, S. (1895). Games of the North American Indians. *Annual Report of the Bureau of American Ethnology*, *24*, 3–846.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, 5(1), 44-48. https://www.jstor.org/stable/40247876

Dashdorj, Ts., & Ganbaatar, A. (1998). *Professional mathematics course – I* [In Mongolian]. Urlakh Erdem Publishing.

David, F. N. (1955). Studies in the history of probability and statistics I. Dicing and gaming (A note on the history of probability). *Biometrika*, *42*(1–2), 1–15. https://doi.org/10.1093/biomet/42.1-2.1

Elbehary, S. G. A. (2022). Rethinking mathematics teachers' professional knowledge for teaching probability from the perspective of probabilistic reasoning: A proposed framework. *International Electronic Journal of Mathematics Education*, *17*(3), em0695. https://doi.org/10.29333/iejme/12145

Gardiner, E. N. (1930). *Athletics of the ancient world*. Clarendon Press.

Gay, G. (2010). *Culturally responsive teaching*: *Theory, research, and practice* (2nd ed.). Teachers College Press.

Jizba, P., & Arimitsu, T. (2004). The world according to Rényi: Thermodynamics of multifractal systems. *Annals of Physics*, *312*(1), 17–59. https://doi.org/10.1016/j.aop.2004.01.002

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mooney, E. S. (2007). Students' probabilistic thinking. In F. K. Lester (Ed.*), Second handbook of research on mathematics teaching and learning* (pp. 909–956). Information Age Publishing.

Itgel, M., & Sugir, T. (2004). *Collection of problems in probability theory* [In Mongolian]. Ulaanbaatar.

Knijnik, G. (2002). Ethnomathematics and education in the landless movement in Brazil. In A. Powell and M. Frankenstein (Eds.), *Ethnomathematics: Challenging eurocentrism in mathematics education* (pp. 253–264). SUNY Press.

Lipka, J., Sharp, N., Adams, B., & Sharp, F. (2005). Math in a cultural context: Two case studies of a successful culturally based math project. *Anthropology and Education Quarterly*, *36*(4), 367–385. https://doi.org/10.1525/aeq.2005.36.4.367

Maystrov, L. E. (1961). Role of games of hazard in the origin of probability. *Acta Classica Universitatis Scientiarum Debrecensis*, *7*, 2.

Miyejav, I. (2007). The use of achievement tests in teaching probability theory to applied mathematics students at Mongolian universities. *Doctoral dissertation*. Moscow State Pedagogical University. https://www.researchgate.net/publication/356970475

Rosa, M., & Orey, D. C. (2011). Ethnomathematics: The cultural aspects of mathematics. *Revista Latinoamericana de Etnomatemattica,* *4*(2), 32-54. https://www.revista.etnomatematica.org/index.php/RevLatEm/article/view/32/378

Rosa, M., & Orey, D. C. (2016). *Ethnomathematics and its diverse approaches for mathematics education*. Springer. https://doi.org/10.1007/978-3-319-59220-6

Shemyakin, A. (2019). Random rules and the ancient history of simulation. *Model Assisted Statistics and Applications*, *14*(4), 347–359. https://doi.org/10.3233/MAS-190473

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465–494). Macmillan.

Simmons, A. H. (1997). *The neolithic revolution in the near east: Transforming the human landscape*. University of Arizona Press.

Tudev, L. (2006). *Mongolian shagaa* [In Mongolian]. Urlakh Erdem Publishing.

Tumurbaatar, T. (2024). Identification of animal anklebones (astragalus) discovered from Xiongnu and Xianbei period graves. *Studia Archaeologica*, *45*(2), 50-61. https://doi.org/10.5564/sa.v45i2.3784

UNESCO. (2014). *Mongolian knuckle-bone shooting*. Retrieved from https://ich.unesco.org/en/RL/mongolian-knuckle-bone-shooting-00959