Journal on Mathematics Education
Volume 16, No. 3, 2025, pp. 981-1000

Prospective teachers' iceberg designs in realistic mathematics education approach: Connecting mathematics and the SDGs

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Received: 30 June 2025 | Revised: 31 August 2025 | Accepted: 4 September 2025| Published Online: 6 September 2025

Abstract

The Iceberg Design framework has been utilized to represent the progression of students' mathematical understanding, moving from informal, contextually grounded reasoning toward formal mathematical abstraction. This study investigates how prospective mathematics teachers develop Iceberg Designs within the Realistic Mathematics Education (RME) framework, a model that enhances contextual learning and supports mathematical literacy. Thirty prospective mathematics teachers from Universitas Negeri Surabaya participated in this qualitative study, collaboratively designing Iceberg models as part of their coursework. Data from document analysis, interviews, and observations were evaluated using content analysis, the research evaluated the depth and coherence of their designs across four key components: situational contexts which evaluates the relevance and variety of real-world situations, model-of representations which examines the assistance of mathematical representation to connect the context into mathematical concept, model-for abstractions which assess the use of mathematical models toward formalization, and formal mathematical concepts which assess the mathematical ideas being explicitly involved. The findings reveal significant variation in the quality and completeness of the Iceberg Designs. Models for equivalent ratios and quadratic equations exhibited strong integration, using multiple, varied contexts to bridge situational and formal mathematical understanding effectively. Conversely, designs for fraction multiplication and quadrilateral area conservation were often surface level, relying on a single, underdeveloped context that hindered abstraction. Importantly, the study underscores the potential of Iceberg Designs to support the Sustainable Development Goals (SDGs), particularly in fostering critical thinking, practical problem-solving, and meaningful contextual learning for high quality of education (SDG 4) and decent work for sustainable economic growth (SDG 8). These insights indicate the need for deeper integration of RME principles in teacher education and curriculum development through sustained investment in this area.

Keywords: Iceberg Design, Mathematical Literacy, Prospective Teachers, Realistic Mathematics Education, SDGs

How to Cite: Sari, Y. M., Fiangga, S., El Milla, Y. I., Shahrill, M., & Yanti, L. P. (2025). Prospective teachers' iceberg designs in realistic mathematics education approach: Connecting mathematics and the SDGs. *Journal on Mathematics Education, 16*(3), 981-1000. http://doi.org/10.22342/jme.v16i3.pp981-1000

Mathematics education is a foundational component of general education, playing a crucial role in developing students' logical reasoning and problem-solving capacities. Beyond its intrinsic value, mathematics serves as a language and tool for other disciplines, fostering creativity, innovation, critical thinking, and logical analysis—competencies essential for cultivating a highly competitive and adaptable workforce. However, researchers have consistently observed that the pursuit of high-quality mathematics education is challenged by persistent systemic issues across many countries (Homavazir & Homavazi, 2024; Llinares, 2021; Priatna et al., 2020; Tanujaya et al., 2017).

For example, results from the Programme for International Student Assessment (PISA) 2022 indicate that students' average performance in mathematics (366 points), reading literacy (359 points), and science (383 points) in some countries remains well below the OECD averages of 472, 476, and 485 points, respectively (OECD, 2022). Alarmingly, these results represent a decline compared to scores from earlier cycles, including those recorded in 2003 (for mathematics and literacy) and 2006 (for science). The decline has been attributed in part to the COVID-19 pandemic, which exacerbated existing educational inequities by limiting access to technology and learning resources during periods of remote instruction (Engelbrecht & Kaiser, 2023; Jana & Rout, 2021; OECD, 2022). In addition to these pandemic-related effects, curricula that underemphasize 21st-century skills and traditional teacher-centered pedagogical practices—where opportunities for critical thinking, inquiry, and problem-solving are limited—have further contributed to low student outcomes (Dilekçi & Karatay, 2023; Martinez et al., 2022; OECD, 2022; Thornhill-Miller et al., 2023). These factors underscore the urgent need for mathematics instructional approaches that systematically cultivate students' critical thinking, creativity, and problem-solving skills.

One promising approach is *Pendidikan Matematika Realistik Indonesia* (PMRI), the Indonesian adaptation of the Realistic Mathematics Education (RME) framework, which has demonstrated potential in improving mathematics learning outcomes (Pramudiani et al., 2022; Sembiring, 2010; Zulkardi et al., 2020). PMRI emphasizes active, student-centered learning within meaningful contexts, enabling learners to construct a deep conceptual understanding of mathematics through guided reinvention (Ariati & Suparman, 2023; Fauziah & Zulkardi, 2022; Prahmana et al., 2020; Rahmawati & Ranti, 2021). By leveraging students' prior knowledge and embedding mathematical ideas in problem-solving activities, PMRI encourages the gradual development of mathematical literacy.

A central component of PMRI is the Iceberg Design framework, which conceptualizes the trajectory of students' mathematical understanding across four interconnected stages (Webb, 2017): situational context where mathematical ideas are introduced through real-life phenomena, model-of where learners construct informal models that represent these contextual situations, model-for where these representations are progressively generalized, supporting abstraction and symbolic reasoning, and formal knowledge where students consolidate and articulate the underlying formal mathematical concepts. This progression—from intuitive, contextually grounded insights to abstract, formal reasoning—has been shown to enhance the quality of mathematics education by supporting deeper conceptual understanding and bridging the gap between everyday experiences and formal mathematics.

Prospective mathematics teachers play a pivotal role in the effective implementation of PMRI across all levels of education. PMRI, as the Indonesian adaptation of RME, promotes contextual learning, enabling students to connect mathematical ideas to meaningful experiences encountered in primary and junior secondary school settings. Preparing these prospective teachers, therefore, requires equipping them not only with content knowledge but also with the pedagogical capacity to design interactive, student-centered learning environments.

A recent systematic review by Risdiyanti et al. (2024) demonstrates that PMRI enhances students' higher-order thinking skills by enabling them to apply mathematical knowledge models to authentic, real-world problems. Consequently, empowering prospective teachers with both conceptual understanding and pedagogical confidence is essential for enabling them to address diverse classroom contexts and student needs. Within this perspective, investigating prospective teachers' construction of Iceberg Designs is crucial to assessing the extent to which PMRI effectively prepares them to foster mathematical literacy, a need emphasized by Webb (2017) and further elaborated by Abrahamson and Zolkower (2020) as well as Zolkower and Gallego (2020).

The importance of strengthening mathematical knowledge and reasoning has grown significantly in the context of today's globalized and knowledge-driven economy (Maass et al., 2019; Novita & Herman, 2021). In line with the newly implemented Indonesian curriculum, PMRI emphasizes conceptual understanding, problem-solving skills, and the application of mathematics to real-life situations. Examining how prospective mathematics teachers integrate PMRI principles—particularly through the Iceberg Design—offers insight into how teacher education can contribute to the achievement of the United Nations Sustainable Development Goals (SDGs). For example, SDG 4 (Quality Education) calls for inclusive and equitable education, which PMRI supports through its emphasis on meaningful, contextually grounded learning experiences (United Nations, 2023). Furthermore, by fostering critical thinking and problem-solving abilities, PMRI contributes to SDG 1 (No Poverty) and SDG 8 (Decent Work and Economic Growth) by preparing a workforce capable of participating in sustainable economic development.

The Iceberg Design is a central framework within PMRI that conceptualizes the progressive development of students' mathematical understanding. In this model, the visible tip of the iceberg represents formal mathematical knowledge, whereas the submerged layers represent the deeper conceptual foundations that must be established before formalization can occur (Palupi et al., 2020). The framework highlights the role of context, models, representations, and problem-solving strategies in scaffolding mathematical reasoning (Boswinkel & Moerlands, 2003; Gravemeijer, 2004).

Consistent with the principles of RME, the Iceberg Design supports a gradual, experiential progression toward abstraction. This process reflects mathematization, defined as the transformation of real-life problems into mathematical representations and the subsequent refinement of those representations to develop deeper mathematical insight (Freudenthal, 1991). RME distinguishes between two types of mathematization, such as horizontal mathematization, in which students translate real-world situations into mathematical representations, and vertical mathematization, in which students work within the mathematical system to refine, generalize, and formalize these representations (Pandra et al., 2021).

Freudenthal (1991) describes this process as one in which learners explore, conjecture, and test mathematical ideas based on lived experience. The Iceberg Design operationalizes this philosophy, providing a structured framework for the development of mathematical literacy and problem-solving competence. Its three key layers include: (1) the base, consisting of real-world contexts that ground students' informal reasoning; (2) the intermediate layer, where students' representations, models, and pre-formal strategies emerge; and (3) the peak, which consolidates formal mathematical knowledge (Boswinkel & Moerlands, 2003).

Empirical evidence supports the effectiveness of the Iceberg Design in mathematics education. For instance, Zulkardi et al. (2020) demonstrated its capacity to bridge the gap between informal reasoning and formal conceptual understanding in contextualized problem-solving tasks. Similarly, Webb (2017) emphasized its role in promoting higher-order thinking through systematic abstraction, while Pramudiani et al. (2022) found that prospective teachers who engaged with Iceberg Design frameworks were better able to develop context-rich tasks tailored to their students' cognitive development. This aligns with the OECD's (2022) definition of mathematical literacy as the capacity to formulate, apply, and interpret mathematics in diverse real-world contexts, thereby enabling effective reasoning and problem-solving. As further supported by van den Heuvel-Panhuizen (2003), the Iceberg Design facilitates progressive abstraction and fosters students' mathematical reasoning.

Moreover, the Iceberg Design can be meaningfully connected to several SDGs. The use of diverse situational contexts directly supports SDG 4 by ensuring that mathematics education is inclusive and relevant. The development of model-of and model-for stages aligns with SDG 9 (Industry, Innovation, and Infrastructure) by nurturing innovative thinking and creative problem-solving. Finally, the transition to formal knowledge contributes to SDG 8 by equipping students with the analytical skills needed for sustainable economic growth. Therefore, the purpose of this study is to examine how prospective mathematics teachers construct Iceberg Designs within the RME/PMRI framework, to analyze the extent to which these designs align with SDGs, and to explore their potential for enhancing mathematics education. By doing so, the study aims to support teacher education programs in preparing future educators who can leverage the Iceberg Design to address educational challenges and contribute to sustainable development.

METHODS

Research Design

This study employed a qualitative research design to explore prospective mathematics teachers' understanding and development of the Iceberg Design within the PMRI framework. Qualitative methods were selected because they enable an in-depth examination of participants' experiences and meaning-making processes in authentic contexts (Denscombe, 2010). This approach is particularly appropriate for investigating the complex, contextual, and subjective nature of teachers' conceptualizations of the Iceberg Design and its pedagogical application. Following Milles and Huberman (2014), the qualitative methodology was used to generate rich, nuanced data capable of capturing participants' reasoning processes, representations, and strategies, thereby providing insights into how prospective teachers conceptualize mathematical literacy through the PMRI approach.

Participants

The study involved 30 prospective mathematics teachers (4 male, 26 female), aged 18–20 years, who were enrolled in PMRI courses at a public university. Participants were selected using purposive sampling, with the goal of including individuals with varying levels of prior experience in designing Iceberg models. The sample size was determined based on the principle of data saturation, ensuring both sufficient depth and diversity of responses while maintaining feasibility for intensive analysis.

To promote collaborative learning and peer interaction, participants were organized into small groups of three to four members. Within these groups, participants discussed ideas, articulated their thinking, and collaboratively refined their Iceberg Designs through iterative dialogue and problem-solving. This group-based approach not only fostered critical reflection on how mathematical concepts could be developed through realistic contexts but also allowed the researcher to identify recurring themes and patterns in participants' approaches. The collaborative setting was essential for eliciting diverse perspectives and deepening participants' understanding of how to design effective, context-rich mathematical learning experiences. In the PMRI framework, an Iceberg Design serves as a visual and conceptual model describing the process of realistic mathematics learning: the visible tip of the iceberg represents the formal mathematical concept, whereas the submerged base illustrates the informal, exploratory activities and contextual situations that support students' progression toward abstraction.

Data Collection

Data were collected through a combination of semi-structured interviews and document analysis of participants' Iceberg Designs.

1. Interviews lasted approximately 45–60 minutes and followed a pre-designed protocol that probed participants' understanding of the Iceberg Design, the mathematical concepts embedded within their models, and the factors influencing their design choices.
2. Document analysis involved collecting written and graphical artifacts created during the design process, including notes, sketches, schematizations, and final models. These documents provided tangible evidence of participants' reasoning processes, use of mathematical notation, and progression from situational contexts toward formal concepts.

This multimethod approach facilitated data triangulation, strengthening the validity of the findings by enabling cross-verification of interview data with participants' actual design work.

Data Analysis

All interviews were audio-recorded, transcribed verbatim, and analyzed alongside the participants' design documents using qualitative content analysis. Coding was conducted inductively to identify emergent patterns, categories, and themes reflecting participants' conceptualization of the Iceberg Design. The analysis was guided by the Hypothetical Learning Trajectory (HLT) framework, which models the developmental pathways through which learners acquire mathematical concepts. As Fiangga et al. (2021) note, the Iceberg Design can be interpreted as a graphical instantiation of HLT, mapping students' progression from situational, context-based reasoning (lower levels) toward formal mathematical knowledge (upper levels). By aligning participants' Iceberg Designs with the HLT framework, the analysis provided insights into both the depth of their pedagogical understanding and the coherence of their designs.

Table 1 presents the analytical framework adapted from Fiangga et al. (2021), detailing the four main components of the Iceberg Design—Situational Contexts, Model-of, Model-for, and Formal Mathematical Concepts—together with their associated indicators and evaluation criteria.

Table 1. Analytical framework for evaluating Iceberg Designs (Adapted from Fiangga et al., 2021)

Iceberg's Component	Indicator	Criteria
Formal	Mathematical Concept	Involvement of the mathematics concepts in the iceberg. 0: no mathematical concepts found, 1: embedded mathematical concepts found but only as an additional. 2: mathematical concepts are irrelevant to the activity objective. 3: mathematical concepts are relevant to completing the designed activity's objective. 4: unique mathematical concepts are created.
Model For	Vertical Mathematization	0: No mathematical representations or symbols are employed, and there is no attempt to lead students to a more formal or abstract level of understanding. 1: Basic mathematical symbols were employed, but there was no demonstration of mathematical relationships or processes and a lack of variety in the representations or models utilized.

		2: Utilize a limited range of mathematical symbols and language in manipulation, but not yet to the formulation and application of formulas. 3: Employs a fair amount of relevant mathematical symbols and language in manipulations during problem-solving, but limited variety, and able to formulate and apply formulas. 4: Employ various mathematical symbols and language effectively in problem-solving manipulations, proficient in formulating and applying formulas in diverse representations
	Horizontal Mathematization	0: No mathematical representations or symbols were used that were appropriate to the context. 1: Basic mathematical symbols were employed but did not indicate mathematical relationships or processes, and there was a lack of variety in the representations or models used. 2: Use contextually relevant mathematical representations or symbols but employ formal mathematical symbols with contextually irrelevant manipulations. 3: Use mathematical representations or symbols related to the context and build relationships with formal mathematical symbols through the process of identifying regularities, relations, and patterns in the context. 4: Use mathematical representations or symbols related to the context and build connections with formal mathematical symbols through the process of identifying regularities, relationships, and patterns in the context. In addition, being able to illustrate them in different contexts
Model-of	Didactical Phenomenology	0: There is no assistive model for thinking. 1: There is a thinking assistance model, but it is relevant to the mathematical concept but does not involve the situational context. 2: The assistance model for understanding the given context is not directly related to the intended mathematical concept. 3: The assistance model for understanding the given context leads to the intended mathematical concept but is only related to one situational context. 4: The assistance model for understanding the given context leads to the intended mathematical concept and is related to more than one of its situational contexts.
	Horizontal Mathematization	0: No context illustration used. 1: There are visual illustrations, but they do not portray the relationship of the given context. 2: Use visualizations and schematizations from context but incompletely. 3: Use visualizations and schematizations from relevant contexts. 4: Use visualization and schematization of relevant contexts and exhibit regularities, relations, and patterns within the context.
Situational	Context	0: No context relevant to the mathematical concept being learned was found. 1: The provided context is not relevant to the mathematical concepts being learned. This can occur if the chosen context does not help students understand or apply the concept being taught.

	2: The context provided is relevant but unvaried. At this level, although the chosen context helps students understand the concept, the lack of variety can reduce the level of student engagement or understanding. 3: The context provided is relevant but only slightly varied. The context helps in understanding the concept, but the variety is limited, so it does not fully facilitate deep understanding or application of the concept. 4: The contexts provided are not only relevant but also varied. At this level, the chosen context effectively helps students understand mathematical concepts and the variety enables students to see how the concepts can be applied in various situations

This analytical framework enabled systematic classification and comparison of participants' designs, providing a robust basis for identifying patterns, commonalities, and differences. The results of this content analysis informed the formulation of research findings and recommendations, offering a deeper understanding of the extent to which prospective teachers' Iceberg Designs reflect PMRI principles and contribute to the development of mathematical literacy.

RESULTS AND DISCUSSION

The findings from participants' Iceberg Designs were analyzed using the RME-based Iceberg Analysis Framework presented in Table 1. Results are presented both as an overall summary and as detailed analyses of each iceberg component, with a particular focus on the situational component, which forms the foundation for students' engagement and subsequent mathematization processes.

Situational Component of Iceberg's Design

Participants collaboratively produced seven Iceberg Designs addressing various junior high school mathematics topics. Two designs focused on geometry (circle area and quadrilateral area conservation), one on algebra (quadratic equations), two on number concepts (fraction multiplication and equivalent ratios), and one on probability (deriving the probability formula). Each situational component was analyzed and rated according to the framework criteria as shown in Figure 1.

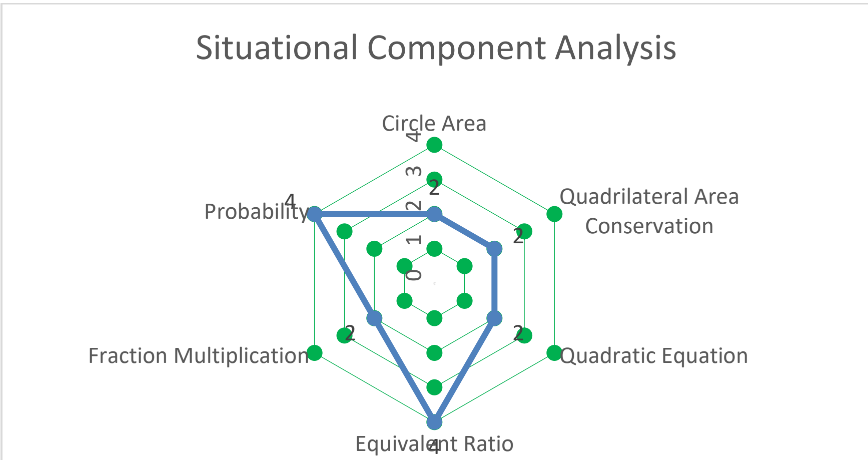


Figure 1. Situational component analysis of Iceberg Design

Overall, participants' situational contexts ranged from Level 2 (relevant but unvaried contexts) to Level 4 (relevant and varied contexts supporting transfer across situations). For example, the design for quadrilateral area conservation was rated at Level 2, as it relied on a single context: determining the lighting needs of rooms in a house presented in Figure 2.

Translated Context:
Mr. Hanan has a house with dimensions 14 m × 12 m. The house is divided into several rooms. Mr. Hanan wants to install lights with identical wattage in each room. Determine the order of rooms from the brightest to the dimmest.

Figure 2. Level 2 Situational Context Example

This context effectively situates the concept of area measurement within a practical household problem, allowing students to explore the relationship between room size and lighting needs. However, its limitation lies in the lack of contextual diversity. According to RME principles, providing multiple situational contexts allows learners to recognize mathematical regularities across settings and strengthens vertical mathematization (Gravemeijer, 2004). Additional contexts—such as calculating the area of a school garden for planting or optimizing space use in urban planning—could broaden students' conceptual exposure and foster transfer of learning to environmental and sustainability-related applications. In contrast, participants' design for equivalent ratios achieved Level 4, demonstrating high relevance and contextual variety illustrated in Figure 3.

Translated Context:
Mrs. Rani produces dresses at a rate of 4 dresses in 4 days. How many dresses can she make if she works for 12 days, assuming a constant work rate?
Mr. Andi's motorcycle travels 77 km using 2 liters of fuel. How far can it travel with 6 liters of fuel?
A student is conducting a study tour from Solo to Semarang. During the trip, the student measures the distance from Solo to Semarang with a map. The actual distance is 1,200,000 cm. If the distance from Solo to Semarang on the map is 6 cm, what is the actual distance from Solo to Semarang?

Figure 3. Level 4 situational context example

Figure 3 presents a Level 4 situational context focusing on the topic of equivalent ratios, characterized by both high relevance and contextual variety. The participants generated three distinct yet complementary scenarios that effectively illustrate the concept of equivalent ratios. The first scenario connects equivalent ratios to a production context, specifically sewing, where students determine how many garments can be produced within a given time frame. This application demonstrates the practical use of ratios in predicting production output based on time constraints. The second scenario links equivalent ratios to fuel consumption and travel distance, thereby highlighting the role of proportional reasoning in determining travel costs or fuel requirements for different distances. The third scenario employs the concept of map scale, enabling students to explore the proportional relationship between distances represented on a map and their corresponding real-world measurements. Collectively, these contexts exemplify the richness and diversity of real-world applications, reinforcing the students' ability to transfer mathematical reasoning across multiple domains. Such variety not only deepens conceptual understanding but also fosters students' capacity for flexible problem-solving and mathematical literacy.

Model-of Component of Iceberg's Design

In the context of Realistic Mathematics Education (RME), the model-of component represents a crucial stage where students construct mathematical representations of realistic situations as a means to deepen their conceptual understanding. Rather than presenting mathematical symbols in isolation, the model-of serves as a mediating tool that enables students to organize their experiences, reason about contextual problems, and progressively move toward formal abstraction (Gravemeijer, 2004; van den Heuvel-Panhuizen, 2003). Within this study, participants were tasked with designing model-of representations that could bridge realistic problem situations with the targeted mathematical concepts. The results of this analysis are summarized in Figure 4.

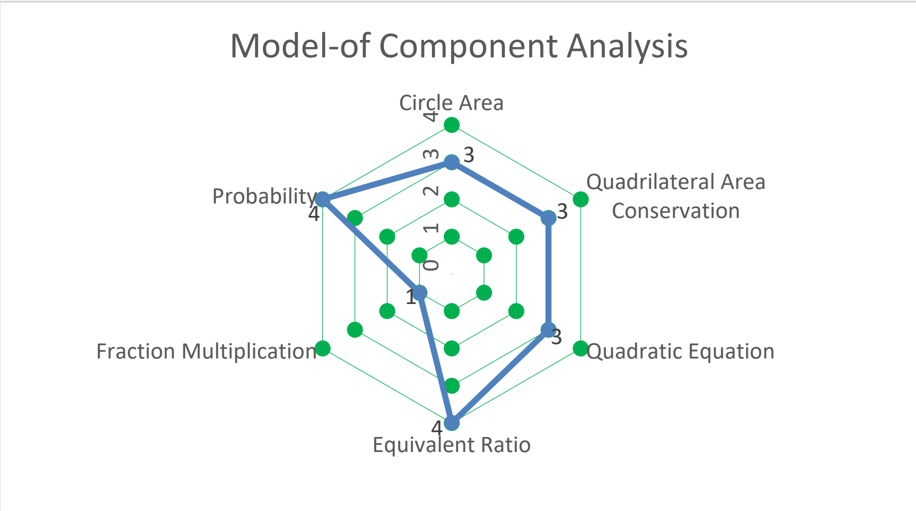


Figure 4. Model-of component analysis in Iceberg Design

Our findings reveal substantial variation in the sophistication of the model-of designs produced by the participants. The levels of horizontal mathematization ranged from Level 1 (minimal contextualization) to Level 4 (richly contextualized and varied representations). For example, in the case of fraction multiplication, several participants achieved only Level 1, as illustrated in Figure 5.

In this example, fraction multiplication was represented using a simple array model, but the situational context—distributing brownie cakes—was omitted. This reveals a pedagogical gap:

participants tended to treat the model as a static mathematical representation rather than as a thinking tool tied to a meaningful situation. Removing the context reduces the model's power to foster realistic reasoning and deprives students of opportunities to conceptualize how fraction multiplication applies in everyday life. Integrating the brownie-sharing scenario would not only ground the model in a concrete experience but also align with RME's guiding principle that mathematical learning should emerge from meaningful situations (Freudenthal, 1991).

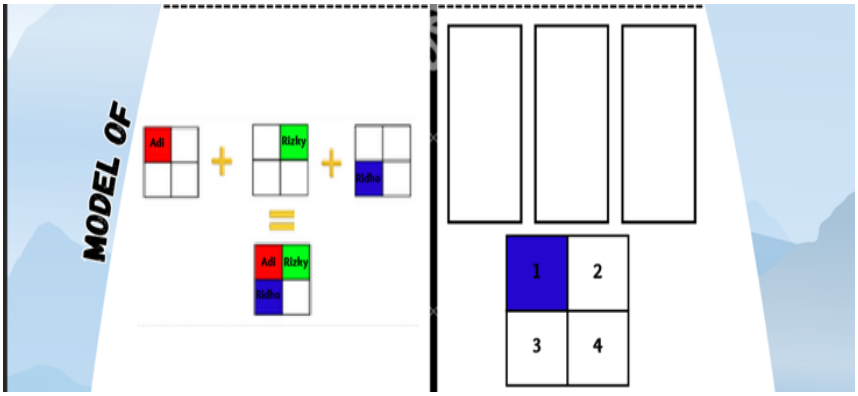


Figure 5. Level 1 Model-of design example

By contrast, Figure 6 illustrates a Level 4 model-of design focusing on equivalent ratios. In this example, participants successfully incorporated multiple pictorial representations—pictograms, tables, and diagrams—directly tied to the situational contexts they had previously developed, including garment production rates, fuel consumption relative to distance, and map scale interpretation. This design exemplifies how model-of representations can function as powerful mediators, linking contextual problems with mathematical generalizations. Notably, participants reported that arriving at this level of sophistication required considerable collaborative discussion and iterative refinement, suggesting that designing effective model-of tasks is itself a complex professional competency.

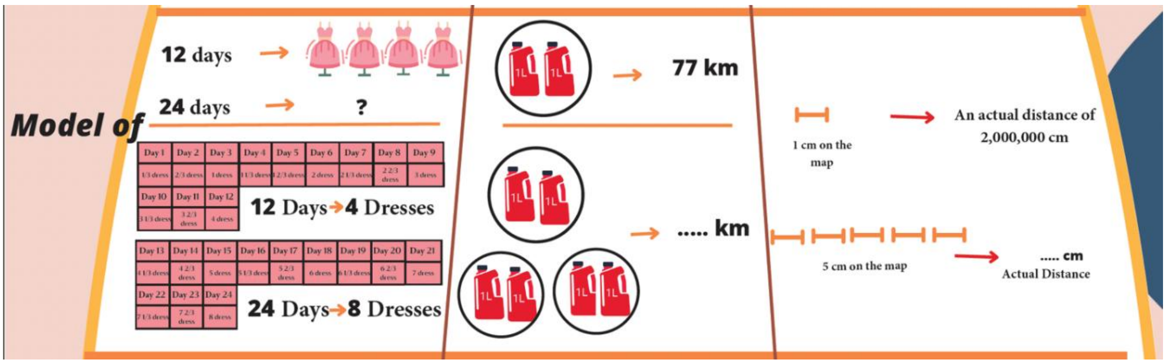


Figure 6. Level 4 Model-of design example

The strength of this Level 4 design lies in its simultaneous variety and coherence. It maintains proximity to the situational contexts while guiding learners toward progressive mathematization, thus serving as an authentic bridge between informal reasoning and formal mathematical understanding. Such designs align strongly with the RME philosophy, which emphasizes that models should evolve from tools-for-situations to tools-for-thinking, supporting students' development of flexible, transferable, and meaningful mathematical knowledge (Gravemeijer, 2004).

Model-for Component of Iceberg's Design

Within the RME framework, the model-for component refers to the use of mathematical models to facilitate understanding of more formal or abstract mathematical concepts and processes. In essence, model-for serves as a bridge that supports students' progression from concrete, context-based reasoning toward generalized mathematical abstraction. Typically, the model-for phase follows model-of, where students first engage with representations grounded in realistic situations. Once students have constructed and explored these initial representations, the model-for extends these models to represent broader or more abstract mathematical concepts, enabling deeper conceptual insight. Figures 7 and 8 present the results of the participants' model-for analyses.

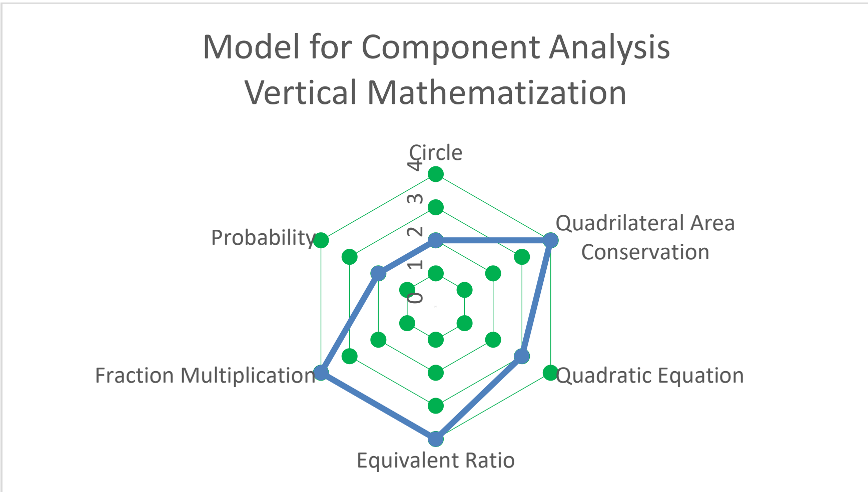


Figure 7. Model-for component analysis "Vertical Mathematization" in Iceberg Design

The radar plots in Figures 7 and 8 illustrate participants' performance across six key mathematical topics—circle area, probability, fraction multiplication, equivalent ratios, quadrilateral area conservation, and quadratic equations—using a 0–4 competency scale. The vertical mathematization plot emphasizes conceptual depth and systematic knowledge acquisition, whereas the horizontal mathematization plot highlights the application of mathematics in varied, contextualized situations.

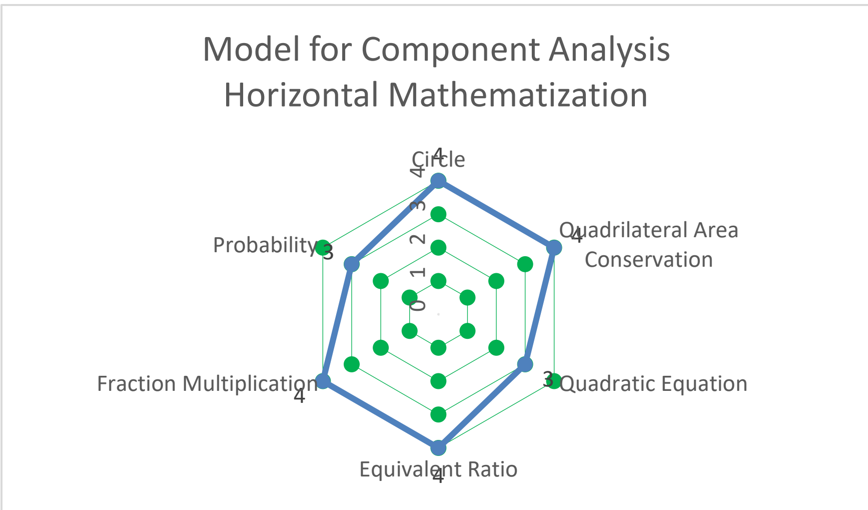
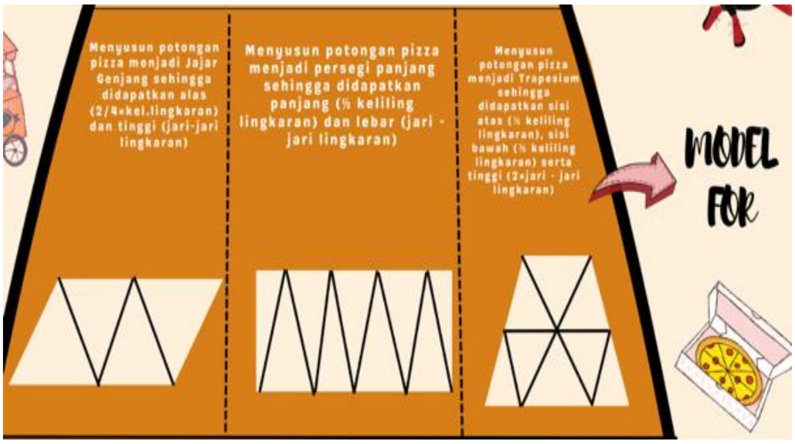


Figure 8. Model-for component analysis "Horizontal Mathematization" in Iceberg Design

The blue lines connecting data points reveal the relative strengths and weaknesses across these dimensions: vertical mathematization demonstrates procedural and formal understanding, while horizontal mathematization captures learners' ability to relate concepts to practical contexts. Furthermore, Figure 9 illustrates a model-for design addressing the circle area. Participants were able to transform the circle into familiar two-dimensional shapes such as parallelograms, rectangles, and trapezoids, demonstrating competence in vertical mathematization at the visual level. However, they encountered difficulty in formula generalization, failing to abstract the area of a circle from the transformed shapes. Thus, while the participants achieved initial visual representation, they did not attain higher-level formal abstraction.



Translated Context:
Arranging the pizza slices into a parallelogram yields a base equal to half of the circle's circumference and a height equal to the circle's radius.
Arranging the pizza slices into a rectangle yields a length equal to half of the circle's circumference and a width equal to the circle's radius.
Arranging the pizza slices into a trapezoid yields a top side equal to half of the circle's circumference, a bottom side equal to three-eighths of the circle's circumference, and a height equal to twice the circle's radius.

Figure 9. Example of a model-for component on circle area topic

In terms of horizontal mathematization, participants effectively linked visual representations to real-life contexts, using a "pizza" scenario to explore relationships between a circle's area and corresponding quadrilaterals. This contextual embedding supported the application of mathematical concepts in everyday situations, reinforcing the RME principle of connecting formal mathematics to meaningful, real-world experiences.

A contrasting result is observed in the model-for design addressing equivalent ratios as shown in Figure 10, which achieved the highest level in both vertical and horizontal mathematization. Participants demonstrated advanced use of mathematical language across multiple representations, including tables, diagrams, and formulas, and successfully linked these representations to diverse real-world contexts such as garment production, fuel consumption, and map scaling. Their reflective notes indicated deliberate efforts to scaffold learning from concrete contextual understanding to formal reasoning, although they reported this as the most challenging design phase.

Vertical mathematization in this design reflects participants' ability to articulate problems in precise mathematical language, apply relevant formulas, and solve contextually grounded problems systematically. Horizontal mathematization, in turn, shows how participants represented and connected mathematical concepts to varied contexts, supporting students' understanding of the equivalent ratio in practical situations. Collectively, these findings illustrate that an effective model-for design integrates formal mathematical reasoning with rich, contextually meaningful representations, embodying the core RME principle that mathematics learning should be both abstract and applicable.

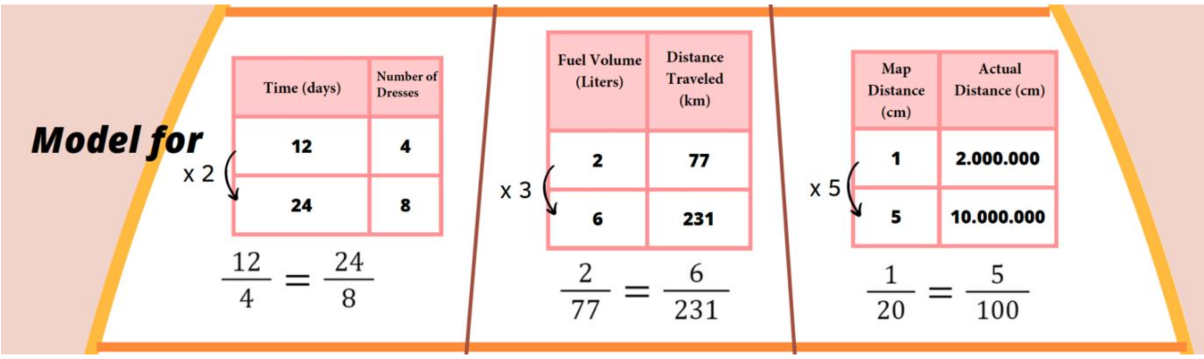


Figure 10. Example of a model component on the equivalent ratio topic

Formal Component of Iceberg's Design

In the RME approach, the formal stage of the Iceberg Design represents the point at which learners abstract mathematical concepts from contextual and representational models into generalized, formalized expressions. At this stage, participants translate insights gained from previous stages—situational, model-of, and model-for—into symbolic representations, formulas, theorems, or definitions. This formalization ensures that students are not merely able to solve specific contextual problems but can also generalize and transfer mathematical reasoning to novel situations, thereby achieving one of the central objectives of mathematics learning. Figure 11 presents the formal analysis outcomes developed by the participants.

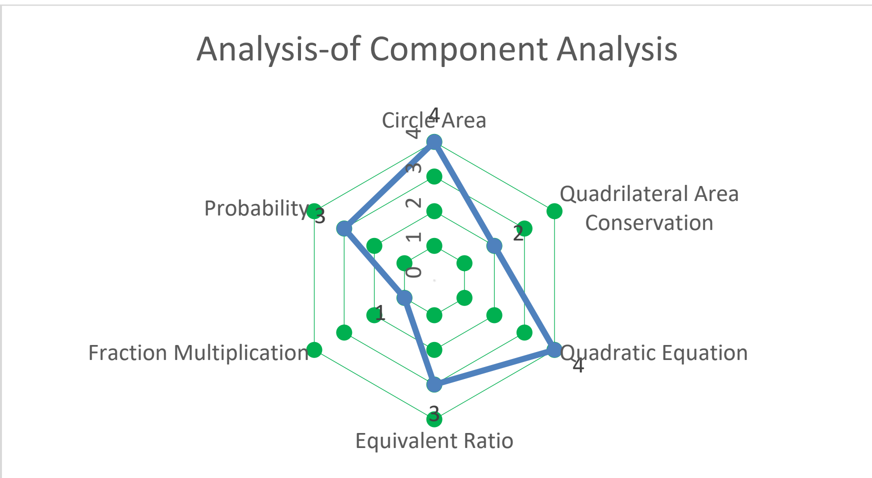


Figure 11. Analysis of formal components in Iceberg Design

The analysis revealed considerable variation in participants' formalization levels, ranging from Level 1 to Level 4. For example, designs addressing circle area (Figure 12) and quadratic equations (Figure 13) achieved Level 4 formalization. These high-level designs indicate that participants successfully formulated concepts through structured, sometimes innovative approaches, demonstrating the capacity to generalize and abstract beyond specific examples.

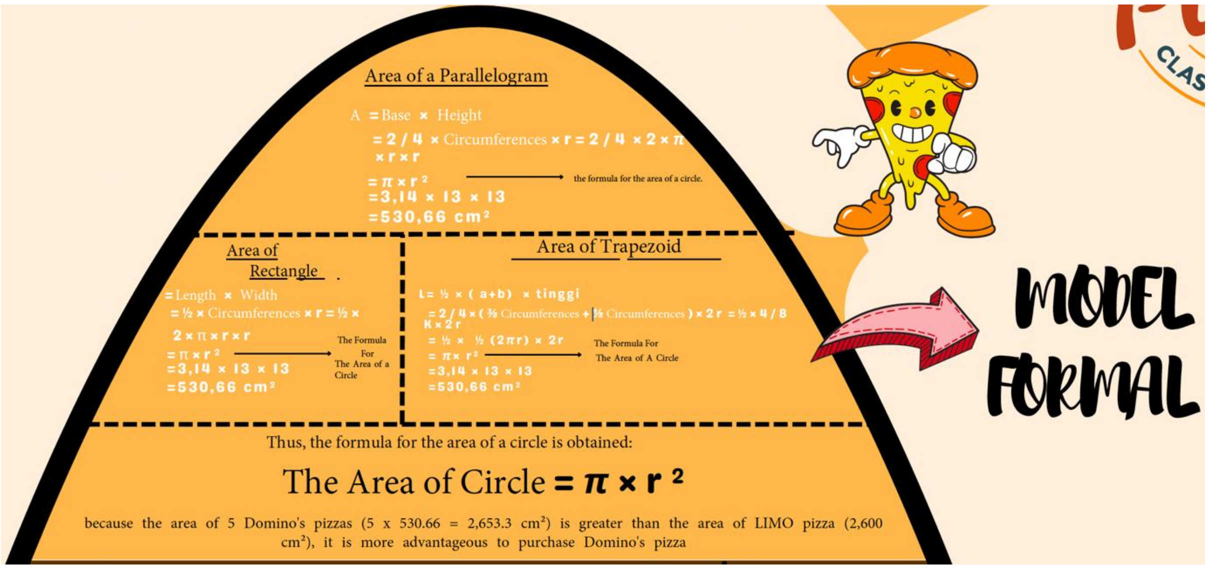


Figure 12. Example of level 4 formal stages on the topic of circles

Conversely, the design for fraction multiplication (Figure 14) reached only Level 1 formalization. Participants primarily provided additional contextual examples without progressing to general rules, formulas, or formal symbolic representations. This suggests a gap in translating practical understanding into abstract, generalizable mathematical knowledge.

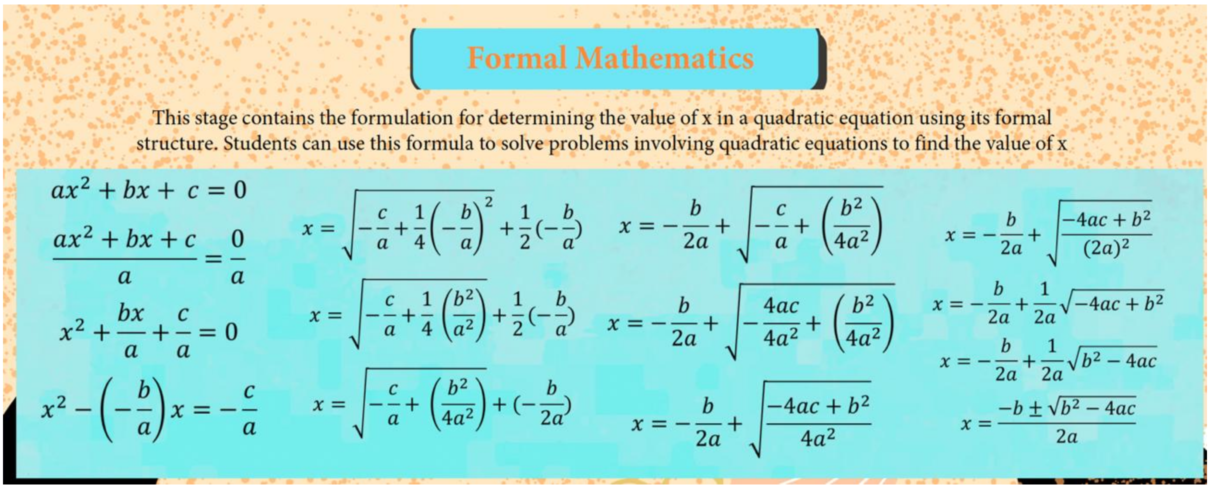


Figure 13. Example of level 4 formal stages on the topic of quadratic equations

Within RME, the formal stage is tightly linked to the model-for process, emphasizing the transition from contextualized representations to symbolic reasoning (Freudenthal, 1991; Gravemeijer, 2004). Achieving high-level formalization reflects deep conceptual understanding, enabling learners to articulate relationships and principles systematically in mathematical language. The participants' success with circle and quadratic equation topics demonstrates their ability to construct formal models and abstract knowledge effectively.

However, the lower performance in fraction multiplication highlights the need for targeted interventions in teacher preparation programs. Prospective mathematics teachers must be trained not only to design contextualized learning activities but also to guide students toward formal generalizations. Enhancing competence in the formal stage strengthens teachers' ability to foster mathematical rigor, conceptual transfer, and problem-solving skills, aligning with the core principles of RME.

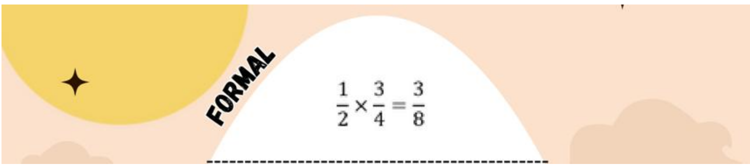


Figure 14. Example of level 1 formal stage on the circle topic

Discussion

In the framework of RME, the iceberg model serves as a conceptual metaphor for the progressive formalization of students' mathematical understanding. It illustrates the developmental trajectory from informal, contextually grounded reasoning to increasingly abstract and formal representations of mathematical ideas (Palupi et al., 2022; Webb, 2017; Webb et al., 2008). Within this approach, context is not merely an illustrative device but a meaningful entry point for problem solving, while models encompass physical manipulatives, visual representations, and mental schemata that mediate learning (Fiangga et al., 2021).

The findings of this study demonstrate that prospective mathematics teachers are capable of constructing comprehensive iceberg designs that span the full spectrum of the RME trajectory—from the selection of rich situational contexts, through the construction of model-of and model-for representations, to the formulation of formal mathematical generalizations (Khairunnisak et al., 2024). The mathematical domains addressed ranged from geometry (e.g., circles, quadrilaterals) to number topics (e.g., equivalent ratios, fractions), extending to algebraic and probabilistic reasoning, with notable attention given to quadratic equations. This diversity highlights the potential of RME as a framework for supporting teachers' pedagogical design capacity and for promoting coherent, meaningful mathematics learning (van den Heuvel-Panhuizen, 2003; Ulfah et al., 2020; Webb, 2017).

Situational contexts function as the foundational layer of the iceberg model, allowing learners to engage with mathematical ideas through scenarios that are experientially meaningful (Webb, 2017). In this study, four of the seven iceberg designs developed by participants employed only a single situational context, whereas three designs successfully incorporated multiple, varied contexts (e.g., in topics such as chance and proportional reasoning). Including multiple contexts offers learners alternative perspectives, fosters critical thinking, and encourages transfer of understanding across problem types. From a pedagogical standpoint, the integration of multiple situational contexts represents a promising strategy for enriching students' conceptual understanding and stimulating productive classroom discourse. Notably, many of the contexts developed by participants aligned with the United Nations' Sustainable Development Goals (SDGs) (United Nations, 2015). For example, a house-plan design for quadrilateral area conservation connects to SDG 7 (Affordable and Clean Energy), while tasks relating fuel consumption to travel distances engage students with issues relevant to SDG 13 (Climate Action). Even everyday contexts—such as comparing pizza prices (circle area) or sharing brownies (fraction multiplication)—can be connected to SDG 12 (Responsible Consumption and Production). Embedding such global issues within mathematics instruction situates learning in socially relevant settings, potentially nurturing students' critical consciousness and preparing them to act as informed global citizens.

The model-of component plays a pivotal role in bridging informal situational reasoning and formal mathematical representation. In this study, most participants successfully developed model-of representations that attained Level 3 sophistication, creating intermediary representations (e.g., arrays, pictograms, tables) that scaffolded students' understanding. However, an important shortcoming was observed in the fraction multiplication design, where the model-of was disconnected from the original context. According to RME principles, model-of should always maintain a reference to the situational context, serving as a representational bridge that guides learners' transition from context-specific reasoning to more generalized mathematical thinking (Fiangga et al., 2021). When this link is absent, models risk being perceived as isolated mathematical artifacts, reducing their power to mediate conceptual understanding.

The model-for and formal stages constitute the higher levels of mathematization in RME, supporting vertical abstraction and generalization. Findings from this study suggest that while some participants successfully used mathematical symbols and language at the model-for stage, many struggled to progress toward full generalization or derivation of mathematical formulas. This was particularly evident in topics such as circles and fraction multiplication, where participants remained at a primarily visual or procedural level rather than articulating generalized rules.

Interestingly, some participants prematurely applied existing formulas during the formal stage, a phase ideally devoted to formulating and justifying such formulas rather than merely applying them. This observation points to a pedagogical gap in understanding the distinct epistemic roles of model-for and formal stages. Strengthening prospective teachers' grasp of how these stages interrelate is crucial for fostering designs that not only contextualize mathematics but also guide students toward conceptual generalization and symbolic reasoning.

While this study offers valuable insights, several limitations must be acknowledged. The relatively small sample of thirty prospective teachers from a single institution limits the generalizability of the findings (Polit & Beck, 2010). The study focused exclusively on junior secondary mathematics topics, which may not represent the full range of mathematical domains. Moreover, while inter-rater reliability measures and detailed rubrics were used, the assessment of abstraction levels inevitably involved a degree of subjective judgment (Polit & Beck, 2010).

Despite these limitations, the study highlights the promise of integrating RME principles into mathematics teacher education. Designing iceberg models that strategically combine multiple contexts, coherent model-of representations, and carefully scaffolded model-for and formal stages can enhance students' conceptual development. Future research should explore longitudinal interventions and examine how prospective teachers refine their designs over time, as well as how these designs impact students' learning outcomes in authentic classroom settings.

CONCLUSION

This study demonstrates that prospective mathematics teachers are capable of developing Iceberg Designs that systematically progress from situational contexts to formal mathematical knowledge. Nonetheless, considerable challenges were observed in the model-for and formal stages. These difficulties primarily stem from the complexity of abstracting context-based knowledge into formal mathematical concepts and symbolic representations, highlighting the need for increased emphasis on vertical mathematization within teacher preparation programs.

The findings have important implications for curriculum development and teacher education. These include the design of specialized training modules focused on abstraction techniques, structured support for transitioning between multiple representations and formal mathematical language, and the development of exemplar Iceberg Designs for key mathematical ideas. Limitations of the study include the small, single-institution sample (30 participants), which constrains generalizability. However, methodological triangulation and researcher reflexivity helped mitigate potential analytical biases. Additionally, the gender composition of the participants (26 women, 4 men) may have influenced perspectives reflected in the results.

This research contributes to SDG 4 (Quality Education) by providing evidence-based frameworks for teacher preparation that emphasize contextualized mathematics learning. It offers practical tools for scaling mathematics education through professional development and curriculum design that connect abstract mathematical concepts with applied problem-solving. Future research should explore the classroom implementation of these Iceberg Designs and examine how they evolve through sustained teaching practice, providing further insight into their impact on student learning outcomes and mathematical literacy development.

Acknowledgments

We would like to express our sincere gratitude to all parties who contributed to completing this research.

Declarations

Author Contribution	:	YMS: Conceptualization, Writing – Original Draft Editing, and Visualization. SF: Writing – Review & Editing, Formal Analysis, and Methodology. YIEM: Writing – Review & Editing and Visualization. MS: Validation and Supervision. LPY: Writing – Review & Editing
Funding Statement	:	This research received no external funding from any source.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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