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Metacognitive-discursive activities in Indonesian mathematics classrooms: A two-stage comparative case study

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Abstract

Classroom discussions are essential for developing students' mathematical understanding. While prior studies have examined teacher-led instruction, there remains a gap in understanding how metacognitive and discursive activities shape the quality of mathematical discussions at the whole-class level. To address this gap, this study proposes a systematic framework for analyzing public classroom discussions with a focus on metacognitive-discursive activities that support mathematical argumentation. The analysis centers on two dimensions: (1) monitoring the logical validity, correctness, and completeness of mathematical arguments, and (2) identifying how discourse quality is enhanced or obstructed by participants' communicative strategies. This qualitative study employed a two-stage procedure: first, a fine-grained micro-level coding of classroom interactions to identify metacognitive and discursive activities, including monitoring (of terminology, methods, and argument consistency), reflection (on representational structures and methodological effectiveness), and discursive actions that either promote or hinder mutual understanding and second, a macro-level evaluation of discussion quality using a standardized rating framework. This methodological approach, applied for the first time in the Indonesian mathematics education context, enabled a more comprehensive analysis of discourse processes in whole-class discussions and helped identify phases in which strategies for enhancing the classroom discussion culture could be developed. The findings indicate that productive mathematical discussions require an environment that encourages students to articulate and critique solution strategies, justify their reasoning, and collaboratively resolve discrepancies with minimal teacher scaffolding. The study contributes to mathematics education research by providing a rigorous analytical model for examining mathematical discourse and offering evidence-based recommendations for cultivating a classroom culture that promotes deeper mathematical understanding.

Keywords: Comparative Case Study, Metacognitive-Discursive Activities, Public Classroom Discussions, Rating System

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Globally, research continues to investigate weaknesses in mathematics teaching that may explain students' persistent underperformance in measures of mathematical achievement. International large-scale assessments, most notably PISA 2022, underscore the urgent challenge of improving the quality of mathematics instruction worldwide (Liu et al., 2024; Genç et al., 2021). Central to this endeavor is a deeper understanding of instructional quality in classroom settings. Among the key dimensions of instructional quality, cognitive activation has been identified as crucial for promoting students' deeper mathematical understanding (Klieme, 2006; Hiebert & Grouws, 2007; Lipowsky et al., 2009; Burge et al., 2015; Li et al., 2021; Alghaeth et al., 2024). Existing theoretical models and measurement instruments,

however, have largely focused on teacher behaviors as the primary indicators of instructional quality.

Recent scholarship has emphasized that effective mathematics instruction cannot be fully understood through teacher actions alone. Rather, it must account for students' engagement, responsiveness, and their co-construction of mathematical meaning through classroom interaction (Quabeck et al., 2023; Stockero et al., 2020; Ayuwanti et al., 2021; Hansen & Naalsund, 2022). Sigurjónsson (2024), for instance, expands the assessment of cognitive activation by documenting patterns of teacher–student and student–student interactions. Nevertheless, two key gaps remain. First, merely documenting interaction patterns does not capture how students actually respond to instructional prompts. Second, there is little systematic attention to the quality of discourse—namely, the extent to which classroom discussions are rigorous, sustained, and conducive to students' independent reasoning.

Empirical evidence across diverse contexts reinforces this concern. In many mathematics classrooms, students tend to passively copy solutions from the board following teacher-led demonstrations rather than engage in genuine problem solving or meaning-making (Turmudi, 2010; Arakaza & Mugabo, 2022). Classroom discourse often reflects a traditional initiation–response–evaluation (IRE) pattern, where teachers pose questions, students respond in chorus with brief or fragmented answers, and teachers selectively affirm correct responses (Aduko & Akayuure, 2025). In Indonesia, Fauzan (2002) documented this pattern, noting that only selected answers—typically those labeled as "good"—receive recognition, discouraging critical thinking and accountability. Students are frequently reluctant to ask clarifying questions or challenge mathematical ideas (Putra et al., 2020), while novice teachers often reinforce these dynamics by tightly controlling lesson flow, thereby reducing opportunities for student agency and participation (Zhang & Wang, 2024).

To analyze these complex classroom dynamics, a purely cognitive perspective is insufficient. Since mathematical learning is socially mediated through dialogue in which teachers and students collaboratively construct and reflect upon solution strategies (Vygotsky, 1978; Sfard, 2008; Mercer & Howe, 2012), a metacognitive–discursive framework offers a more comprehensive lens for investigation. This framework captures how participants monitor the correctness and adequacy of mathematical arguments, reflect on solution strategies, and engage in discursive acts that either promote mutual understanding or hinder it through negative discursivity (Cohors-Fresenborg & Kaune, 2007). Empirical studies (Ate, 2021; Ate et al., 2024; Moza et al., 2024; Napu et al., 2024) have validated this framework as a robust tool for analyzing classroom interaction.

Despite these conceptual advances, the framework has been applied primarily at the micro-level, focusing on localized episodes of interaction. This narrow scope leaves open the question of whether the identified discursive activities contribute to students' conceptual development, foster independent reasoning, or are impeded by specific negative discursive patterns. To address this limitation, Nowińska (2016a; 2016b; 2018) proposed a two-stage rating framework with Guiding Questions (GQs). For example, GQ1 and GQ2 examine how individual metacognitive activities affect students' understanding of mathematical content or methods, while GQ5 assesses the extent to which negative discursive actions obstruct comprehension. GQ6, in turn, evaluates whether student-led debates occur without teacher interruption, thus serving as an indicator of students' discursive autonomy.

Building on this literature, the present study is the first to apply and adapt Nowińska's two-tiered framework to mathematics classrooms in Indonesia. A novel contribution of this study is the introduction of the ND3g code, specifically designed to capture the culturally prevalent phenomenon of chorus responses—a feature not adequately addressed in the original framework. This extension provides a critical lens for examining how culturally specific interaction patterns shape the micro-mechanisms of

instructional quality and constrain or enable metacognitive discourse. Accordingly, this study addresses the following research questions:

1. How do metacognitive and discursive activities manifest in mathematics classroom interaction?
2. How does a holistic evaluation of classroom discourse capture the overall quality of lesson interaction?

METHODS

Research Context and Participants

This study was conducted during the 2024/2025 academic year and involved two seventh-grade mathematics classes from junior high schools in Sumba, Indonesia. The schools and teachers were selected purposively based on two criteria: (a) the teachers' willingness to participate and (b) class sizes that would allow for meaningful observation and analysis of classroom interactions. One class was drawn from a well-regarded school located in the administrative capital of the region, while the second class was selected from a village school approximately 6 km from the capital.

Notably, the teacher in the village school had received specialized professional development in facilitating public classroom discussions, thereby providing a useful contrast for the study's analytical focus. The researcher participated as a non-intrusive observer, ensuring that no intervention or modification of the natural teaching process occurred.

Data Collection Procedures

Data were collected from regularly scheduled mathematics lessons in both schools, each approximately 70 minutes in duration. All lessons were audio-visually recorded to capture verbal and non-verbal aspects of classroom interaction. From the recordings, representative segments were purposively selected and transcribed verbatim for detailed analysis in alignment with the study's research questions.

The transcription process was conducted using Video-Transcript 10.8, which facilitated precise alignment between audio/video data and text and allowed visualization of teacher–student turn-taking. Although software was used to support transcription, all interpretive decisions—including the identification of relevant episodes and coding of interactional features—were made by the researcher. Transcriptions followed established conventions, marking pauses, overlapping speech, and emphatic expressions to ensure analytic rigor.

Analytical Framework

This study used a qualitative design with two stages of analysis based on the framework developed by Nowińska (2016a; 2016b; 2018). In the first stage, each utterance is coded using an alphanumeric system: Monitoring (M), Reflection (R), Discursive (D), and Negative Discursive (ND), as presented in Table 1. The ND3g subcategory was specifically developed for this study. Only categories used in at least one of the two transcripts were included. Utterances were coded as sequences of up to five symbols: a capital letter plus a numeral for the subcategory, optionally followed by a lowercase letter.

Furthermore, up to two italicized prefixes were used to capture pragmatic functions, namely *d* (demand), *r* (reason), *dr* (demand to provide a reason), and *rd* (reason offered in support of a subsequent demand). The results of categorization are presented in a form called a category line, which provides an abstracted presentation of the course of the discussion, in particular the alternation between the teacher's and the pupils' utterances (for examples see Figure 1 and 3 in the "Results and Discussion" section).

Table 1. System for categorizing metacognitive-discursive activities

Code	Description
P	Open Impulse: presents an information without requiring monitoring or reflection
M1	Controlling of a subject-specific activity
M2	Controlling of terminology / vocabulary used for a description / explanation of a concept
M5	Controlling of (consistency of an) argumentation / statement
R1a	Analysis of structure of a subject-specific expression
R2a	Assignment of an object / an issue to a concept
R4	Analysis of the effectiveness and application of subject-specific tools or methods / indication of a tool needed to achieve an intended result
R6a	Evaluation with regard to the issue discussed
D1a	Naming of reference points or persons (in particular to ensure the basis of conversation)
D1d	Activities to improve (e.g. structuring) and facilitate the discourse
D2	Education for discourse, e.g. asserting that rules have not been agreed for the current discourse
ND1a	Asking a self-answering question
ND1b	Repetition of things already said without adding a new point of view to the discourse (also "teacher echoing")
ND2	Inadequate vocabulary (in a description, comment, argumentation, statement)
ND3a	Statements/ questions are not recognizable refer to the things occurred; or were said; or the reference point is not explicit, or the argumentation is fragmentary
ND3b	Shortcomings with regard to grammar or the sentence structure, broken sentences, at first glance comprehensible sentences in which it is not clear what is meant
ND3e	False logical / mathematical structure of an argumentation
ND3g	Students answer in chorus
ND4	No intervention taken against severe disregard of discursivity rules

In the second stage of analysis, the coded classroom discussions were subjected to a holistic evaluation using the Guiding Questions (GQs) within a multi-level rating framework. This stage emphasized not only the frequency but also the quality of metacognitive and discursive activities, thereby offering a comprehensive account of the mechanisms by which classroom discourse supports or constrains mathematics learning.

Table 2 summarizes the possible responses to each GQ, with particular attention to the critical transitions between rating levels, which are essential for distinguishing subtle differences in interaction quality. GQ3, which is designed to capture highly complex metacognitive discussions characterized by synergistic exchanges among multiple participants, was not applied in this study because none of the observed lessons exhibited the level of discursive complexity required for this rating.

Table 2. Guiding questions and criteria for differentiating levels of response

	Guiding Question (GQ)	Level
GQ1	**How far are the metacognitive activities practiced with the effort to an elaborated dealing with the ongoing contents of the scene?**	
	Metacognitive activities with the aim of an elaborate discussion are, if at all, almost exclusively carried out by the teacher. If there is any ambition at all on the part of the teacher to introduce the students to metacognitive thinking processes, then this has not yet been successful insofar as the students make no attempt to undertake the necessary activities in a way that is perceptible to an observer.	1
	Metacognitive activities are not only carried out by the teacher, but also by the students. However, the teacher takes many opportunities to use metacognitive activities personally or often comments personally on the students' contributions with metacognitive activities.	2
	Metacognitive activities are practiced by both the teacher and the students. The teacher rarely uses opportunities personally to use metacognitive activities and comments rarely with metacognitive activities on the students' contributions.	3
	Metacognitive activities are practiced by both the teacher and the students. There are longer phases in which the students are involved in the discourse with metacognitive activities without interference from the teacher. In this way, the students show an effort to make progress in the discourse. The students also use the classroom discussion autonomously to engage in metacognitive activities. This suggests that metacognitive activities are part of the students' habitus.	4
GQ2	**To what extent are metacognitive activities practiced in combination with reasoning with a view to argumentative student participation and student-teacher interactions?**	
	Metacognitive activities with reasoning are almost exclusively carried out by the teacher, if at all.	1
	Metacognitive activities with reasoning are practiced by the students. However, it is not perceptible that reasoning is part of the established teaching culture, e.g. because they rarely occur or because the formulation of imprecise, inadequate reasoning is not punished or because it is not recognizable that an argumentative, critically, elaborated discussion is being sought.	2
	Metacognitive activities with reasoning are practiced by the students. It can be seen that reasoning is part of the established teaching culture, e.g. because they occur frequently or because it is insisted that they are formulated precisely and comprehensibly, or because an argumentative, critical, elaborate discussion is sought. However, the learners do not provide most of the reasoning without being asked to do so by the teacher.	3
	It can be seen that reasoning is part of the established teaching culture. The students give a remarkable number of these reasoning without being asked to do so by the teacher. This suggests that metacognitive activities are part of the students' habitus. It can happen that also students ask for reasoning, check the admissibility of an argument or method, question it or reject it with reasoning.	4
GQ4	**To what extent is there evidence of the use of discursive activities and an effort to achieve a discursive orientation in classroom discussion?**	
	There is (almost) no discursive activity on the part of either the teacher or the students. The rater has the impression that there is a lack of discursive content-related discussion and understanding of the upcoming content and individual contributions to the discourse. It is not apparent that the teacher uses targeted structuring and educational measures to improve the course of the discourse. In particular, it is not apparent that the teacher offers the students formulations in the teaching interventions that they can use precisely as the basis for their own	1

argumentation.	
Discursive activities are evident both on the part of the teacher and the students. The students are not particularly recognizable due to their discursive behavior in terms of analyzing and specifying what is said and meant, or what is presented and imagined. Discursive activities of a particular quality are (nearly) not noticeable on the part of the students (e.g. elaborate and precise differentiation from what has already been said, repetition of an utterance as a basis for further argumentation, measures to improve or facilitate discourse and educational measures). There are two ways to assess level 2: a. It is recognizable that the teacher acts as a role model for the practice of discursive activities. This can be demonstrated by the teacher using targeted structuring measures to improve the course of the discourse or to initiate a discursive discussion of the arguments, assertions, questions or comments presented. The teacher can ensure an orderly course of discourse by leading the classroom discussion back to the intended track. In their teaching interventions, they can also offer the learners formulations which they in turn can use precisely as the basis for their own arguments or with which they can refer to what is meant or presented. This does not necessarily have to be reflected in a coding of discursive activities. However, it is not apparent that the teacher consistently takes educational measures to influence the discursive behavior of the students in the long term. b. It can be seen that the teacher uses appropriate educational measures to educate students to practice discursive activities, demands compliance with previously agreed discourse rules, or consistently admonishes and comments to students that their statements do not refer to what has already been discussed, written, meant or asked. Answer 2b also includes the case where the teacher therefore does not take any educational measures and does not formulate any admonitions because there is no clear reason to do so. However, no occasions are created for a discursive, argumentative discussion of the ongoing content.	2
Discursive activities are evident both on the part of the teacher and the students. These occur frequently and are not concentrated in any significant frequency or form on the part of the teacher. The students stand out through their discursive behavior and thus contribute to the clarity and precision of the content of the classroom discussion. There are two ways to assess level 3: a. The students' discursive activities make it clear who is referring to what. However, discursive activities of a particular quality are not noticeable on the part of the students (e.g. elaborate and precise differentiation from what has already been said, repetition of an utterance as a basis for further argumentation, measures to improve or facilitate discourse and educational measures). b. Discursive activities of particular quality are recognizable on the part of the students (e.g. elaborate and precise differentiation from what has already been said, repetition of an utterance as a basis for further argumentation, measures to improve or facilitate discourse and educational measures).	3
GQ5 To what extent is the conversation disrupted by negative discursive activities and to what extent is an attempt made to counteract this or the consequences thereof?	
Negative discursive activities disrupt contributions by the teacher or the students in a notable form and frequency. There is hardly any targeted and consistent effort to counteract this. The negative discursive activities have a negative effect on the discourse to such an extent that understanding the ongoing subject-specific content or the adequate use and understanding of subject-specific language is made	1

	significantly more difficult, or the comprehension of the course of the lesson or the practicing of a stringent, argumentative discussion is prevented.	
	Negative discursive activities disrupt contributions by the teacher or the students in a notable form and frequency. a. The negative discursive activities have a negative effect on the discourse to such an extent that understanding the ongoing subject-specific content or the adequate use and understanding of subject-specific language is made significantly more difficult or is directly negatively influenced. In several situations, however, a targeted reaction by the teacher can be observed, which suggests that the teacher recognizes rule violations and an attempt is made to ensure that the conversation is focused, or that the teacher is making an effort to counteract the negative discursive activities. However, these measures are not yet globally effective because no clear change in the students' behavior can be seen. b. The negative discursive activities do not have a negative effect on the discourse to such an extent that understanding the ongoing subject-specific content or the adequate use and understanding of subject-specific language is made significantly more difficult or is directly negatively influenced. However, repeated violations of rules for an orderly progression of discourse (e.g. change of reference points, statements are not precisely related to what has already been said) prevent a stringent discussion. This makes the classroom discussion ineffective and meandering. The rater can see that the teacher is keeping an eye on the process of the lesson. After violations, the teacher intervenes locally by correcting or structuring in order to keep the discourse focused. However, the teacher draws no conclusions from this (rules) for further discourses, and no educational measures or admonishments are implemented. It is not to be expected that the students' discussion behavior will change in the long term in the direction of a discursive habitus. c. The negative discursive activities do not have such a negative effect on the discourse that it becomes significantly more difficult to follow the course of the lesson, to understand the current subject content or to use and understand the subject-specific language adequately. It can happen that the students or the teacher use inadequate vocabulary or express themselves imprecisely or with incomplete sentences, but it is still recognizable in the context of the lesson what is meant. In this case, it is not to be expected that the teacher will ask for control after every imprecise utterance or that the teacher will carry out a suitable control activity and thus interrupts a course of thought. If violations of the rules for an orderly course of discussion (e.g. change of reference points, utterances are not exactly related to what has already been said) (almost) do not occur, then there will be no need for the teacher to make an effort to counteract negative discursive activities. However, if the conversation is disturbed by the use of inadequate vocabulary or by local disjunction of the discourse strands, then an effective use of appropriate measures must be clearly recognizable.	2
	Neither of the two sides involved in the discussion shows any notable negative discursive activities. Negative discursive activities, on the other hand, can occur on occasion, but they have no negative impact on discourse, grasp of current subject-specific content, or the students' ability to use subject-specific language effectively. Teacher intervention is not necessary to maintain the discourse.	3
GQ6	**To what extent debates among students without interruption by the teacher are recognizable?**	
	There are no phases in the lesson in which debates among students are recognizable.	1
	There are phases in the lesson in which debates in a simple form are recognizable among students. In this case, metacognitive activities are carried out, but without noteworthy reasoning and without an elaborate discursive character. Negative-discursive activities of subcategory ND3b may not occur.	2

There is a longer phase in the lesson with a stringent and precise discourse on a topic, but this is essentially influenced by the teacher.	3
There is a phase in the lesson with an independent, stringent discussion of a particular topic among the students. This discussion is not significantly influenced by the teacher. The students make their position clear in the discourse; they may take a reasoned stand against what has already been said.	4

Reliability and Validity

The coding process was conducted as an interpretive rather than a purely mechanical task, with careful attention to the contextual meaning of each utterance (Creswell, 2014; Merriam & Tisdell, 2016). All coding decisions were systematically documented, accompanied by explicit justifications, and reviewed in consultation with a domain expert to enhance validity and reduce individual researcher bias. This approach aligns with established strategies for ensuring trustworthiness in qualitative research (Patton, 2015).

Reliability was supported through the systematic application of coding procedures during the first stage of analysis, which enabled consistent replication of results by the same researcher. During the second stage, the holistic evaluation of coded data was guided by the structured use of Guiding Questions (GQs). This procedure ensured alignment between micro-level categorizations and macro-level evaluations while minimizing subjective bias. Validity was further reinforced by comprehensive documentation and expert consultation, ensuring that the resulting categories, subcategories, and reflective assessments accurately represented the observed classroom context and interactional phenomena. Importantly, this study did not seek statistical generalization; rather, it followed an analytic generalization approach aimed at identifying core mechanisms of classroom discourse that may inform theoretical understanding and be transferable to similar educational contexts.

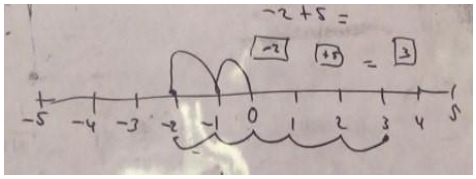
Ethics Statement

The research was conducted in compliance with institutional and school regulations. Formal approval for the study was obtained from the school principal, based on an official authorization letter issued by the academic vice-rector of the researcher's university. Because the study relied exclusively on non-invasive classroom observations and did not involve experimental manipulation or collection of sensitive personal data, review by an institutional ethics committee was not required.

RESULTS AND DISCUSSION

The transcripts were analyzed to investigate how teachers and students engaged in reflection, monitoring, and discursive activities during mathematics lessons, as well as to identify instances where negative discursive behaviors hindered students' conceptual understanding. Table 3 presents excerpts from the first analyzed transcript, which focuses on the topic of addition and subtraction of whole numbers. The following abbreviations are used: T (teacher), S (speaker), P (students collectively), and S1, S2, … (individual students). In this episode, the teacher introduced two distinct strategies for solving problems: the first employed a number line representation, while the second used the metaphor of "money and debt." The full transcript is provided in the Appendix; only selected excerpts are presented here for purposes of focused analysis.

Table 3. Transcript with a justified classification of metacognitive-discursive activities

S	Utterance	Code	Justification
T	Is there any other way? (5 sec). Another way (5 sec). Min two plus five. Is there any other way, besides using the number line? (9 sec). Is there any other way? (6 sec). Min two plus five (5 sec). Which is faster, for example [*Pointing to the previous solution using the number line on the blackboard*].	*d*R4	T asks the students to find an alternative solution strategy other than the number line through a series of explicit questions (*d*R4). Since no response is received after several pauses, T maintains focus on the question by referring to the previous example and providing further explanation through examples involving larger numbers (D1d).
		D1d	
	How about using the largest number? The numbers we use are bigger, whereas on the number line, we just use smaller numbers. What if we are asked to calculate min 100 plus 90 or min 100 plus nine five? If we make a number line like this [*Pointing at the blackboard*], then the *board* will not be enough, or it could be enough if we make it in such a way. But now the question is, is there any other way besides using a number line? The way you learned in elementary school. Is there any other way, besides using a number line? (10 sec)	ND3b	Although T's aim of encouraging students to think about another method is fairly clear, the explanation is delivered with a syntactically disorganized sentence structure, marked by repetition, fragmented sentences, and the use of ambiguous phrases such as "like this… like that… could also be enough…", which hinders students' understanding (ND3b).
	How? (3 sec). S1 [T *asked S1*]. Besides using the number line. Min two plus five. Besides using a number line, is there any other way? (9 sec). S2 [*T asked S2*].		
	Besides using a number line, is there any other way? (5 sec). Okay. We'll try together the way I've learned. Here [*Pointing to the blackboard*] is min two plus five. Here we can conclude minus two. Talking about negative, then we talk about debt and when we talk about positive, then we talk about money. [Some students: Yes.] Then we compare money and debt here [*Pointing to the blackboard*]. We can conclude that when we talk about debt eh negative there is debt and talking about positive, there is money. We just have to compare or fill in the box like this [*Pointing at the blackboard*].	R2a ND3d ND1a	After the students are asked several times with certain pauses and still give no response, T then introduces an alternative representation through a contextual analogy of debt and money (R2a). However, T interprets –2 as debt and +5 as money without taking into account the structure of the operation under discussion (ND3d). T's final question is purely suggestive, as it directs students toward a specific answer already implied in the question (ND1a).

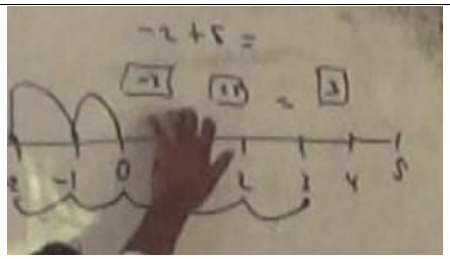
	 The first box is debt; the second box is money. All we have to do is compare. My debt is two and I have five. I pay my debts, two. Do I still have debt, or do I still have money?		
P	Still have money	ND3g	Students answer in chorus (ND3g). The students' answer follows T's argument given before.
T	Still have money. [Some students: Yes.] Yes, how much money do we have left?	ND4	T does not intervene against answering in chorus (ND4).
		ND1b	T repeats a previous statement without providing a new argument (ND1b) and poses a suggestive question by asking students to state the remaining amount of money based on the answer already written on the board (ND1a).
		ND1a	
P	Three	ND3g	Students answer in chorus (ND3g). Students determine the remaining money they have. They can do it by reading the answer from the board. So, it is not classified

Figure 1 shows the category line created for the first transcript. It has to be red from right to left. Each contribution is shown as a short horizontal segment, with the teacher's segment on the upper side (e.g., dR4, D1d, ND3b, …), and the student's segment on the lower side (e.g., ND3g, R4, ND3a, …).

Figure 1. Category line for Transcript 1

A first glance at the category line already shows that almost all of the teacher's utterances or requests for reflection (*d*R4 and *d*R1a) are also additionally rated with negative discursivity (ND3b, ND3d and ND1a). The students only answer in chorus (ND3g). The teacher omits to address such responses (ND4). Furthermore, the answer to the guiding question is presented in Table 4.

Table 4. Response to the guiding question

	Guiding Question (GQ)				
	GQ1	GQ2	GQ4	GQ5	GQ6
Level	1	1	2	1	1

Based on the transcript, on GQ1, there are no metacognitive activities aimed at facilitating an elaborate discussion observable. Metacognitive activity is almost entirely teacher-initiated through questions such as *"Is there any other way, besides using the number line?", "Which one is faster?", or "Where is zero from? Please explain*", which clearly prompted reflection, strategy comparison, and explanation of results, yet students' responses were limited to brief answers such as "*Still have money", "Three", "Zero",* or *"Eight*" without further elaboration, thus showing no observable metacognitive engagement. On GQ2, there are no metacognitive activities with justification recognizable. On GQ4, the teacher organizes the course of the discussion through directions (D1d) and by pointing to positions on the board to link representations (D1a), whereas there is no contribution from the students that indicates discursive engagement, as they do not perform clarification, elaboration, or connect to previous statements, which are characteristic of discursive activity. On GQ5, negative discursive activities disrupt contributions by the teacher. Often the students answered in chorus (ND3g) and the teacher does not comment on this (ND4). Students' contributions are also unclear or repetitive, (ND3a, ND3b), and the lack of clarification prevents the discussion from developing into reflective explanations. Repetition without clarity (ND1b, ND3c) and topic jumps (ND3d, ND3b) further reinforced the disruptive discourse. On GQ6, there is no exchange of views or arguments among students; the discussion only shows a minimal structure triggered by the teacher, without the presence of reflective contributions.

The pattern of simultaneous responses (ND3g) that recurs in this transcript is an important indicator of the lack of justification (GQ2), metacognitive activity (GQ1), handling of unclear contributions (GQ5), weak coherence of discussion (GQ4), and failure to form a debate structure (GQ6). The lesson analyzed here focused on a particularly simple case: as has been shown in the above commentary on the category line, even at first glance it shows that the teacher's entire contribution is dominated by negative discursivity.

Discussion of the Result Concerning First Transcript

The analysis revealed that metacognitive–discursive activities in the mathematics classroom were predominantly teacher-initiated (RQ1). While teacher prompts and interventions were consistently present, students' contributions remained limited, resulting in a discourse pattern that warrants closer examination. The most salient finding was the emergence of a distinctive interaction pattern, coded as ND3g, representing simultaneous student responses. This phenomenon, which was not addressed in the original coding framework, occurred when the teacher posed a question to the entire class and multiple students responded in chorus without individual turn-taking.

Such simultaneous responses significantly constrained the visibility of individual reasoning (Aduko & Akayuure, 2025), making students' solution procedures and mathematical justifications difficult to trace (Sfard, 2008; NCTM, 2000). The lack of discernible individual contributions weakened intersubjectivity by obscuring alternative perspectives and restricting opportunities for productive negotiation of meaning. Moreover, this interaction pattern hindered the teacher's ability to identify and respond to emerging misconceptions in a differentiated manner (Scott et al., 2006; Fauzan, 2002). Consistent with previous findings (Putra et al., 2020), the teacher's dominance over the discussion limited students' active participation, discouraged questioning, and constrained opportunities for critical engagement with mathematical ideas.

Additional patterns—such as repetitive or incomplete answers and unelaborated shifts in meaning—further restricted the potential for meaningful interaction, as they left students' reasoning only partially visible and difficult to build upon (Hofmann & Ruthven, 2018). Although the teacher attempted to

scaffold discussion by pointing to relevant representations on the board and directing attention to numerical features, these strategies were insufficient to mitigate the effects of negative discursivity or to elicit students' explicit articulation of reasoning processes.

Taken together, these findings indicate that mutual listening, progressive idea development, and a supportive dialogic environment were difficult to achieve. The absence of sustained interaction impeded the co-construction of shared and reflective mathematical understanding (Alexander, 2008). Overall, this transcript illustrates how restrictive discourse patterns reduce the quality of classroom interaction, limiting the integration of metacognitive–discursive activities and constraining opportunities for reflective discussion (RQ2). The low level of student engagement in monitoring and reflection underscores the need for discursive conditions that enable students to articulate, clarify, and justify their thinking, thereby supporting deeper mathematical learning.

The second transcript, presented in Table 5, documents a public classroom discussion in a seventh-grade class that had been introduced to a new instructional approach at the beginning of the school year (Kaune & Cohors-Fresenborg, 2021). At the time this transcript was recorded, the students had completed only three mathematics lessons; this discussion took place during the fourth lesson. While the new mathematical content had not yet played a central role, the teacher had already established a didactic contract governing public classroom discussions.

According to this contract, students who wish to contribute after a question is posed must signal their intention to speak and wait to be called upon. Ideally, a student should take the lead in facilitating the public discussion. When mathematical content is addressed, the first priority is to verify the correctness of the information presented. After verification, additional comments should be solicited and considered collectively.

The transcript focuses on the topic of addition and subtraction of whole numbers. Students were given the task shown in Figure 2 and were subsequently invited by the teacher (T) to explain and justify their solutions to the class.

The following account card is incomplete. Please, fill in.

Number	Date	Starting balance	Bookkeeping		Final balance
			Withdrawal	deposit	
3.	12-10-09	250.000		25.000	275.000
4.	17-10-09	275.000			248.000
5.	28-10-09	248.000	130.000		
6.	31-10-09				136.000
7.	02-11-09	136.000			164.000

Figure 2. Student task

Table 5 presents excerpts from the transcript of a public classroom discussion, using the same abbreviations as those introduced in Table 3. For analytical clarity, only selected segments are displayed here, while the full transcript is provided in the Appendix.

Table 5. Transcript with a justified classification of metacognitive-discursive activities

S	Utterance	Code	Justification
T	Who wants to explain? [*S1 raises her hand*] S1.	P	Since the teacher does not specify which metacognitive activities (from the areas of monitoring and reflection) she expects from the students, this is classified as an open stimulus (P).
S1	Third line?	R1a *d*M1 D1d	S1 interprets the structure of the table in the way that the row labeled by the number 3 is the first row, which she has to comment (R1a). She asks whether her interpretation is correct (*d*M1). S1 wants to improve the discourse by reassuring herself that she is talking about the right place. Therefore, it is additionally classified as D1d.
T	The first line.	M2 ND3a	T selects the word based on the visible position of "first row" (M2), but it is not the first row in the table. This could create a misunderstanding; therefore, additionally classified as ND3a.
S1	On the twelfth day of the tenth month of 2009, the initial balance was 250.000. In the bookkeeping, he deposited 25.000. I calculated the 275.000 from 250.000 plus 25.000.	*r*R1a	S1 interprets the structure of transactions in bookkeeping in an elaborated way (*r*R1a).
T	Lamber (Teacher reprimands students who do not listen to their classmate's explanations)	D2	T indicates by calling his name that the students' behavior is not accepted by her (D2).
S1	What do you mean? Is my answer right or wrong? S2.	*d*M5	S1 asks her classmates to check her answer (*d*M5)
S2	That's right	M5	S2 checks S1's argument (M5).
S1	Does anyone have any comments? Does anyone have any questions?	*d*R1a	S1 invites to reflect on her interpretation of the structure of transactions in bookkeeping by asking for feedback or questions (*d*R1a)
T	Please ask questions! This is the first one		T provides opportunities for students to ask questions. Therefore, this is not classified.

Figure 3 illustrates the category line constructed for the second complete transcript, following the same format as Figure 1, with teacher contributions displayed on the upper side of the category line and student contributions on the lower side.

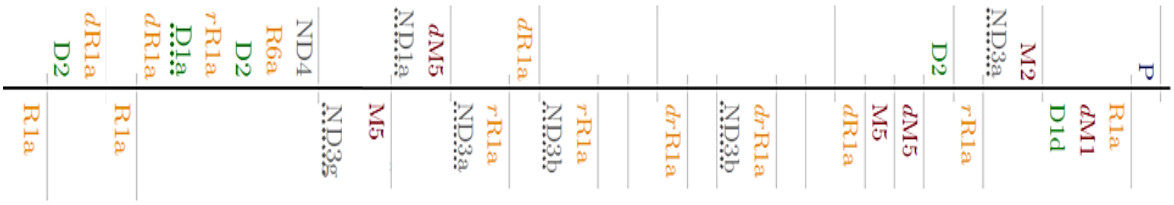


Figure 3. Category line for Transcript 2

A first glance at the category line reveals that within a short time, the focus of the discussion is on the students. It is noticeable that the students engage in metacognitive activities without being interrupted by the teacher. In this phase, they provide a reflection with reasoning (*r*R1a) without being requested to do so by the teacher, and one student even requests a reason from a classmate (*dr*R1a). However, the reasons offered in response are connected with negative discursivity (ND3b and ND3a). After that, the focus is more on the teacher. The teacher asked a suggestive question (*d*M5 connected with ND1a), to which some students responded in chorus (M5 connected with ND3g). The teacher does not respond to this (ND4). However, one can see that the teacher takes the initiative (R6a, *r*R1a, and *d*R1a). In addition, the teacher is present several times with educational measures (D2). The answer to the guiding question derived from the second transcript presented in Table 6.

Table 6. Response to the guiding question

	Guiding Question (GQ)				
	GQ1	GQ2	GQ4	GQ5	GQ6
Level	3	2	2	2c	2

On GQ1, Metacognitive activities are carried out by both the teacher and the students. The teacher guides the discussion with open-ended questions and requests clarification (P, *r*R1a, D1a, *d*R1a). Students, especially S1, demonstrate reflective activity by asking peers to check answers, inviting comments, and interpreting transactions (*d*M5, *d*R1a, R1a, *d*M1, D1d, *r*R1a). On GQ2, Metacognitive activities involving justification are evident in some students (*r*R1a, *dr*R1a), but the justifications provided do not yet reflect the development of argumentative, critical, and elaborated discussion. Although some explanations are detailed, the reasoning remains fragmented and not fully directed towards building a shared understanding (ND3a, ND3b). On GQ4, both the teacher and students engage in discursive activities. The teacher's discursive activity is evident in managing and directing the discussion, such as reprimanding students who are not listening (D2) and pointing to relevant sections in the bookkeeping (D1a). Students show limited discursive activity, such as providing explanations and requesting confirmation from peers (D1d), but their contribution does not stand out. Overall, while discursive practices are present, the students' participation is limited in clarity and impact. On GQ5, a brief simultaneous response from students appears once (ND3g) due to a suggestive question from the teacher, and the teacher does not respond (ND4). Students also give unclear or incorrect answers (ND3a, ND3b). Since these discursive disruptions are minor, the discussion continues, so this scenario is classified as level 2c. On GQ6, students engage in a spontaneous debate among themselves without teacher intervention by giving reasons (*r*R1a), requesting verification (*d*M5), inviting responses (*d*R1a), which were followed by an answer (M5) and a follow-up question (*dr*R1a), thus showing their active involvement.

Discussion of the Results Concerning Second Transcript

The analysis indicates that metacognitive–discursive activity in mathematics learning is primarily student-oriented (RQ1). Observed interactions demonstrate that the teacher initiates discussions through open-ended questions, providing students with the opportunity to interpret and structure the task independently. Students' responses are subsequently evaluated by their peers, followed by requests for clarification and justification from the responding students themselves. These exchanges illustrate that students not only articulate their thinking but also exercise autonomy in directing their cognitive processes, while

simultaneously engaging in monitoring and reflection—key components of metacognitive activation (Cohors-Fresenborg & Kaune, 2007; Cohors-Fresenborg & Nowińska, 2021; Nowińska, 2018).

Open, collaborative, and exploratory exchanges of ideas support the development of conceptual understanding (Alexander, 2008; Hansen & Naalsund, 2022), although the depth of justifications provided was not always consistent. This limitation underscores the teacher's critical role in providing targeted interventions, such as clarifications or highlighting key concepts, without assuming control over students' cognitive work. This approach aligns with Vygotsky's (1978) view that knowledge is socially constructed through interaction.

The transcripts also reveal brief, student-led debates, which, while not yet fully structured, have the potential to develop into more organized and sophisticated discourse (Nowińska, 2016a; 2016b; 2018). These findings reinforce the perspective of Iiskala et al. (2011; 2015), who emphasize that the effectiveness of metacognitive learning depends on the integration of metacognitive activity with high-quality discourse, rather than on the mere presence of such activity.

Overall, this transcript illustrates interaction patterns that support the integration of metacognitive–discursive activities, facilitating elaborative exchanges and collaborative meaning-making, while highlighting the constructive role of teacher interventions in sustaining student engagement (RQ2). Beyond confirming prior findings, the study contributes by demonstrating how student-led evaluation, clarification, and justification create a dynamic discursive environment. In doing so, it advances understanding of how teachers and students collaboratively establish conditions that promote metacognitive activation in mathematics learning.

Discussion of the Results Concerning Both Transcripts

To illustrate the interaction between teacher and student contributions, Table 7 presents the percentage distribution of contributions by activity category. Addressing the first research question (RQ1) regarding the manifestation of metacognitive–discursive activities, the analysis reveals two distinct patterns. Both transcripts include elements of these activities; however, the quality and depth differ substantially. In Transcript A, teacher contributions dominate across nearly all categories, with the majority of interactions classified as non-discursive, thereby constraining opportunities for collective exploration of ideas. In contrast, Transcript B is characterized by a predominance of student contributions, particularly in monitoring and reflection activities, with teacher discursive interventions functioning primarily to support and scaffold student participation. Although negative discursive activities remain present, their frequency is lower than in Transcript A, resulting in more participatory classroom interactions and providing students with greater opportunities to articulate and develop their reasoning.

Table 7. Contributions by category

Category	Transcript A		Transcript B	
	Teacher	Student	Teacher	Student
Monitoring (M)	3%	0%	4%	9%
Reflection (R, *r*R)	13%	3%	9%	26%
Discursive (D)	10%	0%	17%	4%
Negative Discursive (ND)	32%	39%	13%	17%

In response to the second research question (RQ2), holistic evaluation using the Guiding Questions (GQs) captured these differences in overall interaction quality. The low GQ ratings for

Transcript A quantitatively reflect how micro-level patterns—specifically ND3g—systematically constrained individual responsibility of students. Conversely, higher GQ ratings for Transcript B demonstrate that even modest reductions in negative discursivity, combined with an increase in student-initiated activities, produced a significantly more participatory environment conducive to conceptual understanding.

These findings contribute to understanding the integration of metacognitive and discursive activities, indicating that limitations in students' cognitive autonomy may result from restrictive interaction patterns rather than task design alone. This insight is particularly relevant in the context of the Merdeka Curriculum, which seeks to promote active participation and learner autonomy, although structural and contextual challenges continue to affect classroom implementation. Furthermore, these observations align with recent PISA findings (OECD, 2024), which highlight that students' capacity for open argumentation remains limited, particularly in educational environments that emphasize passive reception of information over collaborative idea exploration.

CONCLUSION

This study, based on the analysis of two distinct lessons, concludes that the quality of classroom discourse is determined not by the sheer frequency of metacognitive activities but by their discursive placement and functional integration within classroom interactions. This finding underscores the importance of strategically deploying discursive interventions to support meaningful engagement. Methodologically, the application of Nowińska's two-stage rating system, augmented with the rigorously applied ND3g category, renders qualitative differences in classroom discourse both visible and actionable. As the first implementation of this system in Indonesia, it provides a detailed and empirically grounded analysis of public classroom discussions, identifying critical phases in which strategies to improve discussion culture can be introduced.

The findings highlight the significance of fostering classroom environments in which students independently analyze mathematical problems, engage with diverse perspectives, and participate actively with minimal teacher intervention. Practically, these insights have implications for teacher professional development, including preparing and implementing discourse-supportive strategies, reflectively evaluating student contributions, and designing classroom discussions that cultivate independent participation. While the generalizability of these findings is limited by the small sample size, the study offers a foundational understanding of interaction patterns in mathematics classrooms and provides clear directions for future research, including validation and extension through larger-scale studies. Beyond informing strategies to enhance the quality of classroom interactions, this analysis contributes to understanding mechanisms that foster a constructive discussion culture and their potential impact on student learning outcomes.

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