



## **THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA**

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### **Abstract**

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases, namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used as a basic concept of multiplication. Finally, the students were able to understand the concept of multiplication more easily, and they showed interest in using this learning trajectory.

**Keywords:** multiplication, learning trajectory, design research, rural area

### **Abstrak**

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

**Kata kunci:** perkalian, lintasan belajar, design research, daerah pedesaan

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Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahmana, et al. 2012). It is because learning number operations involves an understanding of symbols, notation, and reference number (or other forms to represent) (NCTM, 2000), and it also plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operations would be one of the prior knowledge that students must have to learn other topics in

mathematics.

The concept of number operations, especially in multiplication, is one of the students' difficulties in understanding mathematics concepts (Ahmad, 2010; bin Syed Ismail, 2010; Drews, et al. 2005; Kilian, et al. 1980; Tanujaya, et al. 2017; Unlu & Ertekin, 2012). Teachers usually teach multiplication operations using symbolic form or something abstract (Unlu & Ertekin, 2012). As a result, students learn multiplication operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010). They also have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010). On the other hands, they also have a poor understanding of the place value (tens and ones) concept concerning multiplication (Drews, et al. 2005; Kilian, et al. 1980).

The result of the previous research explored about number operations is in line with the preliminary classroom observation results of the rural area's students came from, namely Serui, Ambon, and Sorong Selatan. Teachers introduced the concept of division using the formula without involving the concept itself (Prahmana & Suwasti, 2014). Therefore, this research focuses on multiplication operation as one of the concept of number operation that students must be mastered to support their knowledge in learning another mathematics subject.

Several studies indicated that constructivism approach could improve students' understanding of learning multiplication (Ahmad, 2010; Prahmana, et al. 2012; Chang, et al. 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana & Suwasti, 2014; Prahmana, 2015; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This method has been applied to students from rural areas in Indonesia starting from the introduction of integer number and number operations (Prahmana & Suwasti, 2014; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This situation underlies the researchers of this present study to try designing learning trajectory on number operations especially for multiplication operation in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Indonesia. Therefore, the focus of this study is to describe the learning activities on students' performance to do multiplication in Math GASING. It is also because several researcher stated that Math GASING is the suitable method to use in teaching mathematics, especially number operation, more easy, fun, and meaningful.

Finally, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to a formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for the students.

In this research, the literature on Math GASING and number operations are studied as basic knowledge to design sequential activities that will be passed by students ranging from concrete situations to abstract levels. All literature will be explained further in the next section.

**Number Operations**

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys, et al. 1998). The following four relations operation that has a relationship with each other, and students must understand the relationships. Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING, such as:

1. Multiplication and division are inverse operations

$$4 \times 6 = 24 \text{ ----- } 24 : 4 = 6$$

2. Multiplication can be seen as a repeated addition

$$4 \times 6 \text{ ----- } 6 + 6 + 6 + 6$$

3. Division can be seen as a repeated subtraction

$$24 : 6 \text{ ----- } 24 - 6 - 6 - 6 - 6$$

**Mathematics GASING**

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns, and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic math facts is easy if it is based on conceptual learning and visual representations.

The learning process makes students’ learning easy (*GAmpang*), fun (*ASyIk*), and enjoyable (*menyenaNGkan*) in Math GASING (Shanty & Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember — exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). On the other hand, Prahmana (2013) stated that Math GASING shows how to change a concrete sample into an abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING, as one of the innovations in learning mathematics, offers critical point in its learning process. The critical point of GASING means the condition that students must pass during the learning process and studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya & Moss, 2012). The critical point in learning multiplication is that students must master the multiplication concept of  $1 \times 1$  to  $10 \times 10$ . Students could learn various problems of multiplication operations more easily after passing a critical point.

This research uses Math GASING to describe the learning outcomes of rural area's student in learning multiplication as a repeated addition and see student responses. Researchers conducted research on rural area's students because students have experience difficulties in multiplication operations based on the pre-evaluation results. In addition, students are less focused, less accurate in counting, and easy to forget. Therefore, this study could be solved the students' mathematical problem by using Math GASING.

### ***Hypothetical Learning Trajectory***

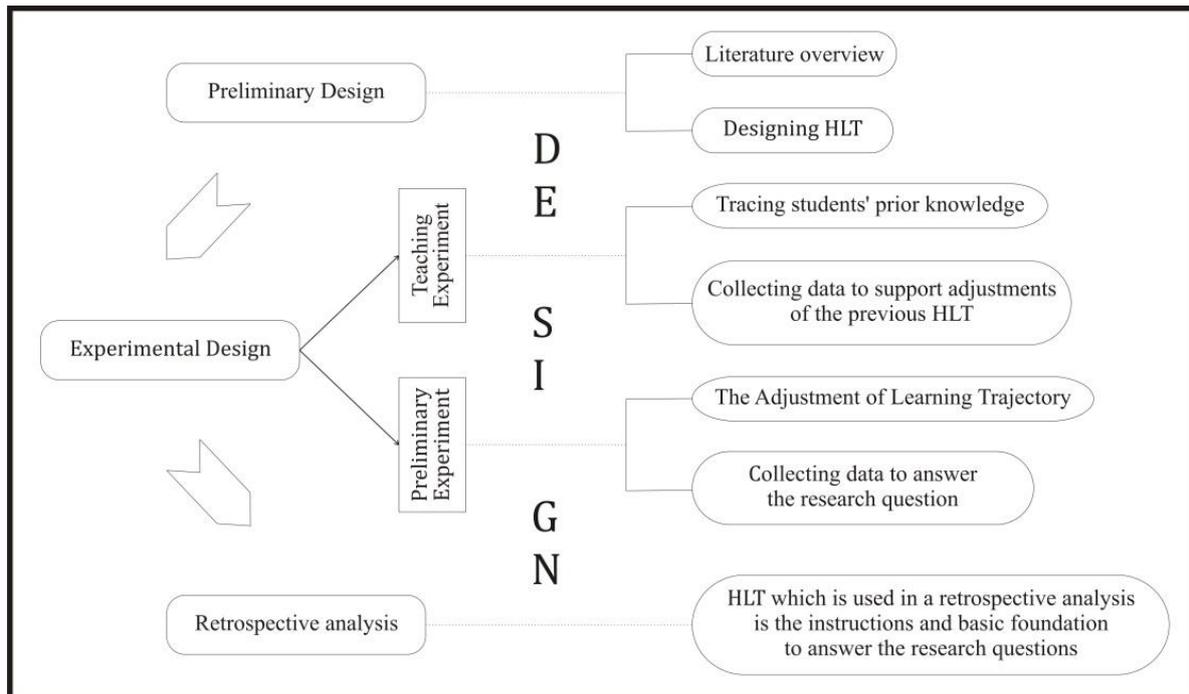
Hypothetical Learning Trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina, et al. 2011). Furthermore, Prahmana (2017) stated that HLT is a hypothesis or prediction of how students' thinking and understanding develop in a learning activity. The HLT in this study had several learning goals expected to be reached by the students during one phase.

### **METHOD**

Design research is used as the research method of this study. Design research consists of five characteristics, such as interventionist nature, process-oriented, reflective component, cyclic character, and theory-oriented (Akker, et al. 2006; Gravemeijer, 2004; Prahmana, 2017). There are two important aspects related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning trajectory taken by students in their learning activities must have HLT and LIT.

The HLT consists of three components (Gravemeijer, 2004). The first component is the purpose of mathematics teaching for the students. Secondly, it is the sequence activity that students must do during the learning process. Lastly, the conjecture is the various answers, strategies, and models that researcher expected from student understanding that emerge and develop when learning activities are carried out in class. Furthermore, there are three phases of design research, such as preliminary design, teaching experiment, and retrospective analysis that can be seen in Figure 1.

The research data came from various data sources. All data sources used aim is to get a visualization of mastery of the basic concepts of student multiplication operations. There are documentation (photos), video, student worksheets, and observation sheets. Furthermore, the data were analyzed retrospectively with HLT as a guide. This research was conducted and completed in 2 days. The research subjects are 11 matriculation teacher candidates at one of the College of Teacher Training and Education in Tangerang. All research subjects came from rural areas in Indonesia, such as Yapen, Ambon, South Sorong, Serui, and also a teacher model.



**Figure 1.** The phase of the design research (Prahmana, et al. 2012).

**RESULTS AND DISCUSSIONS**

The learning activities start from making the same perceptions of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, the students were trained to memorize the multiplication for 1 to 10 using several methods. Lastly, the teacher provides an evaluation to study students' understanding of multiplication by using mental arithmetic activities namely *mencongak* as one of the evaluation processes in these learning activities and exercises using student worksheet and also evaluation sheets. The results show that students master the multiplication operations based on the final evaluation results. On the other hands, the important results is student would like to leave the old way in learning mathematics and change to the Math GASING way. Furthermore, another results indicate that the design of multiplication learning operations in Math GASING has a crucial role as a starting point and increases student motivation in learning. The details would be discussed in the further section.

***Preliminary Design***

The researchers start to do literature review, conduct observation, and design the learning trajectory as a sequence of instructional learning for the learning of multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). The activities are designed by HLT consisting of six activities for two meetings through several easy, fun, and enjoyable activities. Students should be interested and engaged during the learning process. The last activity is evaluation process by using student worksheet and also evaluation sheet to measure the understanding of student in learning multiplication.

**Table 1.** Overview of the learning trajectory of multiplication (adapted from Surya (2011)).

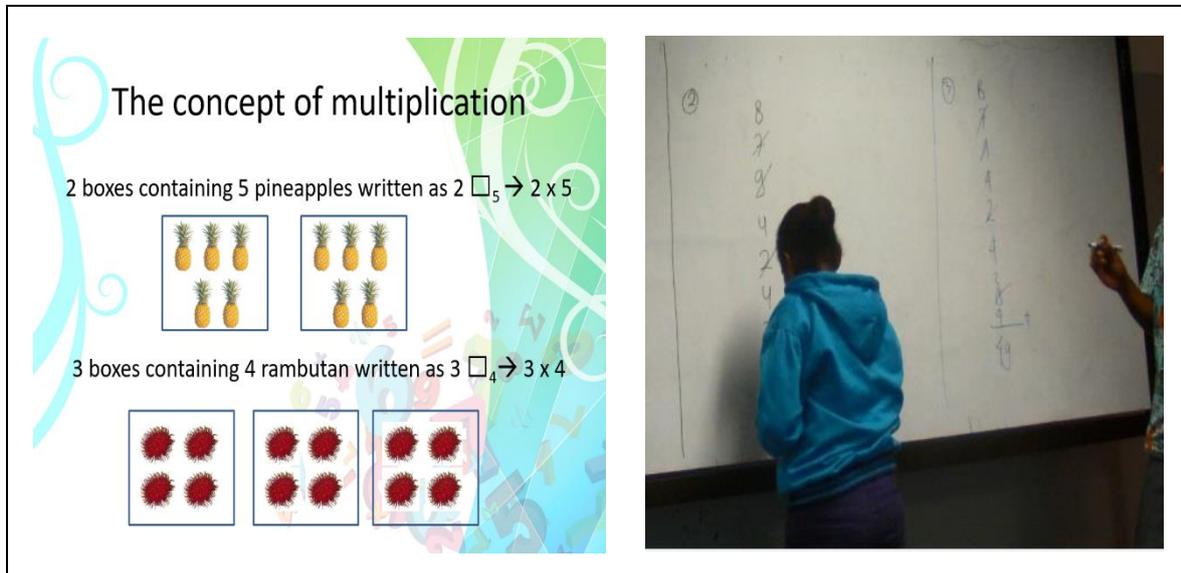
<b>Sequence of activities</b>	<b>Goals</b>	<b>Descriptions</b>
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box," for example $2 \times 3$ means 2 boxes are containing three things in that box, and so on.
Using some method to memorize this part easier	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students can master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as $1 \times 1, 2 \times 2, \dots, 10 \times 10$	Students learn about the same numbers of multiplication, such as $1 \times 1, 2 \times 2, 3 \times 3, \dots, 10 \times 10$ .
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.
Reducing some part in multiplication that already mastered	Memorizing the multiplication of numbers 8, 7 and 6	Students learn multiplication for 8, 7, and 6. The teacher teaches student by using reduce some part in multiplication that already mastered before.
Evaluation	Determining the student ability in learning multiplication	Teacher evaluates the student about the multiplication problem in the formal and informal form.

### ***Teaching Experiment***

Teaching experiment phase consists of several activities that already design in the preliminary stage. In these phases, researchers implement the learning activities using HLT as a teacher guide for the teacher model. The various educational games provided are to make teaching and learning activities more fun and enjoyable for students. This activity is one of the characteristics of learning Math GASING.

The five activities conduct using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box," for example  $2 \times 3$  means 2 boxes are containing three things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students can master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different from the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5.

Furthermore, students learn about the same numbers of multiplication, such as  $1 \times 1$ ,  $2 \times 2$ ,  $3 \times 3$ , ... ,  $10 \times 10$ . Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.



**Figure 2.** Several activities in teaching experiment phase.

### ***Retrospective Analysis***

There are some differences between the multiplication process in Math GASING and the multiplication process in general. These differences are the answer for the research question in this research. The difference is manifested in the learning trajectory to be analyzed retrospectively.

The designing learning trajectory seen in Table 1 is the student-guided activities to mastering the multiplication operations. Therefore, the researcher designed an activity using Math GASING aids. The goal is that students can understand the concrete form of multiplication using the understanding of boxes and something in there. The student must understand that multiplication in the form of repeated addition. The teacher used combination learning tools such as presentation and whiteboard to make learning process effective and efficiency that can be seen in Figure 2. Next, the teacher guides students lead the concept of multiplication as a form of repeated addition during this activity. The teacher uses several methods to remember doubling for one to ten more easily and meaningfully.

On the other hands, teacher makes the order of memorizing the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as  $1 \times 1$ ,  $2 \times$

2,  $3 \times 3$ , ... ,  $10 \times 10$ . After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

The researchers used multiplication learning phases in the Math GASING (Table 1). The introduction activity in learning the basic concept of multiplication have several good discussion. During the discussion, students look like easy, fun, and enjoyable in learning multiplication in Math GASING. Therefore, the learning trajectory guides students to understand the concept of multiplication.

All activities describe the process of students understanding from informal to a formal level according to the multiplication concept. Their experience supported by the Math GASING learning aids can make students pass the critical point of multiplication so that students can master multiplication as a whole. Surya and Moss (2012) stated that student would be able to master the mathematics subject regarding in Math GASING after their pass the critical point of the subject.

The results show that the students can apply the multiplication in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication operations or other words, the design of this study can be used as the starting point of learning multiplication. In the last activities, teacher gives evaluation to measure the students' understanding in multiplication that can be seen in Figure 3.



**Figure 3.** Student evaluation process using student worksheet.

The retrospective analysis has shown that one of the ways making student understanding in learning multiplication makes the learning process can be imaging for students. This result is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hand, learning environment also can support the result of the

learning process (Putri, et al. 2015; Nuari, et al. 2019). Finally, all students can solve several problems and exercises regarding multiplication operation.

## CONCLUSION

The learning of multiplication operation in Math GASING have a significant role as the starting point and improve students' motivation in learning multiplication. Also, the designed students' activities find the multiplication concept. The activities are starting from understanding the concept of multiplication to mastering the multiplication concept of  $1 \times 1$  to  $10 \times 10$ , which is the critical point in learning multiplication in Math GASING. The students solve several multiplication problems more easily after passing the critical point. Lastly, students can do mental arithmetic for any given multiplication problem and answer many multiplication questions very quickly and precisely. Both of evaluation is the characteristics of the assessment in Math GASING.

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