

SENIOR HIGH SCHOOL MATHEMATICS LEARNING THROUGH MATHEMATICS MODELING APPROACH

Bambang Riyanto, Zulkardi, Ratu Ilma Indra Putri, Darmawijoyo

Universitas Sriwijaya, Jalan Padang Selasa 522, Palembang, Indonesia Email: zulkardi@unsri.ac.id

Abstract

By modeling learning students enjoy learning and doing mathematics in new ways. This study aimed firsly to produce senior high school mathematics modeling tasks, lesson plan, and student worksheet for valid mathematical learning; secondly, to produce senior high school mathematics modeling, lesson plan, and student worksheet for practical mathematics learning; lastly, to produce senior high school mathematics modeling tasks, lesson plan, and student worksheet for practical mathematics learning; lastly, to produce senior high school mathematics modeling tasks, lesson plan, and student worksheet for potentially effective mathematics learning. This study used method of development research that consisting of 3 steps, i.e., analysis, design, and evaluation. In the analysis stage, researcher did student analysis, curriculum, and mathematical modeling. Second stage are to design and product. Finally, researchers applied a design of formative evaluation consists of self-evaluation, one-to-one, experts review, small group, and field test. Based on experts review, one-to-one, small groups, and field test were obtained valid, practical, and potentially effective, i.e. mathematical modeling tasks, lesson plan, student worksheet to teach mathematical modeling in senior high school and Mathematical modeling tasks and student worksheets to learn mathematical modeling in senior high school.

Keywords: learning, senior high school mathematics, modeling approach

Abstrak

Dengan pembelajaran pemodelan siswa menyenangi belajar dan *doing mathematics* di dalam cara baru. Tujuan Penelitian ini adalah, pertama, menghasilkan tugas pemodelan matematika sekolah menengah atas, lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP) untuk pembelajaran matematika yang valid, kedua, menghasilkan tugas pemodelan matematika sekolah menengah atas,), lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP) untuk pembelajaran matematika yang praktis, terakhir, menghasilkan tugas pemodelan matematika sekolah menengah atas, lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP) untuk pembelajaran matematika yang praktis, terakhir, menghasilkan tugas pemodelan matematika sekolah menengah atas, lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP) untuk pembelajaran matematika mempunyai efek potensial. Studi ini menggunakan metode penelitian pengembangan yang memiliki 3 langkah, yaitu analisis, desain kemudian evaluasi. Langkah pertama dilakukan analisis siswa, kurikulum, dan pemodelan matematika. Langkah kedua adalah desain dan *product*. Kemudian langkah terakhir, peneliti menggunakan desain evaluasi formatif, yaitu dilakukan *self-evaluation, one-to-one, review* ahli, *small group*, dan *field test*. Berdasarkan *experts review, one-to-one, small group*, dan *field test* diperoleh, yaitu soal pemodelan matematika, lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP), dan yang valid, praktis dan memiliki efek potensial untuk mengajar pemodelan matematika di sekolah menengah atas dan tugas pemodelan dan lembar kerja peserta didik untuk belajar pemodelan matematika di sekolah menengah atas.

Kata kunci: pembelajaran, matematika sekolah menengah atas, pendekatan pemodelan

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Modeling learning makes Mathematics more meaningful and interest for students in this approach (Arseven, 2015). Traditional teaching and separate teaching from problem solving are not enough to prepare students for the 21st century. Bliss et al. (2016) stated that the preparing of 21st century students can be achieved by mathematical modeling because modeling and competency modeling have important contributions to make whether learns able learn and how they must learn in mathematics education. In traditional learning, students are conditioned by "results orientation"

(Kantowski, 1981). The traditional approach also aimed that students produce correct answers validated by the teacher. Students are considered achievers if their actions are the same as the teacher's expectations. The teacher prioritizes the purpose of the following specific procedure instructions to be effective (Cobb et al., 1992). It is very contrary to the modeling learning.

Kilbane and Milman (2014) state that learning in the 21st century is characterized by the use of information and thinking to solve problems. In Problem-Based Learning models, students do both of these, resulting in their development of 21st century skills. It is very compatible with mathematical modeling learning that modeling learning is learning matching the characteristics of 21st century learning, and modeling can access 21st century students' skills (Bliss et al., 2016). Research conducted by Maa β et al. (2018) show that with learning modeling the teachers explained the positive changes in general motivation for mathematics, and stated that their students decided to study mathematics or science, or technology because they enjoyed learning and doing mathematics in new ways. This shows the teacher is very happy to teach mathematics in new ways and students enjoy learning mathematics in new ways.

Yanagimoto (2005) states that traditional mathematics currently taught in schools does not contain subject areas under development (everything is already complete and finished). There is a little examples of modeling in the practice of learning mathematics in many countries. The cause for the abyss between educational programs and facts is that mathematical modeling is not easy for teachers (Arseven, 2015). The looking for of student's problems when modeling is the aerly step before giving teacher intervention or a feedback (Ferri, 2018). This shows that mathematical modeling aiming to diagnose student weaknesses in mathematics and teachers having to have content and pedagogical knowledge to do the mathematical modeling can be done by focusing on modeling tasks. Mathematical modeling taught in school mathematics must be introduced together with strengthening new content (Kawasaki, 2012).

Mathematical modeling methods make learners understand better the relationship between realworld problem and mathematics (Bonotto, 2007; Blum, 2002). Then, studies have shown that mathematical modeling is a approach that teachers do not know well (Frejd, 2012). Quite similarly, introducing modeling into curriculum of mathematics resulted on teachers creating their strategy of learning, causing their learning of mathematics more related to real-world problems (Martinez-Luacles, 2005). Consequently, models have an essential influence in creating mathematics real to learns (Anderson, 2010).

There were three research questions in this Study. Firstly, how senior high school mathematics modeling tasks, student worksheet, and lesson plan were designed valid. Secondly, how senior high school mathematics modeling tasks, student worksheet, and lesson plan were designed practical. Lastly, how senior high school mathematics modeling tasks, student worksheet, lesson plan were designed potentially effective.

METHOD

This study applied method of development research. This method consists of 3 steps, i.e., Analysis, design, then evaluation (Akker, et. al., 2006). On the analysis stage, researcher did student analysis, curriculum, and mathematical modeling. The second stage, researcher did design and produce mathematical modeling tasks, LKPD and RPP. Finally, this research applied a design of formative evaluation (Figure 1). On this stage, researcher did self-evaluation, one-to-one, experts review, small group, and field tests (Tessmer, 1993, Zulkardi, 2006).



Figure 1. Design of Formative Evaluation.

The criteria of success of this research used the form of mathematical modeling asks, student worksheet , and lesson plan in Senior High School that was valid, practical and potentially effective. The validity was got from the comments of experts of mathematics, RME, mathematics education and mathematical modeling. The practicality was got from the students' comments since working mathematical modeling tasks via observations of the one-to-one, small group by video and interview, and the effective was obtained from the field test. Practicality implies simple to implement. It can be interpreted, and is not double meaning.

The collecting of data techniques were, firstly, walkthrough. It based on the comments of experts to produce a valid mathematical modeling tasks, student worksheet, lesson plan on language, content, construct and contexts aspects, secondly, interview and student's solution, it produced from on-to-one, small group and field test to get the practicality and potentially effective of the mathematics modeling tasks, student worksheet, lesson plan. The obtained data were analyzed by implementing method of descriptive analysis, firstly, walkthrough, walkthrough by sheet analysis based on the comments of validation of experts to produce valid mathematical modeling tasks, student worksheet, and lesson plan, secondly, interview and student's solution, it analyze the outcomes of one-to-one and small group to produce practicality and potentially effective of the mathematics modeling tasks, student worksheet, and lesson plan.

RESULT AND DISCUSSION

Preliminary research, i.e. the first cycle of research was done at SMP Negeri 6 Kayuagung with 3 students Grade VIII.2 students as research subjects, namely K. F. P., M. R. A. P. and O. A. In this study the valid and practical mathematical modeling tasks were produced using UBER contexts. Figure 2 shows that the students' commented that the task of mathematical modeling with UBER context was very good and made them think, and reason, and need revisions to terms in the process of modeling that they were not yet familiar with it. It is because the mathematical modeling was new for them.

Saran: Mahani Manurut saya, Allah sudah sangat bagus, karera laisa membuat anak berfikir menalar, tapi bahasa yang digunakan terlalu tinggi untuk arak smip, sehingga anak akan kurang paham dengan saal yang dubenikan

Suggestion:

In my opinion, the material is very good, because it can make children to think reasoning, but the language that used too high for yunior high school, therefore, children do not understand to the tasks that is given.

Figure 2. Students' comments on the modeling task

The first cycle of the study also indicated that learners could create identification of the problems, make assumptions, work mathematically to get results, and provide recommendations even though they were not able to generalize. Figure 3 is a solution for students of UBER context modeling tasks.

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	Model yang Anda Rekomendasikan untuk diimplementasikan dan laporkan hasilnya.
Analisislah dan Evaluasiaya Model dan Solusi yang Anda peroleh Model nya sudah cocole / sesuai	Icesimpulannya: to Bahwa lika yang mengituhi tamanya berjumlah 200 orang maka akan disewa. 25 mobil dergan tan F 1.250.0000-

Do the math: to get solution

 $x = \frac{200 \text{ people}}{8} = 250 \text{ car}$ x = 50.000 × 25 = 1.250.000 Analyze and assess the model and the solutions The model is suitable Iterate as needed to refine and extend the model No Implement the model and report the results Conclusion If there are 250 people on the trip, 25 car will be rented at cost 1.250.000 rupiah

Figure 3. Students Solution

The students' commented that the tasks of mathematical modeling with nutritional context was very good and made them imaginative, but it was quite difficult. This shows that the modeling task is good and needs revision. The comments of student can be seen in Figure 4.



In my opinion, this task is good. It can invite to imagine. But, some fact and problem above out of sync and not clear what's the meaning.

Figure 4. Student comment's on mathematical tasks

The second cycle of the study indicates that students could only identify and simplify the tasks but could not to complete the modeling tasks according to the modeling process. This resulted the fact that mathematical modeling was new to them. Figure 5 shows the solution for students in the task of modeling nutrition contexts.



Identify and specify the problem to be solved

How to solve the problem of malnutrition in Indonesian society? whether by increasing consumption of fruits and vegetables and reducing the consumption of foods that contain high fat and cholesterol

Melakukan	Proses	bermatematika:	Untuk	mendapatkan solusi
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icti 9anduw	••	67	1,070	12,269	2,376
curled borley		123	0,44 m	28.229	2,264

Do the math: get a solution In one serving size White rice: 129 0,285 27,94 2,664 Corn : 132 1,62 29,29 4,966 Wheat bread: 67 1,07 12,269 2,376 Pearled barley: 123 0,44 26,229 2,263

Figure 5. Student's Solution

The Third Cycle of the study took 3 subjects as the subjects of SDIT Bina Insani Kayuagung, namely C. A. M., A., and F. The students' commented that they learn interesting mathematics with mathematical modeling learning. Figure 6 is a student's comment on a mathematical modeling task.

belgiar MateMatika ini Men Yenan 9 kan

This learning mathematics is interesting



In this third cycle of the study, students could only identify and specify problems and make assumptions. They were not able to do a mathematical process to get a solution and cannot evaluate and generalize mathematical models (Figure 7).

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Identify and specify the problem to be solved home, parking lot and playground

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Do the math: get a solution Land size 15 x 40 meter

Figure 7. Student Solution to the Modeling Task

The Fourth Cycle took 6 students as research subjects of SDIT Bina Insani Kayuagung, namely S. A., K. N. I., F. A., M. S. A., M. F. A. H. and D. R. F. The students commented that the mathematics learning became interesting and made them think with mathematical modeling learning. Figure 8 explains a student's comment on a mathematical modeling task. This fourth cycle study used the context of "Jumat Sejahtera".

SOBLATA bagus dan MenaRik MENIDOai aan kitaber Pileit

The problem is good and interesting and makes us think

Figure 8. Students' comments on modeling tasks

In this fourth cycle, students could only identify and specify problems and make assumptions, determine important variables, perform mathematical processes to get solutions, produce mathematical models, but they were not able to generalize and do iterations that can be seen in Figure 9.



Create a mathematical model/formula to calculate only the cost of food from "Prosperous Friday" using the variables (numbers / values / prices) that you have created! Food per box = $3 \times 3000 = 9000$ Drink per box = $1 \times 1000 = \underline{1000}$ = 10.000 rupiah Total of students = 21 people Total of teacher = 2 teachers 23 orang 23 orang x 10.000 = 23.000 rupiah



Create a mathematical model/formula to calculate how long to save from "Prosperous Friday" using the variables (numbers / values / prices) that you have created by assuming how much money you save per day! Total of students = 21 The price of donut = 5000 The money that save in one day = 5000 $5000 \times 21 = 105.000$ rupiah 105/5 = 21 hari

Figure 9. Student Solution to the Modeling Task

This study was conducted at SMA Negeri 1 Palembang. In this study, modeling learning used the context of Musi 2 Bridge, Toll Fee, Online and Conventional taxi, Car Speed on Toll Road, and Water subsciption fee of PDAM. These mathematical modeling tasks and students worksheet was conducted experts review, i.e. Prof. Hendra Gunawan, Prof. Edi Cahyono, Al Jupri, S.Pd., M.Sc., Ph.D., and Dr. Rusdy A. Siroj. Figure 10 show the comment of expert of mathematical modeling, i.e. Edi Cahyono (Figure 10).

Comment [a2]: Pengeluaran per bulan pada struk pembayaran di atas hampir sama. Dengan demikian, akan sulit disusun ekomendasi yang berbeda secara signifikan. Perlu data perbedaan biaya per m³, untuk kelompok pemakaian. Data bisa diperbanyak, misalkan struk selama satu ahun.

Expenditure per month on the payment receipt above is almost the same. Thus, it would be difficult to formulate significantly different recommendations. Data cost per m ^ 3 difference is needed, for usage groups. Data can be reproduced, for example, receipt for 1 year

Figure 10. Expert' comment to the Modeling Tasks on PDAM contexts

After experts review, this study conducted one-to-one. The subjects of one-to-one were the learners of Grade XI IPA 1 and 3 of SMA Negeri 1 Palembang as many as 15 students, namely A. K., A. A., A. K. A. I., A. A. R., A. Y. N., D. T. T., H. K., J. A, M. A. A., M. B. H., M. D. S. N., M. A., N. K., N. S. A, and R. R. A.

One of the comments of students when one-to-one students, named A. K. state that this modeling task was interesting and he never studied modeling tasks like this pervously and this task needed a strong and careful analysis. A. K.'s comments is shown in the Figure 11.

Soci ini menarix, harena sebelum nya saya ti nan pernah menyerianan Soci matematika serumit ini. Soci Untuk mengerianan soci ini dibutuhkan Anarsis Yeng wat an cermat.

This problem is interesting, because before I never solved a math problem this complicated. To do this problem requires a strong and careful analysis.

Figure 11. Student comment's on one-to-one

Judging from the solution of students, the task of modeling with the context of the Toll Road Fee and Musi 2 Bridge, the students were not able to bring up mathematical models even though they did the mathematical processes. But for the other three contexts, the students were able to make mathematical models even though they could not validate and evaluate and do iterations to improve the mathematical models obtained. This is due to new mathematical modeling for them. Figure 12 is a solution for students who work mathematically.

Pengisian ke-n uang pada kartu tol (e-money)	Jumlah total uang yang dikeluarkan untuk pengisian kartu tol (Rp)	Jumlah total biaya administra si (Rp)	Jumlah uang (e-money) pada kartu tol	Jumlah uang yang dibayar untuk perjalanan ke-n (Rp)	Jumlah uang pada kartu tol pada perjalanan ke- n	Uang pada kartu tol yang terisi untuk berapa kali jalan?	Sisa uang (e- money) pada kartu tol pada perjalanan ke- n
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2	tox ada Propision	Br. Kos	11943-500	12020000	pr53,200	2	NP 33, 200
3	Rr50-000	BY 1.500	12940.500	nr 20.000	nr 33, 200	1	119 13,200
4	Fair 202	tr 1.500	Reverso	RP 20.000	nr-61.900	3	Rph1.700
5	the ada	11-1-500	REMB500	126.000	121941.700	2	np 31.300
6	The 202	Re 2-500	12048-500	Rp 20.000	Rp 50.700	2	121030.300
7	12150.000	Rr1.500	12842,500	RP 20-000	120038-200	1	Mp10.700

b)	Berdasarkan	data yang	g telah diketah	ui di atas,	lengkapilah ta	abel di bawah ini:
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charging	The total	The total	The	the amount	The	Money on	Remaining
nth money	amount of	amount of	amount of	of money	amount of	the card	money (e-
on the toll	money	administrative	money on	paid for	money on	filled in	money) on
card (e-	spent on	costs (Rp)	the toll	the nth trip	the toll	for the	the toll
money)	toll card		card (Rp)	(Rp)	card on the	number of	card on the
57	filling		× 1/	× 1/	nth trip	trips?	nth trip
	(Rp)				(Rp)	1	(Rp)
1	Not filling	1.500	48.500	20.000	45.200	2	25.200
2	Not filling	1.500	48.500	20.000	53.200	2	33.200
3	50.000	1.500	48.500	20.000	33.200	1	13.200
4	Not filling	1.500	48.500	20.000	61.700	3	41.700
5	Not filling	1.500	48.500	20.000	41.700	3	31.700
6	Not filling	1.500	48.500	20.000	58.700	2	38.700
7	50.000	1.500	48.500	20.000	38.700	1	18.700

Figure 12. The results of doing mathematics students.

After conducting one-to-one, the small group was also conducted using the context of "Musi 2 Bridge", "Toll Road Fee," Online and Conventional Taxis", "Car Speed on Palindra Toll Road ", and "Water subscription fee of PDAM". The result of small group was that students could solve mathematical modeling problems in the context of "PDAM". The solution to the mathematical model is shown in Figure 13.



Figure 13. Context Mathematical Model PDAM

The comments of students after small group were that they were happy and the material was new and interesting. Student comments are shown in Figure 14.

I enjoy this material and this material is new, and the formulas are unknown to me. Pretty interesting

Figure 14. Student's comment on small group

After that, the study continued to conducting a field test to Grade XI IPA 3 using modeling learning with the context of the PDAM. Then, field test was conducted to Grade XI IPA 3 and IPA 1 with modeling learning of the context of the musi 2 bridge and the toll roads fee. Then, it was conducted to Grade XI IPA 3 using the context of PDAM Fee. Then, it was conducted to Grade XI IPA 3 using the context of Musi 2 Bridge. Then, on Tuesday, April 9, 2019, it was carried out to Grade XI IPA 3 with the modeling learning of the context of the Toll Road Fee.

For the field test, students commented that this modeling task was interesting and they never studied modeling tasks like this before and this task required strong and careful analysis. Figure 15 is the student's comment after the learning process, i.e.

Socil Ini menariu, harena sebelum nya saya tidau pernah menyeriawan Socil Matematika Serumit Ini, Soci Untuk mengeriawan socil Ini dibutuhwan Anavisis Yang Wat dan cermat.

This problem is interesting, because I have never done math problems this complicated before. To do this problem requires a strong and careful analysis.

Figure 15. Students comment's on one-to-one and small group

The modeling learning activities indicate that students are very enthusiastic about learning mathematics which can be seen in the videos and observations. Student solutions in the modeling process also show that students could to create mathematical models for the five contexts of modeling learning that are done even though students have not been able to evaluate the mathematical models they have found. Figure 16 shows the solutions of students for modeling tasks with the context of the PDAM.





Make a mathematical model to calculate the cost and cost of clean water per cubic meter using the variable you created. Determine the monthly cost and water price per cubic meter!

Cost per cubic meter = x, Fixed costs per month = y. $13x + y = 37.800 \ 12x + y = 35.200$ $12x + y = 35.200 \ 12(2.600) + y = 35.200$ $x = 2.600 \ 31.200 + y = 35.200$ y = 4000 Make a mathematical model to estimate the cost that must be incurred by the household in one year based on the cost of water costs and costs for one year!

Usage: a = amount per cubic meter, x = price per meter cubic, y = expense per month bill amount in one month (no expense) = (ax)bill amount in one year (no expense) = (ax + y)12



The same results also show the context of the Car Speed on the toll road. The students could to create mathematical models of problems given. Figure 17 is a student solution.

		\frown
an.	kecepatan mobil :	• Fecepatan rata ? :
altres .	Sa = Sb Va.ta = Vb.tb	$V_{\text{talk}} = \frac{Va + V_0 + Vc + Vd + a + b + d + d + d}{a + b + d + d + d}$
	\rightarrow Va = $\frac{V_{b.tb}}{ta}$ (1)	$L_{\text{s}} V_{\text{colume}} = \underline{\Sigma V}$
	$\rightarrow V_b = \frac{Va.ta}{tb} (11)$	n : jumlah mobil v : becepatan Wobil

Make a mathematical model for the speed of a car that crosses the motorway using the variable you have created! The average speed of all cars and their mathematical models.

 ∇

Car peed:

$$S_a = S_b \rightarrow v_a \cdot t_a = v_b \cdot t_b$$

 $\rightarrow v_a = \frac{v_b \cdot t_b}{t_a} \dots (1) \qquad v_b = \frac{v_a \cdot t_a}{t_b} \dots (2)$
Average speed:

Average speed:

$$\mathbf{v}_{\text{rata}} = \frac{\mathbf{v}_a \cdot \mathbf{v}_b + \mathbf{v}_c + \dots}{a + b + c + \dots} \rightarrow \mathbf{v}_{\text{rata}} = \frac{\sum \mathbf{v}}{n}.$$

n = many cars, and v = car speed

Figure 17. Mathematical model of the "car speed on toll road" context

For the context of Musi 2 Bridge, the students were able to produce two models, namely assuming the arch of the Musi's bridge 2 is a quadratic function and a trigonometric function. Figure 18 is a solution to the mathematical model they found.





Create a mathematical model to calculate the height of the big and small arches of the musi bridge 2 using the variables you have created! y = -0.35(x - 0)(x - 10)y = -0.35 x(x - 10) $y = -0.35 x^2 - 10x$ y_n = nth string height, y = tinggi maksimum x = distance end to end horizontally The bridge arch is a function of sin Sin graph with a hill shape has angle intervals $0 \le x \le \pi$. So, for this bridge

has a difference in the angles
$$\frac{\pi}{15}$$

for each adjacent rope (to determine height) and a small arc. Create a mathematical model to calculate the height of large and small arches of the musi bridge 2 using the variables you have created!

Large arch height:
$$y = 10.\sin\left(n.\frac{\pi}{15}\right)$$
 for its breaking point
The value of $\sin\left(n.\frac{\pi}{15}\right) = 1$, so, $\sin\left(n.\frac{\pi}{15}\right) = \sin\left(\frac{\pi}{2}\right)$

So, n = 15/2. Function for small arches, i.e.

$$y_n = 10.\sin\left(n.\frac{\pi}{8}\right)$$

Figure 18. Mathematical model for "Musi 2 Bridge" context

The context of online and conventional taxi show that students could produce mathematical model derived from the given problem. Figure 19 is the mathematical model found by the student.

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Un = G-S	.00 +	n. 4100			
Eksekutif					
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1	1.6.20	· 12.000	1 n. 2000		

Make a mathematical model to calculate the cost of online and conventional taxis! Online taxis (Jawa, Sumatera, Bali): $a = 0, b = 3.500, U_n + a + nb$ Kalimantan, NTT: The highest: $U_n = n \cdot 3.700$ The lowest: $U_n = n \cdot 6.500$ Bluebird conventional: $U_n = 6500 + n \cdot 4100$ Execitive: The highest: $U_n = 13.000 + n \cdot 7500$ The lowest: $Un = 17.000 + n \cdot 9000$ Express: $U_n = 6500 + n \cdot 3800$

Figure 19. Mathematical model of online and conventional taxi

For the context of the online and conventional taxi fee also shows that students could make mathematical models of the problems given. Figure 20 is a mathematical model found by students.



Figure 20. Mathematical model for "Toll Road Fee" context

After doing field test potentially effect was looked. This can be seen from the comments of the model teacher, the observer teacher and the students themselves as research subjects stating that modeling learning is good to apply in the future because it is interesting, challenging, and meaningful for students and makes students motivated to learn. The learning process of mathematical modeling tasks can produce good mathematical models as expected and there is a validation process as in the modeling cycle. The mathematical model that has not been generated is in the context of the Musi 2 Bridge. The quadratic function model that still has parameters appears. Whereas for trigonometric functions mathematical models have emerged but there are no arguments or validations from the students. Whereas for the contexts of conventional online and taxi taxis, the toll road fee, PDAM and car speed on toll road produced the correct model. Therefore, results of the study shows the learning process on field test is good in accordance with the expectations in accordance with the characteristics of the task and the modeling learning process.

This study produced mathematical tasks, lesson plan, and student worksheets for senior high school mathematics modeling. As a results, the valid, practical and potentially effective senior high school mathematics task of mathematics modeling, lesson plan, and student worksheet were designed valid, practical, and potentially effective to be implemented for more interesting and meaningful mathematics learning to better the quality of learning and mathematical achievement.

The first research problem, how senior high school mathematics modeling tasks, lesson plan, and student worksheet were designed valid?. This research conducted experts review from mathematicians Prof. Hendra Gunawan, experts of mathematical modeling Prof. Edi Cahyono, expert of realistic mathematics education Al Jupri, S.Pd., M.Sc., Ph.D and expert of mathematics education Dr. Rusdy A. Siroj. They state that mathematical modeling tasks in this reseach is good and need revision. After experts review, one-to-one was conducted. Students say that this tasks was interesting and make student to think. Student could make mathematical model in one-to-one. This show that this tasks was valid.

In this research of designing modeling tasks based on the characteristics of the modeling task, the problem comes from the real world. Sullivan et al. (2015) state that designing tasks should consider three aspects of pedagogy, namely, student motivation, task recognition to students, which relates to the teacher wanting learners to have the ability to interpret the demands of the task, and

accessing assignments by all students, on which this issue causes the teacher to think of student motivation, the level of initial knowledge to push with assignments, the current mathematics culture class, expanding tasks that could distinguished to allow all learners to push appropriately. To implement this designed task the teacher can make modeling tasks as an effective way of learning. This modeling tasks can motivate students; with mathematical modeling tasks the students will carry out the process of interpreting assignments and modeling tasks which are valid for all students who are weak, moderate and high in their mathematical abilities.

In desiging tasks of mathematical modeling learning, teachers must make use of everyday contexts. The function of a context is very essential in mathematical modeling because the modeling needs a context for problem frame, and mathematical applications (Lingefjard, 2006; Mousoulides, 2007). A focus characteristic of traditional mathematics learning in many countries is the activity of 'solving story problems' (Schoenfeld, 1992; Mousoulides, 2007).

Saxena et al. (2016) state that sometimes they find it difficult to solve problems and learn formulas and prove theorems. Although a lot of effort and research were carried out in this field but there was a lot of work done at the basic level. However, the level of difficulty gets high at the middle and high levels because we have a lot of content in grades 11 and 12 as well as many new things for students and many formulas, theorem relationships and a few parts of the application. In research it shows that it is very essential to understand the contexta in modeling.

The second research problem is how senior high school mathematics modeling tasks, lesson plan, and student worksheet were designed practical?. This research was conducted small group. In this small group students very active in learning. Students could learn mathematical modeling to get mathematical model. Students also state that this learning is very interesting. This show that mathematical modeling tasks and student worksheet is practical.

It is not easy to implement mathematical modeling lessons in schools due to time constraints (Hino, 2007). Research in mathematics education emphasizes the essential of teacher knowledge about learner thinking (Ball & Cohen, 1999; Ball et al., 2008; Even & Tirosh, 1995; Stillman et al., 2010). But teacher knowledge about students' thinking is not easy matter because it needs them to see, hear, understand, and interpret learners' thinking when driven by mathematics tasks (Ball, 1997). Teachers must be able to estimate difficulties, lack of understand, and predict whether students are appealing in choosing examples (Ball et al., 2008; Hill et al., 2008). It shows the importance of the task of modeling school mathematics. Modeling learning place new challenges on the teacher (Doerr, 2006).

The study conducted by Riyanto et al. (2017) indicates that junior high school learners are very extracted in learning mathematical modeling. Likewise, in the study conducted by Riyanto et al. (2018), students are also interested in learning mathematical modeling, and the research in elementary schools shows the same thing that elementary school pupils can produce good mathematical models and are interested in learning mathematical modeling (Riyanto et al., 2019). Based on the Eric's

(2010) research results, he recommend that future studies be required in the area of mathematical modeling to link theory and practice, to fit between contemporary and approaches of traditional, and role of the teacher as a facilitator. The study conducted by Anthony and Hunter (2010) show that mathematical modeling and application tasks that are discussed in group settings to give a solid basis to learner development of argumentation because they are inherently social experiences with support of effective teacher and communication, collaboration, teamwork, and reflection effectively.

The results of the study conducted by Biccard (2010) show that modeling competencies can not develop by implementing traditional approach and essential teaching to develop mathematics as their own competence. Development of modeling competencies requires support by wise task selection, heterogeneous group formation, and effective teacher knowledge about modeling. The study findings of Wong (2008) can have implications to training of teacher on modeling tasks to improve quality of mathematical achievement. The suppression on mathematical modeling in school level is to focus on the process rather than on the product (Balakrishnan et al., 2010).

The third research problem is how senior high school mathematics modeling tasks, lesson plan, and student worksheet were designed potentially effective?. This study conducted field tests for PDAM, Fee Tol, Musi 2 Bridge, Online and Conventional taxi, and Car speed on Tol road. From field tests, students was very enthusiastic and enjoyed learning mathematics with mathematical modeling and students could make mathematical models using their own methods. From 36 students only one student didn't like this tasks. On field tests also students could make mathematical model, so that couldn't do iteration in modeling process. It is caused that learners are rarely to learn mathematical modeling.

In this study, students studied mathematical modeling to solve real-world problem with emply topic in mathematics. In learning mathematical capital, students must make assumptions, make selections, make variables, mathematization, working mathematically and validating the obtained results both mathematically and in the real world. Similar, the study results of Stillman and Brown (2014) show that the inability to mathematize in the classroom is connected to the inability to apply relevant mathematical skill in the context of modeling than lack of relevant mathematical skill per person, or an orientation view of mathematical applications or persistence on tasks.

Creativity is needed to face an unknown future and the education should complement learners for this purpose (Wessels, 2014). Several skills, such as creativity, spatial, and problem solving, are generally associated with art, but it is also a basic part of mathematics, technology, economics and politics - forming an integral part of a person's daily life (Robinson, 1999; Hendroanto et al., 2018; Ahamad et al., 2018). In mathematical modeling, it is very important for creativity, fluency, flexibility, and novelty. It is very important in facing an increasingly complex life. Furthermore, Saxena et al. (2016) state that the students' performance has not been at the desired level, the students did not solve mathematical problems interestingly. The importance of mathematical modeling in schools is at all levels (Mousoulides, 2007). In this study, the creativity of students in learning

mathematical modeling rises. In addition, metacognitive development and critical thinking skills are seen as importance and must help learners reflect on their answers to modeling tasks (Garofalo & Lester, 1985; Schoenfeld, 1992; Mousoulides, 2007).

In the study conducted by Eric (2013) it concludes that encouraging students in mathematical modeling opens up a strategy to teachers to redesign strategies in which leaners be able solve problems in real-world problems. Modeling tasks of "travel plan" context take a dimension for students to look at agreed phenomena for mathematical thinking, asking them to explain the purpose of modeling situations, evaluating these amounts and variables leads relevance to work with manipulating, developing, and interpreting the model in many strategies to get solutions (Eric, 2013).

In mathematical modeling approach, students should be able to use the many stages in modeling cycle to many open-ended tasks (Haines & Crouch, 2010; Knott, 2014). This approach can be applied to solve other problems successfully and to develop a critical, reflective individual (Knott, 2014). The teachers in his study comment that it is essential for learners to learn mathematics is helpful in extra-mathematical situations (real-world) and one approach to make students learn mathematics with this pupose in thinking must be integrate more modeling activities in mathematics learning (Frejd, 2012; Muhtadi et al., 2017; Mumu et al., 2018).

Good modeling tasks invite learners to encounter the whole modeling process (Balakrishnan et al., 2010). In this study, student was given the entire modeling process. In this study, it can be seen from experts review, student's opinion and student's solution of mathematical modeling tasks. The same results occur in the study using the contexts of Musi 2 Bridge, Toll Road Fee, Car Speed on Toll Roads, online and conventional taxis, and the context of the PDAM whose contexts originating from the real world situation in which the learning uses authentic, opened-ended problems, students assuming, doing election. In this learning, the students produced good mathematical models, active, effective learning processes and they were interested in learning mathematics. The results supported to several research that shown students are interested in realistic material use real-time daily applications for them (Zulkardi, 2002; Saleh et al., 2018; Nuari et al., 2019).

Zulkardi and Putri (2019) said that some new components of the 2013 curriculum have the same characteristics as PMRI. *PMRI* and Learning mathematical modeling are not much different (Zulkardi, 2017). It indicates the importance of modeling learning in the 2013 curriculum. Consequently, teachers should design learning tasks based on the characteristics of mathematical modeling tasks. This statement also correspondends to Blum et al. (2019) stating that the design of activities in mathematical didactics can be in the form of design of tasks, lessons (lesson plan), sequences of teaching, textbooks for mathematical leaning, curriculum, assessments, and ICT-based teaching material or programs for teacher education and must be done by teachers, educators, book writers, curriculum and assessment developers, designer of ICT, and researchers. This statement is in accordance with this study, namely designing modeling tasks, student worksheet, and lesson plan.

CONCLUSION

This study produces the valid and practical high school mathematics modeling tasks, lesson plan, and student worksheet. The products alaso have potentially effective. The findings in this study were that students were very enthusiastic and enjoyed learning mathematics with mathematical modeling and students could make mathematical models using their own methods. Student can apply modeling process, so this promoting student's modeling literacy. Mathematics learning should implement mathematics modeling learning starting from elementary school to college level. Then students individually and in groups solve this mathematical modeling problem by using suitable mathematical topics. There needs to be a change in curriculum that uses real-world problems in mathematics learning mathematics learning, and evaluating mathematical modeling learning in schools and colleges.

REFERENCES

- Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problembased learning in geometry lessons. *Journal of Physics: Conference Series*, 943(1), 012008. https://doi.org/10.1088/1742-6596/943/1/012008.
- Akker, J., Bannan, B., Kelly, A.E., Nieveen, N., & Plomp, T. (2006). *Educational Design Research*. Enschede: SLO
- Anderson, J. (2010). Collaborative problem solving as modelling in the primary years of schooling.In: Kaur, B., & Dindyal, J., (Eds.). *Mathematical Applications and Modelling*. Singapore: World Scientific Publishing Co. Pte. Ltd.
- Anthony, G., & Hunter, R. (2010). Communities of mathematical inquiry to support engagement in rich tasks. In: Kaur, B., & Dindyal, J., (Eds.). *Mathematical Applications and Modelling*. Singapore: World Scientific Publishing Co. Pte. Ltd.
- Arseven, A. (2015). Mathematical modelling approach in mathematics education. *Universal Journal* of Educational Research, 3(12), 973-980.
- Balakrishnan, G., Peng, Y.Y., & Eng, E.G.L. (2010). Mathematical modelling in the Singapore secondary school mathematics curriculum. In: Kaur, B., & Dindyal, J., (Eds.). *Mathematical Applications and Modelling*. Singapore: World Scientific Publishing Co. Pte. Ltd.
- Ball, D.L. (1997). What do students know? Facing challenges of distance, context, and desire in trying to hear children. In: Biddle, B., Good, T., & Goodson, I., (Eds.). *International Handbook on Teachers and Teaching 2*. Dordrect: Kluwer Press.
- Ball, D.L., & Cohen, D.K. (1999). Developing practice, developing practioners: Toward a practicebased theory of professional education. In: Sykes, G., & Darling-Hammond, L., (Eds.). *Teaching as Learning Profession: Handbook of Policy and Practice*. San Fransisco: Jossey Bass.
- Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What make it special? *Journal of Teacher Education*, 59(5), 389-407. https://doi.org/10.1177/0022487108324554.

- Biccard, P. (2010). An investigation into the development of mathematical modelling competencies of grade 7 learners. *Unpublish Doctoral Dissertation*. Stellenbosch: University of Stellenbosch.
- Bliss, K., Libertini, J., Levy, R., Zbiek, R.M., Galluzzo, B., Long, M., Teague, D., Godbold, L., Malkevitch, J., Kooij, H.V.D., Giardano, F., Fowler, K., Pollak, H., Gould, H., Montgomery, M., & Garfunkel, S. (2016). *GAIMME: Guidelines for Assessment & Instruction in Mathematical Modelling Education*. Philadelphia: COMAP & SIAM.
- Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education discussion document. *Educational Studies in Mathematics*, 51(2), 149-171. https://doi.org/10.1023/A:1022435827400.
- Blum, W., Artigue, M., Mariotti, M.A., Sträßer, R., & Heuvel-Panhuizen, M.V.D. (2019). European didactic traditionsin mathematics: Introduction and overview. In: Blum, W., Artigue, M., Mariotti, M. A., Sträßer, R., & Heuvel-Panhuizen, M.V.D. European Traditions in Didactics of Mathematics. Switzerland: Springer.
- Bonotto, C. (2007). How to replace word problems with activities of realistic mathematical modelling. In: Blum, W., Galbraith, P., Henn, H.W., & Niss, M. (Eds.). *Modelling and Applications in Mathematics Education: The 14th Icmi Study*. New York: Springer.
- Cobb, P., Wood, E., Yackel, E., & Mcneal, B. (1992). Characteristics of classroom mathematics traditions: an interactional analysis. *American Educational Research Journal*, 29(3), 573-604. https://doi.org/10.3102/00028312029003573.
- Doerr, M.H. (2006). Examining the tasks of teaching when using student's mathematical thinking. *Educational Studies in Mathematics*, 62(1), 3-24. https://doi.org/10.1007/s10649-006-4437-9.
- Eric, C.C.M. (2010). Tracing primary 6 students' model development within the mathematical modelling process. *Journal of Mathematical Modelling and Application*, 1(3), 40-47.
- Eric, C.C.M. (2013). Students' designing an ideal tourism route as mathematical modelling. In: Stillman, G. A., Kaiser, G., Blum, W., & Brown, J.P. *Teaching Mathematical Modelling: Connecting to Research and Practice*. Brazil: Springer.
- Even, R., & Tirosh, D. (1995). Subject-matter knowledge about students as sources of teacher presentations of the subject-matter. *Educational Studies in Mathematics*, 29(1), 1-20. https://doi.org/10.1007/BF01273897.
- Ferri, R.B. (2018). *Learning How to Teach Mathematical Modeling in School and Teacher Education*. Kassel: Springer International Publishing.
- Frejd, P. (2012). Teachers' conceptions of mathematical modelling at Swedish upper secondary school. *Journal of Mathematical Modelling and Application*, 1(5), 17-40.
- Garofalo, J., & Lester, F. (1985). Metakognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, *16*(3), 163-176. http://dx.doi.org/10.2307/748391.
- Haines, C., & Crouch, R. (2010). Remarks on a modelling cycle and interpretation of behaviours. In: Lesh, R., Galbraith, P.L., Haines, C.R., & Hurford, A. (Eds.). *Modeling Students' Mathematical Modeling Competencies*. New York: Springer. http://doi.org/10.1007/978-1-4419-0561-1
- Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial

visualization. *Journal of Physics: Conference Series*, 943(1), 012029. https://doi.org/10.1088/1742-6596/943/1/012029.

- Hill, H.C., Ball, D.L., & Schilling, S.G. (2008). Unpacking pedagogical content knowledge: conceptualizing and measuring teachers' topic-spesific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.
- Hino, K. (2007). Toward the problem-center classroom: Trends in mathematical problem solving in Japan. ZDM Mathematics Education, 39(5-6), 503-514. http://doi.org/10.1007/S11858-007-0052-1.
- Kantowski, M.G. (1981). Problem solving. In: Fenemma, E. (Ed.) *Mathematics Education Research: Implication for the 80's.* Reston, VA: National Council for the Teaching of Mathematics (NCTM).
- Kawasaki, T. (2012). The problems of mathematical modelling introduction on mathematics education in Japanese school. *Journal of Mathematical Modelling and Application*, 1(5), 50-58.
- Kilbane, C.R., & Milman, N.B. (2014). *Teaching Models Designing Instruction For 21st Century Learners*. New York: Pearson.
- Knott, A. (2014). The Process of Mathematisation in Mathematical Modelling of Number Patterns in Secondary School Mathematics. Unpublish Dissertation. Stellenbosch: Stellenbosch University.
- Lingefjard, T. (2006). Faces of mathematical modeling. *Zentralblatt Fur Didaktik Der Mathematik*, 38(1), 96-112. https://doi.org/10.1007/BF02655884.
- Maaβ, J., O'meara, J., & Patrick, O. (2018). *Mathematical Modelling for Teacher: A Practical Guided to Applicable Mathematics Education*. Switzerland: Springer.
- Martinez-Luaces, V.E. (2005). Engaging secondary school and university teachers in modelling: Some experiences in South American countries. *International Journal of Mathematical Education in Science and Technology*, 36(2), 193–205. https://doi.org/10.1080/0020739051233134003.
- Mousoulides, N. (2007). The Modeling Perspective in the Teaching and Learning of Mathematical Problem Solving. *Unpublish Dissertation*. Cyprus: University of Cyprus.
- Muhtadi, D., Sukirwan, Warsito, & Prahmana, R.C.I. (2017). Sundanese ethnomathematics: mathematical activities in estimating, measuring, and making patterns. *Journal on Mathematics Education*, 8(2), 185-198. https://doi.org/10.22342/jme.8.2.4055.185-198.
- Mumu, J., Prahmana, R.C.I., & Tanujaya, B. (2018). Construction and reconstruction concept in mathematics instruction. *Journal of Physics: Conference Series*, 943(1), 012011. https://doi.org/10.1088/1742-6596/943/1/012011.
- Nuari, L.F., Prahmana, R.C.I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through Math GASING. *Journal on Mathematics Education*, 10(1), 127-142. https://doi.org/10.22342/jme.10.1.6913.127-142.
- Riyanto, B., Zulkardi, Putri, R.I.I., & Darmawijoyo. (2017). Mathematical modeling in realistic mathematics education. *Journal of Physics: Conference Series*, 943(1), 012049. https://doi.org/10.1088/1742-6596/943/1/012049.

- Riyanto, B., Zulkardi, Putri, R.I.I., & Darmawijoyo. (2018). Mathematical learning through modeling task in senior high school: Using nutrition context. *Journal Physics: Conference Series*, 1097(1), 012102. https://doi.org/10.1088/1742-6596/1097/1/012102.
- Riyanto, B., Zulkardi, Putri, R.I.I., & Darmawijoyo. (2019). Learning mathematics through modeling tasks in elementary school: Using growth of population context. *Journal of Physics: Conference Series*, 1166(1), 012033. https://doi.org/10.1088/1742-6596/1166/1/012033.
- Robinson, K. (1999). *Creativity, Culture and Education. Report.* London: National Advisory Commitee on Creative and Cultural Education.
- Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education*, 9(1), 41-54. https://doi.org/10.22342/jme.9.1.5049.41-54.
- Saxena, R., Shrivastava, K., & Bhardwaj, R. (2016). Teaching Mathematical Modeling in Mathematics Education. *Journal of Education and Practice*, 7(11), 34-44.
- Schoenfeld, A.H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In: Grouws, D. (Ed.) Handbook of Research on Mathematics Teaching and Learning. New York: Mcmillan.
- Stillman, G., & Brown, J.P. (2014). Evidence of implemented anticipation in mathematising by beginning modellers. *Mathematics Education Research Journal*, 26(4), 763-789. http://doi.org/10.1007/S1339-014-0119-6.
- Stillman, G., Brown, J., & Galbraith, P. (2010). Researching application and mathematical modelling in mathematics learning and teaching. *Mathematics Education Research Journal*, 22(2), 1-6. https://doi.org/10.1007/BF03217561.
- Sullivan, P., Knott, L., & Yang, Y. (2015). The relationships between tasks design, anticipated pedagogies, and student learning. In: Watson, A., Ohtani, M. Task Design in Mathematics Education: an ICMI Study 22. New York: Springer.
- Tessmer, M. (1993). Planning and Conducting Formative Evaluation. Philadelphia: Kogan Page.
- Wessels, H. (2014). Levels of mathematical creativity in model-eliciting activity. *Journal of Mathematical Modelling and Application*, 1(9), 22-40.
- Wong, K.Y. (2008). An extended Singapore mathematics curriculum framework. *Maths Buzz*, 9(1), 2-3.
- Yanagimoto, T. (2005). Teaching modelling as an alternative approach to school mathematics. *Teaching Mathematics and its Applications*, 24(1), 1-13. http://doi.org/10.1093/teamat/hrh011.
- Zulkardi & Putri, R.I.I. (2019). New school mathematics curricula, PISA, and PMRI in Indonesia. In: Vistro-Yu, C.P., & Toh, T.L. (eds.). School Mathematics Curricula, Mathematics Education – An Asian Perspective. Singapore: Springer.
- Zulkardi. (2002). Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers. *Unpublished Dissertation*. Enschede: University of Twente.
- Zulkardi. (2006). *Formative Evaluation: What, Why, When, and How*. Retrieved Juni 2019. from http://www.reocities.com/zulkardi/books.html.
- Zulkardi. (2017). Pembelajaran pemodelan matematika. *Seminar Nasional Matematika dan Pendidikan Matematika (SNMPM)*. Palembang: Universitas Sriwijaya.