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GEOMETRY REPRESENTATION TO DEVELOP ALGEBRAIC THINKING: A RECOMMENDATION FOR A PATTERN INVESTIGATION IN PRE-ALGEBRA CLASS

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Abstract

The present study is a part of design research in local instructional theory in a pre-algebraic lesson using the Realistic Mathematics Education (RME) approach. The article will focus on recommendations for the type of pre-algebra class that supports elementary school students' algebraic thinking. As design research study, it followed the three steps of preliminary studies, teaching experiment and retrospective analysis. The subject of the study is 32 fifth grade students of MIN 2 Palembang during the teaching experiment phase. The data were gathered from students' worksheets, lesson observation and interviews with the students. Data analysis was done using a constant comparative qualitative method. The results from the study indicate that pattern investigation in pre-algebra class that visualized geometrically supports the students to identify the form of the pattern and construct generalization.

Keywords: Pre-Algebra, Algebraic Thinking, Geometry Representation, Pattern Investigation, Realistic Mathematics Education

Abstrak

Penelitian ini adalah bagian dari *design research* untuk menghasilkan teori pembelajaran lokal di bidang pra-aljabar dengan menggunakan pendekatan Pendidikan Matematika Realistik Indonesia (PMRI). Artikel ini akan memfokuskan pada pembahasan tipe kelas pra-aljabar yang dapat membantu pengembangan kemampuan aljabar siswa sekolah dasar. Sebagai suatu penelitian desain, tahapan yang digunakan dalam penelitian ini adalah studi pendahuluan, uji coba lapangan, dan analisis retrospektif. Subjek penelitian ini adalah 32 orang siswa kelas V MIN 2 Palembang pada saat tahapan uji coba lapangan. Data dikumpulkan melalui lembar jawaban siswa pada lembar kegiatan, observasi dan wawancara selama kegiatan diskusi berlangsung. Data kemudian dianalisis secara kualitatif dengan menggunakan metode komparasi konstan. Dari hasil analisis, diketahui bahwa penelusuran pola pada kelas pra-aljabar yang direpresentasi secara geometri dapat membantu siswa untuk mengidentifikasi pola dan membuat generalisasi.

Kata kunci: Pra-aljabar, Berpikir Aljabar, Representasi Geometri, Penelusuran Pola, Pendidikan Matematika Realistik

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Back to the origin of algebra, Abu Ja'far Muhammad Ibnu Musa al-Khwarizmi defined algebra as "the whole discipline dealing with 'equations'"(Kvasz, 2006). It becomes one of the large mathematical topics which is taught in school. Drijvers, Goddijn, & Kindt (2011) emphasized the role of algebra to develop an algebraic thinking.

Algebraic thinking is the skill to focus on relations among the numbers (Kieran, 2004; Kieran, 2018; Venkat, et al. 2018; Widodo, et al. 2018). It involves the generalization of arithmetic and provides reasoning related to it, the development of mathematical models (mental and formal) in solving algebraic problems, formulate and visualize pattern and the construction of the algebraic language (Dekker & Dolk, 2011; Hendroanto, et al. 2018). It will be essential point for the students in all advance mathematical knowledge (Brawner, 2012). The lack of algebra may contribute to the difficulties in further study such as calculus (Müller, Cury, & de Lima, 2014) and other mathematical skills such as mathematical proof (Güler, 2016) and problem solving (Ferryansyah, Widyawati, & Rahayu, 2018). It is also can become obstacle for the students who pursue higher education and career that use mathematics, even though not mathematics department; such as engineering (Sazhin, 1998).

In spite of the significant role of algebraic thinking, it still be considered as an arduous skill in teaching and learning (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014; Capraro & Joffrion, 2006). Booth, Barbieri, Eyer, & Paré-Blagoev (2014) found that many students have misconception in this topic. The current problem in the teaching and learning algebra is the abstract introduction of algebra in which there is a gap between students' prior knowledge and the emerging algebraic symbols (Dekker & Dolk, 2011; Julius, Abdullah, & Suhairom, 2018). Recently, Wijaya, Retnawati, Setyaningrum, Aoyama, & Sugiman (2019) reported that the students' low understanding of algebra is one of the three most prominent obstacles for students in learning mathematics. The other two problems are still related to the mastery of algebra, which are lack on calculation and represent the problem into mathematical models. Therefore, we may take a look back on how algebra should be introduced for younger students.

Usiskin (1988) proposed four point of view to introduce algebra in school including the arithmetic generalization, strategies in solving problem, relationships between numbers and the structures of the pattern. Therefore, the classroom activities should promote the students' awareness towards patterns, relationships, problem solving abilities and the concept of making generalization. One recommendation to accommodate all of these needs is by having a pre-algebra class (Carraher, Schlieman, Brizuel, & Earnes, 2006).

Pre-algebra class is not aimed to bring the high school mathematics for group of students in their early age (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Indeed, its function is to familiarize the students with the structure (Ball, 2003) and is supposed to provide connections between arithmetic and algebra (Jacobs, et al. 2007).

Lannin (2005) recommend the investigation of the pattern as the center of early algebra activities since it provides "dynamical representation of variables" (p. 233). It can be used as an initial point in which the students will do investigation and identification towards the structure and relation between mathematical objects (Drijvers, Dekker, & Wijers, 2011).

Despite of many benefits of using patterns as initial activity to introduce algebra in earlier ages, its implementation encounters some difficulties. The most common problem is how to foster the students' movement from seeing the pattern as its unit to its generalization (Quinlan, 2001; Lannin, 2005) and how the students making connection between the ideas in mathematics (Kenedi, Helsa, Ariani, Zainil, & Hendri, 2019). Hence, in this study, the pattern investigation was supported by the visual representation embodied in geometrical objects to help the students seeing the structure and make generalization. The research question addressed in this paper is what is the function of the geometry representation to enhance the students' algebraic thinking?

METHOD

This is a design research study with three steps of preliminary study, teaching experiment and retrospective analysis (Bakker & van Eerde, 2015). The subject of the study is 32 fifth grade students of a state junior high school in Palembang who participated in the second cycle of teaching experiment phase on the study. The teacher is the homeroom teacher of the classroom who already teach for more than twenty years.

The researchers discuss the activity with the teacher before, during and after the implementation to adjust the lesson plan with the teacher's experiences based on the students' general condition. The discussion with the teacher also support the retrospective analysis phase in which the researchers classified the strategies employed by the students during the lesson.

The data were gathered from classroom observation during the learning process and students' written work and interview. During retrospective analysis phase, the data was analyzed qualitatively using constant comparative method by continuously testing the data with the conjectures and finding its counterexample.

The instrument of this study is a set of learning trajectory in pre-algebraic class using pattern investigation activities. It was developed under the design heuristic of Realistic Mathematics Education (RME). According to Gravemeijer & Bakker (2006), the design heuristic of RME are including the following items.

1. Guided Reinvention, which refers to the role of students who actively construct their own conceptual scheme.
2. Didactical Phenomenology, which refers to the context that encourage the students to do meaningful learning.
3. Emergent Modelling, which refers to the use of models to connect phenomenon and mathematical concept.

RESULTS AND DISCUSSION

The pre-algebra class using pattern investigation for elementary school students is divided into several topics, including: (1) constant pattern, (2) growing pattern with constant difference and (3) growing pattern with growing difference. This study will focus on the second topic which is growing pattern with constant difference.

The context of the lesson is a dance formation. It takes the form of *V* which is one type of growing pattern with constant difference. The discussion will be narrowed to the plan of doing a flash-mob, which means in every certain amount of time the number of dancers will be added without changing the *V* shape (Figure 1). The learning goals of the activity is to enable students in predicting the number of the dancers in every next formation and assessing the conjecture for generalization of the *V* pattern.

(Source: https://www.travelerbase.com/)

Figure 1. *V* Formation in Balinese Traditional Dance

Growing Pattern with Constant Difference

The number pattern used in this lesson is the odd number start from 3. Hence, the difference is constant, the next term will have two more than the previous one. Mathematically, this pattern can be written as 3, 5, 7, 9, etc. The students were asked to generate the general characteristics of the number that can be listed in the pattern. To begin with, the dance formation in the Figure 1, is elaborated more in the Figure 2, by symbolizing each dancer with a dot. A grid is given to help the students see the position of the dancer.

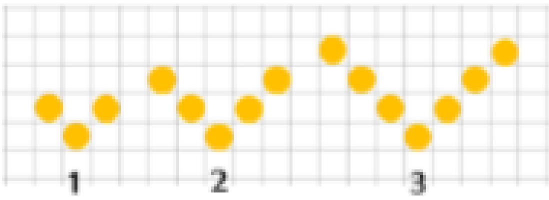


Figure 2. The first-three Number of Dancers of *V* Formation

The students' were working in pair to solve the following questions number 1 to 3; and in a group of four to solve the questions number 4 to 7. The last question was given during the posttest to check the students' transferability in seeing the structure of the number pattern.

1. Make a draw to represent the 4^{th} formation!
2. Calculate the number of dancers in the 6^{th} formation?
3. Make a draw to represent the 17^{th} dancers!
4. Calculate the number of pairs of dancers in the 45^{th} formation?
5. How you find the number of dancers in the 100^{th} formation!
6. Is it possible if a *V* formation has 92 dancers? Explain!
7. Is it possible to combine two *V* formation to get one *V* formation? Explain!

8. Figure 2 is the first-three of *V* formation, while the following Figure 3 is the first-three of *W* formation.

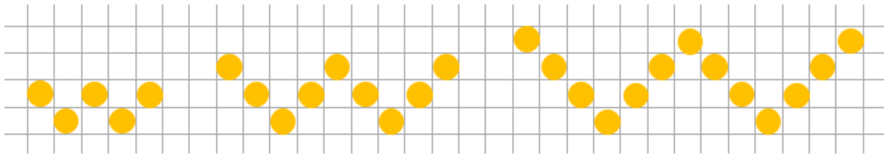


Figure 3. The first-three Number of Dancers of *W* Formation

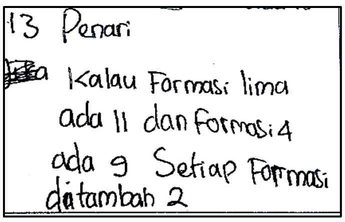
A student concluded that the *W* formation can be obtained from the combination of two *V* formations. Do you agree with the student? Explain your reason!

The strategies and obstacles encountered by the students in solving the pattern investigation problem and how the geometry representative of the pattern help them build the algebraic thinking is discussed in the following section.

Emerging Strategies

Adding Two Strategy

The first-three questions are likely to be solved using the adding two strategy. Most of the students were using "adding two strategy" which is adding two dots (represent dancers) to the previous formation). The following Figure 4 showed the example of the students' work to figure out the number of dancers performed in the 6^{th} formation.



Translation:
There are 13 dancers.
So, there are 11 dancers in the 5^{th} formation and the 9 dancers in the 4^{th} formation.
We add two for each formation.

Figure 4. Adding Two Strategies

Considering Rows

To find the number of dancers using the row consideration methods, the students develop the idea of multiplication by two or doubling. In the same time, they also become able to use the reverse operation to find the number of formations when the total number of dancers are given.

The following Fragment 1 showed students' discussion about the number of dancers in each sides and in the middle to solve the second question. From now and the rest of the paper, the use of initials in the Fragment will be as follows: S refers to student, R to researcher and T to teacher.

Fragment 1. Sides and Middle

[1] S1 & S2 : There are 6 dancers in the right and the other 6 in the left. There is one in the middle.

[2] R : How many dancers will be in each side of the seventh formation?

[4] S1 & S2 : Seven dancers.

[5] R : How about the dancers in the middle?

[6] S1 & S2 : One dancer.

To determine the number of dancers in the 17th formation, the students applied the same approach to solve the third question. However, in the middle of discussion one of the student got confused with the number represented in the *V* formation, especially the single person in the middle. See Fragment 2 to observe the discussion.

Fragment 2. How to Find It?

[1] R : Explain your strategy to find the number of dancers in each side of the formation.

[2] S2 : We can divide the total number of dancers by two and then subtract by one.

[3] Eh, divide it by two and plus one.

[4] Eh.

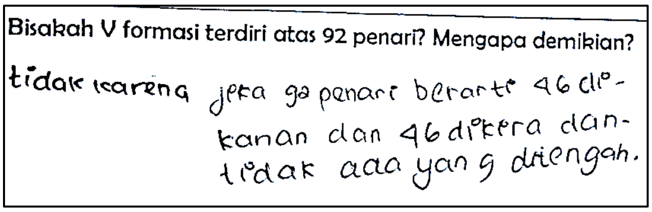
[5] R : What do you mean by divide it by two and then subtract or add by one?

[6] S1 : If we divide the total number of dancers by two, you will have one more dancer left.

[7] R : Where will the left dancer stand?

[8] S1 : The dancer will be in the middle of the formation.

The notion of "someone should be in the middle" play an important role of students' generalization on *V* formation. It can be seen in the example reason the student used to solve the 6th problem as shown in Figure 5.


Bisakah V formasi terdiri atas 92 penari? Mengapa demikian?

tidak karena jika 92 penari berarti 46 di-kanan dan 46 dikiri dan-tidak ada yang g ditengah.

Translation:
It cannot be. If 92 dancers, 46 dancers will be in the right and 46 dancers in the left. No dancer will left to be in the middle.

Figure 5. Someone should be in the Middle

Also, the students were able to state the fact that the addition of two odd numbers will always be an even number and not an odd (see Figure 6). In other words, it cannot perform a *V* formation. Hence they refused the hypothesis given in the 7th problem that two *V* formations can constructed a *V* formation.

Panitia kegiatan berencana untuk menggunakan dua formasi V menjadi sebuah formasi V, bisakah demikian? Mengapa? tidak bisa. karna jumlah dari 2 bilangan ganji adalah bilangan genap.	Translation: It cannot be. The addition of two odd numbers is an even number.

Figure 6. Two odd numbers cannot produce odd

The Function of the Geometry Representation

The students' strategy development in solving pattern investigation related problems showed that the geometry representation played a valuable role in students' algebraic thinking. We can distinguish three major roles of the geometry representation in the pre-algebra classroom.

1. **Context**

   The geometry representation is used as the context of the problem. Mathematically, the *V* formation has the general form of $n = 2m + 1$, where n and m represent the number of dancers and formation respectively. However, the formal algebraic expression is too abstract for the student. In the first prototype of the learning trajectory, we tried a lesson with problem in general form of $n = 2m + 3$. The problem was state descriptively using words, but not be represented in certain geometrical shape. From the observation, the students were hardly find the general relation between n and m and work exhaustedly to count three more from the previous term and so on.

   Differently, pattern investigation activities with geometrical representation helps the students to see the structure of the pattern in more realistic way. It is a good start to set the students' intention in exploring what aspect of the pattern remains constant and what changes. Later, the students can use the support of the geometry visualization to elaborate the change of the mathematical objects and construct the general idea. The similar finding also found by Rivera (2011) that visual representation helps students to establish personal inferences in seeing particular pattern.

2. **Model of and model for situation**

   Besides of the use of context, Realistic Mathematics Education (RME) also well-known for the use of vertical instrument that enable students to perform a guided-reinvention process and construct their own understanding. According to Treffers (1987) models are the most useful bridge to help students to shift from reality to mathematical objects and ideas. Therefore, a mathematical model should give a sense of visualization to the actual condition in phenomena.

   Based on Realistic Mathematics Education (RME) approach, there are two types of model, which are *model of* and *model for* situation. According to Gravemeijer & Doorman (1999), model of situation is used by the students to transfer the context into mathematical statement or object, meanwhile the *model for* situation is used by the students to work with the mathematical ideas.

   In this study, the geometry representation of the *V* dance formation become a bridge to translate the context of dance and the mathematical ideas of constant difference in a growing pattern. Figure 7 showed how the students use the geometry representation as a mathematical model.

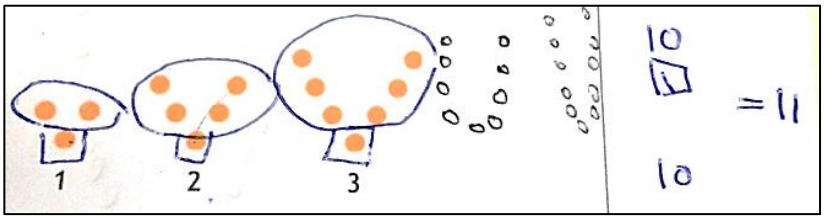


Figure 7. One is single, the rest with pairs

Previously, the pair of students whose written work showed on Figure 7, continue drawing the 4th and 5th formation. After realize that it is exhausted, they gave a mindful thought further and differentiate the formation into “has pair” and the “single” one. The students were using the representation given in the work sheet as a model to start their investigation by circling the “paired” part and squaring the “single” part. On the right side, they use the representation as a model for situation to write the number: the upper part is the number of dancer in pair and one left in the square.

This finding in line with the results of Kusumaningsih, Darhim, Herman, & Turmudi (2018) that emphasize the important of using multiple representation strategy with realistic approach to develop students’ algebraic thinking. The mathematical model is used to express the generalization and to ensure that the students grasp the number of pattern, not merely got the answer by accident or from trial and error process (Radford, 2006).

3. Scaffolding

The use of geometry representation not only beneficial for the students. The teacher can use it as crossed question to enhance students’ critical thinking in observing the pattern. Therefore, the teacher can provide a limited help for the students without directly give the answer and stop the students’ thinking. Consider the following Fragment 3 as the example of the use of geometry representation as the scaffolding.

Fragment 3. What Does The Number Represents: Total Dancers or Formation?

[1] R : What is the information provided in the problem?
[2] Are there 17 dancers or the dancers in the 17th formation?
[3] S4 : There are 17 dancers.
[4] S5 : So, the answer should be 35 dancers.
[5] R : Okay first, can you draw the picture of the formation which has 3 dancers?
[6]S6 : Yes, it will be the first formation.
[7]S3,4,5,6 : (Draw 3 dancers in the 1st V formation).
[8]S4 & S3 : One in the right, one in the left and one in the middle.
[9]R : Okay, so you have 3 dancers. How if you have 17 dancers?

[10]S4 & S5 : 35 dancers in total.

[11]R : Can you show it in picture?

[12]S3 : Eh.

The researcher asked them to draw another formation with 5 dancers and compared to the number of dancers they have in the 5th formation. Then, asked them to re-read the question. Finally, they realize that 17 is not the number of formation but the number of dancers that should be in the formation. This finding can be a meaningful support for the teacher in teaching algebra in the classroom especially to cope with the students' struggles in mathematizing word problems (Jupri & Drijvers, 2016; Salemeh & Etchells, 2016).

4. Students' Mathematical Reasoning and Proof

The students also use the geometry representation to express their mathematical ideas. The example can be seen in the Figure 8 in which the students were working to solve the 8th problem: does two *V* formation can form a *W* formation.

Translation:
I don't think so, because there will be one left [dancer].

Figure 8. Two *V*s cannot be a *W* Formation

There also a student who said that it is possible for two *V* formation to be a *W* formation, but he drew it differently in which he has to remove a dancer. During the interview, he used illustration to support his argument as can be observed in the following Fragment 4.

Fragment 4. Take One

[1] S1 : So, *W* formation is actually the combination of 2 *V* formation.

[2] R : Can you show me how it can be combined?

[3] (*S1 showed his worksheet*)

[4] R : Do you find any strange in the picture?

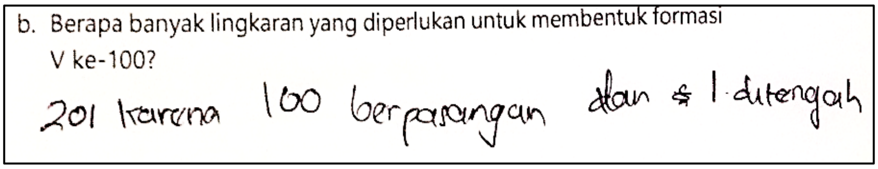
[5] S1 : Yes, the one in the middle (*pointed to the picture*)

[6] R : What do you think about the person in the middle?

[7] Do you have any specific rule to create a *W* from 2 *V* formations?

[8] S1 : Indeed. You have to remove one dancer.

The similar why of thinking also showed in his idea for the next question to determine the total dancers in the 100th formation of *V*. He mentioned the notion of "dancers in pairs and one in the middle" structure and explained as in Figure 9.

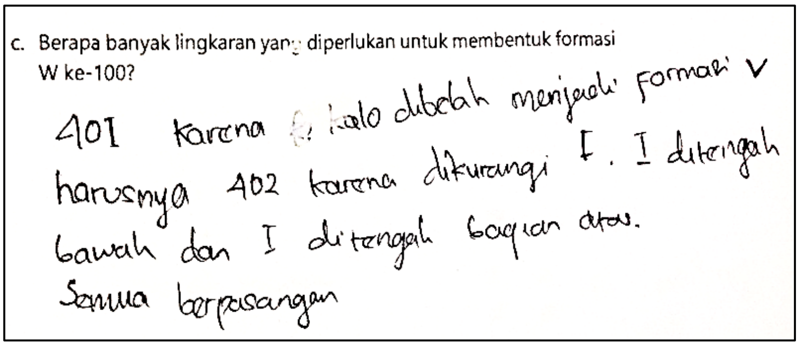

b. Berapa banyak lingkaran yang diperlukan untuk membentuk formasi V ke-100?

201 karena 100 berpasangan dan 1 ditengah

Translation:
201 [dancers], 100 [dancers] have pairs and 1 in the middle

Figure 9. The 100th *V* formation structure

To determine the number of dancer in the 100th W formation, he connected the idea of the *V* and *W* structures. He employed his conclusion that the combination of 2 *V* formations can become a *W* formation when he removes one dancer. Since in the Figure 9 the dancers needed for 100th formation of *V* are 201 then he said it should be doubled and subtract by one. The Figure 10 explained his argument.


c. Berapa banyak lingkaran yang diperlukan untuk membentuk formasi W ke-100?

401 karena kalo dibelah menjadi formasi V harusnya 402 karena dikurangi 1. 1 ditengah bawah dan 1 ditengah bagian atas. Semua berpasangan

Translation:
401. There are 402 [dancers] in 2 V formations, but you remove 1.

Figure 10. The 100th W formation structure

The aforementioned functions of geometry representation enables the students to make connections between the problems, mathematical models, problem solving strategies and see the structure of the pattern algebraically. It provides a meaningful support for the students to work with mathematical objects which will be beneficial for the development of algebraic thinking especially the ability to generalize and reason within algebraic structure (Dekker & Dolk, 2011).

CONCLUSION

A geometry representation is an important support in students' movement from recursive calculations to expressing general formulas. In other words, it helps the students' transition from arithmetic to algebraic thinking. For practice in mathematics classroom, it is recommended for the teachers to conduct a pre-algebra lesson using number investigation embodied in geometry representation. The geometry representation provides an opportunity for the students to see the whole picture of the series of numbers by considering the characteristics of the geometrical object given. Hence, the students will not get lost too early. Also, it creates a challenge for the students to provide a general proof for the figure they started with. Therefore, the existence of the geometry representation of the number pattern is used as a context to start, a model to work with, a scaffolding to additional support and as a reasoning tool to express their mathematical ideas.

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