

Instructional process for the construction of the derivative function: Modelling and simulation in GeoGebra from the Ontosemiotic Approach

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Abstract

The conceptual and procedural understanding of the derivative remains a persistent challenge in undergraduate engineering education, particularly in bridging symbolic, graphical, and applied interpretations. Despite advances in digital tools, few instructional designs systematically integrate interactive exploration with formal mathematical reasoning. This study addresses this gap by proposing an instructional framework that combines functional modeling with GeoGebra-based simulations, grounded in the Ontosemiotic Approach to Mathematical Knowledge and Instruction (OSA), specifically targeting first-year engineering students in Chile. A mixed exploratory–descriptive design was implemented, combining quantitative and qualitative analyses. A three-session intervention involved 102 students who engaged in tasks assessing derivative understanding across multiple representations, including graphical slopes, symbolic differentiation, and applied rate-of-change problems. Data were collected via performance questionnaires and written productions, with validity ensured through expert review and reliability confirmed via pilot testing. Students exhibited strong proficiency in graphical interpretation and procedural manipulation of derivatives, with success rates exceeding 85%. Conversely, tasks requiring formal argumentation, rigorous use of limit definitions, and theoretical justification showed reduced performance at 68%, highlighting the challenge of connecting exploratory simulations with formal mathematical reasoning. The findings demonstrate that integrating functional dependency analysis, interactive simulations, and OSA principles can strengthen comprehension of derivatives, particularly in geometric interpretations and formal rate-of-change reasoning. This research provides a replicable instructional design that enhances both conceptual insight and procedural competence, offering evidence-based strategies for technology-enhanced mathematics instruction in engineering curricula and contributing to broader curriculum development.

Keywords: Derivative, Instructional Process, Interactive Simulation, Mathematical Modelling, Ontosemiotic Approach

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The teaching of calculus remains a persistent challenge in engineering programs worldwide, with high failure rates, limited conceptual understanding, and an overreliance on procedural algebraic techniques being well-documented (Galindo Illanes et al., 2022). In many instructional contexts, teaching continues to follow a transmissive model, privileging the unidirectional delivery of content and providing limited opportunities to connect mathematical concepts with students' prior experiences or professional practice scenarios. The integration of Learning and Knowledge Technologies (LKT) is therefore particularly

relevant, as these tools enable the exploration of mathematical objects across multiple semiotic registers—graphical, symbolic, tabular, and geometric—facilitating representational transitions and supporting deeper conceptual learning (López & Hernández, 2016; Aguilera et al., 2017). Beyond their instrumental value, LKTs serve as strategic didactic resources: they create opportunities to rethink not only how calculus is taught, but also why it is taught, positioning instructors as active mediators between mathematical meanings and professional applications, while simultaneously generating empirical evidence on how students construct such meanings.

The construction of the derivative concept requires the coordinated articulation of several interconnected mathematical objects—including slope, rate of change, tangent, limit, and continuity—across diverse representational registers. This integration constitutes a significant cognitive demand for students and reflects the complexity of derivative-related tasks (Galindo Illanes & Breda, 2023). However, dominant pedagogical practices often privilege algebraic-symbolic manipulation, leaving graphical and conceptual interpretations underemphasized, despite their importance for a comprehensive understanding of the derivative (Galindo Illanes & Breda, 2024; Galindo Illanes et al., 2025). The literature thus reveals a persistent gap: although many studies have documented students' difficulties with derivative concepts, far fewer have developed, implemented, and systematically evaluated replicable instructional processes that combine mathematical modeling, digital simulations, and robust theoretical frameworks such as the Ontosemiotic Approach to Mathematical Knowledge and Instruction (OSA).

The present study is framed within the Ontosemiotic Approach (Godino et al., 2007; 2019; Godino, 2024), which provides a powerful lens for examining the diversity of mathematical meanings associated with objects such as the derivative and for identifying the didactic configurations that promote their emergence in instructional settings. In particular, tasks involving tangents are treated as pivotal contexts for introducing the derivative, as they allow for a coordinated use of geometric, visual, and symbolic representations.

Accordingly, the main objective of this study is to analyze the impact of an instructional process—designed under the OSA, grounded in functional modeling, and supported by interactive GeoGebra simulations—on engineering students' construction of the derivative concept. Specifically, the study addresses two research questions: (1) How does this process foster the construction of meanings of the derivative, particularly its geometric interpretation as a slope or rate of change? and (2) What strengths and difficulties do students display when engaging with tasks centered on the tangent problem field? The instructional design aims to promote the progressive construction of the derivative concept by encouraging students to discover mathematical patterns, articulate multiple representations, and connect empirical exploration with formal definitions.

This research makes a scientific contribution by going beyond the mere description of a teaching experience: it proposes, implements, and rigorously evaluates a replicable instructional design capable of addressing well-documented challenges in calculus education. Its relevance extends internationally, as difficulties in teaching and learning the derivative are not restricted to the Chilean context but are reported globally. The novelty of this study lies in its explicit integration of the OSA, mathematical modeling, and interactive GeoGebra simulations within a coherent and replicable design—an approach that remains underexplored in the literature.

Finally, four conceptual and methodological pillars provide coherence and innovation to the study: (1) the Ontosemiotic Approach as a theoretical and analytical framework, (2) the integration of Learning and Knowledge Technologies as mediators of representational transitions, (3) the use of mathematical modeling as a core instructional strategy, and (4) the incorporation of interactive simulations as a means of supporting exploration and meaning-making. These elements are grounded in prior empirical work demonstrating their

effectiveness in calculus instruction, and together they offer a comprehensive and theoretically informed proposal for re-signifying the teaching and learning of differential calculus in engineering education.

The Ontosemiotic Approach to Mathematical Knowledge and Instruction

This study is framed within the Ontosemiotic Approach to Mathematical Knowledge and Instruction (OSA), which offers a robust conceptual and methodological foundation for analyzing and designing teaching–learning processes in differential calculus, particularly within professional training contexts. This perspective is especially relevant for engineering programs, as it facilitates the articulation of mathematical meanings with professional practices, promoting a deeper, more situated, and functionally meaningful comprehension of calculus concepts.

In what follows, the three axes that structure the theoretical foundations of this study are developed: the fundamental principles of the OSA, its contributions to didactical design and analysis, and its specific implications for the teaching of differential calculus.

Fundaments of the OSA

Developed by Godino et al. (2020b; 2024), the OSA interprets mathematical knowledge as a socially constructed system mediated by norms, tools, and contexts. It integrates ontological dimensions (the nature of mathematical objects and practices) with semiotic dimensions (the representation, communication, and validation of knowledge) (Godino, 2024). Its integrative character establishes a dialogue with semiotics, constructivism, and activity theory, emphasizing that mathematical objects are polysemic entities whose meanings vary across contexts and purposes (Godino et al., 2024; Godino, 2022).

A central construct of the approach is the epistemic configuration, which encompasses the practices, representations, and normative systems associated with a particular mathematical object. This construct provides a lens for analyzing what is taught, how it is taught, and for what instructional purpose (Godino, 2016; Godino et al., 2020a). The OSA adopts a critical and situated stance: rather than offering prescriptive formulas, it promotes context-sensitive understandings, an especially relevant perspective for engineering education where mathematical knowledge must be connected to professional practices (Godino, 2013; Godino et al., 2020b).

Methodologically, the OSA contributes categories such as learning trajectories, semiotic conflicts, and didactical suitability that support both research and instructional decision-making (Godino et al., 2021). In this study, the OSA is used not only as a theoretical lens but also as a methodological guide for examining how students and teachers construct meanings around the derivative.

Design and Didactic Analysis in the OSA

The OSA informs didactical design by providing a framework for planning and evaluating instruction while accounting for multiple dimensions of learning. It goes beyond descriptive analysis, guiding evidence-based interventions concerning content selection, instructional strategies, and assessment practices (Godino, 2024). From this perspective, instructional design involves identifying the meanings to be constructed, the practices to be activated, and the resources that will mediate learning, articulating epistemic, cognitive, affective, interactional, and mediational dimensions (Godino et al., 2007).

Previous research has demonstrated how OSA-based analyses allow the reconstruction of learning trajectories and the identification of semiotic conflicts and conceptual difficulties, highlighting the role of technology in meaning construction (Gusmão & Font, 2022; Godino et al., 2020b). Furthermore, Ontosemiotic reference models have been successfully developed for topics such as proportionality, supporting curriculum design and instructional planning (Burgos & Godino, 2020; Burgos et al., 2020).

The OSA thus assumes an integrative stance between objectivist and constructivist views, positioning the teacher as an active mediator who supports students' progressive construction of mathematical meanings (Godino & Burgos, 2020; Godino et al., 2019). In this article, these conceptual tools are applied to analyze the teaching of the derivative in engineering programs, identifying discrepancies between intended and constructed meanings and informing the design of a contextualized instructional sequence.

Didactics of Calculus with the OSA

The OSA has been extensively used to study fundamental concepts of calculus—such as limit, function, derivative, and integral—emphasizing the multiplicity of meanings that emerge around these objects. For example, Burgos et al. (2021) examined the semiotic complexity of the definite integral, Verón and Giacomone (2021) investigated the meanings of the differential, and Araya et al. (2021) proposed criteria for designing limit-related tasks. Other contributions include analyses of algebraic objects (Sepúlveda-Delgado et al., 2021) and studies of the function within the Chilean curriculum (Pino-Fan et al., 2019).

In the domain of differential calculus for engineering, recent research has emphasized the integration of LKTs, simulations, and contextualized modeling (Galindo Illanes et al., 2022; Galindo Illanes & Breda, 2023; 2024; Galindo Illanes et al., 2023; 2025). These studies range from curriculum reviews to detailed characterizations of problem subfields related to the derivative, underscoring the need to connect mathematical content with authentic professional situations.

In summary, the OSA provides a coherent theoretical–methodological framework for analyzing, designing, and transforming calculus instruction from a critical, situated perspective. Its constructs—Ontosemiotic configurations, didactical suitability criteria, and teachers' didactical–mathematical knowledge—serve as guiding principles in the development of the instructional process proposed in this study, ensuring theoretical coherence and practical relevance in the teaching of derivatives in engineering contexts.

Learning and Knowledge Technologies as Semiotic Mediators

In contemporary higher education—particularly in mathematics teaching and initial teacher training—LKT have assumed a central role in pedagogical innovation. Beyond their instrumental function, these technologies are now conceptualized as semiotic mediators that enable new forms of mathematical representation, exploration, and meaning construction. This perspective is especially pertinent in differential calculus, where the high level of abstraction frequently poses obstacles to students' conceptual understanding.

Research has consistently shown that a deliberate and reflective integration of LKT can transform teaching practices in ways that make learning more dynamic, contextualized, and student-centered. Alarcón et al. (2019) highlight that the strategic use of these tools reconfigures the teacher's role from transmitter of knowledge to facilitator of meaningful processes. Similarly, Valarezo and Santos (2019) emphasize that early exposure to LKT in teacher preparation contributes to the development of pedagogical practices that are responsive to evolving educational contexts. Illescas et al. (2024) stress that the true potential of these technologies lies not in the tools themselves, but in instructors' ability to transform them into powerful learning environments.

Recent developments such as artificial intelligence (AI) and augmented reality (AR) exemplify the evolving potential of LKT. For instance, Torres-Peña et al. (2024) report improvements in students' problem-solving accuracy and motivation when virtual assistants such as ChatGPT and Wolfram Alpha were incorporated into calculus instruction. Similarly, Rahman et al. (2024) developed an AR-based application for teaching solids of revolution, observing gains in academic performance and collaborative engagement. These findings underscore that, when implemented within a coherent didactic plan, LKT

enriches differential calculus instruction by promoting visual, interactive, and professionally relevant approaches that support the development of mathematical meaning.

Mathematical Modeling as a Pedagogical Strategy

In recent years, mathematical modeling has gained prominence as a powerful pedagogical strategy for connecting calculus concepts with situations drawn from professional engineering practice. Rather than being reduced to the routine application of formulas, modeling invites students to mobilize mathematical knowledge in authentic contexts, requiring them to interpret, represent, and solve real-world problems (Ledezma, 2024; Ledezma et al., 2023). In this way, calculus instruction transcends rote algorithmic practice and becomes an avenue for promoting critical thinking and bridging the gap between theory and application.

Scholars such as Kaiser et al. (2015) have emphasized the potential of modeling to foster general competencies, including analytical reasoning, intellectual autonomy, and a structured understanding of mathematical concepts. These claims are corroborated by Spooner (2023), who examined instructional experiences involving modeling projects in differential equations courses, reporting gains in both conceptual comprehension and student motivation. Similarly, Rezaei and Asghary (2024) compared traditional instruction with a modeling-based approach and found that students engaged in modeling not only achieved higher academic performance but also developed stronger competencies for addressing practical problems.

Other studies underscore the motivational and creative dimensions of modeling. Winkel (2023) argues that presenting students with carefully designed real-world challenges facilitates the appropriation of complex mathematical ideas, such as those involved in differential equations. Weinhandl and Lavicza (2021) further highlight that modeling encourages mathematical creativity, particularly when paired with digital technologies and collaborative learning, allowing students to generate original and flexible mathematical solutions that extend beyond purely technical applications.

In sum, mathematical modeling is not merely a didactic technique but a distinctive paradigm for teaching calculus. It seeks to prepare students to use mathematical knowledge as a tool for understanding and transforming their environment. Within engineering education, this approach is particularly relevant, as it enables the design of instructional experiences that connect classroom mathematics with the demands, constraints, and complexity of professional practice.

Interactive Simulation as a Didactic Resource

Interactive simulations have emerged as a central strategy for rendering abstract concepts in differential calculus more tangible and accessible. Dynamic software such as GeoGebra allows students to manipulate mathematical objects in real time—exploring limits, slopes, and derivatives—thus fostering hypothesis testing, conjecture formation, and the development of metacognitive strategies (Colquepisco-Paucar, 2019; Del Río, 2020; Guilcapi et al., 2019). Empirical evidence confirms their effectiveness: Kado (2021) observed both improved performance and more positive attitudes among students using GeoGebra compared to traditional approaches, while Zagoto et al. (2025) reported increased participation and conceptual understanding linked to its visual and interactive affordances.

Structured implementations have also been positively evaluated. Bedada and Machaba (2022) found that more than 70% of students valued a teaching cycle incorporating GeoGebra, citing increased motivation and comprehension of complex content. The use of such tools also promotes a shift in teachers' roles, positioning them as facilitators of active knowledge exploration (Mora-Casasola, 2023). Nevertheless, sustainable implementation requires adequate teacher training, infrastructure, and institutional support (Ardina et al., 2025; Breda et al., 2021).

Finally, in this study, interactive simulation is integrated with the OSA, LKT, and mathematical modeling to form a coherent instructional design. This integration is intended to enhance students' conceptual understanding of the derivative, support the transition between semiotic registers, and promote a deeper engagement with mathematical meaning-making processes.

METHODS

This study employed a mixed exploratory–descriptive design (Creswell & Plano Clark, 2018), combining quantitative and qualitative techniques to capture both the frequency of correct responses and the meanings underlying students' reasoning. The design was particularly suited to exploring how students construct and articulate mathematical knowledge when engaging with modeling tasks and interactive simulations.

Study Context and Participants

The research was conducted at a Chilean university with a full cohort of 102 first-year Industrial Civil Engineering students enrolled in a calculus course. The sample was selected by convenience, corresponding to the class assigned to the principal investigator. Participants were between 18 and 19 years old; 62% identified as male and 38% as female. None had previously used GeoGebra in formal settings, although 15% reported informal familiarity. To ensure reliability, two researchers independently coded students' responses using categories from the OSA, achieving 85% inter-rater agreement. Discrepancies were resolved through discussion and consensus.

Study Phases

The research was structured into four phases summarized in [Table 1](#).

Table 1. Study phases and description of activities

Phases	Description
Phase 1. Instructional design	Construction of a teaching sequence based on modeling and GeoGebra simulations, grounded in the Ontosemiotic Approach. Definition of tasks, development of instruments, and validation through expert judgment.
Phase 2. Implementation	Application of the sequence in three consecutive sessions with a first-year Industrial Civil Engineering class ($n = 102$).
Phase 3. Data collection	Administration of performance questionnaires, collection of written productions, and classroom observations.
Phase 4. Data analysis	Descriptive processing of frequencies and qualitative categorization of responses, articulating both approaches to identify strengths and difficulties.

Instructional Design and Didactic Proposal

The instructional intervention was designed around three core elements that guided the development of students' understanding of the derivative function.

1. Field of the problem: The proposal focused on the problem field of tangents (PT), as characterized by Galindo Illanes et al. (2025), to promote the conceptual construction of the derivative through tasks involving secant and tangent lines.
2. Epistemic configurations: The design incorporated three epistemic configurations—manipulative,



- computational, and algebraic—previously validated in Galindo Illanes and Breda (2024), to ensure a progressive transition from exploratory manipulation to formal symbolic representation.
3. Synchronous work: The learning trajectory was implemented during regular face-to-face class sessions, emphasizing collaborative interaction and real-time feedback to consolidate students’ conceptual development.

Development of the Teaching

The study plan of the calculus subject is of a total duration of 18 weeks. Each week is structured in four chronological hours dedicated to theoretical classes, called lectures, and two additional hours assigned to practical and collaborative activities in a workshop modality.

The teaching of the derivative function was organized into one week-long unit, distributed in 3 synchronic sessions of 120 minutes each. These sessions were carried out in the habitual schedule assigned to lectures and workshops. In the planning of this unit, problems about tangents (PT) were incorporated, which served as a basis to deal with the process of the construction of the derivative function. The distribution of these six sessions is detailed in Table 2.

Table 2. Temporalization and task planning

Session	Didactic Action	Objective	Representations	Epistemic Configurations
1 (Lecture)	Activity 1 Task 1	Introduce the derivative via its geometric interpretation using secant lines.	Graphic, tabular, descriptive	Computational
2 (Workshop)	Activity 2 Task 2	Connect the derivative at a point with the derivative function.	Tabular, graphic, symbolic, descriptive	Computational, algebraic
3 (Lecture)	Activity 3 Task 3	Generalize the derivative function and explore contextual applications (rates of change, optimization).	Graphic, symbolic, descriptive	Computational, algebraic, manipulative

The purpose of session 1, with a duration of 120 minutes, was for students to explore the notion of the derivative in one point starting the approximation through secant lines. For this, tabular activities were developed oriented towards the calculation of slopes of secant lines close to a point of interest in the analyzed function. The session concluded with an interactive simulation in GeoGebra as shown in Figure 1, from which the conclusion was drawn that the derivative in a point corresponds to the slope of the tangent line at that point.

Activity 1 (Session 1): Exploring the concept of the derivative through the secant line.

Instructions:

Use your telephone to scan the following QR code or access the following link <https://www.geogebra.org/m/pPAJ6Zf8>



1. Observe the graph of a function f together with two points P and Q that belong to the curve.
2. Utilize the slider “a” to displace point P and bring it progressively closer to point Q .
3. Pay attention to how the slope of the secant line that joins the points P and Q varies.
4. Note your observations and answer the following questions:

Guide questions:

- (a) How does the slope of the secant line change when point P moves close to point Q ?
- (b) How can you interpret the limit value of this slope?
- (c) What happens with the secant line when P coincides with Q ? What name would you give this line?

Figure 1. Image of Activity 1: Exploring the concept of the derivative through the secant line

Figure 2 depicts the graph of a function f together with a point Q on the curve and its corresponding tangent line L_T . This visual representation is intended to support students' conceptual transition from the secant line, examined in the previous session, to the formal notion of the tangent line as the limiting position of secants when the interval between two points tends to zero. The tangent line L_T is shown as touching the curve at Q and providing the best local linear approximation of f at that point, without intersecting the curve in its immediate neighborhood. This activity is designed to prompt students to identify the tangent line as a unique mathematical object associated with the function at Q , to interpret its slope as the instantaneous rate of change of f , and to articulate the relationship between the curve and its tangent as a predictor of the function's local behavior. By engaging in observation and guided discussion, students consolidate their geometric intuition of tangency, laying a foundation for the symbolic computation of derivatives and the generalization to the derivative function that follows in subsequent sessions.

Activity 2 (Session 2): Analysis of the tangent line of a curve

Observe the following image, which presents a graph of a function f at a point Q belonging to that curve, and the tangent line L_T at f at the point Q .

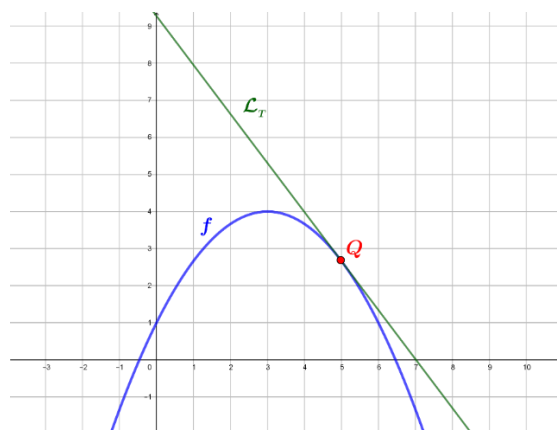


Figure 2. Image of Activity 2: Analysis of the tangent line of a curve

The objective of the third session, with a duration of 120 minutes, was to facilitate the transition from the derivative at one point to the construction of the derivative function. Given that in the previous sessions the students had dealt with the calculation of slopes of secant lines and had explored the geometric meaning of the derivative at one point, it was expected that they would be in condition to deduce the symbolic expression of $f'(x)$ without greater difficulties (Galindo Illanes et al., 2022). For this, activities were proposed in which, starting from the known function $f(x)$, the function that represents the slopes of the tangent lines was built, i.e., the derivative function $f'(x)$ (Font, 2005). This process was developed through the use of an interactive simulation with GeoGebra illustrated in Figure 3. The session finalized with a first approximation to the applications of the derivative, showing examples of contextualized problems like the determination of rates of change, or the optimization of functions, with

the aim of showing the usefulness of the derivative function in the resolution of real problems from the area of engineering.

Activity 3 (Session 3): Exploring the derivative function through interactive simulation.

Instructions:

Utilize your mobile device to scan the following QR code



Or access the applet directly through the link: <https://www.geogebra.org/m/x9wkczt>

Once inside GeoGebra:

- Slide the point Q along the blue curve. Observe how a new graph is simultaneously generated in the lower part.
- Analyze how the slope of the tangent line at point Q varies and how this value represents the new curve being constructed in parallel.
- In your own words, explain the meaning of the derivative function. What does each point in the resulting curve represent? How is it related to the original function?

Figure 3. Image of Activity 3: Exploring the derivative function through interactive simulation

Data Collection Instruments

To analyze student learning, a performance questionnaire was designed consisting of five contextualized tasks. Content validity was reviewed by three experts in mathematics education, who verified the relevance of the statements and their coherence with the study objectives. Internal reliability was tested through a pilot application with 18 students from a parallel course, yielding a Cronbach's alpha coefficient of 0.81, which is considered adequate for exploratory purposes.

In addition, informed consent was obtained from all participants, and approval was granted by the institutional ethics committee, ensuring confidentiality and the academic use of the data. For reasons of space, this section includes only two representative tasks of that instrument. The instrument applied in the first week allowed us to collect information about the specialized knowledge of the student with respect to the concept of the tangent, as it considered the transit from an intuitive or implicit use of its properties towards a conscious and grounded application. In this process, it is expected that the student will be able to explain the conceptual scheme of the tangent line and articulate it with the geometric interpretation of the derivative in one point, in such a way as to be able to adequately argue their answer to the problem.

In this activity, students were presented with the resolution of a contextualized problem as shown in [Figure 4](#) that required the use of the derivative as the main tool in the analysis. To begin, it was expected that the students would obtain the derivative function, be it through the exploration of an interactive applet of GeoGebra or using the formal definition of the derivative from the concept of the limit. The derivative function they should obtain is the following: $f'(x) = 0.3x^2 - 1.8x + 2$. Additionally, they had to interpret that the derivative $f'(x)$ represents the instantaneous slope of the ramp in each point x of the defined domain.

That is to say, it measures how much the height of the ramp changes for each meter it advances horizontally. In a civil engineering context, this slope is crucial for evaluating the inclination of a piece of land or structure, for determining risk zones where the slope is very pronounced (for example, greater than 15%), and for designing technical solutions, such as automatic braking systems, signage, and other safety measures.

Task 2: Designing a ramp with a controlled slope

Context:

In an industrial plant an access ramp is being designed to transport materials via automatized carts. The height of the ramp in function of the horizontal distance covered is given by the function:

$$f(x) = 0.1x^3 - 0.9x^2 + 2x, \text{ con } 1.5 \leq x \leq 4$$

where x represents the horizontal distance (in meters) and $f(x)$ the height of the ramp (in meters).

Objective:

To determine the exact slope of the ramp at a critical design point and obtain the equation of the tangent line, which will permit the installation of safety sensors in zones of maximum inclination.

Instructions

- Calculate the derivative of the function $f(x)$. Interpret its meaning in this context.
- Determine $f'(2)$. What interpretation does this value have?
- Find the equation of the line tangent to the curve $f(x)$ at the point where $x = 2$.
- Represent graphically in GeoGebra or Desmos the function $f(x)$, the point $P(2, f(2))$, and the tangent line found.
- Reflect: In some contexts of road or industrial engineering, a slope is considered a risk if it exceeds 10%, and, in many cases, it is recommended to implement automatic braking systems if the slope is greater than 15%.

Based on the value for the slope you obtained in point $P(2, f(2))$, do you consider it to be necessary to incorporate an additional safety system in this section of the ramp? Justify your answer considering the risks an elevated slope implies in terms of load or structural safety.

Figure 4. Image of Task 2: Designing a ramp with controlled slope

In the following stage, the students had to interpret that the slope of the function at a point indicates how fast and in which direction the height of the ramp changes with respect to the horizontal distance. Then, the students had to evaluate the slope of the ramp at $x = 2$ that corresponds to the value of the derivative of the function at that point, as previously calculated in the activity: $f'(2) = 0.3(2)^2 - 1.8(2) + 2 = -0.4$, which means that at point $x = 2$, the ramp is descending and the negative slope indicates a fall, and the value -0.4 can be interpreted as a decrease of 0.4 meters in height for each meter that it advances horizontally. Finally, the students had to calculate the ordinate of the point whose abscissa is $x = 2$, for that they evaluate $f(2) = 0.1(2)^3 - 0.9(2)^2 + 2(2) = 1.2$, in order to obtain the coordinates of the point at which the tangent line would be drawn.

In the next stage the students used the form point-slope to obtain the equation of the tangent:

$$y - f(2) = f'(2)(x - 2).$$

Thus,

$$y - 1.2 = -0.4(x - 2)$$

Finally, the equation of the line tangent to the curve in $x = 2$ is:

$$y = -0.4x + 2$$

The graph of the situation is now presented in [Figure 5](#). The task finalized with a reflection in relation to the context of the problem, remembering that in some problems in industrial or road engineering, a slope is considered risky if greater than 10%, and, in many cases, it is recommended to implement automatic braking systems if the slope is greater than 15%.

Based on the value of the slope that you obtained in the point $P(2, f(2))$, do you consider that it would be necessary to incorporate additional safety systems in this stretch of the ramp? Justify your answer considering the risks a steep slope implies in terms of load or structural safety.

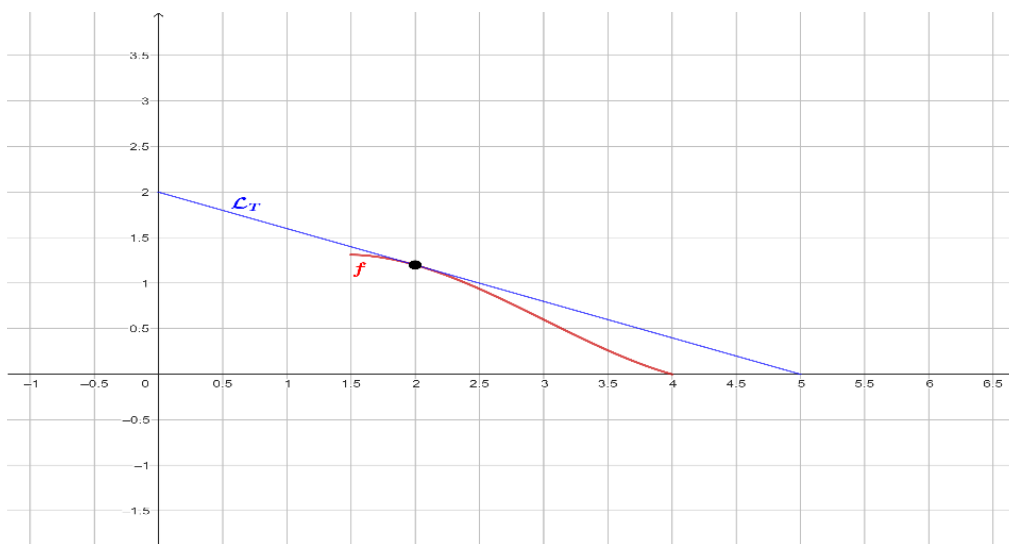


Figure 5. The situation graph of the purposed problem

In next task presented in [Figure 6](#), the students were given a problem of optimization that required the application of the concept of the derivative, using pieces of cardboard of different dimensions as a base material. The objective was to construct a box without a lid from a rectangular piece of cardboard in such a way as to make its volume maximum. For this, it was expected that the students would formulate and resolve the problem following a series of linked steps that reflect the complete mathematical process.

Initially, the student must identify and define the objective function, which, in this case, corresponded to the volume of the box in function an independent variable x , that represented the measurement of the squares that are cut out in the corners of the cardboard to make the box. From this general formulation, they had to substitute the specific dimensions of the available material—for example, if the cardboard measures 30cm wide by 20cm long, the function is expressed as $V(x) = (30 - 2x)(20 - 2x)x$, which would permit them to establish the concrete function that will be optimized.

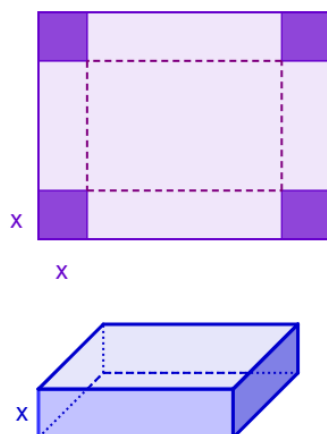
Once the function is defined, the student should develop it through algebraic operations, expanding the products to obtain an explicit expression of $V(x)$. In the following, the first derivative of this function was calculated, which should be done using the GeoGebra applet seen in class and should obtain $V'(x)$,

with the aim of finding the critical values, i.e., those values of x for which the derivative is zero. The resolution of the equation $V'(x) = 0$ provides possible candidates for the maximum and minimum volume. These values must be evaluated in the original function to determine the critical points.

Task 3: Construction of a box without a lid optimized in its volume

Context:

Given a rectangular piece of cardboard of known dimensions (for example, 30 cm $\times$ 20 cm), you need to construct a box without a lid, cutting equal squares in each corner and folding the sides. How big does the cut square need to be for the volume of the box to be maximum?



Objective:

To apply derivatives to resolving problems of optimization in geometric contexts.

Instructions

(a) Define the variable and express the Volume in function of that variable as, for example:

$$V(x) = (\text{length} - 2x)(\text{width} - 2x)x$$

(b) Determine the critical values.

(c) Determine the relative extremes using the criterion of the first derivative.

Reflect:

In the context of the packaging industry and considering that the dimensions of products vary, what other variables do you consider necessary to optimize and protect the product during packaging?

Figure 6. Image of Task 3: Construction of a box without a lid with optimal volume

To establish which of these points provides the maximum volume of the box, the criterion of the first derivative was applied: the sign of $V'(x)$ was analyzed in intervals around each critical value. If the derivative were to change from a positive sign to a negative one, the conclusion would be that a local maximum was reached, which permits the justification that the volume found is effectively the greatest possible under the conditions given.

Data Analysis

The analysis followed a mixed-method approach (Johnson & Onwuegbuzie, 2004). Quantitatively, descriptive statistics and chi-square tests were used to analyze performance frequencies and compare task outcomes. Qualitatively, students correct and incorrect responses were categorized using OSA epistemic and cognitive configurations, allowing for the identification of semiotic conflicts and persistent

reasoning errors. The integration of both analyses provided a comprehensive characterization of students' understanding of the derivative and their ability to apply it to contextualized problems.

RESULTS AND DISCUSSION

Results

This section reports the outcomes obtained in Tasks 2 and 3, completed by 102 first-year students enrolled in the Industrial Civil Engineering program. An exploratory analysis of students' procedures and reasoning is also provided. Table 3 summarizes the results for Task 2, "Designing a ramp with a controlled slope," in which students' performance was evaluated in relation to mathematical actions associated with the primary objects of the OSA. The results reveal generally strong performance.

A total of 94 % of students successfully represented both the function and its tangent line in a digital environment, indicating a robust ability to use technological tools for visualizing mathematical objects. Similarly, 90 % correctly calculated the derivative of the function and interpreted it within the context of the problem, evidencing an appropriate understanding of the derivative as an instantaneous rate of change. Furthermore, 80 % accurately evaluated the slope at $x = 2$ and correctly interpreted its meaning, while 76 % were able to derive the equation of the tangent line using the point-slope form. The action with the lowest success rate was the formulation of an argued reflection regarding the need for a safety system on the ramp (71 %), suggesting a challenge in connecting mathematical results to real-world decision-making.

To strengthen the quantitative analysis, chi-square tests of independence were conducted in addition to frequency calculations, revealing statistically significant differences ($p < 0.05$) between procedural-representational actions and those involving argumentative reasoning.

Table 3. Frequency of accuracy in Task 2 (n=102)

Actions for Task 2: Designing a Ramp with a Controlled Slope	Primary Objects	Absolute Frequency	Relative Frequency (%)
Correctly calculate the derivative of the function and recognize its contextual meaning	Representation / Argument	92	90
Correctly evaluate the slope of the ramp $x = 2$ and interpret it.	Representation / Argument	82	80
Correctly determine the equation of the tangent line using the point-slope form	Procedure / Representation / Definition	78	76
Adequately represent the function and the tangent line in a digital environment	Representation	96	94
Formulate an argued reflection regarding the need for a safety system	Argument	72	71

Table 4 presents the results for Task 3, which required students to model and optimize the volume of a box without a lid. This task assessed their ability to formulate functions, apply derivatives, and justify

optimization procedures. The results indicate relatively strong performance in the initial modeling steps, followed by a decline as tasks required higher levels of formal reasoning and argumentation. Specifically, 86 % of students correctly modeled the volume function, defined the independent variable appropriately, and used the given dimensions of the cardboard, demonstrating a solid understanding of the problem's translation into mathematical language. In subsequent steps, 77 % utilized the GeoGebra applet to explore the function graphically and compute its derivative, indicating effective coordination between dynamic representations and analytical reasoning. However, accuracy dropped to 68 % when identifying critical values, justifying their optimality, and determining the value of x that maximizes the volume using first- or second-derivative tests. Chi-square tests confirmed that this decline was statistically significant ($p < 0.05$), highlighting the need to strengthen students' ability to construct formal mathematical arguments.

Table 4. Frequency of accuracy of Task 3 ($n = 102$)

Actions for Task 3: Construction of a Box without a Lid Optimized in Its Volume	Primary Objects	Absolute Frequency	Relative Frequency (%)
Correctly models the volume function, defining the independent variable and using the dimensions of the assigned cardboard.	Procedure	88	86
Utilizes the simulation applet in GeoGebra to determine the derivative function ($V'(x)$)	Representation	79	77
Correctly identifies the critical values.	Procedure	69	68
Justifies that the critical values are the possible optimums of the function volume	Argument	69	68
Identifies the value of x that allows him or her to determine the maximum, using criteria of first or second derivative	Definition	69	68

Discussion

The discussion below synthesizes the findings of this study, reflecting on their implications for the proposed teaching design and offering considerations for its refinement and future implementation. Furthermore, the results of Task 2 reveal substantial progress in students' comprehension and application of the derivative within a practical context. The very high success rates in the graphical representation of the function and its tangent line in a digital environment (94 %) and in the calculation and contextual interpretation of the derivative (90 %) suggest that students are not only proficient in carrying out formal procedures but are also beginning to establish meaningful connections between algebraic symbolism and its applied significance. This finding underscores the pedagogical value of integrating dynamic mathematical tools such as GeoGebra, which support visualization and foster an embodied understanding of abstract concepts such as the derivative and tangent line. This observation is consistent with previous research (Trouche, 2005; Hohenwarter & Preiner, 2007), which highlights the capacity of

digital environments to strengthen functional thinking and promote coordination across multiple semiotic representations.

Nevertheless, a closer examination of actions that demand higher levels of conceptual integration and argumentative reasoning—such as deriving the equation of the tangent line (76 %) and formulating a reasoned decision about the need for a safety system (71 %)—reveals persistent difficulties. These tasks require students to move beyond procedural fluency to articulate mathematically grounded judgments that bridge formal results with contextual decision-making. Such challenges have been widely reported in the literature as critical points in students' mathematical development, since they involve mobilizing procedural, representational, and argumentative knowledge in an integrated manner (Lithner, 2017). This underscores the importance of designing learning activities that explicitly incorporate argumentation and critical reflection as core objectives, rather than as peripheral tasks.

A comparable pattern is observed in Task 3, which assessed students' ability to model and optimize the volume of a box without a lid. The high rate of success in constructing the volume function (86 %) demonstrates students' competence in translating a real-world problem into mathematical language, representing the essential first step of the modeling cycle. However, accuracy rates decline notably when students are required to identify and justify critical values and to determine the value of x that maximizes the volume (68 %). This decline points to persistent gaps in students' ability to engage with definitional knowledge and to apply formal criteria—such as the first- or second-derivative tests—in a rigorous manner. Similar trends have been reported in prior studies (Galindo Illanes et al., 2022; Galindo Illanes & Breda, 2024), which show that students often exhibit stronger mastery of procedural and representational facets of mathematical knowledge while struggling with definitional and argumentative dimensions. As noted by Pino-Fan et al. (2019), these difficulties are linked to a lack of systematic integration across the different epistemic facets, which limits students' ability to transfer procedural fluency into robust formal reasoning.

The relatively high use of the GeoGebra applet (77 %) further confirms the mediating role of digital tools in supporting students' representational and exploratory activity. However, in line with Drijvers et al. (2009), the findings indicate that the use of technology alone does not ensure deep conceptual understanding. Rather, digital tools should be complemented by carefully designed tasks that promote theoretical validation, encourage students to verbalize and justify their reasoning, and explicitly connect dynamic exploration with formal definitions and theorems.

Taken together, the findings call for a refinement of the instructional design to better balance dynamic exploration, contextualized modeling, and the systematic development of argumentative competence. Specifically, future iterations of the teaching sequence should incorporate opportunities for students to engage in collective validation of results, confront multiple solution strategies, and explicitly connect procedural steps with the underlying theoretical constructs of differential calculus.

No significant differences were observed by gender, although slight, non-conclusive variations were detected between students with higher and lower prior mathematical achievement. This result opens promising directions for future studies involving larger samples and more controlled variables, which may clarify how prior knowledge interacts with the development of representational and argumentative competencies. Furthermore, from the perspective of the OSA, these findings make it possible to identify knowledge configurations that are relatively stabilized around primary objects associated with representation and procedure, whereas those connected to definition and argument remain less consolidated. This observation is consistent with prior research (Verón & Giacomone, 2021) and

reinforces the relevance of employing OSA as an analytical framework for both the design and evaluation of teaching interventions.

The results obtained in the proposed tasks indicate notable advances in first-year engineering students' comprehension and application of the derivative within the context of authentic problems and through the integration of digital environments. Specifically, students demonstrated strong performance in actions related to the use of representations and mathematical procedures, including function modeling, derivative calculation, and the utilization of dynamic tools such as GeoGebra to explore and graphically represent key concepts in differential calculus. These outcomes address the research question by evidencing that a teaching approach grounded in modeling and interactive simulation effectively promotes learning in procedural and representational dimensions, while challenges remain in the development of formal argumentation.

In Task 2, students exhibited a high level of appropriation of the derivative as a measure of rate of change, alongside accurate interpretation of its significance in practical contexts. Furthermore, the strong performance in digital graphical representations suggests a successful integration of technological resources as mediators of learning. Nevertheless, actions requiring higher-order contextual reasoning—such as formulating judgments regarding the necessity of a safety system—revealed persistent difficulties, reflecting gaps in connecting mathematical procedures with contextualized decision-making. This observation aligns with prior research documenting students' challenges in transitioning from procedural fluency to formal justification (Lithner, 2017).

Task 3, which focused on an optimization problem, showed that students were competent in the initial stages, particularly in modeling the problem functionally. However, as the task progressed to steps demanding formal interpretation and argumentative justification, performance declined, indicating a need to strengthen the systematic application of definitions, theorems, and rigorous mathematical criteria beyond procedural execution or the instrumental use of digital tools. From the perspective of the OSA, these results highlight that knowledge configurations associated with representation and procedure are more readily consolidated than those linked to definition and argument, underscoring the relevance of OSA as an analytical framework for examining both the teaching and learning of calculus concepts.

Finally, two limitations of the study must be acknowledged, such as the short duration of the intervention (three sessions over one week), which constrains the ability to infer long-term learning effects, and the implementation in a single institutional context, which limits generalizability. These limitations highlight the need for future research that replicates the study across diverse educational settings and extends the intervention to examine the sustained impact on students' reasoning and argumentation skills.

CONCLUSION

This study confirms the value of combining digital simulation, mathematical modeling, and the OSA in teaching differential calculus, demonstrating that such integration fosters significant advances in procedural and representational learning. Students exhibited strong capabilities in modeling real-world situations, calculating derivatives, and graphically representing functions and tangent lines, with GeoGebra serving as an effective mediator of conceptual understanding. Importantly, the research illustrates that these tools are most effective when complemented with reflective, argumentative, and theoretically grounded activities, which support the development of critical reasoning and the contextual application of mathematical concepts.



The study's limitations must be acknowledged to contextualize the findings. The intervention was short in duration, encompassing only three sessions, and was implemented within a single institutional context. The absence of a control group and longitudinal follow-up further restricts the generalizability of the results. Consequently, the conclusions should be interpreted with caution, and replication in varied educational settings is necessary to confirm the robustness and sustainability of the observed learning outcomes.

Finally, several directions for future research are recommended. First, replicating the instructional design across multiple institutions and over extended periods would provide a stronger evaluation of its efficacy. Second, incorporating comparative designs with control groups could rigorously assess the specific impact of modeling, simulation, and OSA on student learning. Third, examining potential differences in performance according to student subgroups—such as prior mathematical achievement, gender, or technological familiarity—could provide insights for differentiated instruction. Finally, longitudinal studies investigating the transfer of learning to subsequent calculus courses and professional engineering contexts would clarify the long-term impact of this integrative approach. Collectively, these recommendations highlight the potential of combining theoretical frameworks, digital tools, and modeling activities to foster a comprehensive, contextually grounded, and transferable understanding of calculus concepts in engineering education.

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Declarations

- Author Contribution : MKGI: Conceptualization, Writing – Original Draft, Editing, and Visualization.
DCM: Writing – Review & Editing, Formal Analysis, and Methodology.
AB: Validation, Supervision, and Coordination of Fieldwork.
GS-S: Methodological Support, Critical Review of Content, and General Supervision.
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REFERENCES

- Aguilera, C., Manzano, A., Martínez, I., Lozano M., & Casiano, C. (2017). El modelo de aula invertida. Universidad de Almería. *Revista Internacional de Psicología del Desarrollo y de la Educación*, 4(1), 261-266. <https://doi.org/10.17060/ijodaep.2017.n1.v4.1055>

- Alarcón, A., Vázquez, A., & Marimón, C. (2019). Las tecnologías del aprendizaje y el conocimiento en la formación docente. *Revista Conrado*, 15(68), 180-186. http://scielo.sld.cu/scielo.php?script=sci_arttext&pid=S1990-86442019000300180
- Araya, D., Pino-Fan, L. R., Medrano, I., & Castro, W. F. (2021). Epistemic criteria for the design of tasks about limits on a real variable function. *Bolema*, 35(69), 179-205. <https://doi.org/10.1590/1980-4415v35n69a09>
- Ardina, G., Bautista, G., & Santos, M. S. D. (2025). GeoGebra integration in mathematics teaching: Bridging research and classroom practice. *Journal of Interdisciplinary Perspectives*, 3(8), 845-856. <https://doi.org/10.69569/jip.2025.505>
- Bedada, T. B., & Machaba, M. F. (2022). Investigation of student's perception learning calculus with GeoGebra and cycle model. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(10), em2164. <https://doi.org/10.29333/ejmste/12443>
- Breda, A., Pochulu, M., Sánchez, A., & Font, V. (2021). Simulation of teacher interventions in a training course of mathematics teacher educators. *Mathematics*, 9(24), 3228. <https://doi.org/10.3390/math9243228>
- Burgos, M., & Godino, J. D. (2020). Modelo ontosemiótico de referencia de la proporcionalidad. Implicaciones para la planificación curricular en primaria y secundaria. *Avances De Investigación En Educación Matemática*, 18, 1–20. <https://doi.org/10.35763/aiem.v0i18.255>
- Burgos, M., Bueno, S., Godino, J. D., & Pérez, O. (2021). *Onto-semiotic complexity of the Definite Integral. Implications for teaching and learning Calculus. REDIMAT – Journal of Research in Mathematics Education*, 10(1), 4-40. <https://doi.org/10.17583/redimat.2021.6778>
- Burgos, M., Castillo, M. J., Beltrán-Pellicer, P., Giacomone, B. Y., & Godino, J. D. (2020). Análisis didáctico de una lección sobre proporcionalidad en un libro de texto de primaria con herramientas del enfoque ontosemiótico. *Bolema*, 34(66), 40–68. <https://doi.org/10.1590/1980-4415v34n66a03>
- Colquepisco-Paucar, N. T. (2019). Aprendiendo el cálculo diferencial e integral con GeoGebra. *Killkana Técnica*, 3(2), 11–16. https://doi.org/10.26871/killkana_tecnica.v3i2.484
- Creswell, J. W., & Plano Clark, V. L. (2018). *Designing and conducting mixed methods research* (3rd ed.). SAGE.
- Del Río, L. S. (2020). Recursos para la enseñanza del Cálculo basados en GeoGebra. *Revista Do Instituto GeoGebra Internacional De São Paulo*, 9(1), 120–131. <https://doi.org/10.23925/2237-9657.2020.v9i1p120-131>
- Drijvers, P., Kieran, C., Mariotti, M., Ainley, J., Andresen, M., Chan, Y. C., Dana-Picard, T., Gueudet, G., Kidron, I., Leung, A., & Meagher, M. (2009). Integrating technology into mathematics education: Theoretical perspectives. En *New ICMI studies series* (pp. 89-132). Springer. https://doi.org/10.1007/978-1-4419-0146-0_7
- Font, V. (2005). Una aproximación ontosemiótica a la didáctica de la derivada [An ontosemiotic approach
- Galindo Illanes, M. K., & Breda, A. (2023). Significados de la derivada en los libros de texto de las carreras de Ingeniería Comercial en Chile. *Bolema*, 37(75), 271–295. <https://doi.org/10.1590/1980-4415v37n75a13>

- Galindo Illanes, M. K., & Breda, A. (2024). Instructional process of the derivative applied in business engineering students in Chile. *Uniciencia*, 38(1), 1–23. <https://doi.org/10.15359/ru.38-1.17>
- Galindo Illanes, M. K., Breda, A., & Alvarado, H. A. (2023). Diseño de un proceso de enseñanza de la derivada para estudiantes de ingeniería comercial en Chile. *Revista Paradigma*, 44(4), 321–350. <https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p321-350.id1386>
- Galindo Illanes, M. K., Breda, A., Alvarado, H. A., & Sala-Sebastià, G. (2025). Characterization of sub-fields of derivative problems in engineering textbooks. *Eurasia Journal of Mathematics, Science and Technology Education*, 21(3), em2591. <https://doi.org/10.29333/ejmste/15987>
- Galindo Illanes, M. K., Breda, A., Chamorro, D. D., & Alvarado, H. A. (2022). Analysis of a teaching learning process of the derivative with the use of ICT oriented to engineering students in Chile. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(7), em2130. <https://doi.org/10.29333/ejmste/12162>
- Godino, J. D. (2013). Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. *Cuadernos de Investigación y Formación en Educación Matemática*, 11, 111–132. https://enfoqueontosemiotico.ugr.es/documentos/Godino_2013_idoneidad_didactica.pdf
- Godino, J. D. (2016). *La idoneidad didáctica como herramienta de análisis y reflexión sobre la práctica del profesor de matemáticas*. [Videoconferencia]. Vimeo. <https://vimeo.com/175426315>
- Godino, J. D. (2022). Emergencia, estado actual y perspectivas del enfoque ontosemiótico en educación matemática. *REVIEM*, 2(2), 1–24. <https://doi.org/10.54541/reviem.v2i2.25>
- Godino, J. D. (2024). *Enfoque ontosemiótico en educación matemática: Fundamentos, herramientas y aplicaciones*. McGraw-Hill.
- Godino, J. D., & Burgos, M. (2020). ¿Cómo enseñar las matemáticas y ciencias experimentales? Resolviendo el dilema entre transmisión e indagación. *Paradigma*, XLI 80–106. <https://doi.org/10.37618/PARADIGMA.1011-2251.2020.p80-106.id872>
- Godino, J. D., Batanero, C., & Font, V. (2019). The onto-semiotic approach: Implications for the prescriptive character of didactics. *For the Learning of Mathematics*, 39(1), 38–43. <https://www.jstor.org/stable/26742011>
- Godino, J. D., Batanero, C., & Burgos, M. (2024). Understanding the onto-semiotic approach in mathematics education through the lens of the cultural historical activity theory. *ZDM Mathematics Education*, 56, 1331–1344. <https://doi.org/10.1007/s11858-024-01590-y>
- Godino, J. D., Batanero, C., & Font, V. (2007). The ontosemiotic approach to research in mathematics education. *ZDM*, 39(1-2), 127–135. <https://doi.org/10.1007/s11858-006-0004-1>
- Godino, J. D., Batanero, C., & Font, V. (2020a). El enfoque ontosemiótico: Implicaciones sobre el carácter prescriptivo de la didáctica. *Revista Chilena de Educación Matemática*, 12(2), 47–59. <https://doi.org/10.46219/rechiem.v12i2.25>
- Godino, J. D., Batanero, C., Burgos, M., & Gea, M. M. (2021). Una perspectiva ontosemiótica sobre problemas y métodos de investigación en educación matemática. *Revemop*, 3, e202107. <https://doi.org/10.33532/revemop.e202107>

- Godino, J. D., Burgos, M., & Wilhelmi, M. R. (2020b). Papel de las situaciones adidácticas en el aprendizaje matemático. Una mirada crítica desde el enfoque ontosemiótico. *Enseñanza de las Ciencias*, 38(1), 147-164. <https://doi.org/10.5565/rev/ensciencias.2906>
- Godino, J. D., Giacomone, B., Batanero, C., & Font, V. (2017). Enfoque ontosemiótico de los conocimientos y competencias del profesor de matemáticas. *Bolema*, 31(57), 90-113. <http://dx.doi.org/10.1590/1980-4415v31n57a05>
- Guilcapi, J. R., Martínez, J. M., Saavedra, M. C., & Guilcapi, L. E. (2019). Aplicación del software Matlab, como estrategia metodológica en la enseñanza-aprendizaje de cálculo de una variable a nivel superior de ingeniería de telecomunicaciones de la UTA. *Explorador Digital*, 3(3.1), 30-40. <https://doi.org/10.33262/exploradordigital.v3i3.1.863>
- Gusmão, T. C. R. S., & Font, V. (2022). Análisis metacognitivo de un aula de matemática sobre medida de superficies. *Revista Latinoamericana De Investigación En Matemática Educativa*, 25(2), 169–196. <https://doi.org/10.12802/reime.22.2522>
- Hohenwarter, M., & Preiner, J. (2007). Dynamic mathematics with GeoGebra. *Journal of Online Mathematics and its Applications*, 7(1), 2-12.
- Illescas, M. S., Illesca, T. L., Enriquez, M. del C., Riera Cartuche, D. R., Salazar, M. A., Hidalgo, L. E., & Bernal, A. P. (2024). Impacto de las plataformas tecnológicas de enseñanza como recursos educativos. *Ciencia Latina Revista Científica Multidisciplinar*, 8(4), 11401-11419. https://doi.org/10.37811/cl_rcm.v8i4.13307
- Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, 33(7), 14–26. <https://doi.org/10.3102/0013189X033007014>
- Kado, K. (2021). Impact of GeoGebra on the students' conceptual: Understanding of limit of a function in Bhutan. *International Journal of Asian Education*, 2(4), 539-548. <https://doi.org/10.46966/ijae.v2i4.140>
- Kaiser, G., Blum, W., Ferri, R. B., & Greefrath, G. (2015). Anwendungen und modellieren. En *Springer eBooks* (pp. 357-383). https://doi.org/10.1007/978-3-642-35119-8_13
- Ledezma, C. (2024). Mathematical modelling from a semiotic-cognitive approach. *Journal of Research in Mathematics Education*, 13(3), 268-292. <https://doi.org/10.17583/redimat.15066>
- Ledezma, C., Breda, A., & Font, V. (2023). Prospective teachers' reflections on the inclusion of mathematical modelling during the transition period between the face-to-face and virtual teaching contexts. *International Journal of Science and Mathematics Education*, 22, 1057–1081. <https://doi.org/10.1007/s10763-023-10412-8>
- Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM Mathematics Education*, 49, 937-949. <https://doi.org/10.1007/s11858-017-0867-3>
- López, R. R., & Hernández, M. W. (2016). Principios para elaborar un modelo pedagógico universitario basado en las TIC. Estado del arte. *Episteme Uniandes. Revista digital de Ciencia, Tecnología e Innovación*, 3(4), 575-593. <https://www.redalyc.org/pdf/5646/564677242009.pdf>
- Mora-Casasola, M. F. (2023). Implementación de recursos educativos digitales, una revisión sistemática desde la enseñanza del cálculo diferencial. *Revista Digital Matemática, Educación e Internet*, 24(1), 1-18. <https://doi.org/10.18845/rdmei.v24i1.6709>

- Pino-Fan, L. R., Parra-Urrea, Y. E., & Castro-Gordillo, W. F. (2019). Significados de la función pretendida por el currículo de matemáticas chileno. *Magis. Revista Internacional de Investigación en Educación*, 11(23), 201–220. <https://doi.org/10.11144/Javeriana.m11-23.sfpc>
- Rahman, M. A., Sook Ling, L., & Yin, O. S. (2024). Sistema de aprendizaje interactivo para el aprendizaje de cálculo. *F1000Research*, 11, 307. <https://doi.org/10.12688/f1000research.73595.2>
- Rezaei, J., & Asghary, N. (2024). Teaching differential equations through a mathematical modelling approach: The impact on problem-solving and the mathematical performance of engineering undergraduates. *International Journal of Mathematical Education in Science and Technology*, 56(5), 899–919. <https://doi.org/10.1080/0020739X.2024.2307397>
- Sepúlveda-Delgado, O., Suárez-Aguilar, Z. E., & Pino-Fan, L. R. (2021). Significados de referencia del objeto grupo. *Revista de Investigación, Desarrollo e Innovación*, 11(2), 297-318. <https://doi.org/10.19053/20278306.v11.n2.2021.12757>
- Spooner, K. (2023). Using mathematical modelling to provide students with a contextual learning experience of differential equations. *International Journal of Mathematical Education in Science and Technology*, 55(2), 565–573. <https://doi.org/10.1080/0020739X.2023.2244472>
- Torres-Peña, R. C., Peña-González, D., Chacuto-López, E., Ariza, E. A., & Vergara, D. (2024). Updating calculus teaching with AI: A classroom experience. *Education Sciences*, 14(9), 1019. <https://doi.org/10.3390/educsci14091019>
- Trouche, L. (2005). An instrumental approach to mathematics learning in symbolic calculator environments. In D. Guin, K. Ruthven, and L. Trouche. (eds.), *The Didactical Challenge of Symbolic Calculators* (pp. 137-162). Springer. https://doi.org/10.1007/0-387-23435-7_7
- Valarezo, J. W., & Santos, O. C. (2019). Technologies for learning and knowledge in teacher education. *Revista Conrado*, 15(68), 180-186. <https://conrado.ucf.edu.cu/index.php/conrado/article/view/1003>
- Verón, M. A., & Giacomone, B. (2021). Análise dos significados do conceito de diferencial de uma perspectiva ontosemiótica. *Revemop*, 3, e202109. <https://doi.org/10.33532/revemop.e202109>
- Weinhandl, R., & Lavicza, Z. (2021). Real-world modelling to increase mathematical creativity. *Journal of Humanistic Mathematics*, 11(1), 265–299. <https://doi.org/10.5642/jhummath.202101.13>
- Winkel, B. (2023). Using modelling to motivate and teach differential equations. *International Journal of Mathematical Education in Science and Technology*, 55(2), 193-197. <https://doi.org/10.1080/0020739X.2023.2289794>
- Zagoto, M. M., Musdi, E., Arnawa, I. M., Fauzan, A., Bentri, A., & Dakhi, O. (2025). Effectiveness of Geogebra-based learning on students' cognitive and affective participation in mathematics. *Salud Ciencia y Tecnología*, 5, 2001. <https://doi.org/10.56294/saludcyt20252001>

