

Culturally responsive approaches to geometric translation: Exploring *Songket* motifs and students' proving trajectories

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Abstract

Research on students' proving processes in geometry has largely emphasized formal reasoning, with limited exploration of cultural contexts as scaffolds for mathematical understanding. Addressing this gap, this study investigates the integration of South Sumatera *Songket* motifs as culturally relevant tools to support students' proving processes in learning geometric translation. Using a Design Research methodology, a validation study was conducted with 30 junior high school students in Palembang. The research progressed through three phases: preparation, design experiments (preliminary and main teaching experiments), and retrospective analysis. Learning tasks were designed based on Habermas' Construct of Rationality—epistemic, teleological, and communicative—to structure the proving trajectory. Culturally grounded tasks facilitated students' progression from intuitive exploration to formal justification. In the first activity, the *Songket* Durian motif supported recognition of translation as an isometric transformation through visual pattern analysis. Subsequent tasks introduced algebraic reasoning with coordinate shifts and vector notation, leading to replication of the *Perahu Kajang* motif across Cartesian quadrants to formulate general transformation rules. These findings reveal the effectiveness of cultural artifacts in supporting both intuitive and formal dimensions of proof. Embedding cultural artifacts in mathematics instruction fosters culturally responsive and proof-oriented learning, enhancing conceptual understanding while strengthening connections between mathematics and cultural identity. This study contributes a novel approach by systematically employing cultural motifs to design proof-based learning trajectories in geometry, offering a model for integrating cultural heritage with mathematical reasoning in diverse educational settings.

Keywords: Culturally Responsive Pedagogy, Design Research, Geometric Translation, Proving Processes, *Songket* Motifs

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Geometric translation is a fundamental transformation in mathematics, defined as the uniform displacement of all points in a figure along a given vector while preserving both direction and magnitude (Lunardon, 2022a). Unlike reflection or rotation, translation maintains the orientation of an object throughout the process (Judge & Gutkin, 2000; Prahmana & D'Ambrosio, 2020). From a structural perspective, translation is grounded in the algebraic framework of elementary abelian groups, which formalizes its role in affine and linear spaces (Lunardon, 2022b). Within coordinate geometry, vector-based representations further enable precise modelling of displacement and provide students with powerful tools for analysing transformational properties (Song & Wang, 2016).

Beyond its formal structure, translation plays a central pedagogical role in developing students' spatial reasoning and understanding of transformational geometry. However, research shows that many students experience persistent difficulties in this domain. They often rely on visual heuristics or rote procedures rather than algebraic reasoning, which can result in misconceptions and fragmented conceptual understanding (Ikhsan et al., 2022). Trial-and-error approaches are frequently observed in problem-solving, especially in numeric translation tasks, limiting students' ability to generalize and reason abstractly (Hoiriyah et al., 2023). Moreover, the lack of meaningful contextualization reduces student engagement and hinders knowledge transfer, as conventional textbooks rarely embed translation tasks within authentic or culturally relevant settings (Cahyaningrum & Pradipta, 2021; Ogbonnaya et al., 2024). Consequently, students struggle not only to recognize and perform translations but also to articulate proofs of their properties using formal mathematical language (Wintarti & Chofifah, 2023; Nurrahmawati et al., 2021).

To address these difficulties, researchers have proposed frameworks that scaffold the transition from intuition to formal reasoning. Habermas' construct of rationality provides a particularly robust foundation, emphasizing epistemic, teleological, and communicative dimensions of reasoning that can guide students toward deeper mathematical engagement (Urhan & Bülbül, 2022a; 2022b). This framework enables teachers to diagnose students' conceptual gaps while structuring activities that foster proof-oriented thinking. Yet, as several studies note, even when rationality-based scaffolds are provided, students' learning remains limited if tasks are detached from meaningful or familiar contexts (Haj-Yahya, 2021). This indicates a need for instructional designs that combine structured reasoning with cultural relevance, enabling students to see mathematics as both formally rigorous and contextually meaningful.

Culturally responsive pedagogy has been widely recognized as a promising approach to bridge this gap (Payadnya et al., 2024). Embedding cultural practices and artifacts into mathematical tasks has been shown to improve inclusivity, sustain engagement, and promote meaningful connections between abstract ideas and lived experiences (Kalinec-Craig et al., 2019; Sevgi & Erduran, 2020; Güçler et al., 2018). In particular, South Sumatera *Songket* motifs—renowned for their symmetrical and repetitive designs—provide an authentic context for learning geometric translation. Motifs such as *Durian* (King of Fruit) and *Perahu Kajang* (Traditional boat typical of South Sumatra, especially from the Kayu Agung area in Ogan Komering Ilir Regency) naturally embody repeated displacement patterns, offering students opportunities to explore congruence, identify translation vectors, and connect cultural forms with formal mathematical reasoning (Sari et al., 2024; 2025). By working with these motifs, students can engage in epistemic and teleological reasoning while also negotiating understanding through communicative rationality, thereby supporting the development of proof-oriented learning trajectories.

Against these problems, the present study explores how culturally grounded instructional tasks can be designed to support students' proving processes in geometric translation. Specifically, it investigates how South Sumatera *Songket* motifs can be systematically integrated into classroom activities to scaffold students' progression from intuitive visualization toward formal justification. The study therefore seeks to answer the research question: in what ways can culturally contextualized tasks based on *Songket* motifs foster students' reasoning and proof construction in geometric translation? By addressing this question, the study contributes a novel model of culturally responsive mathematics instruction that leverages local cultural heritage to strengthen conceptual understanding, formal reasoning, and students' ability to engage in proof.

METHODS

This study adopted a validation design within the framework of design research (Bakker, 2018). The



research was carried out over two iterative cycles, each comprising three stages: preliminary design, design experiment, and retrospective analysis. This approach enabled systematic refinement of instructional tasks and the development of a validated learning trajectory to support students' proving processes in geometric translation.

Participants

The study involved 30 ninth-grade students (approximately 14 years old) from a public junior high school in Palembang, South Sumatera, Indonesia. The school was selected based on its voluntary agreement to participate. Expert reviewers, including mathematics education specialists, were also involved in validating the instructional tools prior to classroom implementation.

Instruments and Materials

The instructional design was guided by Habermas' construct of rationality (Urhan & Bülbül, 2022b), emphasizing epistemic, teleological, and communicative reasoning. This framework served as both a design tool for structuring mathematical tasks and an analytical lens for evaluating students' reasoning. Instructional materials included lesson plans, student worksheets, and ICT-based media, all of which were validated by experts. South Sumatera *Songket* motifs, particularly *Durian* and *Perahu Kajang*, were used as culturally contextualized resources for designing translation tasks.

Procedure

In the preliminary design stage, the researchers developed a Hypothetical Learning Trajectory (HLT) specifying learning objectives, anticipated student strategies, and potential difficulties. A literature review on *Songket* motifs and a preliminary student study informed the design.

In the design experiment stage, two forms of implementation were conducted: a pilot teaching experiment in Cycle 1 with a small group of students, and a classroom teaching experiment in Cycle 2 with an intact class. The pilot experiment tested the feasibility of the activity sequence, while findings informed the revision of the HLT. In Cycle 2, the revised trajectory was implemented collaboratively with the classroom teacher. Pre-activity discussions and post-activity reflections were conducted to ensure alignment and improvement. Data sources included classroom observations, student activity sheets, field notes, and semi-structured interviews.

In the retrospective analysis stage, data from both cycles were examined to evaluate the validity of the HLT and refine the instructional design. The analysis focused on students' reasoning patterns, the role of cultural artifacts in facilitating conceptual understanding, and the extent to which the tasks supported progression from intuitive visualization to formal justification.

Data Analysis

Collected data—including validation sheets, observations, interviews, and student work—were analyzed descriptively. The analysis emphasized how students engaged with translation tasks, the strategies they employed, and the ways in which culturally grounded activities supported proof-oriented reasoning. This iterative process produced a validated learning trajectory demonstrating how *Songket*-based instruction can strengthen students' proving processes in geometry.

RESULTS AND DISCUSSION

In the preliminary stage, the instructional worksheet was systematically developed to facilitate students' proving processes in geometric translation through culturally contextualized and cognitively sequenced

activities. Traditional South Sumatera *Songket* motifs, specifically the Durian and *Perahu Kajang* patterns, were employed as the pedagogical entry point to situate abstract mathematical concepts in familiar and meaningful contexts. The design of the HLT was guided by Jürgen Habermas' three constructs of rationality: epistemic, teleological, and communicative (Urhan & Bülbül, 2022a; 2022b). Within this framework, epistemic rationality was operationalized through activities that encouraged students to observe, recognize, and hypothesize translational patterns from *Songket* motifs; teleological rationality was fostered through problem-solving tasks requiring the purposeful application of translation vectors in coordinate geometry; and communicative rationality was emphasized through collaborative discussions and peer justification of mathematical arguments. This structured integration of cultural artefacts with rational thinking processes enabled students to transition from intuitive visual recognition to formal symbolic representation, thereby strengthening their conceptual understanding and reasoning in transformational geometry.

Epistemic Rationality (Conceptual Understanding through Cultural Visualization)

In the initial stage, students were introduced to the *Songket* Durian motif from Lubuklinggau as a culturally grounded context for exploring geometric translation. This activity engaged students' perceptual and intuitive faculties by directing their attention to the repetition and symmetry embedded in the traditional woven textile. Learners were encouraged to recognize recurring patterns and investigate whether the motif preserved its form when repositioned, thereby formulating preliminary hypotheses regarding translational invariance. Guiding questions, such as determining the number of steps required to shift the motif horizontally or vertically, facilitated students' internalization of translation as a geometric transformation characterized by direction and magnitude while maintaining congruence in shape and size. By embedding mathematical exploration within a familiar cultural artefact, this stage effectively bridged abstract geometric ideas with everyday visual experiences, fostering epistemic rationality through the transformation of intuitive observations into structured mathematical reasoning.

Teleological Rationality (Purpose-Oriented Mathematical Practices)

Building upon the initial stage of intuitive exploration, the subsequent phase focused on goal-oriented tasks designed to formalize students' understanding of geometric translation. In this phase, learners examined a sequence of coordinate points representing the transformation of the motifs ABCDEFGH to KLMNOPQR and subsequently to STUVWXYZ. They were tasked with identifying vector displacements between corresponding points and deriving general transformation rules based on these relationships. Through systematic practice, students articulated translation rules using vector notation, thereby quantifying horizontal and vertical shifts with mathematical precision. These activities cultivated teleological rationality by situating students' reasoning within purposeful problem-solving, reinforcing the conceptualization of translation as a function that preserves both congruence and structural integrity within the Cartesian plane.

Communicative Rationality (Mathematical Justification and Design Application)

The final phase of the worksheet emphasized reflective dialogue and symbolic justification through collaborative, design-based tasks. Utilizing the culturally significant *Perahu Kajang* motif from Ogan Komering Ilir, students were instructed to apply translation rules systematically across all four quadrants of the Cartesian plane. They were guided to articulate the impact of each transformation on the position of individual points while ensuring consistency across successive repetitions. Learners were further required to derive general translation rules, verify the uniformity of spacing between motifs, and describe

the underlying geometric properties. This stage fostered communicative rationality by encouraging students to justify their reasoning through written explanations and diagrammatic representations. By reconstructing cultural motifs through mathematical operations, students engaged in purposeful discourse, strengthened spatial reasoning, and deepened their conceptual clarity regarding key properties of translation—namely, preservation of shape, directionality, and congruence.

Hypothetical Learning Trajectory (HLT) for Translation

The development of this HLT for teaching translation transformations was designed to guide students through an inquiry-based and culturally contextualized learning experience. By incorporating the *Songket Durian* and *Perahu Kajang* motifs, learners are introduced to mathematical translations in a setting that is both familiar and analytically rich. The instructional design positions students within a sequence of observational, computational, and argumentative tasks that gradually support their progression from recognizing visual repetition to formulating general translation rules expressed through vectors. Central to this trajectory is the integration of Habermas' three dimensions of rationality—epistemic, teleological, and communicative—which collectively foster deep reasoning and the ability to construct and justify mathematical claims. Embedding traditional textile motifs within this framework enables students to anchor abstract mathematical operations in tangible and culturally meaningful representations. [Table 1](#) provides a detailed trajectory outlining the aims, activities, and conjectures about students' thinking.

Table 1. The HLT for translation learning using *Songket* motifs

Aims	Activity	Conjecture of Students' Thinking
Recognize repetition in cultural patterns	Observe the <i>Songket</i> Durian motif and identify repeated forms and directions of repetition.	Students identify recurring visual patterns and infer that motifs are reproduced through consistent shifts, indicating the concept of translation.
Understand translation as movement with direction and magnitude	Estimate how the motif shifts horizontally or vertically and describe the distance and direction of movement.	Students begin to conceptualize translation as movement with fixed direction and magnitude, forming an informal understanding of vector translation.
Apply coordinate-based translation	Analyze changes in coordinates between motifs (e.g., ABCDEFGH → KLMNOPQR → STUVWXYZ).	Students recognize consistent coordinate changes and formulate translation vectors (e.g., $T = (5, 0)$).
Generalize and visualize translation rules	Apply the translation vector to replicate the <i>Perahu Kajang</i> motif across all quadrants of the Cartesian plane.	Students extend motifs across quadrants using translation rules, demonstrating understanding of directionality, congruence, and periodicity.
Justify properties of translation	Verify that translated figures retain shape, size, and orientation, and reflect on how translations function geometrically.	Students conclude that translation is an isometric transformation and articulate its properties using coordinate geometry.

This trajectory highlights translation not only as a mechanical skill but also as a visual, communicative, and reasoned process embedded within meaningful contexts. By situating the learning pathway in culturally relevant artefacts and grounding tasks in Habermas' rationality framework, the

design enables students to transition from intuitive pattern recognition toward formal mathematical justification. Such integration ensures that the learning environment simultaneously promotes knowledge construction, goal-oriented problem solving, and collaborative discourse, all of which are critical for advancing mathematical literacy in transformational geometry.

Design Experiment

The design experiment on translation transformations was implemented in two iterative phases: Cycle 1 (Pilot Implementation) and Cycle 2 (Classroom Implementation). The pilot phase involved a small group of students who engaged with the activities outlined in the HLT, which embedded South Sumatera's traditional *Songket* motifs—specifically the motif of *Durian* and *Perahu Kajang*—as culturally meaningful contexts for introducing translation in geometry. The primary aim of this phase was to assess the feasibility of the instructional design and to examine how effectively students could engage with and internalize mathematical concepts of translation through culturally grounded artefacts.

The findings from the pilot revealed both strengths and challenges. Students responded enthusiastically to the incorporation of cultural motifs, demonstrating curiosity and engagement when exploring repetitive patterns. For instance, in observing the *Durian* motif, several students intuitively identified the repeated arrangement of shapes, describing it informally as a “*geser pola*” (pattern shift). This language reflected an emergent conceptualization of translation, yet it also exposed a key limitation: students' reliance on perceptual recognition rather than formal mathematical reasoning. While they recognized that shapes were shifted, they encountered difficulty expressing these shifts in terms of translation vectors or coordinate transformations. Similarly, in coordinate-based tasks (e.g., ABCDEFGH $\rightarrow$ KLMNOPQR $\rightarrow$ STUVWXYZ), students initially concentrated on absolute positions of figures rather than relative displacements, underscoring the need for explicit scaffolding to support abstraction.

Furthermore, while students successfully replicated the *Perahu Kajang* motif across quadrants of the Cartesian plane, many were unable to articulate the underlying translation rules or verify the consistency of distances between successive motifs. These challenges highlighted the critical need to design tasks that explicitly bridge the gap between intuitive visualization and formal mathematical representation. In response, the HLT was refined to incorporate more structured guidance in constructing translation vectors, increased opportunities for reflection on displacement and direction, and targeted prompts to promote justification within coordinate geometry. This iterative refinement strengthened both teleological rationalities, by directing students toward purposeful task completion, and epistemic rationality, by enabling learners to progressively link informal intuitions with formal abstraction (Prahmana et al., 2024).

Teaching Experiment

The revised HLT was implemented in Cycle 2 with a whole class setting as shown in Figure 1. Prior to instruction, the researcher collaborated with the classroom teacher through joint planning sessions to align the pedagogical goals of each activity with the expected mathematical reasoning outcomes. After the implementation of each task, reflective discussions were conducted to analyze students' engagement and responses, allowing for adaptive adjustments to subsequent activities. Data were collected through classroom observations, student worksheets, and informal interviews to evaluate how effectively the HLT supported students' understanding of translation, vector representation, and periodicity in geometric transformations.





Figure 1. Activity I – Observing repetition in the *Songket Durian* motif (Left) and Activity II – Analyzing coordinate shifts and translation vectors (Right)

In this sequence of activities, translation was introduced through a culturally grounded context, beginning with the *Songket Durian* motif from Lubuklinggau, South Sumatra. This motif, noted for its symbolic meaning and symmetrical repetition, functioned as a perceptual stimulus to foster observation and conceptual engagement. Students were first invited to examine the arrangement of motifs and respond to the guiding question: “*Do the motifs appear to be copied or shifted in a particular direction?*” This prompt elicited epistemic rationality by encouraging the formulation of initial hypotheses through informal reasoning. Many students recognized horizontal repetition and expressed it in vernacular terms such as *geser ke samping* (shifted sideways). A subsequent question—“*How far is each motif moved from the next?*”—shifted attention toward estimating distance and direction, enabling students to reconceptualize translation as systematic displacement rather than simple recurrence.

The second activity advanced toward formalization within a coordinate system. Students analyzed the transformation of a motif (ABCDEFGH) into its translated forms (KLMNOPQR and STUVWXYZ) on a Cartesian plane, identifying and quantifying changes in coordinates. This task highlighted teleological rationality, requiring purposeful reasoning about vector displacement and congruency preservation. Initially, students described translation as mere movement; however, guided scaffolding helped them articulate displacement using ordered pairs (e.g., $(+5, 0)$). Structured prompts such as “How can we express this shift using coordinate pairs?” directed them to bridge spatial intuition with symbolic representation, reinforcing their conceptualization of translation vectors.

The culminating task employed the *Perahu Kajang* motif in a design-based challenge. Positioned in the first quadrant, the motif was replicated across the remaining quadrants through explicit translation rules (e.g., shift left nine units, then down five units). This activity emphasized communicative rationality, as students collaborated to verify accuracy, measure displacement, and ensure congruence. By framing translation as a reflective, culturally embedded, and dialogic practice, the task integrated mathematical precision with meaningful cultural context. Furthermore, an excerpt from a small-group discussion illustrates this process:

- Student R* : “If we shift the motif to the left by nine units, I think it should end up in the second quadrant. The shape stays the same, just moved over.”
- Student D* : “Yeah, but how do we know it’s exactly nine units? What if we move it more or less? Won’t the spacing between motifs change?”
- Student A* : “That’s a good point. The key is the vector. The worksheet says it should move left by nine and down by five for the third quadrant. If we apply the same vector, the spacing stays consistent.”
- Student R* : “So $(-9, -5)$ is the rule we follow each time?”

- Student D* : “Right! And the shape doesn’t change—it just moves. The pattern repeats neatly.”
- Student L* : “But how can we be sure the new position is correct?”
- Student A* : “By checking each coordinate. If every point is shifted by exactly $(-9, -5)$, the translation is accurate.”

This exchange demonstrates students’ progression from intuitive visualization to rule-governed reasoning, highlighting how communicative rationality emerged as they challenged assumptions, negotiated meaning, and co-constructed understanding.

Overall, Cycle 2 affirmed the effectiveness of embedding ethnomathematical artefacts within a structured HLT. Students were able to internalize translation not as a rote algorithm but as a culturally meaningful, collaboratively reasoned, and mathematically coherent transformation. The interplay between visual intuition, symbolic formalization, and dialogic interaction underscored the significance of Habermas’ rationality framework in cultivating both mathematical literacy and culturally responsive pedagogy.

Retrospective Analysis

The retrospective phase of this study investigated how the revised HLT supported students’ conceptual development in geometric translation. This stage consolidated insights from earlier design cycles, shedding light on both the strengths of the instructional design and aspects that required further refinement. A distinctive feature of the design was the integration of cultural artefacts, particularly *Songket* motifs from South Sumatra, which provided a meaningful and engaging context for exploring spatial reasoning and vector-based transformations.

The use of *Durian* and *Perahu Kajang* motifs illustrated the potential of situating abstract mathematical ideas within culturally familiar contexts. These motifs enabled students to progress from intuitive recognition toward more formal reasoning. Learners were able to observe repeated shifts in motif arrangements, identify directions of displacement, and describe movements in terms of distance. This gradual transition reflected the development of epistemic rationality, as students constructed increasingly systematic interpretations of translation through observation, reflection, and logical justification (Setianingsih et al., 2025).

Despite these promising outcomes, several challenges arose during the coordinate geometry phase. Many students experienced difficulty generalizing translation rules using vector notation, particularly when shifting from everyday expressions such as *geser ke kiri* (shift to the left) to algebraic representations like $(-9, 0)$ or $(-9, -5)$. This revealed the need for stronger scaffolding to bridge concrete spatial intuition with abstract symbolic formalism. In response, worksheets were redesigned to include guiding prompts, visual supports, and structured reflection tasks aimed at supporting these representational transitions.

The design research methodology played a crucial role in facilitating iterative refinement. Through cycles of preliminary design, pilot testing, classroom enactment, and retrospective analysis, the HLT evolved in alignment with students’ learning trajectories (Adha et al., 2024). Insights from the pilot phase, for instance, revealed conceptual gaps that informed adjustments such as clearer visual aids and explicit modelling of translation vectors during classroom implementation.

Habermas’ framework of rationality provided an effective lens for interpreting student learning (Urhan & Bülbül, 2022a; 2022b). Learners demonstrated epistemic rationality when hypothesizing about motif displacements, teleological rationality when engaging purposefully with quadrant-based tasks, and



communicative rationality through peer interaction, feedback, and shared meaning-making. These dimensions collectively supported a shift from procedural engagement to deeper conceptual understanding. The *Perahu Kajang* quadrant task was particularly effective in advancing teleological rationality, as it required systematic application of translation vectors, reinforcing the recognition of translation as an isometric transformation that preserves size, orientation, and shape.

Overall, the findings highlight the pedagogical value of embedding mathematical concepts within culturally meaningful contexts. This approach resonates with the principles of *Pendidikan Matematika Realistik Indonesia* (PMRI) (Zulkardi et al., 2020) and ethnomathematics (D'Ambrosio, 2001), both of which emphasize connecting learning with cultural practices to enhance relevance and motivation. While cultural integration enriched engagement and understanding, the findings also point to the need for scaffolding to facilitate transitions from intuitive to formal reasoning. Future refinements should focus on strengthening students' ability to articulate mathematical relationships, formalize vector notation, and engage collaboratively in meaning-making. Such a design, grounded in constructivist and sociocultural perspectives, fosters higher-order mathematical thinking that is both contextually grounded and culturally responsive.

Discussion

The findings of this design research highlight the potential of culturally contextualized learning trajectories to support students' proving processes in geometric translation. By embedding tasks within the traditional *Songket* motifs of South Sumatera—specifically *Durian* and *Perahu Kajang* motifs—abstract mathematical concepts were anchored in familiar visual and cultural artefacts. This motivational contextualization functioned as an effective cognitive scaffold, particularly during the early phases of conceptual development, enabling students to connect their prior cultural knowledge with formal mathematical inquiry.

Students exhibited strong visual intuition when recognizing repetitive patterns in the *Songket* motifs, suggesting an initial understanding of translation as structured movement. Nevertheless, this perceptual insight was not automatically translated into formal reasoning. The transition from informal descriptions to formal representations such as coordinate shifts and vector notation required carefully designed instructional interventions. These results are consistent with previous findings (e.g., Prahmana, 2022; Azmi et al., 2025), which emphasize that cultural artefacts can serve as powerful heuristic tools, but only when supported by structured mathematical tasks and reflective discourse that facilitate abstraction.

The application of Habermas' Construct of Rationality provided a robust analytical lens for interpreting students' reasoning progression (Urhan & Bülbül, 2022a; 2022b). Activities that promoted epistemic rationality, such as identifying repetition and estimating displacement, enabled learners to formulate initial conjectures from empirical observation (Urhan & Bülbül, 2022a). Subsequent tasks situated in the Cartesian plane activated teleological rationality, where students applied vector rules purposefully to reproduce motif patterns (Urhan & Bülbül, 2022b). Communicative rationality was evident in collaborative discussions, as students refined their understanding through peer explanations and verbal justifications. These findings align with prior research on the sociocultural dimensions of mathematical reasoning, which stresses the importance of discourse and interaction in shaping conceptual understanding (Lerman, 2001).

Importantly, the observed progression from culturally intuitive observation to formal geometric justification resonates with the principles of the PMRI approach, which emphasizes realistic, meaningful,

and student-centered mathematics learning (Sari et al., 2025). The incorporation of local cultural artefacts enhanced students' engagement and provided a relevant frame for interpreting geometric translation. This reinforces the ethnomathematical perspective that mathematics learning should bridge abstract theory with students lived experiences (D'Ambrosio, 2001; Wiryanto et al., 2022; Sari et al., 2024; Azmi et al., 2025).

Despite these positive outcomes, the study also identified persistent challenges in supporting students' transition from spatial intuition to symbolic reasoning. While many students could visually replicate translation patterns, not all were able to consistently articulate the underlying rules using formal vector notation or precise mathematical language (Luzano, 2025). This finding indicates the need for greater scaffolding in algebraic representation and language development, particularly for students with limited exposure to symbolic reasoning.

Overall, this study contributes to the growing body of research advocating for culturally responsive and theory-informed approaches to mathematics education. By integrating local cultural contexts with Habermas' rationality framework, the design offered both a pedagogical model and a theoretical tool for analyzing students' reasoning in transformational geometry. The implications extend to curriculum design and teacher professional development, underscoring the importance of embedding geometric concepts in culturally meaningful contexts to foster engagement, conceptual understanding, and mathematical communication skills. Future research should investigate the longitudinal effects of such culturally grounded learning designs and explore their adaptability across diverse cultural, curricular, and technological settings.

CONCLUSION

This study demonstrates that embedding geometric translation within culturally meaningful contexts—specifically through South Sumatera's *Songket* motifs of *Durian* and *Perahu Kajang*—can significantly enhance students' conceptual understanding, spatial reasoning, and proving processes. By situating abstract mathematical concepts within familiar ethnomathematical artefacts, students were able to progress from intuitive recognition of repetitive patterns to the formal articulation of translation as an isometric transformation involving direction and magnitude. The implementation of the HLT, structured through Habermas' Construct of Rationality, proved effective in scaffolding this progression. Epistemic rationality supported the initial exploration of cultural patterns, teleological rationality guided purposeful application of vector representations, and communicative rationality facilitated peer interaction and mathematical justification. Collectively, these processes provided a coherent pathway from informal intuition to formal reasoning, affirming the pedagogical value of culturally responsive mathematics education.

Nevertheless, several limitations must be acknowledged. The study was conducted in a single school with a relatively small group of participants, which restricts the generalizability of its findings. While the integration of *Songket* motifs provided a strong scaffold for visual and intuitive reasoning, some students encountered persistent challenges in transitioning from spatial visualization to algebraic notation and vector-based representation. This suggests the need for more explicit scaffolding strategies that support the representational shift from informal to formal reasoning. Future research should therefore explore the longitudinal impacts of culturally contextualized learning designs on students' mathematical development across grade levels and investigate their adaptability in diverse cultural and curricular contexts. Comparative studies employing a wider range of cultural artefacts, as well as research

integrating digital technologies with ethnomathematical contexts, may further advance the scalability, inclusivity, and global relevance of mathematics education.

This study underscores the potential of integrating local cultural artefacts, such as South Sumatera *Songket* motifs, into the teaching of geometric translation. Embedding mathematics within culturally meaningful contexts enhances student engagement, supports intuitive reasoning, and fosters the transition toward formal mathematical abstraction. The use of the HLT, guided by Habermas' Construct of Rationality, provides teachers with a systematic framework to design tasks that promote epistemic, teleological, and communicative rationality in the classroom. For educators and curriculum developers, the findings highlight the importance of culturally responsive mathematics education that is both contextually relevant and mathematically rigorous.

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Declarations

- Author Contribution : AS: Conceptualization, Data Curation, Project Administration, Formal Analysis, Investigation, Resources, Writing - Original Draft, Editing and Visualization.
RIIP: Supervision, Methodology, Conceptualization, Writing – Review & Editing, Validation.
Z: Methodology, Validation, Formal Analysis, Resources, and Writing – Review & Editing.
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