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A prospective mathematics teacher's lesson planning: An in-depth analysis from the anthropological theory of the didactic

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Abstract

This paper aims to explore an Indonesian prospective mathematics teacher's lesson planning and identify its characteristics of mathematical task design from the viewpoint of the anthropological theory of the didactic. The well-documented activities concerning lesson planning developed by the prospective mathematics teacher in her experiment to conduct a study on mathematics teaching were used as the primary data to be analyzed. Part of the anthropological theory of the didactic, namely mathematical praxeology, was used as the theoretical framework to analyze and explore the mathematical tasks design, their techniques, theoretical arguments of the techniques used, and theories underpinning the theoretical discourses. The study points out mathematical praxeology for each mathematical task included in the lesson plan analyzed. In addition, this study also figured out the characteristics of the mathematical praxeology of the lesson design developed by the prospective teacher.

Keywords: Anthropological Theory of the Didactic (ATD), Lesson Plan, Mathematical Praxeology, Prospective Mathematics Teacher

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Lesson planning has long been recognized as essential in teaching practices that ultimately impact students' learning opportunities (Clark & Peterson, 1986; Floden et al., 1981; Stigler & Hiebert, 2009). Several studies show that lesson planning is necessary for effective teaching (Akyuz et al., 2013; Gonzalez Thompson, 1984; Ozogul & Sullivan, 2009; Ruys et al., 2012). High-quality instruction as showed by Japanese teachers has certain characteristic as providing greater opportunities for students to think deeply about richer mathematics and develop a solid understanding of mathematical ideas (Corey et al., 2010; Stigler & Hiebert, 2009). To continuously improve the quality of their instructions, Japanese teachers have intensively involved in lesson study activities and shared instructional practices to their educational communities through lesson observations and after lesson reflections (Doig et al., 2011; Fujii, 2015). To answer a research question about how Japanese teachers achieve such a high-quality instruction, Melville (2017) examined what Japanese educators do during their daily *kyozaikenkyu* (instructional materials research) which is related to their lesson planning processes, and why they do it. His study points out that through the instructional materials research, Japanese teachers are possible to do their continuous professional development and are better prepared to teach from an inquiry-based approach.

Recognizing the critical role of lesson planning for both mathematics teachers in their daily professional activities and prospective mathematics teachers within the process of their preparation to be professional teachers, several researchers focused their studies on such issues as teachers' and prospective teachers' lesson planning. Concerning lesson planning by teachers, some studies point out the following results. Precise lesson planning relates to successful ways of managing and controlling instructional activities (Gonzalez Thompson, 1984). Due to the difficulties in preparing a lesson plan, Rusznyak and Walton (2011) suggested that some teachers still need to be helped by providing them guidelines for preparing better lesson plans. A study conducted by Panasuk et al. (2002) revealed that among some parts of lesson planning processes regarding mathematics instruction for middle school, preparing a lesson plan that focuses on mathematical concepts is considered the most challenging. In attempting to provide appropriate suggestions for educators outside Japan who are seeking to improve the quality of lesson study processes, Fujii's (2019) study revealed that lesson planning not only plays an important role and function in lesson study activities but also serves a crucial function. Furthermore, it has specific characteristics that those who wish to engage in lesson study need to be understood. A comparative study between lesson plans developed by U.S. and Turkish teachers, conducted by Cicek and Tok (2014), addressed that both countries have similar policies regarding yearly and daily lesson plans that are all provided by their educational systems in both countries.

Studies on lesson planning by prospective teachers revealed the following results. Fraser et al. (2011), in a study focusing on enhancing lesson planning and quality of classroom life for mathematics students' use of technology, addressed that technology enhanced students' quality of life by facilitating their lesson planning, helping them stay on track, reducing their stress, and making it easy for them to adjust, modify, and reuse their lessons. Sharil and Kyriacou (2015) conducted an explorative study regarding the impact of reflective practice in developing pre-service teachers' professional development, particularly on instructional planning, by focusing on what entails reflective practice and instructional planning, the dimensions that underpin reflection, and how the convergence between reflection and instructional planning may contribute to pre-service teachers' professional development. To investigate how prospective secondary mathematics teachers design lesson plans using their pedagogical content knowledge, Emre-Akdogan and Yazgan-Sag (2018) study points out three main findings, namely prospective teachers preferred to use student-centred and technology-based teaching activities, they took the students' understandings and prior knowledge into consideration, and they experienced difficulties since their mathematical content knowledge is not in comply with the conceptions included in the curriculum. A study conducted by Fernández (2005) provides an illustration of a reform-oriented program for improving the quality of prospective teachers' preparation through a Micro-teaching Lesson Study (MLS). By this program, the prospective teachers have cyclical experiences to do 'planning-implementing-reflecting-revising' during the program's implementation. This study revealed that participants perceived the experience and its components as beneficial in their development as teachers. In Baldry and Foster (2019) study regarding Lesson Study in Mathematics Initial Teacher Education in England, an important point to be highlighted for future program of prospective mathematics preparation, is the need to do collaborative lesson planning as part of lesson study activities.

Studies previously figure out on lesson planning neither by mathematics teachers nor by prospective mathematics teachers, are not yet focus on exploring the thinking processes regarding both practical as well as theoretical block that underpinned the process of developing lesson plans. From the viewpoint of the Anthropological Theory of the Didactics, mathematical praxeologies play very important roles in exploring the thinking processes and their justifications regarding certain aspects of educational activities (Bosch & Gascón, 2006a, 2014a; Chevallard, 2019a). As exemplified by some studies (Asami-Johansson, 2021; Lundberg & Kilhamn, 2018a), praxeological analyses could be used to explore and identify the thinking processes and their justifications within lesson planning development by mathematics or prospective mathematics teachers. In addition, praxeological analyses could also be implemented in other human actions, as exemplified in several studies (Artigue & Winsløw, 2010; Pansell & Bjorklund Boistrup, 2018; Takeuchi & Shinno, 2020; Wijayanti & Winslow, 2017). This paper aims to explore an Indonesian prospective mathematics teacher's lesson planning and identify its characteristics of mathematical task design from the viewpoint of the anthropological theory of the didactic.

Theoretical Framework

As Chevallard (2006a) explained, ' A praxeology is, in some ways, the basic unit into which one can analyse human action at large.' Proponents of the Anthropological Theory of the Didactic (ATD), introduced first time by Chevallard in the early nineteen eighties, exemplified such human actions as production, diffusion, or acquisition of knowledge (Bosch & Gascón, 2014b) the way knowledge is conceived, modified and transmitted between social institutions (Bosch et al., 2020), mathematical modelling activities (Ärlebäck, 2011), proportional knowledge in school textbooks (Lundberg & Kilhamn, 2018b), and problem-solving oriented-approach (Asami-Johansson, 2011). Lesson planning as an integral part of the instructional activity is a teacher or prospective teacher's action that is potentially analysed using an epistemological approach such as praxeology, including mathematical praxeology (MP). The following will be outlined to determine what, how, and why concerning MP in terms of a prospective teacher's lesson planning activities.

Principles and Key Constructs of Praxeologies

According to Chevallard (2006b) that 'one fundamental principle of the ATD is no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast'. Based on this principle, lesson planning as an integral part of teaching-learning activities accordingly existed as a teacher's action that can be explained, justified, made intelligible, and accounted for in relation to the teaching-learning processes. Another principle of the ATD, according to Chevallard (2006b), is 'no human doing goes unquestioned'. Therefore, the mathematical praxeology of a lesson plan developed by a teacher could be questioned, especially regarding the practical and theoretical blocks of the praxeologies.

As explained by some scholars (Bosch & Gascón, 2006b, 2014b; Chevallard, 2006b, 2019b; Winsløw, 2012), that a praxeology consisted of two main components namely, practical component or praxis block and a theoretical component or logos block. Each of the components is broken down into two elements. The practical block is divided into two elements, namely types of tasks as well as their ways of doing the tasks (a set of techniques) to carry out provided type of the tasks. While the theoretical block is also divided into two elements, namely technology, which is etymologically meaning as discourse of the techniques being used to carry out the tasks, and theory that functioned as a basis and support of the technological discourse used to justify the techniques of solving the tasks. Therefore, a praxeology is an entity formed by four components, namely a type of tasks, a set of techniques, a technological discourse, and a theoretical basis that underpinned the technological discourse used. In term of a mathematics lesson planning, there are two parts of the lesson plan need to be considered: the design of mathematical tasks provided for students to step by step respond the tasks by doing a series of mental actions, constructing ways of thinking, and formulating a way of understanding (Suryadi, 2019; Harel, 2008); and a didactical design that inherently related to each part of the mathematical tasks design. Accordingly, there are two main praxeological analysis need to be adopted, namely mathematical and didactic praxeology analysis.

Mathematical Praxeology Analysis

In terms of a mathematical tasks design, there are two main elements of the design, namely mathematical objects (concepts, theorems, proofs, mathematical problems, or problem solutions) as the goal of mathematics learning process, and a series of mathematical tasks that facilitates the development of students' mental actions, ways of thinking, and ways of understanding in relation to certain mathematical objects (Suryadi, 2019; Harel, 2008). As explained before, that a praxeology is an entity formed by four components, consisting of a type of tasks, a set of techniques, a technological discourse, and a theoretical basis that underpinned the technological discourse used. Consider T is a set of mathematical tasks and ti is an element of T. A mathematical tasks design is a series of tasks in T so that $t1 \wedge t2 \wedge t3 \wedge .. tn\text{-}1 \wedge tn = T$. Every ti in T has its own set of techniques τ, a technological discourse θ, and theoretical discourse Θ that underpinned the technological discourse so that (ti, τ , θ , Θ) will form a praxeology. Accordingly, all praxeologies related to T will eventually form a way of thinking in accordance with the construction of a way of understanding related to certain mathematical object being studied. Therefore, to analyse a mathematics praxeology as a whole (related to a mathematical task design), it is necessary to analyse all of mathematical praxeology being considered, discover and examine their relationships, and figure out the characteristics of all the mathematics praxeology and their relationships related to the mathematical tasks design.

Background and Research Questions

The Bachelor of Mathematics Education curriculum at the Indonesia University of Education adheres to a concurrent system, namely mathematics and education courses are carried out in the same period of time, every semester. However, the system is still structural in nature, in which students gain experience in learning the fields of education and mathematics at the same time. Substantially, the relationship between mathematics and education subjects is not explicitly visible. For example, advanced mathematics courses such as real analysis are not related to educational courses. Thus, a general understanding emerges in the community, that the Mathematics Professional Education Program (PPG) can be followed by mathematics undergraduates in just one year, namely by adding several educational courses. This view is not to blame, because in reality a concurrent system only occurs structurally, not substantially.

To overcome the problem of interpreting the concurrent system which is more structural in nature and trying to change it to be more substantial. Suryadi (2019) have succeeded in formulating a transpositional model that can be applied to various mathematics courses. Based on this study, it is also exemplified that the transpositional model can be applied to educational subjects, including mathematics lesson planning. However, in order to see the clarity of the contribution of the research to be carried out, it is first necessary to elaborate on the fundamental problems associated with the lesson planning course.

The mathematics lesson planning course is one of the compulsory subjects for prospective mathematics teachers. This subject has the characteristics of accumulating a variety of previously acquired knowledge both mathematics and education subjects. In the mathematics courses, both the content and the process of each lecture appear to be independent. If there is a relationship between mathematics courses, the only relation is that one subject to become a prerequisite for another course. The mathematics courses more emphasize the aspects of mastery of the material compared to provide the experience of mathematical thinking in the context of constructing mathematical objects including concepts, rules, proof of rules, mathematical problems, and solving mathematical problems. Meanwhile, learning experiences that emphasize the relationship between mental action, ways of thinking, and ways of understanding, as explained by Harel (2008), currently need to be prioritized in mathematics lectures.

Harel (2008) explains that the meaning of mathematics from a pedagogical point of view is not only related to and focuses on mathematical objects but instead describes the tripartite relationship of three mental activities, which include mental actions (MA), ways of thinking (WoT), and ways of understanding (WoU). According to Suryadi (2019), the mathematical thinking experience that involves the tripartite relationship basically describes the relationship of the thinking process between two different knowledge areas, namely the area of mathematical knowledge as a posteriori knowledge (MA and WoT) and mathematics as a priori knowledge (WoU). Learning experiences that connect these two areas of knowledge in various mathematical studies are very important for prospective educators, especially when it viewed from a transpositional concept (didactic and pedagogical). The learning experience will ultimately lead to and be actualized in the mathematics lesson planning course.

Considering these conditions, the goal of this study, as stated previously, and the theoretical framework of the study, it is necessary to formulate a research question: What is the nature of mathematical tasks designed within a lesson plan developed by a prospective mathematics teacher and its mathematical praxeology in the viewpoints of the anthropological theory of the didactic?

METHODS

As presented previously, this study proposes a research question. In attempting to answer the research question, we used the following methodology:

Subjects of the Study

The subject of the study was a prospective mathematics teacher who had completed her undergraduate studies.

Data Analysis

The data used in the study was a well-documented activity regarding lesson planning by the prospective mathematics teacher in her experiment to conduct a study on mathematics teaching. The document containing those activities includes mathematical task design, didactic and pedagogical considerations related to the task design, and a theoretical framework that underpins factors related to mathematical task design and their didactic preferences. The lesson design, which was analysed, consisted of three parts: introducing and constructing the meaning of a variable; solving linear equations with one variable in two forms, that is, $x \pm b = c$ and $ax \pm b = c$; and solving linear equations in two forms, that is, $ax \pm b = cx$ and $ax \pm b = cx + d$. The lesson plan was oriented to be implemented for grade seven at the Junior Secondary School in Bandung, Indonesia.

In relation to the research question, we used data regarding all task designs developed by the prospective teacher and relevant techniques related to each part of the mathematical tasks, as well as technological-theoretical considerations that derive the development of the mathematical task designs and their relevant techniques related to each of the mathematical tasks. As explained before, these kinds of data were analysed using a mathematical practice approach. By this approach, we analysed all of the mathematical practices being considered, discovered and examined their relationships, and figured out the characteristics of all the mathematical practices and their relationships related to the mathematical task design.

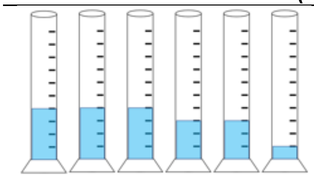
RESULTS AND DISCUSSION

Results of the Study

The results of this study will include the depiction of every praxeology within each mathematical task; the relationship among praxeologies within mathematical tasks; and figure out the characteristics of all the mathematics praxeology as well as their relationships related to the mathematical tasks and relation of didactic-pedagogical predictions and anticipations.

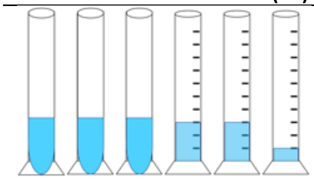
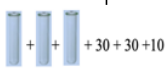
The research question to be answered is ‘What is the nature of mathematical tasks design within a lesson plan developed by a prospective mathematics teacher and its mathematical praxeology in the viewpoints of anthropological theory of the didactic?’. Firstly, we will present the mathematical tasks and their praxeologies. As mention before that the lesson design developed by the prospective teacher being the subject of this study comprises the following parts: introducing and constructing the meaning of a variable; solving linear equations with one variable in two forms that is x ± b = c and ax ± b = c; and solving linear equations in two forms that is ax ± b = cx and ax ± b = cx + d. In terms of the first part of the tasks design, it includes three consecutive tasks T_1, T_2, and T_3. Accordingly, there are three praxeologies need to be analysed. Based on the available data regarding each task, its techniques considered by the subject, theoretical reasoning that underpinned the techniques expected, and theoretical basis that derived all the considerations within the process of developing the lesson plan, it is revealed the following depiction of the mathematical praxeology $P_1(T_1, \tau, \theta, \Theta)$ which T_1 is a set of mathematical tasks, ti is a mathematical task,τ is a technique for solving the task ti,θ Is a technology underlying the technique used, Θ is a theory underlying the technology used . In the praxeology P_1, the subject provided a mathematical task and her expected solutions as predicted students’ responds. According to her explanation, the task T_1 was designed to encourage students to have experience on transitional process from arithmetic to algebraic thinking based on the contextual problem provided. She also highlighted that the theoretical basis for developing the design was derived by the concept of variable, as well as algebraic representation and operation. The following Table 1 consisting of the mathematical task (T_1), the predicted solutions depicted possible techniques that could be used, justification related to the task and its techniques, and the theoretical basis as the main consideration.

Table 1. Mathematical Praxeology $P_1(T_1, \tau, \theta, \Theta)$

Mathematical Tasks (T_1)	Techniques (τ)	Technology (θ)	Theory (Θ)
 If one strip of glasses above containing *10 ml*, how much blue liquid in all glasses and how it carried out.	The expected solutions are as follow. • 40 + 40 + 40 + 30 + 30 + 10 = 190 • (3 x 40) + (2 x 30) + 10 = 190 • 40 + 40 + 40 + 30 + 30 + 10 = (3 x 40) + (2 x 30) + 10 = 190	Facilitating students contextually to do transitional process from arithmetic to algebraic thinking	The concept of variable; algebraic representation and operation

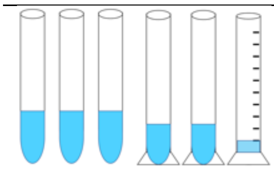
In terms of the second praxeology P_2, the subject constructed a task design T_2 which likely be developed as a modified version from the first task T_1 by changing three glasses with the same unknown amount of blue liquid inside the glasses. The expected solutions considered by the subject were of five kinds as alternative techniques that could be used to solve the problem. The problem and its possible solutions are provided in the hoped that it could encourage students to use their own representation of unknown amount of liquid. This reason was primarily based on theoretical consideration related to algebraic representation and operation. The complete depiction of a mathematical praxeology $P_2(T_2, \tau, \theta, \Theta)$ is presented in the following Table 2.

Table 2. Mathematical Praxeology $P_2(T_2, \tau, \theta, \Theta)$

Mathematical Tasks (T_2)	Techniques (τ)	Technology (θ)	Theory (Θ)
 If one strip of glasses above containing *10 ml* and the first three glasses contained the same amount liquid, how much blue liquid in all glasses and how it carried out.	The expected solutions are as follow. • Using picture to represent the unknown amount of liquid  • Using symbols or points to represent the unknown amount of liquid □+□+□+ 30 + 30 + 10 … + … + … + 30 + 30 + 10 • Using letters to represent the unknown amount of liquid such as the following t + t + t + 30 + 30 + 10 a+ a + a+ 30 + 30 + 10 • Using x in a representation $x + x + x + 30 + 30 + 10$ • Using a technique taken from the first task $2x + (2 \times 30) + 10$	Facilitating students through contextual problem to encourage them to use their own representation of unknown amount of liquid	Algebraic representation and operation

The second task T_2 also was modified by changing two more glasses with the same unknown amount blue liquid but different from the previous three glasses. This problem, that was developed contextually and modified from two previous problems, was designed in an attempt that students can learn from solving previous problems and implement their understanding to solve this problem using proper algebraic representation. As explain and justify by the prospective mathematics students, the problem T_3 as a mathematical task, expected solutions to the problem as techniques expected used, explanation regarding the problem and its alternative solutions as a technology, and the theoretical basis that underpinned all the considerations as the theory, we then present all of these as a mathematical praxeology $P_3(T_3, \tau, \theta, \Theta)$ (Table 3).

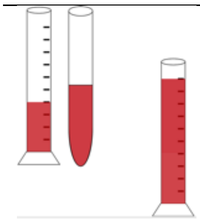
Table 3. Mathematical Praxeology $P_3(T_3, \tau, \theta, \Theta)$

Mathematical Tasks (T_3)	Techniques (τ)	Technology (θ)	Theory (Θ)
 If the first three glasses containing the same amount liquid, the next two glasses containing the same amount liquid, and one strip containing *10 ml*, how much blue	The expected solutions are as follow. • $x + x + x + y + y + 10$ • $3x + 2y + 10$	Although the problem was developed contextually, it was hoped that students could learn from solving previous two problems and implement their understanding to solve this problem using proper	Algebraic representation and operation

Mathematical Tasks (T_3)	Techniques (τ)	Technology (θ)	Theory (Θ)
liquid in all glasses and how it carried out.		algebraic representation	

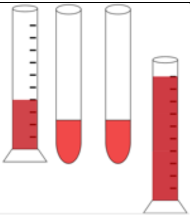
Regarding the second part of the mathematical tasks, two tasks need to be considered, T_4 and T_5. Accordingly, two praxeologies need to be analysed. Based on the available data regarding each task, its techniques considered by the prospective teacher, theoretical reasoning that underpinned the techniques expected, and theoretical basis that derived all the considerations within the process of developing the lesson plan, it is points out the following depiction of the mathematical praxeology $P_4(T_4, \tau, \theta, \Theta)$ and $P_5(T_5, \tau, \theta, \Theta)$. In the praxeology P_4, the prospective teacher provided a mathematical task and her expected solutions as predicted students' responds. According to her explanation, the task T_4 was designed to encourage students to have experience on transitional process from arithmetic to algebraic thinking based on the contextual problem provided. She also highlighted that the theoretical basis for developing the design was derived by the concept of variable, as well as algebraic representation and operation. The following Table 4 consists of the mathematical task (T_4), the predicted solutions depicted possible techniques that could be used, justification related to the task and its techniques, and the theoretical basis as the primary consideration.

Table 4. Mathematical Praxeology $P_4(T_4, \tau, \theta, \Theta)$

Mathematical Tasks (T_4)	Techniques (τ)	Technology (θ)	Theory (Θ)
 If one strip of glasses above containing *10 ml*, and if the liquid from two glasses at the left side the same amount with that of the third glass, how much red liquid in the centre glass and how it carried out.	The expected solutions are as follow. • Using number facts • Using cover up approach • Using counting techniques • Working backward • Trial and error substitution • Change the order • Adding the same number to each side of equation	Facilitating students contextually to use previous experiences to solve this problem with various possible techniques of algebraic operations	Algebraic representation and operation

T_5 was developed by the prospective teacher in an attempt to strengthen learning experience attained from solving the previous problem by modified the glass at the centre into two glasses containing the same unknown amount red liquid. In the praxeology P_5, the prospective teacher provided a mathematical task and her expected solutions as predicted students' responds. According to her explanation, the task T_5 was designed to encourage students to have experience on transitional process from arithmetic to algebraic thinking based on the contextual problem provided. She also highlighted that the theoretical basis for developing the design was derived by the concept of variable, as well as algebraic representation and operation. The following Table 5 consisting of the mathematical task (T_5), the predicted solutions depicted possible techniques that could be used, justification related to the task and its techniques, and the theoretical basis as the main consideration.

Table 5. Mathematical Praxeology $P_5(T_5, \tau, \theta, \Theta)$.

Mathematical Tasks (T_5)	Techniques (τ)	Technology (θ)	Theory (Θ)
 If one strip of glasses above containing *10 ml* and the first three glasses (from the left) contained the same amount liquid with that of the fourth, how much red liquid in each of the glasses at the centre and how it carried out.	The expected solutions are as follow. • $4 + x + x = 10$ $2x = 10 - 4$ $2x = 6.$ $x = 6/2 = 3$ • $4 + 2x = 10$ $2x = 10 - 4$ $2x = 6$ $x = 6/2 = 3$	Facilitating students to represent a contextual problem algebraically and solve the algebraic equation using appropriate rules of algebraic operations	Algebraic representation, operation, and equation

The third part of the mathematical design just include a mathematical tasks, T_6. Accordingly, only one praxeology needs to be analysed. Based on the available data regarding the task, its techniques considered by the prospective teacher, theoretical reasoning that underpinned the techniques expected, and theoretical basis that derived all the considerations within the process of developing the lesson plan, it is points out the following depiction of the mathematical praxeology $P_6(T_6, \tau, \theta, \Theta)$. In the praxeology P_6, the prospective teacher provided a mathematical task and her expected solutions as predicted students' responds. According to her explanation, the task T_6 was designed to encourage students to have experience on using all previous learning experiences regarding transitional process from arithmetic to algebraic thinking, in a contextual problem solving. She also highlighted that the theoretical basis for developing the design was derived by the concept of variable, as well as algebraic representation and operation. The following Table 6 consisting of the mathematical task (T_6), the predicted solutions depicted possible techniques that could be used, justification related to the task and its techniques, and the theoretical basis as the main consideration.

Table 6. Mathematical Praxeology $P_6(T_6, \tau, \theta, \Theta)$

Mathematical Tasks (T_6)	Techniques (τ)	Technology (θ)	Theory (Θ)
 In the first week of school semester, Marsha and Nobita upload their photograph in her own Instagram. Marsha put 9 photos and Nobita put six photos. If every week Marsha add 2 photos, and Nobita add 3 photos, in what week the number of their photos will be the same?	The expected solutions are as follow. • Using two tables Week / Marsha: 1 / 11; 2 / 13; 3 / 15; 4 / 17; 5 / 19; 6 / 21 Week / Nobita: 1 / 6; 2 / 9; 3 / 12; 4 / 15; 5 / 18; 6 / 21 • Using one table Week / Marsha / Nobita: 1 / 11 / 6; 2 / 13 / 9; 3 / 15 / 12; 4 / 17 / 15; 5 / 19 / 18; 6 / 21 / 21 • Using algebraic equation $9 + 2x = 3 + 3x$ $9 - 3 = 3x - 2x$ $6 = x$	Facilitating students to use all previous experiences regarding representation of contextual problems algebraically and to solve a contextual problem solving	Algebraic representation, operation, and linear equation

Mathematical Tasks (T_6)	Techniques (τ)	Technology (θ)	Theory (Θ)
	$9+2x=3+3x$ $9+2x-9=3+3x-9$ $2x=-6+3x$ $2x-2x=-6+3x-2x$ $0=-6+x$ $6=x$ $9+2x=3+3x$ $9+2x-3=3+3x-3$ $6+2x=3x$ $6+2x-2x=3x-2x$ $6=x$		

Looking deeply all the previous mathematical praxeologies and their relationship, we could point out the following characteristics:

- The first praxeology depicted a simple contextual task that potentially facilitates students to use previous learning experience, so that they could represent the task in early algebraic ways. This situation can provide students to experience a transitional process from arithmetic to algebraic thinking.
- The second praxeology comprises a modified mathematical task from the first problem, so that the students can go through as they have experience in the first problem. Meanwhile, since there are three glasses with unknown amount of blue liquid inside the glasses, the students need to do such different things as using pictures, geometric figures, question mark, or something else to represent something that unknown. This problem plays very important foundation to introduce the meaning of variable in a mathematical expression.
- The third praxeology depicted a modified problem from the two previous problems. By these problems, the students can strengthen the previous experience through using two different variables in one mathematical expression.
- The fourth and the fifth praxeology figured out the possibilities for the students to implement previous experiences in solving very simple algebraic equation from contextual problems.
- In the sixth praxeology, the students are challenged to solve more difficult problem than the previous ones, so that they have good experience to do a nonroutine mathematical problem solving by connecting several experiences in solving a series of mathematical problems that totally formed a mathematical learning trajectory.

The students' experiences of doing a series of interrelated problems as already designed by the prospective mathematics teacher, depicted a very important characteristic of a mathematical learning trajectory as explained and exemplified in several studies (Clements et al., 2004; Clements & Sarama, 2004; Gravemeijer et al., 2003; Szilágyi et al., 2013).

Discussion

All the findings related to all mathematical praxeologies are inextricably linked to the process of knowledge constructions through teaching and learning activities proposed. As explained in studies of theory of knowledge or epistemology, that knowledge is defined as justified true belief (Audi, 2010; Pritchard, 2023). Accordingly, the processes of knowledge construction are also related to how things could be considered as something true, what are the sources of beliefs that something being considered is true, and how to justify that true belief. True beliefs can be constructed through such causes as testimony by experts, observation perceptually, memorial analyses using available knowledge or relevant theoretical framework, and introspective consciousness. Interrelated true belief coming from several sources could be analysed to form justification related to corresponding true belief (Audi, 2010; Chisholm et al., 1989; David, 2004; Pritchard, 2023; Rice, 1970). In term of the study results regarding mathematical praxeology, which is related to mathematical knowledge constructions, it is points out such important aspects needed to be discussed as the model of mathematical tasks design, predicted students' solutions to each task of the design, theoretical arguments of related tasks as well as their predicted techniques, and the theoretical basis of all parts of the lesson design activities.

The tasks design which includes six types of mathematical problems depicted a structural and functional relationship in the sense of starting from developing arithmetic-algebraic representation skills, using the skills to represent more difficult problem, implementing the skills in solving linear algebraic equations, and implementing the skills to solve more difficult contextual problem related to linear algebraic equation. It is likely, that the design could facilitate the students to develop their justification based on perceptual, memorial, and introspective consciousness aspects related to corresponding mathematical knowledge being developed. Having opportunities for the students to work independently could bring up genuine ideas related to strategies of solving the problems. When various ideas come up from the students, it seems that they will also have opportunity to compare and validate their solutions, to refine their ways of thinking, and to reconstruct their solutions to be more favourable and understandable. Comparing and validating their own thinking and solutions to others are very important in learning process as suggested by some studies (Chapin et al., 2009; Rittle-Johnson et al., 2019; Stein et al., 2008). Refining students' ways of thinking and reconstructing their solutions are also very important regarding the solutions to be more reasonable and justified, as addressed in some studies (Baker, 2019; Swanson, 2019; Yeager et al., 2019).

Predicted students' solutions to each task of the design and their anticipations are imperative and influential when roaming the class (Vale et al., 2019). Their study also suggested that including the prediction and anticipation activities in lesson planning, could help teachers to do better possibilities on facilitating students' learning such as noticing and selecting students' solutions for hole-class discussion; eliciting, supporting and challenging prompts for each anticipated solution enabled a teacher to attend to misconceptions or misinterpretations of the task and to elicit students' reasoning when interacting individually with students and during the whole-class discussion of solutions. Compared to the main results of their study, the technological discourses of each mathematical praxeology developed by the prospective teacher are likely reasonable and potential to facilitate students experiencing better learning and better opportunities to be more independent learners especially in terms of constructing new mathematical knowledge.

This study also points out such important aspects needed to be highlighted as the model of didactic tasks design, their technological discourse, and theoretical basis of the discourses. As depicted previously, in order to broaden better possibilities on facilitating the process of students learning, the prospective teacher provided predicted students' responses to each of mathematical task and their didactic and pedagogical anticipations. The technological discourses provided for all didactic tasks, and their didactical-pedagogical anticipations are all interrelated so that she could facilitate any possible ways of students thinking in forming certain mathematical object being studied. The characteristics of technological discourses proposed by the prospective teacher are in line with some results of previous studies. Lewis et al. (2019) study addressed that predictions provide opportunity for teachers to check out how well they know individual learners. It also enables a teacher to address how each student will move from their initial response and grasp the key ideas of the lesson. Llinares et al. (2016) study revealed that good predictions could help teachers to provide better anticipations concerning students' ways of thinking. Yilmaz et al. (2017) study highlighted that anticipating and interpreting students' thinking is among teachers' tasks in which teachers must generate hypotheses about how students' mathematical thinking could be developed.

CONCLUSION

The lesson plan developed by the prospective teacher being studied could facilitate students actively involved in constructing mathematical knowledge, including the concept of variables as well as algebraic representation, operation, and linear equations. From the view of the anthropological theory of the didactics, it is pointed out that the epistemic nature of the mathematical tasks within the lesson plan could facilitate students to independently use their perceptual and memorial potentials to be involved in situations of action, formulation, and validation. Due to the epistemic learning process, the impact of their independent process of mental actions, ways of thinking, and ways of understanding, students have opportunities to formulate their own new mathematical objects.

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