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Realistic Mathematics Engineering for improving elementary school students' mathematical literacy

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Abstract

Mastery of mathematical literacy is essential for developing life skills in the 21st century. Mathematical literacy is even more critical in elementary schools as it forms the basis for mastery at the junior and senior high school levels. Elementary school students differ in their characteristics from students at the higher education level. They, therefore, require an appropriate learning model to improve their mathematical literacy. This research aims to develop a learning model, termed Realistic Mathematics Engineering (RMEng), that combines the Realistic Mathematics Education approach with the steps of the Engineering Design Process and determines the model's effectiveness. The model was validated by seven experts from three universities in Indonesia and had an Aiken validity index value of 0.786, indicating that it was valid. The RMEng model contains the following steps: understanding realistic problems, solving problems in informal ways, developing formal mathematics, developing products, and drilling. Discussions and presentations can be incorporated into each of these five steps. The RMEng model was subjected to the stages of preliminary and main field testing and was revised based on the suggestions of teachers and observers. Through experimental research compared to a control group, the RMEng model was proven more effective in increasing elementary school students' mathematical literacy at the 0.000 significance level.

Keywords: Engineering Design Process, Elementary School, Mathematical Literacy, Realistic Mathematics Education, Realistic Mathematics Engineering

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Mathematical literacy is essential to build 21st-century skills (Rizki & Priatna, 2019). It serves as a bridge connecting mathematics with its application in daily life. Literacy instruction in science and math in elementary education lays a substantial foundation for subsequent literacy development in middle and high school (Yang et al., 2019). However, the way to teach mathematical literacy remains an ongoing issue within the mathematics education research community (Haara, 2018; Sfard, 2014). Three main challenges have been identified regarding the learning of mathematical literacy in schools: (1) neither researchers nor teachers are certain about the action needed to develop students' mathematical literacy, (2) special efforts to work directly with mathematical literacy through mathematics alone are not successful, and (3) teaching for mathematical literacy requires efforts beyond traditional mathematics teaching (Haara et al., 2017).

The Programme for International Student Assessment (PISA) 2012 defined mathematical literacy as an individual's ability to formulate, employ, and interpret mathematics in various contexts (OECD, 2013). It included mathematical reasoning and employed mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. Aligned with PISA, the Indonesian Center for Assessment and Learning (Pusmenjar, 2020a) equated numeracy with mathematical literacy and defined it as the ability to think using mathematical facts, concepts, procedures, and tools to solve everyday problems in various contexts relevant to individuals as citizens of Indonesia and the world. In this research, the mathematical literacy of elementary school students concerns their ability to formulate, employ, and interpret mathematics to solve everyday problems in various contexts relevant to their age and stage of development.

The Trends in International Mathematics and Science Study (TIMSS) assessed the mathematical abilities of grade 4 students. In 2015, Indonesian students achieved an average score of 397, far below the international average score of 505 (Mullis et al., 2015). Meanwhile, the results of PISA 2018 showed that the math literacy score of Indonesian students was 379, far below the average OECD score of 489 (Schleicher, 2019). PISA 2018 assessed the cumulative education and learning results of students aged 15, with the students in the sample also required to have been enrolled in educational institutions in grade 7 or higher. The low mathematical literacy of junior and senior high school students does not exclude the possibility of the impact of less effective learning in elementary schools (Firdaus et al., 2017). Therefore, students should be well-trained in basic mathematical concepts at the elementary level.

Our previous research (Nurmasari et al., 2022) highlighted low mathematical literacy in primary school students (n = 112). The average score on the mathematical literacy test was 44.036 out of 100. A total of 56% of the students scored below 46, and only 17% achieved more than 65. According to the grades 5 and 6 mathematics teachers (n = 20), the students experienced greater difficulty with word problems than with mathematical problems that were devoid of context. The questionnaire results additionally showed that 67% of the teachers focused more on conveying material in the curriculum than developing students' reasoning abilities, which may also have contributed to low mathematical literacy.

The learning process and student achievement are strongly influenced by the type of assessment used to test students' abilities (Jürges et al., 2005; Gibbs, 2010). The Indonesian Education Assessment Center has stated that the competencies assessed in the grade 5 school examination should be limited to the ability to solve questions according to mathematical concepts and not extend to their application to everyday life (Pusmenjar, 2020a). At the same time, the results from PISA and TIMSS provide essential data that concern the government with regard to improving the matter and replacing the national exam with a minimum competency assessment that measures two types of literacy: reading literacy and mathematical literacy (or numeracy). Both types of literacy are included because all students need basic abilities or competencies, regardless of their profession and future careers (Pusmenjar, 2020b).

We conducted a simple survey in August 2020 to determine what needed to be developed to improve mathematical literacy, and 75% of the teacher respondents identified the learning model. Among their reasons was that learning models provide the most comprehensive opportunities for students to express their mathematical ideas, develop their thinking skills, and solve problems given by the teacher. Teachers believe that learning models capable of improving mathematical literacy will introduce mathematical issues that are closely related to everyday life and be realistic, contextual, and based on problem-solving.

From the results of preliminary studies and the literature review, Realistic Mathematics Education (RME) has been identified as a promising method of improving mathematical literacy. In RME, learning mathematics means doing mathematics, based on the fundamental component of solving everyday-life (contextual) problems. Another central principle is that students must be able to reinvent mathematical concepts, while the teaching and learning process must be very interactive (Fauzan et al., 2002). Fauzan et al.'s research illustrated that the RME approach could be introduced in elementary schools in Indonesia. It may also overcome several problems, notably with regard to transforming the classroom atmosphere, and provides guidelines on developing and implementing quality curriculum materials for teaching mathematics. Prior studies have found that students who studied with RME performed better than those who learned through conventional methods (Apsari et al., 2023; Laurens et al., 2018).

RME has three main characteristics: (1) the activities and contributions of the students themselves, (2) are related to reality and focus on application, and (3) the level of understanding (Van den Heuvel-Panhuizen, 1996). In RME, students use contexts and models and proceed through various levels of mathematization to develop their mathematics. As such, the type of mathematics studied is not that of a closed system, but rather mathematics as an activity, a mathematical processing of reality, and even, if possible, the mathematization of mathematics itself (Freudenthal, 1968). Mathematization is an organizing activity that refers to the essence of mathematical activity, which is directed at acquiring factual knowledge, learning concepts, acquiring skills, and using language and other organizing skills in solving problems, whether placed in a mathematical context or not (Treffers, 1987). In mathematization, concrete or non-mathematical issues are transformed into mathematical problems.

Aspects of RME support the development of students' understanding (Van den Heuvel-Panhuizen, 2001). First, it emphasizes progressive schematization over progressive complexity. Children can immediately work with large numbers depending on the context of the problem. Second, context is used as a means to grow. An appropriate context is one that can be developed into a model to support mathematical thinking. Third, models built from context become the basis of progress. Such models serve as a bridge for students to progress from informal to more formal mathematical knowledge. Fourth, interactive learning facilitates students' understanding. By listening to and discussing others' findings, students obtain ideas for improving their strategies.

From the description above, it can be concluded that RME is a mathematics education approach that emphasizes mathematics as a human activity, emphasizes process over results, and involves authentic problems and problem-solving that can be interpreted realistically according to the context. Realistic in this context denotes that issues and problem-solving can be understood experientially in students' minds. De Lange (1996) stated that the development of mathematical concepts and ideas from a real-world starting point can be termed "Conceptual Mathematization and described as a cycle comprising the following stages: (1) the real world, (2) mathematization and reflection, (3) abstraction and formalization, and (4) mathematization in the application. This cycle reflects the mathematizing process in RME and was used in building the RMEng learning model syntax.

Elementary school students are in the concrete operational stage (Piaget, 1964; Snowman et al., 2012). Here, while children can logically solve problems, abstract mathematics is often complicated for them to understand. Learning activities involving actual or concrete objects are therefore required. Cognitive abilities and students' imaginations will be trained through the application of mathematics to design and solve problems, which may positively impact mathematical literacy. Students will learn to apply the steps that engineers follow when designing a product or a material, which are often referred to as the Engineering Design Process (EDP). Based on a meta-analysis study by Fidai, et al. (2020), EDP was proven to have a positive and statistically significant effect size on students' mathematics achievement, especially in elementary schools. With the steps of EDP, students are helped to apply abstract mathematics in a concrete design. EDP provide students to learn mathematics by applying it in real life, not only playing numbers but doing something about their mathematical calculations (Firdaus et al., 2020).

In education, design activity is increasingly recognized as a means by which learners can learn and experience and as a general framework for school education (Li et al., 2019). Engineering assignments offer opportunities for students to apply and expand their thinking to solve problems and achieve goals (Brakoniecki et al., 2016). Engineering design and mathematical modeling are the main tools or techniques in science, technology, and innovation (Nwulu & Muchie, 2019). Engineering design is concerned with creating functional and innovative products and processes, whereas mathematical modeling seeks to utilize mathematical principles and concepts to describe and control real-world phenomena.

The engineering design framework offers a structure that demands fundamental skills and processes driven by national standards for mathematics and science (NCTM, 1989). These include problem-solving, the use of reasoning and proof, understanding and recognizing the relationship between mathematics and science, contexts outside mathematics and science, the use of inquiry and technological design in science, and the use of communication and representation to model physical understanding (Ortiz, 2008). EDP can thus be employed as a pedagogical strategy in which students follow steps to create the most effective solutions that are iteratively tested and justified by mathematical and scientific concepts (Hafiz & Ayop, 2019). While the definition of EDP varies widely, the education community has identified several main characteristics: (1) the design process begins with a definition of the problem; (2) a design problem has many possible solutions, and engineers must identify a systematic approach to selecting one; (3) design requires modeling and analysis, and (4) the design process is iterative (Berland et al., 2014).

The EDP stages used to develop the RMEng learning model refer to those proposed by ITEA (2007), namely: (1) defining a problem, (2) generating ideas, (3) selecting a solution, (4) testing the solution, (5) making an item, (6) testing the item, and (7) presenting the results. Steps 1–6 can be repeated until a solution or product meets the intended needs. In mathematical literacy, the problem identification stage helps students to formulate real world problems in a mathematical context. The steps of generating ideas, choosing a solution, testing the solution, and making a product provide opportunities for students to apply facts, procedures, and mathematical concepts in solving concrete problems. Testing and presenting the product helps students interpret mathematical results in the real world. Through these substantial activities, learning becomes more meaningful for students. In this study, the EDP steps will be synthesized using the RME approach with the expectation that this will help to increase mathematical literacy.

The RMEng learning model is essential to develop. First, research on RME that focused on increasing mathematical literacy in elementary schools was still rare. Previous experimental research related to RME used varying dependent variables, for example, mathematics learning achievement (Mulbar & Zaki, 2018), reasoning ability (Saleh et al., 2018), mathematical representation ability (Fauzan et al., 2018), and mathematical literacy in junior high school (Fauzana et al., 2020). Second, although there were several studies on RME, the learning steps still vary. RMEng makes the mathematization process in RME into more concrete learning steps. Third, the combination with EDP will further facilitate students to formulate, employ, and interpret mathematics back into the real world. In addition, this combination makes RMEng more suitable for elementary school students who are at the concrete operational stage.

Various researchers have previously developed RME-based learning models (Rusdi, Fauzan, et al., 2020; Sumirattana et al., 2017; Wahyudi et al., 2017); however, they have yet to focus on increasing mathematical literacy. Developing a learning model that integrates the RME approach with EDP in mathematics teaching and learning is challenging. However, this integration is expected to affect mathematical literacy significantly. This research aims to develop a new Realistic Mathematics Engineering (RMEng) model that combines RME and EDP. The model development consists of creating a theoretical product, validating the product, improving the product based on expert suggestions, conducting preliminary field testing, improving the product based on preliminary field testing, conducting main field testing, and improving the product based on the main field-testing result. It also aims to determine the effectiveness of the RMEng learning model in increasing students' mathematical literacy in elementary schools. The effectiveness is determined based on the results of the product testing stage.

METHODS

Research Steps

The RMEng learning model was developed through the following stages of research and development design: (1) preliminary study, (2) product development, (3) product testing, and (4) product dissemination and implementation (Budiyono, 2019). Before the product development, a preliminary study was conducted comprising initial research in the form of a mathematical literacy test (Nurmasari et al., 2022), a literature study on mathematical literacy (Nurmasari et al., 2023), a literature study on learning approaches or steps to increase mathematical literacy, and a needs analysis via questionnaires and interviews with teachers. This article focuses on describing and discussing the product development and testing stages with the steps shown in Figure 1.

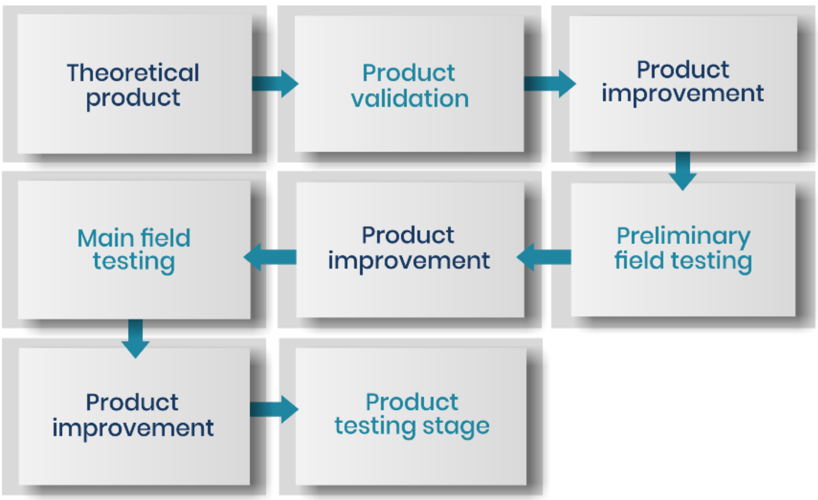


Figure 1. Product development and testing steps

- Theoretical product
  The authors created the theoretical product through a review of the literature. It comprised the RMEng learning model draft and lesson plan. The learning model included syntax, social systems, principles of reaction, support systems, and instructional and accompaniment impacts (Joyce et al., 2016). The lesson plan was created to apply or test the RMEng learning model. The lesson plan created in this

research was still limited to one content in sixth grade, namely geometry. It comprised two cycles; the first cycle contained nine meetings, with six meetings for the second cycle. The content in Cycle I consisted of spatial geometry (prism, cylinder, pyramid, cone, sphere), with compound geometric shapes in Cycle II.

- Product validation
  Seven experts validated the learning model draft and the lesson plan. The validation process lasted for three months. The experts comprised one professor in mathematics education, one professor in curriculum and learning, three associate and assistant professors in mathematics education, and two associate and assistant professors in pedagogy. The seven experts were lecturers from three universities in Indonesia who were selected purposively based on their expertise and had a Google Scholar H-index of at least 6. The scores for each item ranged from five (very appropriate) to one (inappropriate). Expert judgment was used to determine the Aiken validity index. According to Aiken's table (1985), the statistical significance of the Aiken validity index for a questionnaire with five options, at a significance level of 0.05, is 0.75. Thus, the product was deemed valid if the index validity was ≥ 0.75. Valid means that the learning model was developed based on development theory and is appropriate to the demands of the model characteristics. Furthermore, expert comments were adopted to improve the learning model.
- Product improvement
  The authors revised the learning model draft and lesson plan and then discussed it with each expert until they all agreed. The revision process lasted for two months.
- Preliminary field testing
  Following revision to the RMEng learning model based on expert input, preliminary field testing was conducted to obtain suggestions from teachers and observers in order to improve the model and determine its effectiveness. This took place in grade 6 of an elementary school with 24 students as the subjects. The One Group Pretest-Posttest Design was used as the research design for the preliminary field testing. The research data were analyzed statistically using the sample-related t-test.
- Product improvement
  After each meeting in the preliminary field testing, a discussion was held with the teachers and observers to improve the learning model draft and lesson plan. Revisions were made immediately based on the results of the discussion.
- Main field testing
  The main field test was carried out following revisions to the RMEng learning model based on the suggestions from teachers and observers. The testing was performed in three elementary schools, with 50 students in grade 6. The One Group Pretest-Posttest Design was again employed as the research design. The research data were analyzed statistically with the sample-related t-test.
- Product improvement
  After completing the main field testing, a Forum Group Discussion (FGD) was held between the researchers, teachers, and observers from the three schools. The results of the FGD were used to improve the draft learning model.
- Product testing stage
  Samples for the product testing stage were taken from the population using cluster random sampling techniques. First, from 24 districts, two were randomly selected. One school was then selected

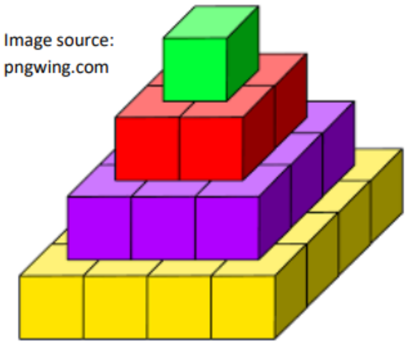
randomly from each district. One class was the control group (n = 15), and one class at another school was the experimental group (n = 30). The product testing stage aimed to determine whether the learning model developed was more effective than the existing one. The research design for this stage was the Control Group Pretest-Posttest Design. The results from the product testing stage were analyzed statistically with the independent sample t-test.

Research Instrument

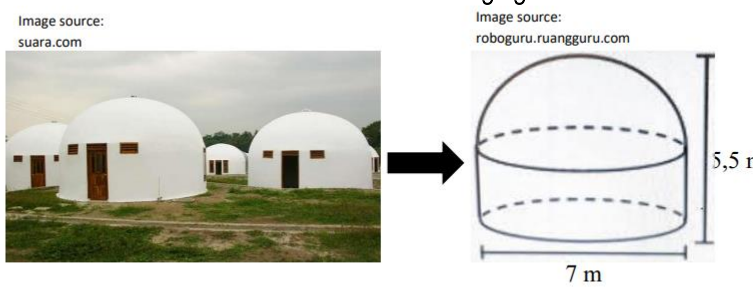
The pretest and posttest instruments used in the field testing were validated by six experts, comprising three mathematicians or learning mathematics experts, two evaluation experts, and one language expert. The scores from five experts (mathematics and evaluation experts) were used to calculate the Aiken validity index. Items were said to be valid if V Aiken ≥ 0.8 (according to the Aiken table). Based on the calculation results, all items were valid. The test instruments validated and revised according to the expert input were tested on 42 grade 6 students from two elementary schools. Based on the analysis results using Iteman software, 24 questions were taken from 30 questions that met the criteria of $r_{pbis} \geq 0.3$ and $0.30 \leq P \leq 0.70$. The results of the analysis with Iteman Version 4.3 software showed that the reliability coefficient of the mathematical literacy test instrument for spatial material (Alpha/KR-20 value) was 0.838. The reliability coefficient was 0.838 ≥ 0.70, meeting the specified criteria.

Based on the cognitive process of mathematical literacy, the pretest and posttest instruments comprised the following proportions of questions: 21% on the formulate aspect, 50% on the employ aspect, and 29% on the interpret aspect. For the context, they were 67% personal, 25% socio-cultural, and 8% scientific. Examples of the questions used are shown in Table 1.

Table 1. Examples of questions in the pretest and posttest

Cognitive Process	Content	Context	Question
Formulate	Compound geometric shapes	Personal	My little brother is playing assembling a tower of small cubes with a side length of 1 cm for each cube as shown in the following picture.  The volume of the tower is ... cm^3. a. $(4 + 3 + 2 + 1)$ b. $\big((4 \times 4) + (3 \times 3) + (2 \times 2) + 1\big)$ c. $(4 \times 3 \times 2 \times 1)$ d. $\big((4 \times 4) \times (3 \times 3) \times (2 \times 2) \times 1\big)$
Employ	Compound geometric shapes	Scientific	The dome house has the appearance of an igloo that resembles a half sphere. The dome house design is built with integrated walls and roofs so that it has stronger resistance to earthquakes. An architect

designed the dome house, which is a combination of a tube and a half ball with the dimensions shown in the following figure.



The entire outer wall of the dome house will be painted white. If the surface area of the doors and windows is 2 m^2, then the surface area of the wall to be painted is … m^2.

a. 42
b. 75
c. 119
d. 158

Interpret	Cylinders	Socio-cultural	A factory produces sterile milk with several package sizes. The factory plans to develop tubular packaging. The volume of each product is shown in the following table. (1 ml = 1 cm^3)

No.	Product	Volume (ml)
1	A	150
2	B	200
3	C	500
4	D	1,000

An example of a suitable packaging size for a product with a volume of 200 ml is ….

a. diameter of 4 cm and height of 16 cm
b. diameter of 6 cm and height of 16 cm
c. diameter of 6 cm and height of 18 cm
d. diameter of 7 cm and height of 18 cm

RESULTS AND DISCUSSION

Product Development Stage

The mathematical literacy of elementary students in Indonesia may be nurtured by reforming the strategy and practices of teaching and learning in class. The RMEng learning model was developed based on the literature review to improve mathematical literacy. A matrix of the relationship between the syntax of the RMEng learning model, RME, EDP, and mathematical literacy is shown in Figure 2.

Students will likely be trained to formulate situations mathematically by identifying realistic problems and solving them (*formulate*). The steps of solving problems, developing formal mathematics, and developing products facilitate students' application of mathematical concepts, facts, procedures, and reasoning in solving problems (*employ*). Developing products makes learning more meaningful and will ensure it remains in students' long-term memory. Students can interpret, apply, and evaluate mathematical results back into the real world (interpret) at this stage. The problem practice and discussion stages are expected to strengthen mathematical literacy in all three aspects: formulate, employ, and interpret.

RMEng has covered all stages of the RME cycle. The first stage in RMEng is for students to understand realistic problems. Realistic problems derive from the real world, the constantly developing world in which students live (Freudenthal, 2002). The first stage of RMEng thus corresponds to the first stage of the RME cycle, which is the real world. The second stage of RMEng, solving problems in informal ways, corresponds to the stage of mathematizing and reflections in RME. At the problem-solving stage, students need a process of horizontal mathematization and reflection on how the solutions are achieved. The third stage of RMEng, namely developing formal mathematics, follows the abstraction and formalization stages of RME. At the stage of formal mathematics development, an abstraction process occurs, and students move from more concrete to more abstract situations (Filloy & Sutherland, 1996). At this stage, they also progress from solving problems informally to a more formal problem-solving method (formalization). The fourth and fifth stages of RMEng, developing the product and drilling, respectively, correspond to the mathematizing in the application stage in the RME cycle. Presentations or discussions are integral to each stage of RME (principle of interactivity) and can also be carried out at each stage of learning in RMEng.

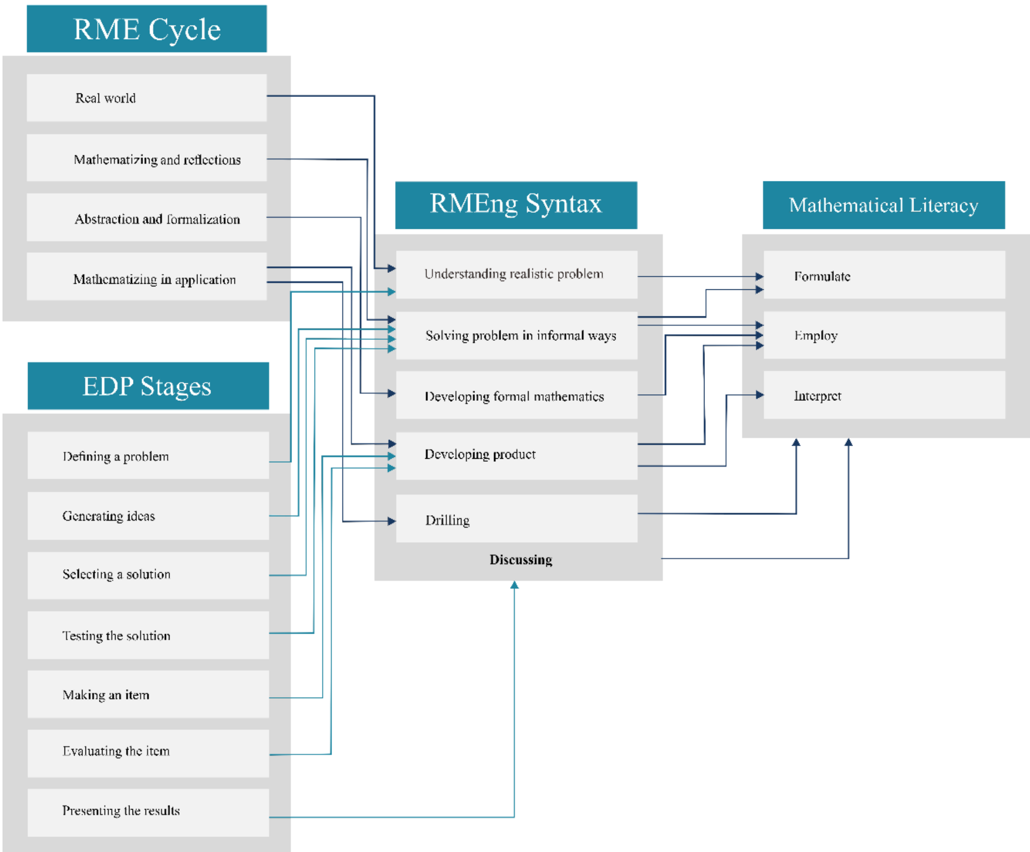


Figure 2. RMEng development framework

The first stage of RMEng, understanding realistic problems, corresponds to the first step of EDP, namely defining or understanding the problem. The second stage of RMEng, solving problems in informal ways, aligns with the EDP steps of generating ideas, selecting a solution, and testing the solution. The fourth stage of RMEng, developing the product, corresponds to the EDP stage of making and evaluating the item. Discussions and presentations can be included in each step of the RMEng, while the stage of presenting results in EDP is included in the RMEng discussion stage. So, the three phases of RMEng have covered all the steps of the EDP. Two phases of RMEng, namely developing formal mathematics and drilling, are adopted from the RME approach.A draft of the learning model was prepared based on the development framework and then assessed by seven experts. The results of the expert assessment are shown in Table 2.

Table 2. The results of the RMEng learning model assessment by experts (maximum score = 5)

No.	Study Criteria	Average Score
Syntax		4.04
1.	The syntax conforms to the principles of the RME approach.	4.29
2.	The syntax contains EDP steps.	3.71
3.	The syntax is appropriate for students' development.	4.29
4.	The syntax helps students achieve new knowledge, skills, and self-understanding.	4.14
5.	The syntax is easy for teachers to understand.	3.86
6.	The syntax helps students learn how to learn.	3.86
7.	The syntax helps learners take responsibility for learning and supports their efforts.	4.14
Social System		4.43
1.	Learner-centered learning.	4.43
2.	Teacher's role as a facilitator.	4.43
Principles of Reaction		4.33
1.	Teachers' responses to students conform to the principles of the RME approach.	4.40
2.	Students are given the broadest possible scope to develop problem-solving procedures.	4.40
3.	Students are allowed to improve the product to match their expectations.	4.20
Support System		4.20
1.	Support system appropriate for the syntax of the learning model.	4.20
2.	Complete support system appropriate for the need to apply the learning model.	4.20
Impact of Learning and Accompaniment		3.90
1.	Instructional impact appropriate for the learning model developed.	4.00
2.	Accompaniment impact appropriate for the developed learning syntax.	3.80
Average		4.15

The average score of the expert assessment was 4.15, indicating that the learning model was in the appropriate category. The assessment was then used to calculate the Aiken validity index, which was 0.786, indicating that the learning model was valid. However, this was obtained before the learning model was repaired or revised. The experts provided feedback on each component, sub-component, and the entire learning model. Discussions with validators were also conducted via email and WhatsApp, and the validation process lasted for three months. Table 3 summarizes the feedback from the experts on the overall learning model.

Table 3. Summary of experts' advice on the RMEng learning model draft

No.	Experts' Suggestions
1.	Looking at the existing model, if you follow the syntax, the teacher can implement it. However, they will need help with the design evaluation process and making constraints because this needs to be visible in the designs that have been created. In addition, the vertical mathematization process will be quite challenging for teachers because mathematical concepts in everyday life must be transformed into mathematical models. This is part of the EDP, which is more often related to solving concrete problems. From the steps that have been made, the relationship between the RMEng model and the increase in students' mathematical literacy still needs to be clarified. This can be studied further by looking

for papers on increasing mathematical literacy to see the relationship more explicitly. In particular, how do students understand questions according to specific contexts?

2. The theoretical product is relatively easy to use. Still, when the teacher sees the lesson plan in the RMEng syntax section, they will likely find it difficult because the syntax editor needs to be consistent. Example: how to apply the syntax "Understanding realistic problems, solving problems"? The syntax language should be in the form of rational student activities.
In the prototype, the researcher intended for the model to impact literacy, but literacy should have been mentioned in the lesson plan. In the lesson plan, indicators of learning and literacy should be interrelated (literacy is embedded in learning).
3. It must be implemented first, and then it can be answered. Still, theoretically, if the learning model can be implemented by all teachers from good-quality elementary schools or regular ones, or teachers who have higher- or lower-order thinking skills, then it can be assumed that the level of practicality is high and vice versa. Still, theoretically, the effectiveness of the RMEng learning model can be seen from the mathematical literacy of students at a high level. Individual and classical learning mastery is achieved. Students respond positively to applying the RMEng learning model and vice versa.
4. The distinctive feature of RME, besides its three principles and five characteristics, is horizontal and vertical mathematization. All of these characteristics still need to be clarified in the developed syntax. So, if the core value model is based on RME, it is still being developed. However, if the focus is only on contextual and problem-based learning, it seems to be more in that direction. For this reason, a more comprehensive study is needed on the characteristics of RME originating from primary sources to improve the model in the future.
The relationship with students' mathematical literacy abilities needs to be seen. This is because the developer still needs to provide an in-depth study regarding this matter, at least of the role of RME in fostering students' mathematical literacy skills, even though several studies have proven this. It would be better if the developer undertook a fundamental revision of how to develop a learning model by reading a book entitled "The Networking of Theories," edited by Bikner-Ashbahs and Prediger, and published by Springer in 2014, which conducts an in-depth study of RME. Finally, create a correlation diagram between the developed model syntax and the capability indicators to be improved.
5. Theoretically, what needs to be re-examined when combining a model with another theory is whether it is a development of the existing syntax or is new, and this is a separate step in the current syntax. This model will be able to improve students' mathematical literacy if the implementation of the RMEng model is adequately prepared to train students to do horizontal and vertical mathematization. Hence, it needs an initial context as the problems students have to solve are familiar and can be solved informally. The students' reasoning process will be awakened when running vertical mathematization. For this reason, the support system for this model must be well-prepared.

The suggestions from the experts were used to continuously improve the learning model draft. Discussions and revisions continued for two months until all of the experts agreed on a draft learning model to be tested. The following sections are from the primary draft of the learning model that the seven experts approved for testing.

1. Syntax

The stages or cycles of RME learning (De Lange, 1996) were combined with the EDP steps to improve mathematical literacy in elementary school students. This merger produced the RMEng model syntax. The developed learning syntax must also consider the psychological development of elementary school students in the concrete operational stage (Piaget, 1964; Snowman et al., 2012). The stage of developing a product facilitates them to study through actual or concrete objects. The syntax of the learning model developed is shown in Figure 3.

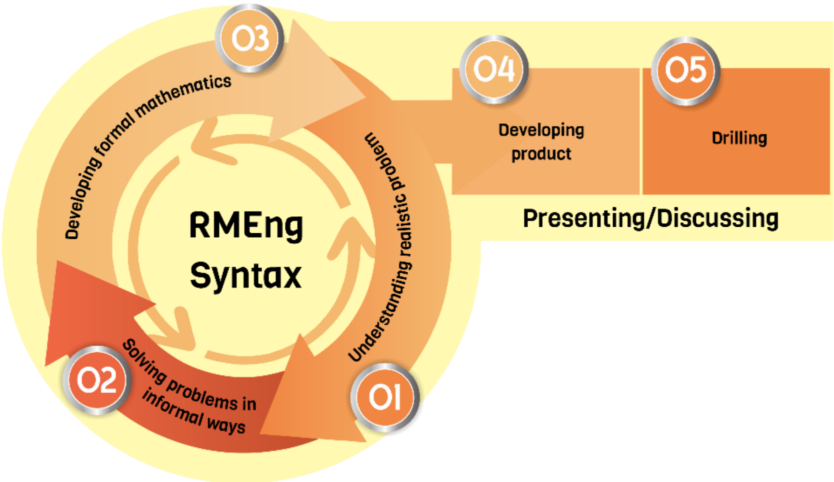


Figure 3. RMEng learning model syntax

RMEng starts with realistic problems and proceeds to the solving of issues in students' own (informal) way before developing formal mathematics. These three steps can be performed repeatedly according to the concept or material that students must master. If the idea or material has been conveyed, the teacher again provides realistic problems that facilitate students to develop products to apply and reinforce their mathematical understanding. The product development stage starts with designing, making, testing, revising, presenting, and further revising, if necessary, until a good product is obtained. Presentations or discussions can be included at all stages of the RMEng. The RMEng syntax is equipped with drilling mathematical literacy questions related to the material studied.

2. Social System

In RME, students become subjects who discover mathematics for themselves while the teacher merely guides them (Treffers, 1987). Through EDP, teachers provide opportunities for students to consult, participate, negotiate, work together, and review. When faced with a realistic problem, students are challenged to develop strategies to solve it and discuss it with others (Wubbels et al., 1997). The social system constructed in RMEng entails forming an attitude of independence, cooperation, critical reasoning, and creativity. This aligns with the national education goal in Indonesia of realizing the Pancasila student profile. The latter includes various character traits and competencies that students are expected to achieve based on the noble values of the Pancasila (i.e., the ideological pillars of the Indonesian state). The roles of teachers and students in the RMEng learning model are described in Table 4.

Table 4. The social system of the RMEng learning model

Phase	Teacher Activities	Student Activities
Understanding realistic problems	The teacher outlines prior perceptions about the problems, exploring students' knowledge or experience. The teacher offers realistic problems.	Students try to understand the problems given. Students discuss in their groups to gain a more comprehensive understanding. Students can ask the teacher if they need help.
Solving problems in informal ways	The teacher guides the class and allows students to discuss the problems given in groups. The teacher ensures that the discussion runs smoothly and that all	Students discuss in groups to transform problems into mathematical sentences and solve the problems given in their own way (informal ways). Students reflect on the

	students are active. The teacher can provide a stimulus if the discussion does not run as planned.	method or problem-solving procedure they use.
Developing formal mathematics	After each group finishes its presentation, the teacher guides the discussion so that a mathematical negotiation occurs. The teacher guides the class discussion after all groups with different problem-solving procedures have presented their problem-solving strategies. The teacher facilitates discussion to enable students to develop more formal problem-solving methods.	Several groups present the results of problem-solving in front of the class. Another group responds. After all the groups with different problem-solving procedures have finished their presentations, class discussions are held to discuss the problem-solving strategies. At this stage, students practice developing more formal problem-solving techniques.
Developing a product	The teacher provides realistic problems that facilitate students' production of a product or work. The teacher monitors each group's work and ensures that all group members are active. The teacher allows each group to design, manufacture, and test their product, and they provide input if needed. The teacher guides the presentation and class discussion.	Each group creates a design based on the discussion results and then develops a product based on their design. Students test the products they have created. If the product is yet to meet expectations, students can repeat the necessary steps. Each group then presents their product and other groups respond or provide input.
Drilling	The teacher poses math literacy questions based on the students' learned material. The exercise questions cover aspects of the formulation, employment, and interpretation. The teacher provides guidance for children having difficulty. At the end of the lesson, the teacher can facilitate class discussion to discuss questions that students consider difficult.	Students work on the exercises individually. Once complete, these can be continued with class discussions, especially if students have struggled to answer the questions.

3. Principles of Reaction

In RMEng, the teacher is a facilitator, moderator, and evaluator. Teachers provide opportunities for students to reinvent mathematical ideas and concepts and apply mathematics in their own way. At each stage, the teacher also provides students with opportunities to discuss or negotiate. The teacher evaluates learning to monitor each student's progress, identifies any difficulties encountered, and improves the quality of learning.

When students face a realistic problem, they can solve it in their own way. If they require help in understanding the context of the problem, the teacher can slowly guide them by posing trigger questions and encouraging the discussion process. The teacher should not blame students if they make mistakes; rather, they teach the students to correct themselves if there are errors in the problem-solving procedures developed. Instead of providing explanations, the teacher guides students to find answers to their questions independently. Teachers encourage students to collaborate, work together, and communicate with their friends. If certain students are less active, the teacher provides a stimulus to encourage them to become active. In terms of developing students' vertical mathematization, teachers must foster their interest in mathematics by demonstrating that they value their contributions and ensuring that student input plays a role in the reasoning of the whole class and, if possible, in subsequent activities (Gravemeijer, 2010).

4. Support System

The RMEng learning model support system includes a syllabus, lesson plans, evaluation instruments (pretest, posttest, quizzes), and media or learning aids following the prepared lesson plans. Other support systems consist of teachers who understand the goals and steps of RMEng learning and students who are active in solving the realistic problems given. The existing supportive learning environment, such as physical space and class setting, is also essential.

5. Impact of Learning and Accompaniment

The instructional impact is the direct impact of learning. The expected instructional impact of the RMEng learning model is to increase students' mathematical literacy in formulating, employing, and interpreting. The desired accompanying effects from the application of this model include: (1) Students can communicate ideas in solving problems and applying mathematics in everyday life, (2) students' reasoning abilities increase because they discover mathematical procedures and concepts for themselves, and (3) students become more active learners in constructing knowledge and respecting others' opinions through a mathematical negotiation process.

Lesson Plan

Lesson plans were prepared according to the learning model built and validated and the draft learning model. The lesson plan created in this research was still limited to one content in sixth grade: geometry. The lesson plan used for the preliminary field testing, main field testing, and product testing stage was almost the same, although it was refined before progressing to the next stage. The differences lay in the field-testing subjects. The learning objectives of each meeting are shown in Table 5.

Table 5. The learning objectives of each field test meeting

Cycle	Meeting	Learning Objectives
I	1	Find and analyze the characteristics of prisms, cylinders, pyramids, cones, and spheres.
	2	Find the formula and determine the surface area of a prism.
	3	Find the formula and determine the surface area of cylinders and pyramids.
	4	Find the formula and determine the surface area of a sphere.
	5	Find the formula and determine the volume of prisms and cylinders.
	6	Find the formula and determine the volume of pyramids and cones.
	7	Find the formula and determine the volume of a sphere.
	8	Design a product based on an understanding of geometric shapes to solve everyday problems.
	9	Create a product based on the design and solve everyday problems about geometric shapes (drilling).
II	1	Determine the surface area and volume of the combined geometric shape.
	2	Determine the surface area and volume of the combined geometric shape.
	3	Determine the surface area and volume of the combined geometric shape.
	4	Design a product based on an understanding of combined geometric shapes to solve everyday problems.
	5	Create products based on an understanding of combined spatial shapes to solve everyday problems.
	6	Solve everyday problems about compound geometric shapes (drilling).

Table 5 indicates that the steps of the RMEng learning model cannot be completed in a single meeting. The steps require several meetings within a cycle. The lesson plan was composed of two cycles in geometric shape content. The lesson plan for each meeting included basic competencies, learning indicators, objectives, learning steps, worksheets, and daily assessment questions. Table 6 contains examples of the learning steps in a meeting. The example given is from the lesson plan in the fourth meeting of Cycle I. This meeting covered only three of the five steps of RMEng. The other two steps, developing the product and drilling, were considered in the eighth and ninth meetings.

Table 6. Examples of learning steps in a meeting

No.	Activity	RMEng Stage
1.	Group discussion to understand the problems in Worksheet 1 (find the formula for calculating the surface area of a ball using orange peel).	Understanding realistic problems
2.	Students discuss solving the problems in Worksheet 1.	Solving problems in informal ways
3.	One or two groups present their discussion results, and the other groups respond.	
4.	Classical discussion on how to calculate the surface area of a ball.	Developing formal mathematics
5.	Student discussion to understand the realistic problems on Worksheet 2 (problems about the surface area of a ball).	Understanding realistic problems
6.	Students discuss solving the problems in Worksheet 2.	Solving problems in informal ways
7.	One or two groups present their discussion results, and the other groups respond.	
8.	Classical discussion to determine the simplest or easiest way to solve a problem.	Developing formal mathematics

The lesson plan validator was the same as the learning model validator. The validation results are shown in Table 7. The average score of the expert assessment was 4.39, indicating that the lesson plan was in the appropriate category. The assessment was then used to calculate the Aiken validity index, which showed a value of 0.857, indicating that the lesson plan was valid, although this was obtained before the lesson plan was revised.

Table 7. The results of the RMEng lesson plan validation by experts (maximum score = 5)

No.	Study Criteria	Average Score
Learning Objectives		4.53
1.	Clarity of core competencies and basic competencies.	4.83
2.	Compatibility of core competencies and basic competencies with learning objectives.	4.67
3.	Suitability of indicators for basic competencies.	4.50
4.	Suitability of indicators for learning objectives.	4.50
5.	Appropriateness of indicators for the level of students' development.	4.17
Content		4.10
1.	Lesson plan systematization.	3.83
2.	Sequence appropriateness of mathematics learning activities for the RMEng learning model.	3.83
3.	Appropriate description of student and teacher activities for each stage of learning mathematics with the RMEng model.	3.67
4.	Clarity of learning scenarios (stages of learning activities: beginning, core, closing).	4.50
5.	Completeness of assessment instruments (questions, answer keys, scoring guidelines).	4.67
Language		4.67
1.	The use of language is in accordance with the Indonesian language guidelines.	4.67
2.	Language effectiveness.	4.67
3.	Effectiveness of sentence structure	4.67

Time Allocation	4.34
1. Appropriateness of time allocation with learning activities.	4.50
2. Time breakdown for each learning stage.	4.17
Average	4.39

The experts provided feedback on the lesson plan. Discussions with validators were also conducted via email and WhatsApp, and the validation process lasted for three months. The input provided by the validators included: (1) the learning steps must ensure that the EDP component is in line with the concept of vertical mathematization in RME, (2) the learning process needs to be adapted to the theoretical framework, (3) the lesson plan systematization should be adjusted to the existing regulations, (4) the lesson plan should refer to the syntax being developed, and (5) the adequacy of the time allocation in the lesson plan needs to be considered. The lesson plan was revised until all of the validators agreed; it was then used in the preliminary field testing.

Preliminary and Main Field Testing

The preliminary field-testing stage was carried out in grade 6 of an elementary school with 24 students. The students undertook the pretest before the field testing, achieving an average pretest score of 41.542. This then increased to 65.042 after being given the RMEng model for 15 meetings. The field-testing results were analyzed by paired sample t-test and showed a Sig (2-tailed) value of 0.000. This value was lower than the cut-off value of 0.05, which means H_0 was rejected. Thus, it can be concluded that there was an average difference between the pretest and posttest, indicating there was an effect of using the RMEng learning model in increasing mathematical literacy. The suggestions obtained in the preliminary field testing included: (1) The syntax pictures needed to be made more precise and more enjoyable, (2) some of the group work questions were too difficult, so they needed to be revised. After the revision process was complete, the next phase was carried out, namely main field testing.

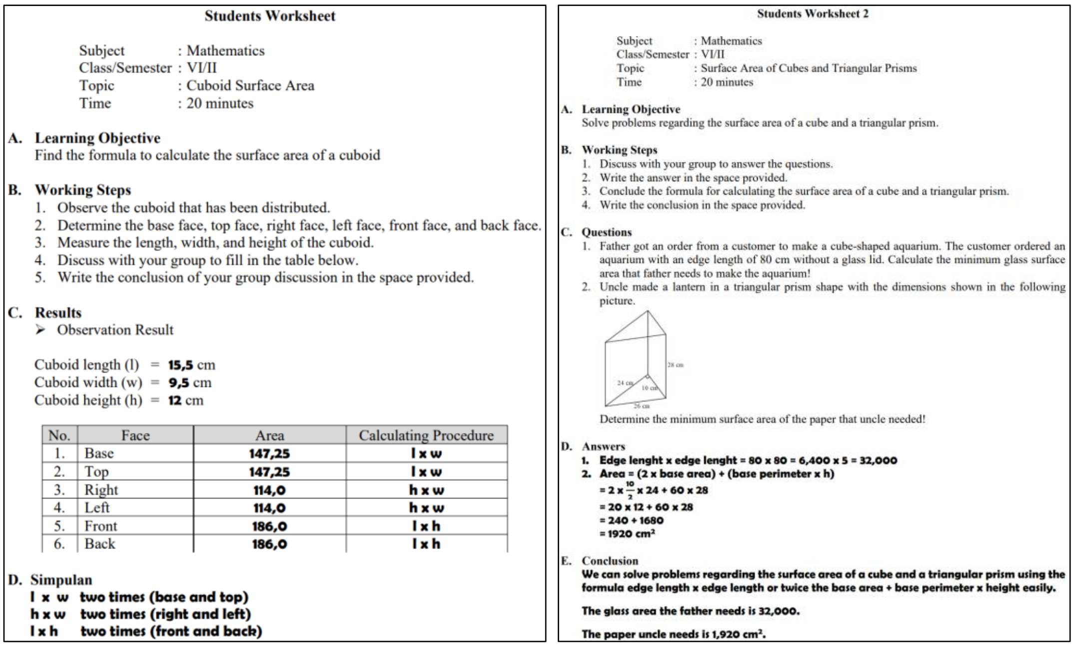
The main field testing was carried out in three classes in three elementary schools with 50 students in grade 6 as the subjects. At the beginning of the main field testing, an FGD was conducted with the grade 6 teachers. At this stage, the teachers were given a briefing about the purpose of the field testing and material about the RMEng learning model. They were also given a draft of a model book and complete learning tools. The students again completed the pretest before the main field testing, achieving an average pretest score of 42.960. The score increased to 67.340 after participating in mathematics learning for 15 meetings. To assess the effectiveness of the RMEng learning model, a statistical analysis was carried out with the sample-related t-test. The analysis results showed a Sig (2-tailed) value of 0.000. Thus, it can be concluded that there was an average difference between the pretest and posttest, indicating an effect of using the RMEng learning model in increasing mathematical literacy.

The teacher filled out a practicality questionnaire at the end of each preliminary and main field-testing stage. This comprised three main questions, namely concerning the practicality of the RMEng learning model, the practicality of the RMEng learning model book, and the practicality of the RMEng lesson plans. The practicality of the learning model was determined based on three indicators: the ease of understanding the learning syntax, the ease of applying the learning syntax, and the completeness of the support system. The practicality of the RMEng model book was assessed on five indicators: the ease of understanding the description of the material, the accuracy of the language, the suitability of the book size, the suitability of the size of the letters, and the attractiveness of the layout. The practicality of the lesson plan was determined based on four indicators: the completeness of the lesson plan, the ease of understanding the learning steps, the ease of carrying out the learning steps, and the accuracy of the language. A summary of the results of the practicality questionnaire is shown in Table 8.

Table 8. The results of the practicality questionnaire of the RME learning model

No.	Activity	Score	
		Preliminary Field Testing	Main Field Testing
1.	The practicality of the RMEng learning model	4.33	4.44
2.	The practicality of the RMEng learning model book	4.20	4.47
3.	The practicality of the RMEng lesson plans	4.25	4.50
	Average Score	4.26	4.47

After completing the main field testing, another FGD was held between the researcher, the teachers, and the observers. All of the teachers and observers viewed the RMEng learning model positively. The advantages of the RMEng model that they cited included: (1) Students become accustomed to solving realistic problems in their lives so that their ability to understand word problems increases, (2) students become more active and independent in solving math problems, (3) the learning atmosphere is fun, (4) students are facilitated to explore their potential when solving problems, and (4) students have experience in finding a mathematical formula so that the knowledge gained becomes more rooted. However, the teachers also expressed the following weaknesses: (1) It takes quite a long time, so the teacher must be competent in allocating time when using the RMEng learning model, and (2) students with low abilities still need help with exercises with higher-order thinking skills (HOTS) questions. Examples of student worksheets and answers (in English translation) can be seen in Figure 4.



Students Worksheet

Subject : Mathematics
Class/Semester : VI/II
Topic : Cuboid Surface Area
Time : 20 minutes

A. **Learning Objective**
Find the formula to calculate the surface area of a cuboid

B. **Working Steps**
1. Observe the cuboid that has been distributed.
2. Determine the base face, top face, right face, left face, front face, and back face.
3. Measure the length, width, and height of the cuboid.
4. Discuss with your group to fill in the table below.
5. Write the conclusion of your group discussion in the space provided.

C. **Results**
➢ Observation Result

Cuboid length (l) = **15,5** cm
Cuboid width (w) = **9,5** cm
Cuboid height (h) = **12** cm

No.	Face	Area	Calculating Procedure
1.	Base	**147,25**	**l x w**
2.	Top	**147,25**	**l x w**
3.	Right	**114,0**	**h x w**
4.	Left	**114,0**	**h x w**
5.	Front	**186,0**	**l x h**
6.	Back	**186,0**	**l x h**

D. **Simpulan**
l x w two times (base and top)
h x w two times (right and left)
l x h two times (front and back)

Students Worksheet 2

Subject : Mathematics
Class/Semester : VI/II
Topic : Surface Area of Cubes and Triangular Prisms
Time : 20 minutes

A. **Learning Objective**
Solve problems regarding the surface area of a cube and a triangular prism.

B. **Working Steps**
1. Discuss with your group to answer the questions.
2. Write the answer in the space provided.
3. Conclude the formula for calculating the surface area of a cube and a triangular prism.
4. Write the conclusion in the space provided.

C. **Questions**
1. Father got an order from a customer to make a cube-shaped aquarium. The customer ordered an aquarium with an edge length of 80 cm without a glass lid. Calculate the minimum glass surface area that father needs to make the aquarium!
2. Uncle made a lantern in a triangular prism shape with the dimensions shown in the following picture.

28 cm
24 cm
10 cm
26 cm

Determine the minimum surface area of the paper that uncle needed!

D. Answers
1. **Edge lenght x edge lenght = 80 x 80 = 6,400 x 5 = 32,000**
2. **Area = (2 x base area) + (base perimeter x h)**
$= 2 \times \frac{10}{2} \times 24 + 60 \times 28$
$= 20 \times 12 + 60 \times 28$
$= 240 + 1680$
$= 1920\ cm^2$

E. Conclusion
We can solve problems regarding the surface area of a cube and a triangular prism using the formula edge length x edge length or twice the base area + base perimeter x height easily.

The glass area the father needs is 32,000.

The paper uncle needs is 1,920 cm^2.

Figure 4. Examples of student worksheets and answers (English translated version)

Figure 4 shows the worksheet of a second meeting in Cycle I on the prism surface area topic. In worksheet 1, students faced a realistic problem of finding the cuboid surface area. They discussed in groups and started by measuring a real cuboid's length, width, and height. The table helped them to find the formula. A group of students concluded that the surface area of the cuboid was 2 × (l × w), 2 × (h × w), and 2 × (l × h). When presenting the results, they received input from other groups to add a plus sign, so the formula became 2 × (l × w) + 2 × (h × w) + 2 × (l × h) and to write the cuboid surface area in the conclusion section. The teacher suggested that the formula could be written more concisely, such as 2 lw + 2 hw + 2 lh. Activities on worksheet 1 helped students to move from informal ways to formal mathematics.

In worksheet 2, students were asked to answer questions through group discussion. A group of students could answer the first question correctly, although there was a slight error in the notation ($80 \times 80 \neq 6400 \times 5$). They could answer the second question well. They have been able to use formulas to solve problems related to the surface area of a prism. In conclusion, there were still a few errors in the arranged sentences. During the presentation, they received input from other groups that the surface area of the cube was not only edge length × edge length but 6 × edge length × edge length. The teacher also suggested that students write sentences more clearly so it is understood. In the conclusion section, students also write conclusions about the area of glass needed by the father and the area of paper needed by the uncle. It shows that they can interpret mathematics back into the real world.

Figure 5. Student activity in RMEng

Towards the end of the cycle I, students were asked to design and make a product in a solid geometry shape. Examples of products made by students were birthday hats, cupboard replicas, and pencil cases. At the end of cycle II, students were asked to design and create a product in a combined solid geometry shape. Examples of products made by students were house replicas, vehicle replicas, ice cream replicas, and pencil replicas. Designing and creating products facilitated students to formulate, employ, and interpret mathematics back into the real world. At the end of each cycle, students worked on practice questions (drilling). Drilling helped them strengthen all aspects of mathematical literacy. Examples of student activities during study with RMEng can be seen in Figure 5.

No further suggestions for the RMEng learning model draft emerged during the main field-testing stage. However, there was a suggestion about the learning device, namely, to provide answer keys in group discussions so that the teacher can more easily monitor the progress of the discussion. Thus, the product testing stage was carried out following the revision of the learning device.

Product Testing Stage

The product testing stage aimed to determine whether the developed learning model was better than the existing one. This stage was carried out in two schools. One class was the experimental group, and one class was the control group. Before being given instructional activities, the students in the two groups were given a pretest, and a balance test was conducted. The analysis results showed a 2-tailed significance value of 0.983 > 0.05. Thus, the students' mathematical literacy was balanced before the two groups were given instructional activities. The students worked on the posttest after each group had been given instructional activities for 15 meetings. Data on the pretest and posttest results before and after the students were given instructional activities are shown in Table 9.

Table 9. Comparison of the pretest and posttest results of the experimental and control groups

Group	n	Pretest	Posttest	Sig. (2-tailed)
Experimental	30	42.33	69.37	0.000
Control	15	42.27	57.07	0.000

Based on the paired sample t-test, both the experimental and control groups showed a significant increase in mathematical literacy. The average value of the experimental group was higher than that of the control group. To determine whether the mean differences between the experimental and control groups were significant, an independent sample t-test was performed. As shown in Table 10, the 2-tailed significance value was 0.000 < 0.05. It can thus be concluded that the posttest scores in the experimental group were significantly higher than the control group.

Table 10. Independent sample test of the experimental and control groups

		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Posttest	Equal variances assumed	1.630	.209	3.801	43	.000	12.300	3.236
	Equal variances not assumed			3.517	23.111	.002	12.300	3.497

A comparison of the pretest and posttest in the experimental and control groups is shown in Figure 6. In both groups, scores were higher on the formulate aspect compared to the employ aspect, and scores on employ were higher than interpret, both before and after learning was given. These results are in line with certain previous studies (Ahyan & Juandi, 2019; Utaminingsih et al., 2021), but different from our prior study (Nurmasari et al., 2022) and various other research results (Ambarita et al., 2018; Fadillah, 2019; Solikhah et al., 2022), where students performed best on the employ aspect and worst on the interpret aspect. However, comparisons between the formulate, employ, and interpret aspects are inconsistent and vary widely. Several studies have found the highest scores for the interpret aspect and better scores for employ than formulate (Almarashdi & Jarrah, 2023; Dewantara et al., 2015). Other studies have found the highest scores for the formulate aspect, followed by interpret and then employ (Rusmining & Sawitri, 2022). Comparisons of the three aspects of mathematical literacy can also vary according to the class level (Jailani et al., 2020).

In Figure 6, we can see that in both groups, the highest increase was in the employ aspect. The most visible difference is in the formulate aspect, where the score in the experimental group increased by 26 points, while in the control group, it only increased by 8 points.

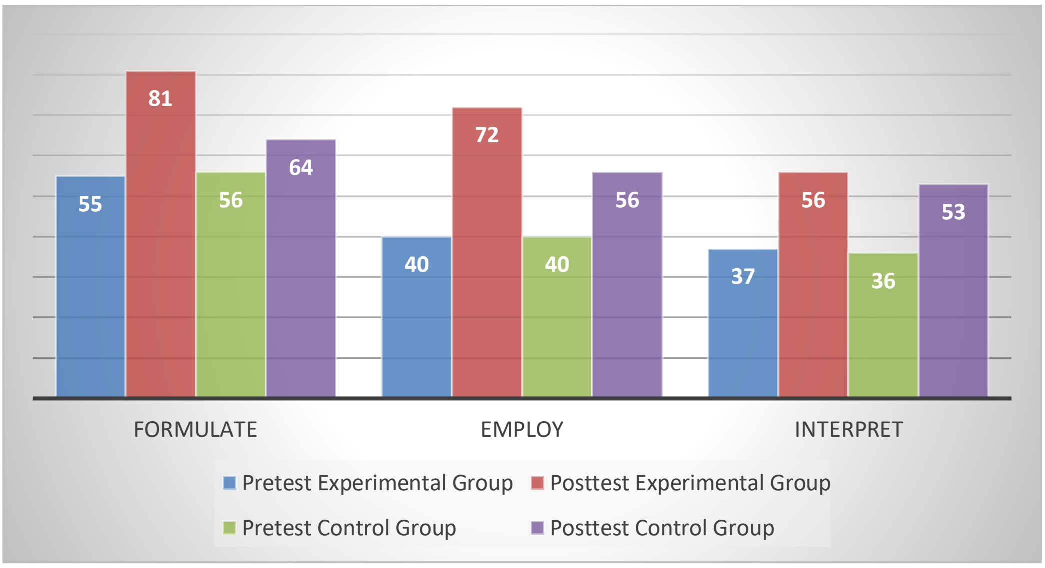


Figure 6. Comparison of mathematical literacy in the experimental and control groups

Some researchers have developed RME-based learning models that have been effective in increasing mathematical literacy or learning mathematics (Rusdi, Arnawa, et al., 2020; Rusdi, Fauzan, et al., 2020; Sumirattana et al., 2017; Wahyudi et al., 2017). The most visible difference between the RMEng learning model and previously developed models lies in the discussion or presentation stage. In RMEng, discussions or presentations are not discrete steps but are instead integrated into the five existing steps. They can be conducted at each stage: (1) understanding realistic problems, (2) solving problems in informal ways, (3) developing formal mathematics, (4) developing products, and (5) drilling. Additionally, the three models described above were developed in secondary schools and colleges, while learning models aimed at improving mathematical literacy have yet to be developed in elementary schools. Aside from learning models, other products such as web-based RME (Lisnani et al., 2023) and learning modules based on RME (Aulia & Prahmana, 2022; Mattoliang et al., 2022; Yuliana et al., 2023) have also shown effectiveness in increasing students' mathematical literacy.

CONCLUSION

The RMEng learning model was built using the RME approach and EDP steps to improve the mathematical literacy of elementary school students. The model employs the following syntax: (1) understanding realistic problems, (2) solving problems in informal ways, (3) developing formal mathematics, (4) developing products, and (5) drilling. It also has the following characteristics: start from realistic problems; students build their knowledge in informal ways; there is a process of mathematical negotiation that enables students to develop formal mathematics, where they are facilitated to implement mathematics in developing a product to solve everyday problems; process discussions or presentations are held at each stage; and students are allowed to practice solving mathematical problems that are contextual with daily life (mathematical literacy). The RMEng learning model can be used according to needs and adjusted to the applicable curriculum.

Experts validated the RMEng learning model; it achieved an Aiken index value of 0.786 (before revision), indicating that it was valid. The model was subsequently revised based on expert advice to ensure approval. After approval by experts, the RMEng learning model was used in the preliminary and main field-testing stages and further improved based on suggestions from teachers and observers. The product testing stage proved that students who studied with the RMEng model had significantly higher mathematical literacy scores than the control group at the 0.000 significance level. Teachers in elementary schools, especially in the upper grades, are therefore advised to use the RMEng approach to improve students' mathematical literacy. Further research on the RMEng learning model, with other content or different grade levels, is needed.

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Declarations

Author Contribution	:	LN: Conceptualization, Writing - Original Draft, Formal analysis, Editing and Visualization. B: Conceptualization, Writing - Review & Editing, Methodology, Validation and Supervision. JN: Writing - Review & Editing, Validation and Supervision. MR: Conceptualization, Writing - Review & Editing, Validation and Supervision, Funding acquisition.
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Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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