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Learning obstacle of probability learning based on the probabilistic thinking level

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Abstract

This study aims to determine students learning obstacles in probability material based on their probabilistic thinking (PT) level using the theory of didactical situation (TDS) perspective. This is qualitative research with the case study method. The subject consisted of 23 grade 9 students in junior high school who had studied the material and had taken the test. The test results were used to classify students depending on their PT level using the framework developed by Graha A. Jones. Furthermore, interviews were performed with three representative students from each PT level. The interviews indicated that students in each PT level continue to face learning obstacles, which include instrumental ontogenic and epistemological obstacles. The lowest PT level demonstrated a more complex and comprehensive learning obstacle in all constructs of PT. From the TDS perspective, all students who are at various levels of PT have reached an action situation. Only 33% of the students at the subjective level and 83% of the students at the transition level reached the formulation situation. Still, students at that level needed help to reach the other two situations. Unlike the two previous levels, students at the level of quantitative informal thinking can achieve situations of validation and institutionalization.

Keywords: Learning Obstacle, Probability, Probabilistic Thinking, Theory of Didactical Situation

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Today, in modern society, probability is the most common problem encountered in daily activities. Probability is a domain in Mathematics that studies measurements of an event's uncertainty, such as the concepts of chance, risk, prize, and randomness, which are directly related to the amount of data obtained and must be decided in a situation of uncertainty (Koparan & Rodríguez-Alveal, 2022). Knowledge of probability can empower humans to make wise decisions when facing various situations (Wijaya et al., 2021; Bryant & Nunes, 2012; Kennedy & Johnson, 2007; Serradó et al., 2005). Probability is not about predicting whether a particular event will occur but about determining how that probability is distributed across the possible events (Yusuf et al., 2021; Baltaci, 2016; Galavotti, 2015).

According to that information, the importance of the material and the benefits in daily life are evident, which is why probability is included in the curriculum of various countries. This can be seen in the last two decades, where much attention in school curriculum has been given to probability and statistics (Koparan & Rodríguez-Alveal, 2022; Inzunza Cazares & Rocha Ruiz, 2021; Batanero et al., 2014). Probability is an essential material for every student. Bagehot (in Sharma, 2016) said, "Life is a school of probability." This quote suggests that probability pervades almost everything we do. Generally, probability offers a tool for modeling and simulating reality, which can link mathematics and the real world (Shodiqin et al., 2022).

In dealing with probabilistic situations, a person will carry out thinking activities, considering whether things that can affect that situation can occur or not. Mental activity associated with contexts that contain elements of uncertainty is called probabilistic thinking (Lamprianou & Lamprianou, 2003). Probabilistic thinking is the attempt to predict the probability of a situation occurring by using mathematical and logical tools. In this universe, probabilistic is infinitely complex; probabilistic thinking helps determine the most likely outcomes. Applying probabilistic thinking causes decisions to be more precise and effective. Therefore, it is essential for students to develop these skills (Koparan & Rodríguez-Alveal, 2022).

Probabilistic thinking is a high-level thinking ability that is important in human life subjects such as health, economics, logistics, and insurance, which involve situations of uncertainty (Dooren, 2014; Shodiqin et al., 2022). The large amount of information currently expressed in terms of uncertainty has resulted in the importance of probabilistic thinking abilities. Previous research related to probabilistic thinking centered on determining the level of probabilistic thinking at various levels of education, starting from elementary school to college (Fa'ani et al., 2016; Raya, 2017; Raya et al., 2021; Sari et al., 2023; Sari et al., 2017; 2018a; Shodiqin et al., 2021; 2022). The research results found that students' probabilistic thinking tends to be low and is often at a level below their age.

These studies reveal that many factors influence a person's level of probabilistic thinking, like mathematical abilities and their conception of probability materials (Maher & Ahluwalid, 2014; Sari et al., 2018b; Wijaya et al., 2021). These are related to the mathematical learning process experienced by students. But, many research found that mathematical learning, especially on probability material, still has problems (Gürbüz et al., 2012; Koparan & Rodríguez-Alveal, 2022; Sezgin Memnun et al., 2019; Yusuf et al., 2021).

Fauzi and Suryadi (2020) stated that learning obstacle is one of the problems in mathematical learning that is important to explore. The study of learning obstacles is part of the didactic situation theory developed by Brousseau (2002). Learning obstacle is one of the realities that impact didactic study design (Suryadi, 2019b). Learning obstacles can be seen from behavior describing that learning obstacle. Koparan and Rodríguez-Alveal (2022) stated that the obstacle was because the preparations made by both teachers and students were not good enough. Thus, this study was conducted to identify the learning obstacles experienced by students who have studied probability materials based on their level of probabilistic thinking using the theory of didactical situation perspective.

Probabilistic Thinking

Probability is the science of randomness and uncertainty (Hokor, 2023; Armiati et al., 2022). Probability is also related to an event's sample point and sample space (Prihartini et al., 2020). It is a numerical quantification of the occurrence or non-occurrence of an event under uncertainty. Randomness is the lack of pattern or predictability in events or when a sequence of events has no order and does not follow a reasonable pattern. Random events cannot be predicted with certainty (Hokor, 2023).

Uncertainty is a situation that involves unknown facts. It is concerned with predictions of future events, which have elements of doubt. An example of the application of this material is when someone decides to go to college to continue their education. Suppose university A has more applicants than B, then it is likely that individuals will place A as their first choice (Armiati et al., 2022; Sya'bani et al., 2021). An event is a set of outcomes of an experiment. Also, uncertainty is where some possible outcomes have an undesired effect or significant loss. The reasoning about uncertainties is termed probabilistic reasoning. Probabilistic reasoning is the "way people reason about uncertain situations and make decisions based on the likely outcomes" (Hokor, 2023). Bryant & Nunes (2012) asserted that "this reasoning allows us to work out the probability of particular outcomes, and thus to understand the risks and possible benefits of acting in one way rather than another." The ability to identify the most likely outcomes of uncertainty constitutes probabilistic thinking. Figure 1 presents a model for probabilistic thinking (Hokor, 2023).

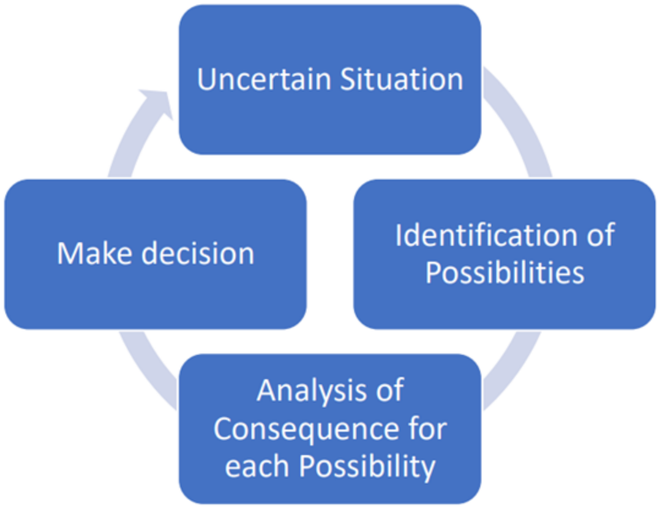


Figure 1. Probabilistic Thinking Model

Jones et al. (1997) conducted a two-year study of children's probabilistic thinking and postulated that four types of probabilistic thinking exist: subjective, transitional, informal, quantitative, and numerical. Nikiforidou (2018) described this study as pioneering research, thus making a summary of each of these forms of thinking. Subjective thinking involves making intuitive judgments based on imagination and personal preferences. Transitional thinking refers to inflexible attempts to begin to quantify probabilities. Informal quantitative thinking describes the use of more generative strategies in quantifying probabilities. Finally, numerical thinking relates to the use of valid numerical measures to describe probabilities. Jones et al. (1997) presented these types of probabilistic thinking as levels through which children progress as they develop more complex understandings of probability. In the absence of a more recent framework for describing the development of young children's probabilistic thinking, empirical studies in the field of probability have continued to draw on this framework since it was first established over twenty years ago (Nikiforidou, 2018). The current study used the work of Jones et al. (1997) to examine the types of thinking used by young children when making probabilistic judgments. There are four key constructs of probabilistic thinking to capture the manifold nature of probabilistic thinking and its interconnections; sample space, probability of an event, probability comparison, and conditional probability.

Research conducted by HodnikČadež & Škrbec (2011), found that elementary school children in Athens who had not studied the probability material were at level 2 (transitional) where children were able to distinguish between certain events that might or might not occur. Research conducted by Kurniasih & Sujadi (2017) states that the probabilistic thinking level of junior high school students who have received the basic probability material is at levels 1 and 2. Shodiqin et al. (2021) mentions that probabilistic thinking behavior is influenced by cognitive factors. Students' knowledge of probabilistics and personality in the form of self-regulated learning is a factor that influences probabilistic thinking. This follows the findings of a study conducted by Shodiqin et al. (2021). The research shows that the probabilistic level of prospective teacher students is still at level 2 for those with middle-level self-regulated learning. Students in mathematics learning materials experience various problems.

The Theory of Didactical Situation

Guy Brousseau initiated the theory of didactical situation (TDS). TDS takes a systemic perspective on mathematical didactics to study the conditions for mathematical knowledge dissemination and appropriation through educational institutions (Artigue et al., 2014). In this theory, learning begins with an action situation. This situation is critical to provide space for children to use previous experience and knowledge. When a new mental object begins to form, Brousseau refers to it as a formulation situation, which is essentially the occurrence of abstraction between students and between teachers and students. This allows the negotiation of meaning so that arguments, statements (claims), or representations develop and encourage internal and external validation. External induces individuals to apply concepts in different contexts (Suryadi, 2019b). In simple terms, the stages of the didactical situation of TDS are presented in Figure 2.

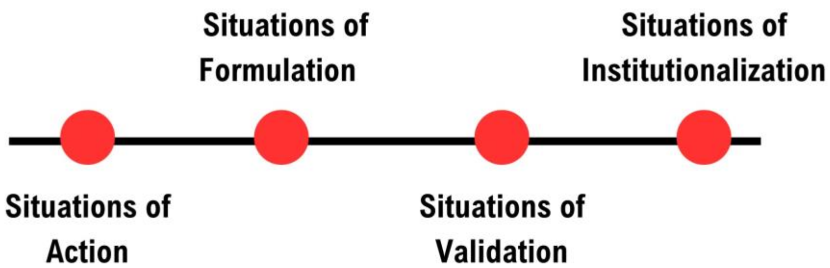


Figure 2. Representation of TDS

Learning Obstacles

The learning process is only sometimes as expected. The developed situation and learning flow are sometimes the cause of difficulties for children. The teacher tries to overcome this problem by providing didactic interventions, which are also possible causes of difficulties or obstacles to the learning process. In this study, obstacles are defined as a condition experienced by students that makes it impossible or difficult for them to solve problems. In the theory of didactic situation (TDS), difficulties caused by external factors are known as obstacles. Brousseau (Suryadi, 2019a) suggests three types of learning obstacles, specifically ontogenic, epistemological, and didactical. An ontogenic obstacle is related to children's readiness to learn. There are three types of ontogenical obstacles, which are psychological, instrumental, and conceptual. Unpreparedness produced by psychological factors such as inadequate motivation and interest in the content being studied is referred to as a psychological ontogenic obstacle. Technical unpreparedness in a child causes the child to be unable to completely follow the situation that occurs in learning due to not comprehending essential technical matters in a learning process. Conceptual ontogenic obstacle is a type of difficulty related to the conceptual level contained in the design that is not suitable for the child's condition, as determined by learning experiences (Suryadi, 2019b)

The second obstacle is the epistemological obstacle, which is caused by the limited context in which an idea is first learned. Subsequent learning obstacles are those caused by the state of the didactic design used or by the teacher's didactic interventions, which result in the Jordan effect (the occurrence of an outside learning flow that the teacher validates) or by inappropriate didactic interventions.

Several prior research such as Maharani et al. (2022), have found learning obstacles that occur in probability material due to a didactical design implemented in Indonesia, revealing that junior high school students face the three learning obstacles. Nur (2021) found that there are epistemological obstacles in three parts of the probability material: determining sample points and sample space, the theoretical probability value of an event, and determining empirical probability/relative frequency to solve real problems. In Malaysia, the most common learning obstacle experienced by students is related to the use of language in probability material (Yusuf et al., 2021). Serradó et al. (2005) conducted a learning obstacle analysis based on the book Secondary Education in Spain. They discovered three learning obstacles in the student book: epistemological, ontogenic, and didactic, in all probability material.

METHODS

This research is didactical design research (DDR) developed by Suryadi (2010), which is classified as qualitative research with a case study method. DDR consists of three stages, namely (1) didactic situation analysis, (2) metapedadidactic analysis, and (3) retrospective analysis (Suryadi, 2019a). This method was used because it is in line with the aim of the study to examine and reflect in depth a special case of a particular phenomenon (Koparan & Rodríguez-Alveal, 2022). In this case, the learning obstacle, which is the impact of didactic design, which is part of the first stage in DDR. The Hermeneutic Phenomenology approach was used in this study, with hermeneutics relating to the study of meaning, and phenomenology is a study related to one's experience in the process of forming meaning (Suryadi, 2019a).

Participants

This study was conducted in a public school in a border area in one of Indonesia's provinces. The subjects of this study were class IX students at an Indonesian junior high school. The research sample includes 23 children, with eight male students and 15 female students. The research subjects were students in class VIII who had received probability material learning at the same school.

Data Collection and Analysis

Data collection in this research was carried out in various stages by applying the principle of method triangulation as a form of applying the precautionary principle of researchers in collecting data, namely carrying out written tests, interviews, and document studies. Moustakas (1994) mentions that there are five essential stages in phenomenological analysis, namely (1) compiling a list of expressions from answers; (2) performing reduction and grouping of expressions; (3) consistent expressions are grouped and made into themes; (4) the expressions and themes are validated and labeled regarding the interview transcripts; (5) representatives from groups who have the same expression have in-depth interviews to review more comprehensive reasons regarding the child's answers for further analysis of the learning barriers they have.

In this study, the research subjects were given a test in the first stage that contained materials related to constructs in probabilistic thinking, namely the sample space, the probability of an event, and the comparison of probabilities. There was one construct that was not tested, namely conditional probability. This construction was not included in the test because the purpose of this research is to look at learning obstacles caused by didactic designs that have been experienced by students, whereas conditional probability material has not become the content of Indonesia's curriculum for class VIII. After completing the test, students' responses were grouped and analyzed to gather data on students' probabilistic thinking levels for each construction. The framework for analyzing the level of students' probabilistic thinking was developed by Jones in 1997 (Shodiqin et al., 2021), as shown in Table 1.

Table 1. A Framework for Assessing Students' Level of Probabilistic Thinking

Thinking Levels	Sample Space	Construction Probability of an event		Comparison of probability
Subjective (Level 1)	The list of members of the set that may occur in a single event is not yet complete	Predicting the most/least likely event to occur based on subjective judgment	Distinguish "certain", "impossible" and "possible" events subjectively based on the sample space	Comparing the probability of an event in two different sample spaces, usually based on various subjective considerations
Transition (Level 2)	Complete in terms of registering members of the set that may occur in one event, but still relying on subjective opinion	Predicting the most/least likely event based on quantitative judgment but can return to subjective judgment	Distinguishing "certain", "impossible" and "possible" events within reasonable parameters	Making probability comparisons based on inconsistent quantitative assessments
Informal Quantitative (Level 3)	Complete in listing the members of the set that may occur in a single occurrence	Predict the most/least likely event based on quantitative judgments including situations involving contiguous outcomes	Distinguishing "certain, "impossible" and "possible" events and justifying them quantitatively	Making probability comparisons based on quantitative assessments related to the concept of opportunity that is consistently justified by valid quantitative reasoning but in a limited way
Numeric (Level 4)	Complete in terms of registering members of the set that may occur in a single event and able to use the strategy in two or more events	Predict the most/least likely event based on the numerical probability of an event	Distinguishing "certain," "impossible" and "possible" events based on the membership of the overall sample space	Provides numerical probability measures and compares them

After obtaining an analysis of students' probabilistic thinking levels, the next step was to conduct in-depth interviews and then analyze the learning obstacles experienced by students based on their probabilistic level. In this analysis process, didactic situation theory was applied to the flow of learning, namely situations of action, formulation, validation, and institutionalization, as seen in Figure 3.

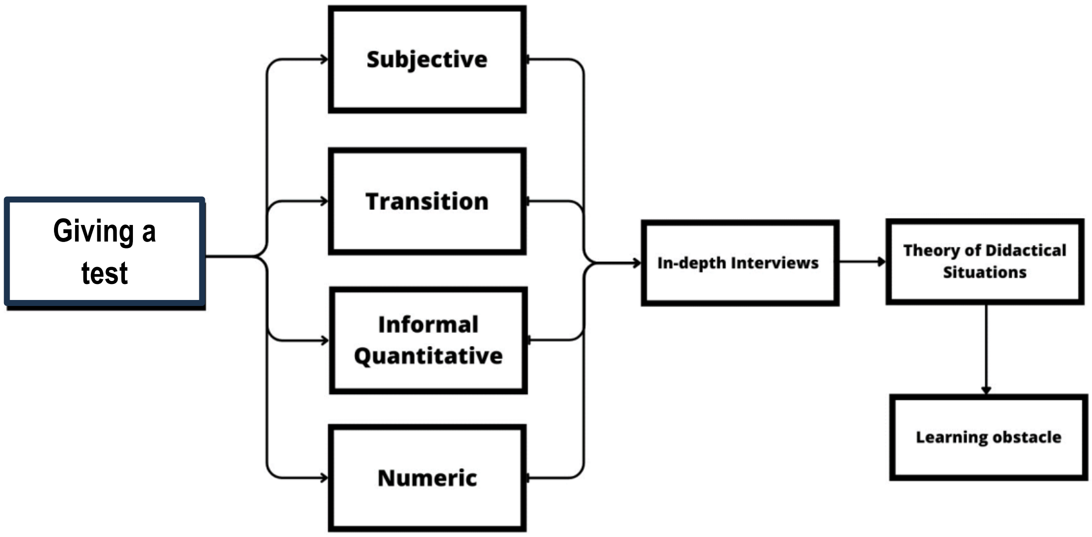


Figure 3. Data Analysis Framework

In analyzing learning obstacles, the theory of didactic situation framework was used. This theory explains the flow of learning experienced by students. In this study, learning obstacles were examined down to the probabilistic thinking stages experienced by students, namely action situations, formulation situations, validation situations, and institutionalization situations based on the test that has been given. The criteria listed in Table 2 are used to simplify the analysis process.

Table 2. Criteria in The Theory of Didactical Situations

Category	Characteristics
Action	• Respond to questions from the interviewer • Write answers even if they are wrong • Write answers without accompanying reasons • Distinguishing the meaning of impossible, possible, and certain based on personal experience
Formulation	• Be able to explain logical reasons for written answers • Be able to define the meaning of probability • Be able to distinguish the meaning of impossible, likely, and bound to happen using the concept of uncertainty
Validation	• Associate the concept of opportunity in the answers given • Being able to give a quantitative value in the meaning of impossible, possible, and certain to occur with logical reasons
Institutionalization	• Solve daily problems utilizing the concept of opportunity • No flow of thought is subjective to problems related to the opportunity material (the flow of thinking is at the numerical stage)

RESULTS AND DISCUSSION

The data collection process began by administering the respondent's ability test to 23 class IX students who had studied the probability material. In this test, students answered questions related to problems in the concept of probability. The test was awarded based on the development of the level of probabilistic thinking developed by Graha A Jones in 1997 (Jones et al., 1997), precisely the sample space of an event, the probability of an event, and the comparison of probabilities. The distribution of questions can be seen in Table 3 below.

Table 3. Distribution of Questions Based on the Construction of Probabilistic Thinking

Question	The type of problem about Probability	Construct Related to PT
1.	Is the coach wrong?	PE
2.	A six-sided die is rolled once	SS, PE
3	Marbles bag	CP
4	Spinner round	PE
5	Who will I choose?	CP
6	What is uncertainty?	PE

with;
SS = Sample Space
PE = Probability of an Event
CP = Comparison of Probabilities

Analysis of Student's Probabilistic Thinking

The students' test results were analyzed based on the probabilistic thinking framework in Table 2. After the test was analyzed, students' probabilistic thinking level profiles were obtained, as shown in Table 4. In Table 4, each data was described and given a unique code. For example, the code label SS-2-L2 shows the construction of the sample space (SS) found in problem no 2 (2), think probabilistically at level 2 (L2). Labeling the code aims to make it easier when describing the results of student work. In addition, each number at the end of the code indicates the number of students who meet the description of the code. Thus, in the sample space construction, namely in question number 3, 4 students were at level 2 thinking probabilistically (transition).

Table 4. Profile of Students' Probabilistic Thinking Level

Construct	Question	Thinking Levels	Code
Sample Space	2	L1	SS-2-L1 (14)
		L2	SS-2-L2 (4)
		L3	SS-2-L3 (5)
Probability of an event	1	L1	PE-1-L1 (15)
		L2	PE-1-L2 (6)
		L3	PE-1-L3 (2)
	2	L1	PE-3-L1 (14)
		L2	PE-3-L2 (4)
		L4	PE-3-L4 (5)
	4	L1	PE-4-L1 (14)
		L2	PE-4-L2 (5)
		L4	PE-4-L3 (4)
	6	L1	PE-1-L1 (17)
		L2	PE-1-L2 (4)
		L3	PE-1-L3 (2)

Construct	Question	Thinking Levels	Code
Comparison of probabilities	3	L1	CP-3-L1 (4)
		L2	CP-3-L2 (19)
	5	L1	CP-5-L1 (5)
		L2	CP-5-L2 (17)
		L4	CP-5-L4 (1)

Based on Table 4, many students are still at level 1 thinking probabilistically for the sample space construction. This indicates that students are still determining the sample space of a single event by utilizing the results of their subjective thinking and cannot mention all the possibilities that will happen. Similarly, with PE construction, of the three questions that contain these constructions, most students are still at level 1 (subjective). This indicates that students are still using their subjective side, such as their own experiences in solving problems related to the probability of an event. For the last construction, namely the comparison of probabilities, it can be observed from the two questions given that students were able to apply quantitative comparisons, but the results of the comparisons are still not separated from the subjective side. The majority of students have not been able to capitalize on the concept of probability.

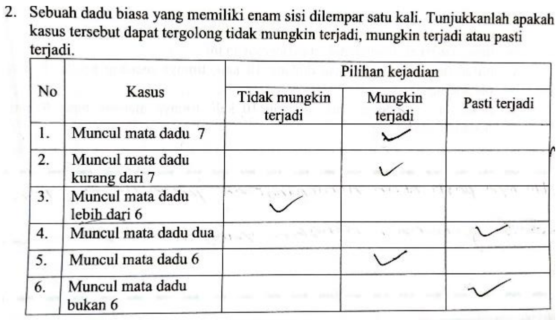
Seeing the results of the analysis from the students' level of probabilistic thinking, further analysis is needed to find out the causes or obstacles that cause students' high subjectivity in solving problems related to probability. In this study, these obstacles were analyzed using didactical situation theory.

Students' Learning Obstacle of Solving Probabilistic Problems That Demand Probabilistic Thinking Ability

After obtaining the level of students' probabilistic thinking that was described in the previous section, several important points were revealed: 1) students had various ways of responding to probabilistic problems, 2) students still did not understand the concept of probability properly as seen in the majority of students still used their subjective views in determining the probability of an event, 3) students still could not compare opportunities by utilizing the concept of the probability of an event, that the probability value of an event is between 0 and 1. Students tend to compare the number of sample spaces in an event.

The problems above indicate that students do not have a good understanding of the concept of probability. Therefore, it is important to explore the possibility of learning obstacles experienced by students in solving probability problems that require probabilistic thinking skills. Based on the test results, the researcher chose students who tended to always be at the same level in each construction to carry out in-depth interviews. Due to the uneven level of constructional thinking, the researcher decided to divide students into three categories. S1 participant was the first participants who become representatives for students who tend to always be in L1 for all constructions. The second participant, namely S2, is a representative student who tends to be at L2 and S3 is a representative student who tends to be at L3. The interviews were conducted using didactic situation theory to find out whether there are learning obstacles and what are the categories of those learning obstacles.

First, students in the subjective thinking level category (level 1) had difficulty applying the concept of probability in determining the probability of an event. There are two questions related to this construct. Problems in question 2 require students to categorize the event as impossible, possible to occur, and certain to occur. Students tend to group these events depending on their own experience (subjective). The results of student answers can be seen in Figure 4.



2. Sebuah dadu biasa yang memiliki enam sisi dilempar satu kali. Tunjukkanlah apakah kasus tersebut dapat tergolong tidak mungkin terjadi, mungkin terjadi atau pasti terjadi.

No	Kasus	Pilihan kejadian		
		Tidak mungkin terjadi	Mungkin terjadi	Pasti terjadi
1.	Muncul mata dadu 7		✓	
2.	Muncul mata dadu kurang dari 7		✓	
3.	Muncul mata dadu lebih dari 6	✓		
4.	Muncul mata dadu dua			✓
5.	Muncul mata dadu 6		✓	
6.	Muncul mata dadu bukan 6			✓

2. A six-sided die is rolled once. Indicate whether the following cases could be classified as impossible to occur, possible to occur, absolutely to occur

Question	Case	Classified		
		Impossible	Possible	Absolutely
1	Getting seven number		✓	
2	Getting less than seven number		✓	
3	Getting more than six number	✓		
4	Getting two number			✓
5	Getting six number		✓	
6	Getting number except six			✓

Figure 4. S1's answer

The following are some excerpts from in-depth interviews with student (S1) at a subjective level.

Intv: In the 4th part, S1 chooses something that will happen. According to S1, why must this absolutely happen?

S1 : Because mmmmm two. Because a lot of the appearance of the dice that rolled once is two like that, Ma'am

Intv: Can you explain more about what that means?

S1 : When I play that it appears more two than seven, ma'am. I always get two. So if one six-sided die is rolled once you will get two

Intv: The next section appears six. Here S1 selects possible to occur. Why is it possible? What's the difference with two?

S1 : (S1 thinks more than 1 minute) The answer should be impossible, ma'am. Because every S1 play this game, six never appears, it's different from the two which always appear.

From the interview excerpts, it can be seen that S1 classifies events that are impossible, possible, and certain to occur based on his perspective (subjective) experience. S1 said he never got "six" when he played the game. This condition showed he use a subjective perspective about uncertainty, and S1 has not been able to relate probability material. Based on the previous criteria related to the thinking process in the didactic situation theory, S1 is in the action stage. At the action stage, students can think according to previous experience. This means that there are obstacles, namely students do not understand the probability material. Students show inability applied the concept of probability to a different problem representation than that provided by the teacher. According to Brosseau (2002) and Suryadi (2019b), this condition indicates students have epistemological obstacle.

Based on Table 2, to facilitate the process of analyzing the results of the interviews, the researcher represents them in Table 5. The rubrics in the instrument have previously been tested and validated with the help of expert judgment. Table 5 summarizes the S1 interview using the perspective of didactic situation theory and learning obstacles experienced by students (remarks ○ stage has been reached, × has not been reached).

Table 5. S1 Interview Results

No	Action	Formulation	Validation	Institutionalization	Learning Obstacle
1	○	○	×	×	Instrumental Ontogenic Obstacle
2	○	×	×	×	Epistemological obstacle

No	Action	Formulation	Validation	Institutionalization	Learning Obstacle
3	○	×	×	×	Instrumental Ontogenic Obstacle
4	○	×	×	×	Instrumental Ontogenic Obstacle, Conceptual ontogenic obstacle
5	○	×	×	×	Epistemological obstacle
6	○	○	×	×	Instrumental Ontogenic Obstacle

The second representative was a student who tend to be at the level of transitional thinking (level 2) who has the initial S2. A unique answer was given by S2 on the probability comparison construction. S2 has provided quantitative responses while forming conclusions on the subjective side. The probability comparison construction is found in question no 5, where students are implicitly asked to compare opportunities. S2's problems and answers related to question number 5 can be seen in Figure 5.

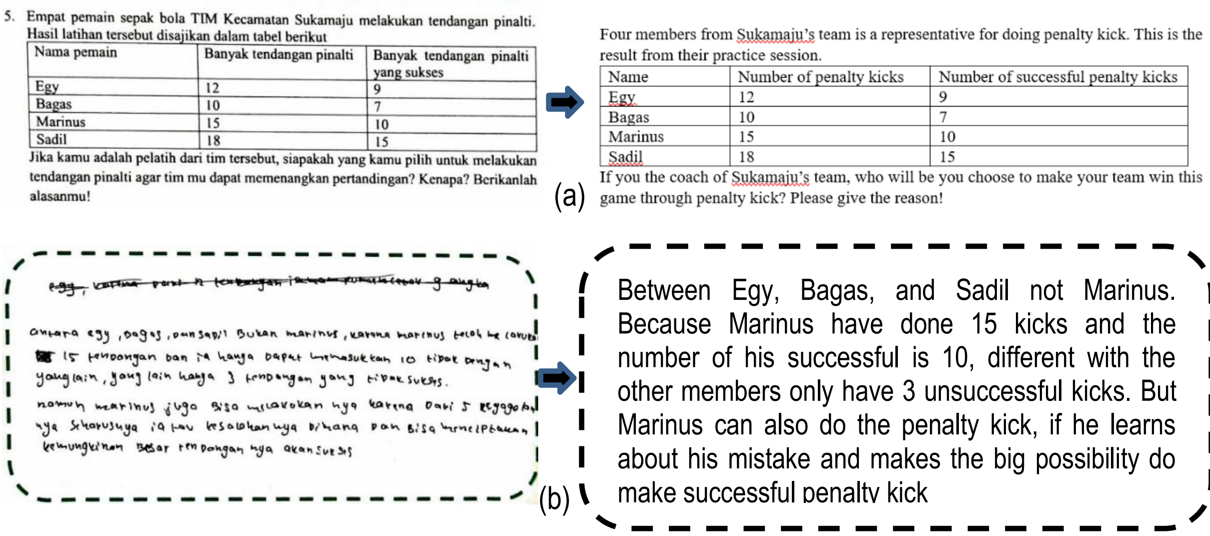
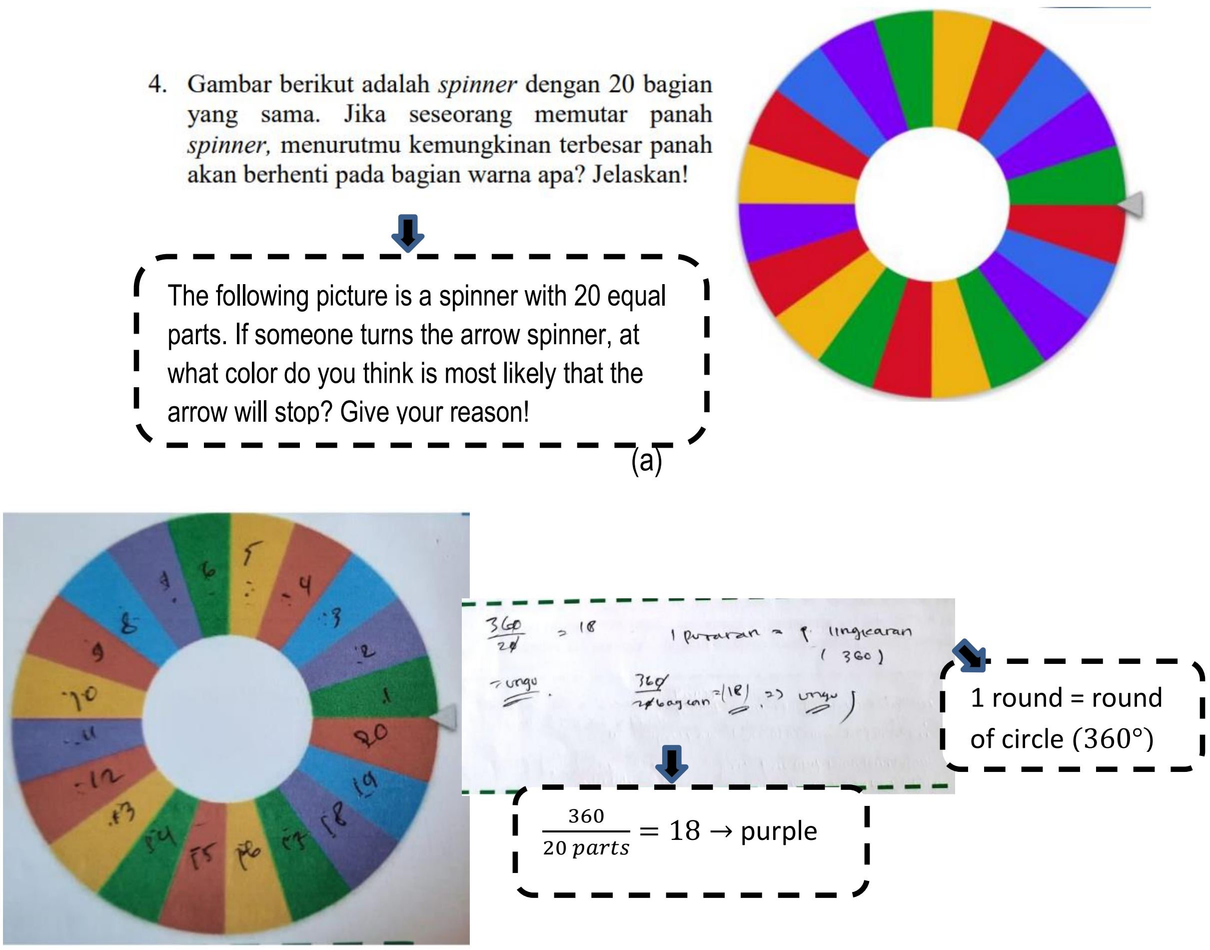


Figure 5. (a) The Problem given in Question 5, (b) S2's answer

In Figure 5 it can be seen that S2 initially used a quantitative approach, namely finding the difference between the number of penalties taken and the number of successful penalties. Then three names of players were obtained, specifically Egy, Bagas, and Sadil. However, it can be seen that S2 was not able to decide which player to choose to take the penalty kick because the value of the differences was the same, that is 3. In the last part, S2's answer again discusses one name he did not choose before, namely Marinus, because he had the biggest difference, which is 5. S2 shows his subjective side. S2 concludes that Marinus can be chosen if he can correct his mistakes.

In the construction of the probability of an event contained in the 4th problem, S2 again shows an answer that describes the level of transitional thinking. In problem 4, students are asked to implicitly determine the probability of an event. The problems given and S2's answers can be seen in Figure 6.



(b)

Figure 6. (a) The Problem given in Question 4, (b) S2's answer

In Figure 6 it can be seen that S2 uses a quantitative approach even though it is not related to the context of the opportunity material. S2 applies previous knowledge to determine that the number of angles on a circle is 360°. After that, S2 divides 360° with 20 because the spinner has 20 parts. The results are 18. In the conclusion step, S2 shows his subjective thoughts. To explore the thought processes experienced by S2, interviews were conducted with the following interview transcripts:

Intv: On the S2 answer sheet for question no 4, the answer is purple. Can you explain to Ms why S2 answered purple?

S2 : Hmm... Choose purple, yesterday it was just playing logic, Ma'am.

Intv: Can you explain more about what that means? Here is the number 360, where did it come from?

S2 : From all the degrees of the circle, Ma'am, because the spinner is a circle. One circle is 360°, then divided by 20 and I get 18. And the 18th color, I try to count in the spinner and I get purple, Ms

Intv: Then what color is the first and why?

S2 : Hmmm… It's green, Ms. Because the initial playing was from the green color, and the green color was near the arrow, Ma'am. So S2 considers starting from there.

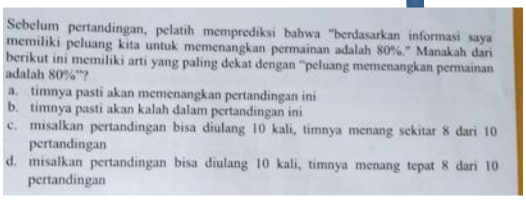
From the transcripts of the interview, it can be seen that S2 has not been able to relate the problem given to the concept of probability. Based on the preceding criteria connected to the thinking process in the didactic situation theory, S2 is in the action stage in the fourth problem. During the action stage, students might think based on prior experience. S2 did not realize he had to choose one individual in question number 4 after tracing through interviews (S2 needs to comprehend what to do from that question). Brosseau (2002) and Suryadi (2019b) claim that this condition indicates that students experience instrumental ontogenic obstacles. At the same time, the 5th problem is at the formulation stage. At this stage, S2 has been able to provide logical reasons, but it still needs to be related to the concept of probability. This means that there are obstacles; students need help understanding the probability material. Students need to be able to apply the concept of opportunity with a different problem representation than what the teacher has given. According to Brosseau (2002) and Suryadi (2019b), this condition indicates that students experience epistemological obstacles.

Based on Table 2, to facilitate the process of analyzing the results of the interviews, the researcher represents them in Table 6. Table 6 summarizes the S2's interviews using the perspective of the theory of didactical situations and learning barriers experienced by students (remarks ○ stage has been reached, × has not been reached)

Table 6. S2 Interview Results

No	Action	Formulation	Validation	Institutionalization	Learning Obstacle
1	○	○	×	×	Conceptual ontogenic obstacle
2	○	○	×	×	Instrumental Ontogenic Obstacle
3	○	○	×	×	Instrumental Ontogenic Obstacle
4	○	×	×	×	Instrumental Ontogenic Obstacle, Epistemological obstacle
5	○	○	×	×	Epistemological obstacle
6	○	○	×	×	Conceptual ontogenic obstacle, Epistemological obstacle

Students who are the third representative are students who tend to be at the level of informal quantitative thinking (level 3) and who have the initial S3. The test results from S3 show that he has been able to predict the most/most unlikely event based on the numerical probability of an event as seen from the PE construction in question number 6 which can be seen in Figure 7(b). In question no 4 there were no students who were at level 3, but immediately at level 4. Similarly, in S3, even though the dominant answers were at level 3 but for question number 6 answers were assessed as having occupied L4 (numeric).

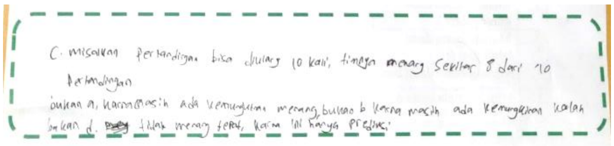

Sebelum pertandingan, pelatih memprediksi bahwa "berdasarkan informasi saya memiliki peluang kita untuk memenangkan permainan adalah 80%." Manakah dari berikut ini memiliki arti yang paling dekat dengan "peluang memenangkan permainan adalah 80%"?

a. timnya pasti akan memenangkan pertandingan ini
b. timnya pasti akan kalah dalam pertandingan ini
c. misalkan pertandingan bisa diulang 10 kali, timnya menang sekitar 8 dari 10 pertandingan
d. misalkan pertandingan bisa diulang 10 kali, timnya menang tepat 8 dari 10 pertandingan

Before a match, the coach predicted that "based on the information I have our chance of winning the game is 80%." Which of the following has the closest meaning to the "chance of winning the game is 80%"?

a) his team will certainly win this match
b) his team will certainly lose this match
c) suppose that the match could be repeated 10 times, his team wins around 8 out of the 10 matches
d) suppose that the match could be repeated 10 times, his team wins exactly 8 out of the 10 matches

(a)



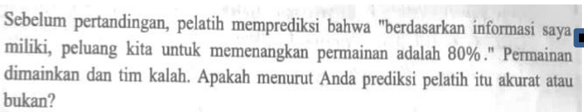
c. misalkan pertandingan bisa diulang 10 kali, timnya menang sekitar 8 dari 10 pertandingan
bukan a, karena masih ada kemungkinan menang, bukan b karna masih ada kemungkinan kalah
bukan d. tidak menang tepat, karna ini hanya prediksi

c) suppose that the match could be repeated 10 times, his team wins around 8 out of the 10 matches
Not A, because there is still possibility to win
Not B because there is still possibility to lose, and not d. not exactly will win because this is only prediction

(b)

Figure 7. (a) The Problem given in Question 6, (b) S3's answer

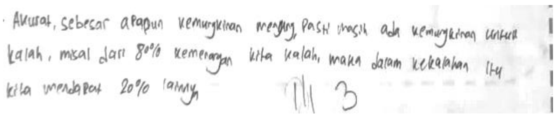
In Figure 7, it can be seen that S3 can interpret opportunity/probability as uncertainty. S3 has provided logical reasons for the answers he thinks are correct. It can be seen that S3 has been able to distinguish the meaning between "around" and "certainly" and added the argument that "this is a prediction, so it cannot be said to be certain." Based on the theory of the didactical situation for question no 6, the S3 is in an institutionalization situation. S3 can provide logical reasons for his answers by applying the concept of probability correctly, especially regarding the meaning of uncertainty. The next exciting thing happened to other questions with the same context but different diction. This question is question number 1, which is still in the same construction as question number 6. The problems given and the answers to S3 can be seen in Figure 8.



Sebelum pertandingan, pelatih memprediksi bahwa "berdasarkan informasi saya miliki, peluang kita untuk memenangkan permainan adalah 80%." Permainan dimainkan dan tim kalah. Apakah menurut Anda prediksi pelatih itu akurat atau bukan?

Before the game, the coach predicts that "based on my information, we have an 80% chance of winning". The game is over, and the team loses. Do you believe the coach's prediction is correct?

(a)



Akurat, sebesar apapun kemungkinan menang, pasti masih ada kemungkinan untuk kalah, misal dari 80% kemenangan kita kalah, maka dalam kekalahan itu kita mendapat 20% lainnya

Accurate, no matter how big the chance of winning is, there is still a possibility of losing. For example, from 80% chance of wins, we still have 20% chance for lose

(b)

Figure 8. (a) The Problem given in question 1, (b) S3's answer

It can be seen that S3 already understands well the meaning of uncertainty. However, S3 revealed that the results were accurate. To explore the thinking processes experienced by S3, interviews were conducted with the following interview transcripts:

Intv: In question number one, S3 answered that the coach's prediction was accurate. Why does S3 think it's accurate?

S3 : Hmm... Accurate because the problem says that predictions are based on information, ma'am.

Intv: Oo I see, so if there are words "based on information", it is correct. Isn't it S3?

S3 : Not really ma'am. Hmm.. Because the maximum value of the opportunity is 100%, ma'am. If it's 80%, it means that there are 20% failures, so it doesn't matter if it fails ma'am. Unless it is said that it will win 100% but in fact it fails, it is not accurate, ma'am. Because there is no percent (%) failure.

From the interview excerpts, it can be seen that S3 did not understand the meaning of accurate. Accurate can be interpreted as conformity with what is happening in the real conditions of the data. Cambridge Dictionary defines accurate as correct, exact, and without any mistakes. Meanwhile, according to S3, accuracy in probability is part of the uncertainty that has a source. Brosseau (2002) and Suryadi (2019b) said this condition indicates that students experience instrumental ontogenic obstacles, especially in the meaning of accurate words. Based on the didactic situation theory for question no 1, S3 is in a formulation situation. S3 can provide logical reasons related to the answers he conveys.

Based on Table 2, to facilitate the process of analyzing the results of the interviews, the researcher represents them in Table 7. Table 7 summarizes the S3 interviews using the perspective of the theory of didactical situations and learning obstacles experienced by students (remarks ○ stage has been reached, × has not been reached)

Table 7. S3 Interview Results

No	Action	Formulation	Validation	Institutionalization	Learning Obstacle
1	○	○	×	×	Instrumental Ontogenic Obstacle
2	○	○	○	○	-
3	○	○	×	×	Epistemological obstacle
4	○	○	×	×	Instrumental Ontogenic Obstacle
5	○	○	○	○	-
6	○	○	○	○	-

CONCLUSION

The results of this research show that the learning obstacles experienced by students can be affected by students' level of probabilistic thinking. Students who are at the subjective (L1) and transitional (L2) levels have more evenly distributed learning obstacles on all tasks compared to the two other levels. This is supported by the research results of Sari et al. (2023) that students' probabilistic thinking abilities are influenced by students' ability to understand mathematical concepts. The research results of Nurhayati et al. (2023) show that the ability to understand mathematics is related to the learning obstacles they experience. The lower the ability to understand mathematics, the more complex it is. Learning obstacles experienced by these students. The results of this research also show that probabilistic thinking abilities can describe the mental actions experienced by students in learning. Students who reach the numerical thinking level (L4) have reached the action stage in TDS, namely institutionalization. To make the explanation more accessible, the summary results related to the probabilistic level and the didactic situation process can be seen in Figure 9. Suggestions for further research are expected to be able to create didactic designs that can facilitate students reaching the institutionalization stage so that it can overcome the learning obstacles found and can develop students' probabilistic thinking abilities.

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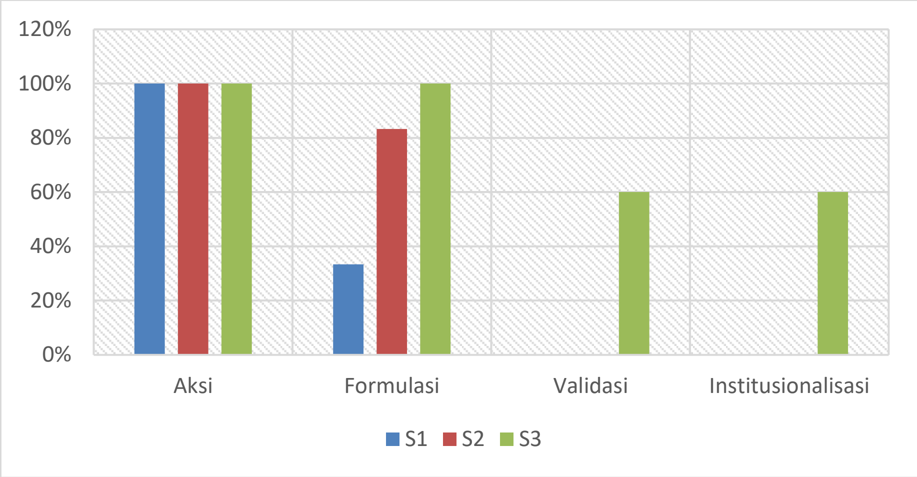

Figure 9. The Result of TDS participants

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