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Pre-service teachers' perspective toward problematic word problems

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Abstract

Despite extensive research indicating that students often fail to apply real-world knowledge and common sense when solving word problems, the underlying causes remain underexplored. Teacher behavior and instructional methods are potential factors contributing to students' tendency to provide unrealistic answers to such problems. The current study aims to address this gap by examining the cognitive processes and perspectives of pre-service teachers when solving problematic word problems. A group of 146 pre-service teachers (97 females, 49 males) in Iran participated in the study, which consisted of two phases. In the first phase, participants were given three problematic word problems to solve and were subsequently asked to evaluate four different student responses. A significant correlation was found between the participants' responses in the initial test and their evaluations in the second phase. In the second phase, the study employed a phenomenographic approach to explore the thinking processes and perspectives of the pre-service teachers while solving the problems. The analysis of interview data led to the identification of two primary categories of unrealistic problem-solving: "inattention" and "ignoring." In the "inattention" category, the problem solver fails to recognize the relevance of real-world knowledge, while in the "ignoring" category, the solver acknowledges real-world factors but deliberately chooses not to integrate them into the solution. In the end, a model of unrealistic problem-solving is proposed and discussed, with implications for teacher training and pedagogical practices.

Keywords: Common Sense, Mathematics, Phenomenography, Pre-Service Teacher, Problematic Word Problems

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Applied mathematical problem-solving constitutes a fundamental educational objective in numerous countries. This approach seeks to integrate the theoretical foundations of pure mathematics with practical applications, thereby serving as a bridge between abstract mathematical concepts and their real-world implementations in applied sciences. Within the national mathematics curriculum, spanning both primary and secondary education levels, the development of competencies such as "solving every day and abstract problems" and "mathematical modeling" is emphasized as a core learning goal (National Curriculum Document, 2011). The research literature on mathematical modeling presents various iterations of the modeling cycle, among which the widely referenced framework by Blum and Leiß (2007) is particularly notable (Figure 1). These representations collectively illustrate the notion that a mathematical model serves as a simplified abstraction of a real-world phenomenon, structured through mathematical formalization to facilitate problem-solving within practical contexts (Doerr et al., 2017).

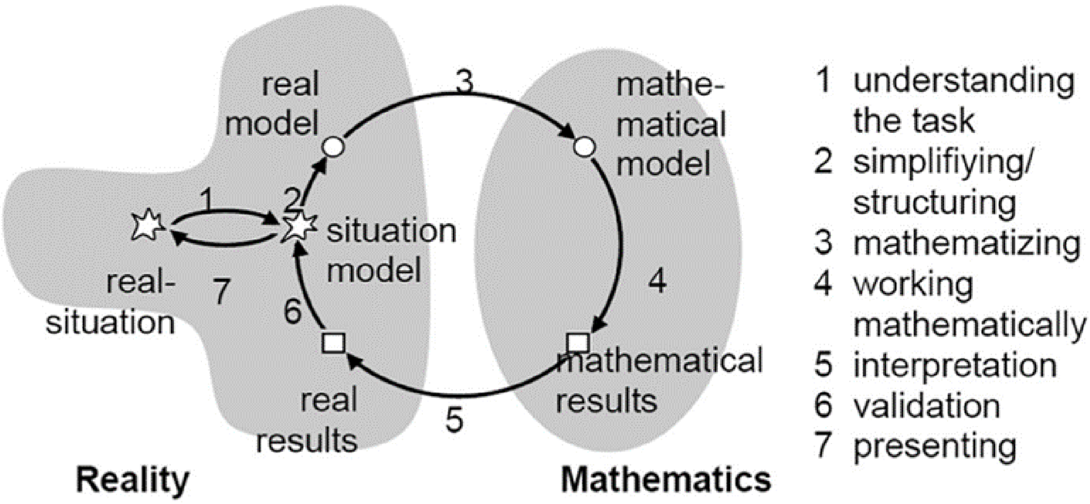


Figure 1. The modelling cycle depicted by Blum and Leiß (2007, p. 225)

According to Verschaffel et al. (2000), the teaching and learning of mathematical modeling and real-world problem-solving are predominantly facilitated through word problems. A word problem is a mathematical task presented in textual form, incorporating numerical data that requires one or more arithmetic operations to determine a solution. Word problems play a significant role in mathematics education, serving multiple functions. In addition to providing practice in modeling and solving real-life situations—referred to as the application function of word problems—they are also employed to assess students' mathematical proficiency, foster creative thinking, enhance motivation, and support the development of new mathematical concepts and skills (Verschaffel et al., 2000). Among these, the application function is particularly crucial, as the ability to solve problems encountered in daily life is considered essential. By engaging with these representations of real-world scenarios beyond the classroom, students are expected to develop the mathematical competencies necessary for addressing practical challenges in their present and future lives (Verschaffel et al., 2000).

Nevertheless, growing concerns have been raised regarding the extent to which conventional word problems in school mathematics effectively cultivate students' capacity for mathematical modeling and applied problem-solving. Instead of serving as authentic contexts that align with the mathematical modeling cycle—where students integrate commonsense reasoning and everyday experiences across different stages of the process—word problems in school settings are increasingly perceived as artificial, puzzle-like exercises with limited relevance to real-world applications (Verschaffel et al., 2000).

Researchers have examined the underlying causes of the "suspension of sense-making" that occurs when students engage with school word problems. There is a general consensus that the strategies and perspectives employed in solving such problems are often acquired implicitly, gradually, and tacitly as students become immersed in the culture and practices of mathematics classrooms (Schoenfeld, 1991; Verschaffel et al., 2000; 2010). For instance, a study by Sarkar Arani et al. (2017) explored the role of classroom culture regarding mistakes—specifically, how teachers and students perceive and learn from errors. Their findings indicate that in Malaysia, both students and teachers exhibit a low inclination toward learning from mistakes, leading to a high likelihood of errors being repeated.

Conversely, in Japan, teachers typically introduce mistakes to the entire class, encourage discussions to correct them, and view errors as opportunities for collaborative learning.

A teacher's limited emphasis on real-world applications in mathematics instruction can result in students developing misconceptions about the subject. Specifically, students may come to perceive mathematics as solely concerned with numerical computations and formulas, disconnected from practical, real-life contexts. Such misconceptions can diminish their motivation and engagement in learning mathematics. This acculturation process appears to stem from two primary aspects of education: the nature of the problems presented to students and the instructional approaches adopted by teachers (Verschaffel et al., 2000). However, despite these theoretical critiques, empirical evidence supporting this analysis of classroom practices and culture remains relatively scarce.

The role of teachers in fostering a learning environment where students feel both challenged and supported is crucial for enhancing engagement and academic achievement (Fraser & Walberg, 2005, as cited in Morrison et al., 2021). Additionally, teachers' mathematical flexibility—specifically, their ability to interpret and make sense of solution strategies that differ from their own—represents a key aspect of pedagogical content knowledge in mathematics (Jacobson et al., 2018). Morrison et al. (2021) further observed that students in their study experienced both support and intellectual challenge while working on their projects, as their teachers acted as collaborative partners in the learning process. This finding highlights the significant influence of teachers on students' educational experiences.

The present study seeks to empirically examine the second critical educational factor: the extent to which teachers comprehend and address the realism of word problems. More specifically, this research focuses on analyzing the cognitive processes and perspectives of pre-service teachers when solving interpretive word problems, employing a phenomenological approach. This aspect of inquiry remains largely unexplored in prior studies.

Theoretical and Empirical Background

Numerous examples illustrate that students often fail to incorporate real-world knowledge when solving word problems. One of the most well-known examples in this regard comes from an earlier French study, which presented primary school students with the following problem: "There are 26 sheep and 10 goats on a ship. How old is the captain?" The vast majority of students provided a numerical answer, most commonly 26 + 10 = 36, whereas only a small fraction questioned the validity or meaningfulness of the question (Verschaffel et al., 2010).

To empirically examine this phenomenon, Greer (1993) and Verschaffel et al. (1994) conducted studies in which students were asked to solve two distinct categories of word problems: standard and problematic. Standard word problems (S-items) required the direct application of arithmetic operations to the given numerical data. For example: "A man cuts a piece of rope 12 meters long into sections of 1.5 meters each. How many sections does he obtain?" In contrast, problematic word problems (P-items) necessitated the integration of real-world knowledge for an appropriate response. An example is: "A man needs a rope long enough to span the distance between two poles that are 12 meters apart. However, he only has pieces of rope that are 1.5 meters long. How many pieces must he tie together to bridge the gap?" In their study, Verschaffel et al. (1994) administered a paper-and-pencil test to 75 fifth-grade students, consisting of ten matched pairs of S-items and P-items. The findings indicated that only 128 out of the 750 responses to P-items (17%) were deemed "realistic," a result consistent with Greer's (1993) earlier research. Overall, both studies demonstrated that students tend to disregard real-world knowledge and realistic considerations when solving word problems (Verschaffel et al., 1994).

These findings have been replicated in multiple countries, including China, Germany, Japan, Switzerland, the Netherlands, and Venezuela, with remarkably consistent results across different cultural and educational contexts. Researchers were often surprised and disappointed by these outcomes, having initially expected less pronounced effects than those observed in the original studies (Verschaffel et al., 2010). In contrast to the results of the IREM (1980) study, Radatz (1983), in a study involving over 300 German students, found that the proportion of students attempting to provide a solution to insoluble problems increased with years of schooling. He concluded that students' problem-solving behavior is significantly shaped by the extent of mathematics instruction they have received. In response to these findings, researchers have explored various strategies to increase students' realistic responses when solving P-items by modifying the formulation or presentation of the problems.

General Warning about the Problematic Nature of Some Test Items

A notable study by Yoshida et al. (1997) investigated the same ten pairs of word problems utilized in the research by Verschaffel et al. (1994), examining two experimental conditions with Japanese fifth-grade students. In the first condition, participants received neutral instructions at the beginning of the test. In the second condition, students were additionally provided with a general cautionary statement regarding the potentially problematic nature of some test items. However, this additional instruction led only to a slight, statistically non-significant increase in the proportion of realistic responses (15% under the neutral condition compared to 20% in the warning condition).

Similarly, Xin et al. (2007) examined the same ten pairs of standards (S-items) and problematic (P-items) word problems from Verschaffel et al. (2010) under two instructional conditions. The first condition replicated the warning instruction used in Yoshida et al. (1997), while the second condition employed a process-oriented instructional approach. In this approach, participants were explicitly encouraged to reflect on two key questions: (a) What real-life situation does the problem statement represent? and (b) Is it appropriate to solve these problems using direct arithmetic operations? The findings indicated that process-oriented instruction resulted in a marginally significant improvement in the percentage of realistic responses, increasing from 21% under the warning condition to 28%.

Describing the Problem Situation using Representational Illustrations

Another investigation by Dewolf et al. (2015) explored the impact of visual illustrations on upper primary school students' ability to incorporate realistic considerations when solving problematic word problems (P-items). The study, conducted in both Turkey and Belgium, aimed to determine whether representational illustrations would help students develop a richer mental image of the problem context, thereby fostering more realistic responses. Contrary to expectations, the findings indicated that students in both countries continued to disregard real-world knowledge when solving school word problems, despite the presence of illustrations.

To further examine why these illustrations did not enhance students' realistic reasoning, Dewolf et al. (2015) conducted an eye-tracking study, which revealed that students devoted minimal or no attention to the visual representations. As a follow-up, the researchers implemented a modified experimental condition in which students were required to observe the illustration for five seconds before reading the problem statement. However, even with this enforced viewing, the results remained unchanged, indicating that visual aids alone were insufficient in prompting students to incorporate real-world reasoning into their problem-solving processes.

Rewording the Problem by Students and Dyadic Interaction to Solve the Problem

Mellone et al. (2017) investigated whether rewording problematic word problems (P-items) and encouraging dyadic interaction could enhance students' realistic reasoning. In their study, students were instructed to rewrite the problems themselves, aiming to promote a deeper understanding of the problem context and facilitate more meaningful engagement. However, despite these interventions, the percentage of realistic responses remained low, at approximately 23%, which was consistent with findings from prior studies. This suggests that merely altering the wording of problems and fostering peer discussion may not be sufficient to significantly improve students' ability to incorporate real-world knowledge into their mathematical reasoning.

Presenting the P-items in Humorous Condition

Van Dooren et al. (2019) explored an alternative approach to fostering realistic reasoning by embedding P-items within humorous contexts that explicitly highlighted the artificial and non-realistic nature of conventional school word problems. In their study, P-items were presented alongside cartoon characters engaging in jokes that emphasized the tension between strict mathematical norms and common-sense reasoning. For instance, one joke featured a teacher asking, “There are five birds on the roof, and you shoot two. How many are left?” to which a student humorously responded, “None, miss, the others have flown away.” This humorous intervention led to a 16.5% increase in the percentage of realistic responses, suggesting that humor may serve as an effective tool in prompting students to critically evaluate and apply real-world knowledge when solving word problems.

Presenting the P-items in Authentic Settings

Previous experimental studies examining P-items largely maintained the traditional scholastic context in which these problems were presented. However, alternative studies explored the impact of presenting P-items within more natural or authentic settings. For instance, DeFranco and Curcio (1997) investigated students' interpretations of division by remainder (DWR) problems, a specific type of P-item. They conducted an experiment with 20 sixth-grade students, presenting them with two different versions of the well-known bus problem. In the first version, students solved the problem individually as a conventional school mathematics task: “328 old-timers want to travel. Forty people can sit on each bus. How many buses are needed for them to travel?” The second version framed the same problem in a more authentic context—making a phone call to order minivans for class transportation. While only 2 out of 20 students provided a situationally appropriate response in the restricted version, 16 students gave realistic answers in the authentic version. A study by Dorri and Rafiepour (2016) yielded similar findings, reinforcing the potential of authentic contexts to promote realistic reasoning in problem-solving.

While most investigations employed paper-and-pencil tests, a few small-scale studies adopted individual interviews to gain deeper insights into students' non-realistic responses to P-items (e.g., Caldwell, 1995; Hidalgo, 1997; Inoue, 2001; Reusser & Stebler, 1997; Selter, 2001). Many students reflected on their solutions, acknowledging that they had answered automatically without hesitation. For instance, one student stated, “I know all these things, but I would never think to include them in a math problem. Math isn't about things like that. It's about getting sums right, and you don't need to know outside things to get sums right” (Caldwell, 1995). Others reported considering the complexities of modeling but ultimately adhered to the norms of “the word problem game” (Verschaffel et al., 2000). One student explained, “I did think about the difficulty, but then I just calculated it the usual way. (Why?) Because I just had to find some sort of solution to the problem, and that was the only way it worked. I've got to have a solution, haven't I?" (Reusser & Stebler, 1997).

Although the majority of research on P-items focused on upper elementary school students, some studies examined how older students approached these problems. Verschaffel et al. (1997) investigated this phenomenon among 18- to 21-year-old Flemish pre-service teachers enrolled in a professional bachelor's program for elementary education. To understand how teachers consider realistic reasoning in word problem instruction, the researchers administered two tests. First, participants solved seven pairs of P- and S-items, mirroring the original study by Verschaffel et al. (1994). In the second phase, the pre-service teachers evaluated four different handwritten student responses to the same 14 problems, assigning scores of 1 point (totally correct), ½ point (partially correct), or 0 points (totally incorrect). The findings revealed that only 48% of pre-service teachers provided realistic responses in the first test. In the second test, they rated only 47% of realistic answers (RA) with a score of 1, while 6% received a ½-score and 47% were scored as 0. Conversely, non-realistic answers (NA) were scored with a 1 and ½ in 56% and 26% of cases, respectively, while only 18% received a 0-score.

The present study aims to investigate the tendency of pre-service teachers to suspend common sense when solving problematic word problems, similar to the study by Verschaffel et al. (1997). In the Iranian national elementary mathematics curriculum, students are expected to solve real-world problems (National Curriculum Document, 2011, p. 35). As previously discussed, teachers' perspectives on P-items and their instructional approaches significantly influence students' suspension of common sense when solving these problems. Therefore, it is essential to examine this issue among prospective teachers, as in the study by Verschaffel et al. (1997). The study is guided by the following research questions:

1. How do Iranian pre-service teachers respond to P-items?
2. How do Iranian pre-service teachers reason and act when solving problematic word problems? Specifically, what factors contribute to their unrealistic responses?

Mathematics education is influenced by multiple disciplines, including mathematics, psychology, sociology, and philosophy (Higginson, 1980). Considering local educational contexts, this study investigates the responses of Iranian pre-service teachers, replicating Verschaffel et al.'s (1997) survey study. Since no prior research has examined this phenomenon in Iran, and given the passage of time since 1997, it is essential to determine whether this trend persists in pre-service teachers' performance. Following confirmation of the phenomenon's existence, the study further explores how pre-service teachers think when solving these problems. Unlike previous studies, the primary focus here is to analyze the cognitive processes and perspectives underlying their problem-solving approaches. To address the first research question, a survey using a paper-and-pencil test was conducted, while a phenomenographic approach was employed for the second question to provide deeper insights into the phenomenon.

METHODS

Participants

This study involved a total of 146 second-year undergraduate students (97 females and 49 males) enrolled in elementary school teacher training programs at two universities located in different provinces of southeastern Iran. Participation was voluntary. In the primary education undergraduate curriculum, a total of 150 credits are required, of which 6 credits are allocated to mathematics courses. This credit allocation is consistent across all universities in the country that offer teacher training programs. The participants had an average age of 19.5 years and had previously completed several mathematics-related courses, including a mathematics pedagogy course emphasizing the importance of connecting mathematical education with students' real-world experiences. However, none of their coursework had explicitly addressed the topic of (un)realistic mathematical modeling in word problems.

Materials and Methods

In the first phase of this study, three problem items (P-items) were selected from the seminal works of Greer (1993) and Verschaffel et al. (1994) presented in Table 1. These P-items were designed to assess the participants' ability to differentiate between realistic and non-realistic mathematical modeling approaches, aligning with prior research in this domain.

Table 1. The three P- items involved in Part 1 of the study

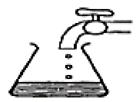
Name	Problem
Runner	John's best time to run 100 meters is 17 seconds. How long will it take to travel 1 kilometer?
Flask	This flask is being filled from a tap at a constant rate. 1f the depth of the water is 3.5 cm after 10 seconds; how deep will it be after 30 seconds (This problem was accompanied by a picture of a partly filled cone-shaped flask)
Rope	A man wants to have a rope long enough to stretch between two poles 12 meters apart. But he has only pieces of rope 1.5 meters long. How many of these pieces would he need to tie together to stretch between the poles?

Following the methodology of Verschaffel et al. (1997), these three problems were presented to the participants twice, with different tasks assigned in each instance. In the first test (Test 1), the participants were required to independently solve the problems and subsequently provide an incorrect answer that they anticipated a student might give. Figure 2 illustrates an example of the Flask problem as presented in Test 1.

This flask is being filled from a tap at a constant rate.

If the depth of the water is 3.5 cm after 10 seconds.

How deep will it be after 30 seconds?



Correct answer:

Expected incorrect answer:

Comments:

Figure 2. Sample problem in Test 1

In the second test (Test 2), the pre-service teachers were instructed to evaluate four different hand-written responses, purportedly from hypothetical students, for each of the three-word problems from Test 1. The evaluation process involved assigning scores of 1 point, ½ point, or 0 points to each response. The scoring instructions, adapted from Verschaffel et al. (1997), were provided at the beginning of Test 2 as follows:

> *"This test consists of the same three problems as in Test 1. However, this time, each problem is accompanied by four distinct answers observed in a previous study involving fifth-grade students. Please evaluate each response by assigning a score in the designated box at the*

bottom-right corner of each response alternative. Assign 1 point for an answer you consider to be (absolutely) correct, 0 points for an answer that is (completely) incorrect, and ½ point for responses that you believe are partially correct but contain elements of incorrect reasoning. Each problem allows for all three scoring categories to be used multiple times, while some scores may remain unused. Additionally, a designated space is provided at the bottom of each problem for comments or explanations. If you wish to comment on a specific response, please indicate the corresponding letter (A, B, C, or D)."

Figure 3 illustrates an example of the Flask problem as presented in Test 2.

This flask is being filled from a tap at a constant rate. If the depth of the water is 3.5 cm after 10 seconds, how deep will it be after 30 seconds?

A) 3 × 3.5 cm. = 11.5 cm.
After 30 seconds the depth of the water will be 11.5 cm.

B) 3 × 3.5 cm = 10.5 cm.
After 30 seconds the depth of the water will be 10.5 cm

C) 3.5 cm. + 20 cm. = 23.5 cm.
After 30 seconds the depth of the water will be 23.5 cm

D) It is impossible to give a precise answer.

Figure 3. Sample problem in Test 2

Scoring Criteria

The four responses provided for evaluation in Test 2 were categorized based on the classification framework established by Verschaffel et al. (1997). These categories are as follows:

1. Realistic Answer (RA): A response that demonstrates an effective and appropriate application of real-world knowledge in interpreting the problem context (e.g., Answer D).
2. Non-Realistic Answer (NA): A response that results from an uncritical and direct application of arithmetic operations without considering real-world constraints (e.g., Answer B).
3. Technical Error (TE): A response derived from a straightforward arithmetic application, but containing an execution mistake in the calculation process. Despite being a computation error, TE responses are still classified as non-realistic solutions (e.g., Answer A). Technical errors are common in mathematical problem-solving, which is why this category was explicitly included in the study.
4. Other Answer (OA): A response that does not fit into any of the aforementioned categories, such as an incorrect solution arising from the selection of an inappropriate mathematical operation (e.g., Answer C).

Following the classification system proposed by Verschaffel et al. (1994; 1997), the responses provided by pre-service teachers in Test 1 were categorized into five groups. These included the four categories listed above—Realistic Answer (RA), Non-Realistic Answer (NA), Technical Error (TE), and Other Answer (OA)—along with an additional category for unanswered problems (NOA, No Answer).

Furthermore, if a pre-service teacher's response included a critical remark about the problem itself or a justification for their answer (e.g., "This is an approximate solution"), it was classified as a realistic response. Conversely, a non-realistic response (NR) was characterized by a complete absence of real-world considerations in both the answer and the accompanying comments.

Table 2 presents the distribution of pre-service teachers' responses to the three P-items in Test 1. The findings indicate that only 4.1% of all responses were classified as realistic, while the majority (79.45%) were categorized as non-realistic. Additionally, 8.9% of responses contained technical errors, and 3.88% fell under other answer categories. These results highlight a predominant inclination toward non-realistic responses among pre-service teachers.

Table 2. Percentage distribution of pre-service teachers' responses to Test 1 for each problem

Problem	Name	Realistic Answer (RA)	Nonrealistic Answer (NA)	Technical Error (TE)	Other Answers (OA)	No Response (NOA)
1	Runner	2.73	78.76	12.32	2.05	4.10
2	Flask	5.47	78.08	10.27	2.73	3.42
3	Rope	4.10	81.50	4.10	6.84	3.42
Total		4.10	79.45	8.90	3.88	3.65

Furthermore, Table 3 provides an overview of the percentage and absolute frequency of scores (1, ½, and 0) assigned to realistic and non-realistic responses for the three P-items in Test 2. The results indicate that 87.67% of pre-service teachers assigned a score of 0 to realistic responses, signifying that they perceived these responses as entirely incorrect. Conversely, 81.05% of the scores assigned to non-realistic responses were 1, demonstrating that pre-service teachers overwhelmingly considered non-realistic responses as correct.

Table 3. Percentage and absolute numbers of scores of 1, 1/2, and 0 for the realistic and non-realistic answer to three P-problems of Test 2

Problem	Name	Realistic Answer			Non-Realistic Answer		
		1	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0
1	Runner	4.80 (7)	8.22 (12)	86.98 (127)	71.23 (104)	21.92(32)	6.85 (10)
2	Flask	4.80 (7)	7.53 (11)	87.67 (128)	86.3 (126)	11.64 (17)	2.06 (3)
3	Rope	4.11 (6)	7.54 (11)	88.35 (129)	85.61 (125)	6.17 (9)	8.22 (12)
Total		4.57 (20)	7.76 (34)	87.67 (384)	81.05 (355)	13.24 (58)	5.71 (25)

Relationship between the Two Tests

Following the approach of Verschaffel et al. (1997), the study further examined the alignment between pre-service teachers' responses in Test 1 and their evaluations of responses in Test 2. A strong correlation was expected between the solutions provided in Test 1 and the way pre-service teachers assessed student responses in Test 2. To analyze this relationship, responses from participants who provided realistic answers in Test 1 (4.1% of total responses) were compared to their assigned scores in Test 2, as presented in Table 4.

As anticipated, the results demonstrate a clear and strong relationship between pre-service teachers' responses in Test 1 and their evaluations in Test 2. Specifically, 61.1% of participants who provided a realistic answer in Test 1 assigned full credit to the realistic response in Test 2. Similarly, 84.7% of those who initially provided a non-realistic response in Test 1 assigned full credit to the non-realistic answer in Test 2. These findings are consistent with previous studies (Verschaffel et al., 1997), reinforcing the notion that pre-service teachers exhibit a strong preference for non-realistic problem-solving approaches.

Table 4. Percentage of scores (1, ½ or 0) given to the realistic and unrealistic answer in Test 2 by Pre-Service Teachers who gave a realistic and a non-realistic answer to the corresponding P-item in test 1

PST's Score for the Student's Answer in Test 2	Realistic reaction in Test 1 (4.1% of all cases)		Non-realistic reaction in Test 1 (95.9% of all cases)	
	Realistic Answer	Non-realistic Answer	Realistic Answer	Non-realistic Answer
1	61.11 (11)	22.2 (4)	3.33 (14)	84.76 (356)
$\frac{1}{2}$	33.33 (6)	44.4 (8)	6.42 (27)	10.47 (44)
0	5.55 (1)	33.33 (6)	90.23 (379)	4.76 (20)

RESULTS AND DISCUSSION

Due to the inherent limitations of a pen-and-paper test, the study employed the Phenomenography Method, which involved conducting interviews with seven volunteers to gain a deeper understanding of the phenomenon. The primary criterion for selecting participants was that they had provided unrealistic answers to at least one of the problems in Test 1 or had assigned full scores to unrealistic responses in Test 2. These participants were identified based on their performance in the first phase of the study and willingly volunteered to participate in the interviews.

As presented in Table 5, Interviewee 1 provided realistic answers to two out of the three problems in Test 1 and was the only participant to assign a score greater than zero to a realistic response in Test 2. The majority of interviewees had demonstrated a tendency toward unrealistic responses in the first test. In total, seven participants took part in the interview process, contributing data from 21 cases: two cases of realistic answers (RA), three cases of other answers (OA), and 16 cases of nonrealistic answers (NA). Following the interviews, the researchers determined that theoretical saturation had been reached. Prior to conducting the interviews, participants were asked for their consent to audio-record the sessions, and they were assured of complete confidentiality. The average interview duration was approximately 20 minutes, with a minimum of 10 minutes and a maximum of 45 minutes.

Initially, participants were presented with the three problems from Test 1 (refer to Table 1) and were instructed to read each problem aloud while verbally explaining their thought process and solution. If a participant provided an unrealistic response and did not incorporate real-world knowledge or common sense relevant to the problem, the interview proceeded with a predetermined set of specific questions and prompts. These were designed to encourage the participant to activate and apply their real-world knowledge while simultaneously uncovering the rationale behind their initial unrealistic response.

For instance, in the case of the third problem (rope), participants were asked the following questions (see Figure 4):

1. Please read the problem aloud again. (Re-reading the problem)
2. One of your peers suggests that "It is impossible to determine an exact answer to this question." Do you agree with their reasoning? Is your answer correct, or is theirs? (Confronting the participant with a realistic perspective)
3. Did you experience any uncertainty regarding your answer or solution while responding to the question? (Assessing doubt in the response)
4. Imagine you need to fasten a clothesline between two trees in a dormitory yard, but you only have a 1.5-meter-long rope. How many rope segments would be required to accomplish this task? (Introducing an authentic, real-world problem)
5. Can you reformulate this problem in a way that would make it easier for children to understand while maintaining the same solution? (Rewording the problem)
6. Can you illustrate the situation described in the problem by drawing a diagram?
7. Now, examine this illustration—what are your thoughts on it?

Table 5. Interviewee's response for each problem in test 1 and scores they allocated to RA and NA solutions in test 2

Interviewee	Runner Problem			Flask Problem			Rope Problem		
	Test 1	Test 2 Score		Test 1	Test 2 Score		Test 1	Test 2 Score	
		RA	NA		RA	NA		RA	NA
1	RA	0	½	RA	½	1	NA	0	1
2	OA	0	½	OA	0	1	OA	0	0
3	NA	0	1	NA	0	1	NA	0	1
4	NA	0	1	NA	0	1	NA	0	1
5	NA	0	1	NA	0	1	NA	0	1
6	NA	0	1	NA	0	1	NA	0	1
7	NA	0	1	NA	0	1	NA	0	1

These questions aimed to prompt participants to reflect on their reasoning, reconsider their initial responses, and engage in a deeper cognitive process that incorporated real-world applications.

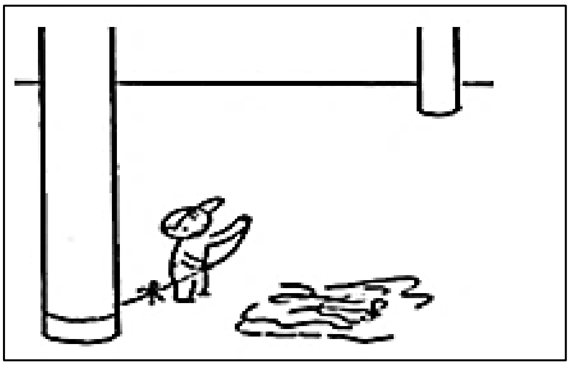


Figure 4. Representational illustrations related to the rope problem

Similar sets of guiding questions and prompts were developed for the remaining two problems.

These questions and hints were informed by the methodologies employed in previous qualitative studies on P-items (e.g., Dewolf et al., 2015; Mellone et al., 2017; Vos, 2011).

In phenomenological research, key sentences or meaningful statements are systematically identified and highlighted. Each of these significant statements—whether an entire interview, a segment of a participant's thought process, or an essential component of their reasoning—is carefully selected and subsequently compared (Gall et al., 1942). The process of analyzing phenomenographic data follows key stages outlined by McCosker et al. (2004), which include:

1. Familiarization – Researchers review the data multiple times to gain a deep understanding of its details.
2. Condensation – The most representative statements are selected to identify emerging patterns.
3. Comparison – The similarities and differences in responses are analyzed to determine sources of variation.
4. Grouping – Statements are categorized based on their commonalities.
5. Articulation – The essence of these similarities is extracted, classified, and described.
6. Labeling – The identified categories are linguistically represented.
7. Contrasting – The categories are systematically compared and contrasted (Han & Ellis, 2019).

Although different models may present variations in their methodological steps, they share common core elements. The frameworks of Gall et al. (1942) and McCosker et al. (2004) served as analytical guidelines for this study. Following the completion of the interviews, the recorded conversations were transcribed verbatim and repeatedly reviewed to comprehend participants' experiences, thoughts, and perspectives. The data analysis was conducted in two phases:

1. Examining the process by which participants arrived at unrealistic solutions.
2. Investigating their reasoning or perspectives underlying such responses.

During the analysis, researchers marked and took notes on key phrases that provided insight into participants' cognitive processes. For instance:

1. "I was not paying attention" or "I didn't think about it" (Interviewee 6 – Rope problem).
2. "I realized", "I considered that… but did not take into account the real-world implications" (Interviewee 1 – Runner problem).
3. "I did not pay attention to the form of..." (Interviewee 3 – Flask problem).
4. "It would be better to specify in the problem: 'without considering fatigue'" (Interviewee 5 – Runner problem).

Subsequently, responses exhibiting similar characteristics were grouped into preliminary categories. For example, Interviewee 3 (Flask problem) and Interviewee 6 (Rope problem) were categorized under inattention, as both exhibited a failure to recognize the role of real-world knowledge in their answers. Likewise, Interviewee 1 (Runner problem) and Interviewee 5 (Runner problem) were grouped under ignoring, where participants acknowledged realistic considerations but consciously disregarded them.

Within these categories, specific phrases were further arranged to highlight key themes. For example, statements like "I just realized" were classified under inattention, whereas phrases such as "Of course, this is not an accurate answer" were categorized under ignoring.

In the second phase of analysis, the underlying reasons for participants' unrealistic responses were examined. The data was re-read multiple times, and researchers annotated key terms reflecting participants' beliefs about the nature of mathematical problems. Phrases such as "rational", "accurate", and "calculative", which emphasize a procedural or numerical approach to problem-solving, were commonly identified. Similar phrases, including "numerical answers", were grouped within this conceptual category.

To ensure the reliability and validity of the findings, two measures were employed. First, the extracted key components of participants' reasoning were shared with them for verification, allowing them to provide feedback on the accuracy of the researchers' interpretations. Second, two independent experts conducted separate coding of the data, with an inter-rater reliability exceeding 90% agreement. The following section presents two overarching categories derived from participants' approaches to unrealistic problem-solving.

Based on the responses of the interviewees, the unrealistic answers they provided to mathematical problems can be categorized into two primary causes: inattention and deliberate ignoring. Inattention occurs when pre-service teachers fail to recognize the problematic nature of a word problem from a realistic modeling perspective. The following interview excerpts illustrate this phenomenon:

Extract 1 (Rope problem)
Interviewee 6 : "I was not paying attention; I didn't think about it. I just did the basic calculation."

Extract 2 (Flask problem)
Interviewer : "See the bottles in the picture."
Interviewee 3 : "It doesn't matter to me whether the shape is there or not. I can answer, but it may be easier for the child to answer with a shape. I work with numbers; I don't work with shapes."

The second cause of unrealistic responses is ignoring the realistic aspects of the problem despite being aware of them. This occurs when pre-service teachers recognize the complexity of realistic modeling but choose to disregard it in their problem-solving process. The following excerpts exemplify this behavior:

Extract 3 (Flask problem)
Interviewer : "You paid attention to these issues, for example, that the diameter of the bottle is important, or in the third problem, that the knot is necessary, but you did not consider them in solving the problems?"
Interviewee 4 : "Yes, because in general, in mathematics and physics, they ask questions that may not have a practical basis at all or are not correct. It is important that the answer be given."

Extract 4 (Runner problem)
Interviewee 5 : "In this problem, a runner's fatigue should be considered, as it affects the time taken to complete the route. However, we assume the runner maintains a constant speed, so we use 100 meters in 17 seconds and derive 170 seconds as the answer. It would be better if the problem explicitly stated, 'without considering fatigue.'"

Following the provision of hints, some participants (9 out of 19) reconsidered their initial unrealistic answers and acknowledged the real-world connection within the problem. This shift in understanding is exemplified in the following interview extract:

Extract 5 (Rope problem)
Interviewer : "Imagine you want to attach a clothesline to two trees in the dormitory area, but you only have pieces of rope 1.5 meters long..." (the interviewer had not finished the question before the

interviewee provided a more realistic response).

Interviewee 5 : "Well, we have to tie these together. If we want to do that, we need more rope. As a precaution, we take an extra rope. If we calculate precisely, we need eight ropes, but in practice, we need more because tying knots shortens the ropes."

At the conclusion of the interviews, participants reflected on why they neglected real-world knowledge in problem-solving. Some admitted that they had not carefully read or analyzed the problem situation, which aligns with Hidalgo (1997), who identified the absence of critical heuristic and metacognitive skills—such as careful reading, problem analysis, and verification of the computational outcome—as factors leading to unrealistic answers. The following extract provides an example:

Extract 6 (Rope problem)
Interviewer : "Why do you think you did not consider this issue initially?"
Interviewee 6 : "I was not paying attention; I didn't think about it. I just did the basic calculation."

Other interviewees cited the influence of ingrained assumptions on their problem-solving approaches, as demonstrated in the following excerpt:

Extract 7 (Flask problem)
Interviewee 3 : "We were always given simple examples with no exceptions. I had a fixed mental image of the bottle's shape and diameter. Students who provide unrealistic answers also respond based on an image in their mind rather than the actual problem context."

This contrast between perceptual and conceptual understanding may explain why representational illustrations are ineffective in resolving word problems (Verschaffel et al., 2016). Additionally, some interviewees indicated that a fundamental belief in the dichotomy between mathematical problems and real-world scenarios contributed to the exclusion of real-world considerations in their solutions. Mathematical problems are perceived as requiring numerical, precise, and unique answers derived through arithmetic operations, particularly in written assessments. The following excerpts highlight these beliefs:

Extract 8 (Rope problem)
Interviewee 5 : "Scientific mathematics is rational, accurate, and based on calculations. We do not consider errors. We know that 12 divided by 1.5 is 8. We cannot say that 8.5 meters of rope is needed. If the answer is 8, then the answer is 8."

Extract 9
Interviewee 4 : "It is important to get a numerical answer. If this were a test question, I would have provided the unrealistic answer even if I personally believed it was incorrect."

Another common belief among interviewees was that knowledge about the problem context is unrelated to mathematics and therefore should not influence calculations. This perspective is evident in the following extracts:

Extract 10 (Runner problem)

Interviewee 2 : "Factors such as runner fatigue do not belong in mathematics. These are psychological aspects, not mathematical ones."

Interviewee 3 : "You know, I did not really solve the problem—I just answered it using mathematics."

Interviewee 4 : "It is pointless to consider these aspects. They apply in real life, not in mathematics."

Interviewee 7 : "The goal of the problems we are given is not to reflect reality but to focus on calculations."

Additional explanations emerged from the interviews. Some participants suggested that students might struggle with certain problem situations due to a lack of familiarity. For example, regarding the flask problem, a student may not realize that a narrower diameter results in a faster rise in water level, a finding also reported by Hidalgo (1997). Another participant noted that some students hesitate to express their understanding due to a lack of confidence:

Interviewee 1 : "The student may know the answer but lacks the courage to express it. In my future classroom, I plan to foster a close teacher-student relationship to encourage open expression, even when students provide incorrect answers. This is very important."

Research by Laine, Ahtee, and Näveri (2020) found that students in classrooms emphasizing silent individual work were less accustomed to explaining and justifying their thinking. In contrast, students in classrooms that fostered open discussions were more likely to articulate and defend their perspectives (Newstead, 1998, cited in Laine et al., 2020). Ultimately, students' problem-solving strategies and beliefs are shaped by their understanding of the didactical contract and the socio-mathematical norms that dictate expected behavior, thinking processes, and teacher-student communication in mathematics classrooms (Verschaffel et al., 2020).

Numerous studies have demonstrated that students generally do not incorporate real-world knowledge or common sense when solving complex word problems (e.g., Verschaffel et al., 2010; Dewolf et al., 2015). A key factor contributing to this phenomenon is the perspective of teachers regarding problematic word problems (P-items) and their instructional approach to handling these problems in the mathematics classroom (Verschaffel et al., 1997). The present study aimed to investigate this issue by first analyzing the responses of Iranian pre-service teachers to P-items and subsequently conducting a qualitative inquiry into their cognitive processes while solving these problems, as well as the underlying reasons for their unrealistic responses.

In the first phase of the study, a total of 146 pre-service teachers were asked to solve three problematic word problems (P-problems), which have been widely used in prior research (see Verschaffel et al., 2000; 2010), across two test sessions. In the initial test, only 4.1% of the responses to the three P-items demonstrated realistic reasoning. In the second test, participants were tasked with evaluating four types of student responses to the same set of P-items. The findings from both tests aligned with previous research, revealing a strong correlation between the pre-service teachers' unrealistic responses in the first test and their evaluation of realistic and unrealistic responses in the second test. Specifically, among those who provided realistic responses in Test 1, 61.1% assigned full scores to realistic answers in Test 2. Conversely, 84.7% of those who initially provided unrealistic answers assigned full scores to unrealistic responses in Test 2. This suggests that most pre-service teachers tended to evaluate responses in a manner consistent with their own problem-solving approach, giving the highest score to the response that aligned with their own answer. These findings provide empirical support for the argument that teachers play a crucial role in shaping students' beliefs and strategies regarding the integration of real-world elements into mathematical word problem solving and mathematical modeling more broadly. More specifically, if teachers do not incorporate real-world knowledge when solving mathematical word problems, students who perceive a contradiction between the mathematical solution and real-world knowledge are likely to disregard real-world considerations and rely solely on mathematical reasoning.

To further explore this phenomenon, a phenomenographic approach was employed to examine the tendency to disregard real-world knowledge when solving P-items. The qualitative analysis classified the reasons behind pre-service teachers' unrealistic responses into two categories: "inattention" and "ignoring." This section of the study highlights how qualitative findings help explain the quantitative results obtained in Tests 1 and 2. In the first category, "inattention," problem solvers failed to recognize the relevance of real-world knowledge in formulating their responses (Figure 5 – Inattention). In such cases, they proceeded directly to mathematical modeling and arithmetic computations without considering the situational context or interpreting the results. For instance, Interviewee 6 stated, "I was not paying attention, I didn't think about it. I just did the basic calculation." This pattern of reasoning aligns with the model proposed by Verschaffel et al. (2000). The second category, "ignoring," represents a novel pattern identified in this study, wherein problem solvers acknowledge realistic considerations but deliberately choose to disregard them (Figure 5 – Ignoring). In this case, the solver interprets the problem within its real-world context but consciously decides not to incorporate these considerations into the solution process.

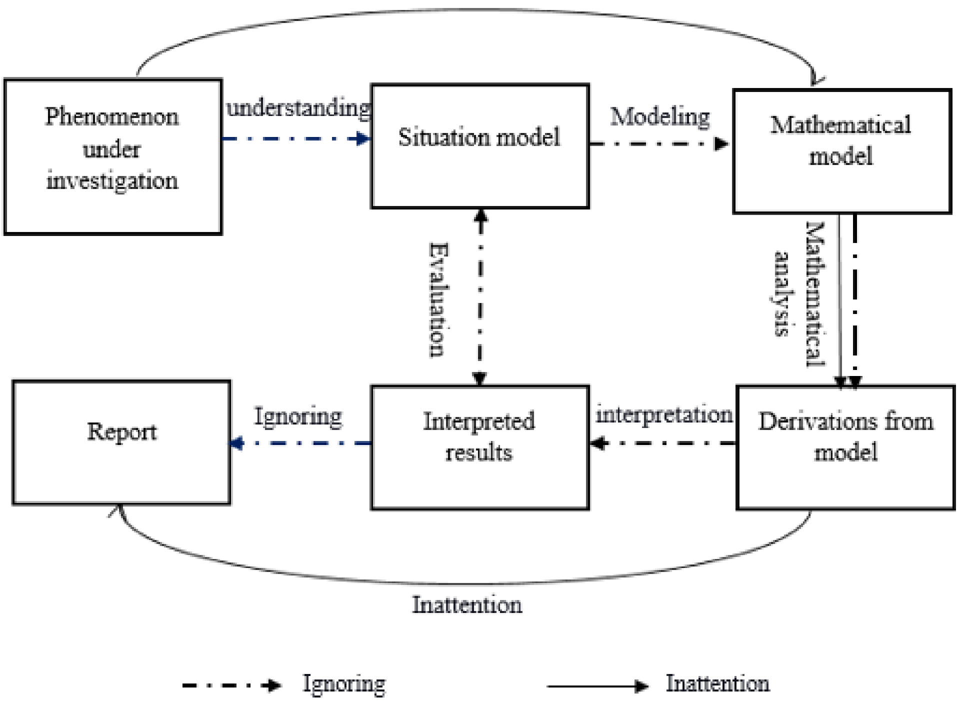


Figure 5. The superficial solution of word problems

The statements provided by the interviewees suggest that beliefs are a critical factor contributing to unrealistic responses. Failure to accurately read and analyze the given problem may be linked to pre-existing beliefs about mathematical problems, as individuals' expectations and assumptions regarding the nature of mathematical problems can influence their attention to different aspects of a problem. Another reason cited by a pre-service teacher pertains to a cognitive disconnect between visual perception and conceptual understanding. This issue is also associated with beliefs, as illustrated in the following statement: "We were always given simple examples with no exceptions. I had a fixed mental image of the bottle's shape, assuming the entire diameter was constant…" (Interviewee 3). These findings underscore the significance of mathematical beliefs, highlighting their importance for further research. However, it appears that the nature of textbook problems, exam questions, and even the accompanying visual representations significantly influence the formation of these beliefs.

Two additional belief-related factors were identified through the interviews. The first concerns the perceived nature of mathematical problems, in which mathematics is viewed as a discipline where answers must be numerical, unique, and derived through one or more arithmetic operations. The second belief is the perceived separation between the mathematical domain and real-world applications. To address these misconceptions, school culture should foster greater opportunities to illustrate that mathematics is not rigidly predetermined, that problems do not always have a single correct solution, and that mathematical concepts have historically emerged, at least in part, through real-world observations (Fitzpatrick et al., 2019).

CONCLUSION

The findings of this study highlight the importance of incorporating real-world contexts into mathematical problem-solving to enhance pre-service teachers' ability to apply practical knowledge in word problems. By making mathematics more relevant to everyday experiences, this approach fosters a deeper conceptual understanding and strengthens problem-solving skills. Additionally, promoting a classroom culture that embraces mistakes as learning opportunities can encourage students to engage more confidently with real-world applications. Establishing an environment that values exploration and critical thinking empowers students to develop a more intuitive and flexible approach to mathematical problem-solving. These findings contribute to the ongoing discourse on effective mathematics instruction, particularly in teacher education programs aimed at bridging the gap between theoretical knowledge and practical application.

Despite its contributions, this study has several limitations that should be acknowledged. The relatively small sample size in the interview phase may limit the generalizability of the findings, and a larger, more diverse participant pool could provide a more comprehensive understanding of pre-service teachers' problem-solving strategies. Additionally, this study focused on a specific group of pre-service teachers, which may not fully represent educators with varying levels of experience and educational backgrounds. Future research should explore these variations by incorporating diverse research methodologies and analyzing the perspectives of in-service teachers. Moreover, comparative studies on different teacher education programs could provide insights into how real-world knowledge is integrated into mathematics instruction. Longitudinal studies examining the influence of classroom culture on students' problem-solving approaches and investigations into the role of digital tools in enhancing mathematical comprehension could further inform innovative instructional strategies. These future directions can strengthen the applied and student-centered focus of mathematics education, ultimately improving pedagogical effectiveness.

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Declarations

Author Contribution	:	AR: Project Administration, Funding Acquisition, Conceptualization, Writing - Original Draft, Writing – Review Editing, Data Curation, Formal Analysis, Investigation, Methodology, Validation, and Visualization. ZK: Conceptualization, Writing - Original Draft, Writing – Review Editing, Data Curation, Formal Analysis, Investigation, Methodology, Validation, and Visualization. LV: Conceptualization, Writing – Review Editing, Formal Analysis, Investigation, Methodology, Validation, and Visualization.
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REFERENCES

Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics* (pp. 222–231). Chichester: Horwood. https://doi.org/10.1533/9780857099419.5.221

Caldwell, L. (1995). Contextual considerations in the solution of children's multiplication and division word problems. *Unpublished Master's thesis*. Queen's University, Belfast, Northern Ireland.

DeFranco, T. C, & Curcio, F. R. (1997). A division problem with a remainder embedded across two contexts: Children's solutions in restrictive versus real-world settings. *Focus on Learning Problems in Mathematics*, *19*(2), 58-72.

Dewolf, T., Van Dooren, W., Hermens, F., & Verschaffel, L. (2015). Do students attend to representational illustrations of non-standard mathematical word problems, and, if so, how helpful are they?. *Instructional Science*, *43*, 147–171. https://doi.org/10.1007/s11251-014-9332-7

Doerr, H.M., Ärlebäck, J.B., & Misfeldt, M. (2017). Representations of Modelling in Mathematics Education. In: Stillman, G., Blum, W., Kaiser, G. (eds) *Mathematical Modelling and Applications. International Perspectives on the Teaching and Learning of Mathematical Modelling*. Springer, Cham. https://doi.org/10.1007/978-3-319-62968-1_6

Dorri, M., & Rafiepour, A. (2016). Suspension of Sense-Making in Solving Word Problem: The Role of Authenticity. In *the national conference on Primary Education*, pp: 575-586. Birjand, Iran. (in Persian).

Fitzpatrick, C. L., Hallett, D., Morrissey, K. R., Yıldız, N. R, Wynes, R., & Ayesu, F. (2019). Response sentences, examples, and authenticity do not help children solve real wor(l) d problems. *Learning and Instruction, 61*, 111–125. https://doi.org/10.1016/j. learninstruc.2018.10.002.

Gall, M., Gall, J., & Borg, W. (1942). *Educational Research: An Introduction*. Pearson.

Greer, B. (1993). The mathematical modelling perspective on problems. *Journal of Mathematical Behavior*, *12*, 239-250.

Han, F., & Ellis, R. (2019). Using phenomenography to tackle key challenges in science education. *Frontiers in Psychology, 10*, 1414. https://doi.org/10.3389/fpsyg.2019.01414

Hidalgo, M. C. (1997). L'activation des connaissances a propos du monde réel dans la résolution de problémes verbaux en arithmétique. *Unpublished doctoral dissertation*, Université Laval, Québec, Canada.

Higginson, W. (1980). On the foundations of mathematics education. *For the Learning of Mathematics, 1*(2), 3–7. http://www.jstor.org/stable/40247706.

Inoue, N. (2001). *The role of personal interpretation in mathematical problem solving: Enhancing the relevance of mathematical learning to everyday experience* (Internal report). Teachers College, Columbia University.

IREM de Grenoble. (1980). *Bulletin de l'Association des professours de Mathématiques de l'Enseignement Public*, n. 323, 235-243.

Jacobson, E., Lobato, J. & Orrill, C.H. (2018). Middle school teachers' use of mathematics to make sense of student solutions to proportional reasoning problems. *International Journal of Science and Mathematics Education*, *16*, 1541–1559. https://doi.org/10.1007/s10763-017-9845-z.

Laine, A., Ahtee, M. & Näveri, L. (2020). Impact of teacher's actions on emotional atmosphere in mathematics lessons in primary school. *International Journal of Science and Mathematics Education*, *18*, 163–181. https://doi.org/10.1007/s10763-018-09948-x.

McCosker, H., Barnard, A., & Gerber, R. (2004). Phenomenographic study of women's experiences of domestic violence during the childbearing years. *Online Journal of Issues in Nursing*, *9*(1), 1-16.

Mellone, M., Verschaffel, L., & Van Dooren, V. (2017) The effect of rewording and dyadic interaction on realistic reasoning in solving word problems. *Journal of Mathematical Behavior*, *46*,1-12. https://doi.org/10.1016/j.jmathb.2017.02.002

Morrison, J., Frost, J., Gotch, C., McDuffie, A. R., Austin, B., & French, B. (2021). Teachers' role in students' learning at a project-based STEM high school: Implications for teacher education. *International Journal of Science and Mathematics Education*, *19*, 1103–1123. https://doi.org/10.1007/s10763-020-10108-3

National Curriculum Document. (2011). *Organization for Research and Programing*. Ministry of Education, Iran.

Radatz, H. (1983). Untersuchungen zum Lösen eingekleideter Aufgaben. *Journal für Mathematik Didaktik*, *4*(3), 205–217. https://doi.org/10.1007/BF03339231

Reusser, K., & Stebler, R. (1997). Every word problem has a solution: The suspension of reality and sense-making in the culture of school mathematics. *Learning and Instruction*, *7*, 309—328. https://doi.org/10.1016/S0959-4752(97)00014-5

Sarkar Arani, M. R., Shibata, Y., Sakamoto, M., Iksan, Z., Amirullah, A. H., & Lander, B. (2017). How teachers respond to students' mistakes in lessons: A cross-cultural analysis of a mathematics lesson. *International Journal for Lesson and Learning Studies*, *6*(3), 249-267. https://doi.org/10.1108/IJLLS-12-2016-0058

Schoenfeld, A. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J.F. Voss, D. N. pekins & j. w. Segal (Eds.), *Informal reasoning and education* (pp.311-343). Erlbaum.

Selter, Ch. (2001). "1/2 Busse heißt: ein halbvoller Bus!" Zu Vorgehensweisen von Grundschülern bei einer Textaufgabe mit Rest ["1/2 bus means a bus half-full!"—Solution strategies of elementary school children for DWR problems]. In C. Selter, & G. Walther (Eds.), *Mathematik lernen und gesunder Menschenverstand* (pp. 162 173). Leipzig: Klett.

Van Dooren, L., Lem, S., De Wortelaer, H., & Verschaffel, L (2019). Improving realistic word problem solving by using humor. *Journal of Mathematical Behavior*, *53*, 96-104. https://doi.org/10.1016/j.jmathb.2018.06.008.

Verschaffel, L., De Corte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modeling of school word problems. *Learning and Instruction*, *4*, 339–359. https://doi.org/10.1016/S0959-4752(97)00008-X

Verschaffel, L., De Corte, B., & Greer, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction*, *4*, 273–294. https://doi.org/10.1016/0959-4752(94)90002-7

Verschaffel, L., Lehtinen, E., & Van Dooren, W. (2016). Neuroscientific studies of mathematical thinking and learning: A critical look from a mathematics education viewpoint. *ZDM*, *48*, 385-391. https://doi.org/10.1007/s11858-016-0781-0

Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM*, *52*, 1–16 (2020). https://doi.org/10.1007/s11858-020-01130-4

Verschaffel, L., Van Dooren, W., Greer, B., & Mukhopadhyay, S. (2010). Reconceptualising word problems as exercises in mathematical modelling. *Journal für Mathematik-Didaktik, 31*, 9-29. https://doi.org/10.1007/s13138-010-0007-x

Vos, P. (2011). What is 'authentic' in the teaching and learning of mathematical modelling? Trends in teaching and learning of mathematical modelling. *International Perspectives on the Teaching and Learning of Mathematical Modelling*. Springer. https://doi.org/10.1007/978-94-007-0910-2_68

Xin, Z., Lin, C., Zhang, L., & Yan, R. (2007). The performance of Chinese primary school students on realistic arithmetic word problems. *Educational Psychology in Practice*, *23*, 145–159. https://doi.org/10.1080/02667360701320853

Yoshida, H., Verschaffel, L., & De Corte, E. (1997). Realistic considerations in solving problematic word problems: Do Japanese and Belgian children have the same difficulties? *Learning and Instruction*, *7*, 329–338. https://doi.org/10.1016/S0959-4752(97)00007-8