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Evaluation of procedural and conceptual knowledge of mathematical functions: A case study from Morocco

Safâ Qetrani, Naceur Achtaich

Laboratory of Analysis, Modelling, and Simulation (LAMS), Hassan II University of Casablanca, Casablanca, Morocco
*Correspondence: safa.qetrani@gmail.com

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Abstract

To measure the procedural and conceptual knowledge of functions and to clarify the relationship between them in the context of mathematics education in Morocco, a structural equation modeling (SEM) analysis was used. In addition, correlation tests between students' grades at their Mathematical Analysis Assessments and their procedural and conceptual knowledge of functions scores were established to investigate whether students' grades mainly reflect their performance on procedural knowledge or conceptual knowledge. The sample consisted of 337 high school Moroccan students. The study findings indicated that a large group of participants scored high in procedural tasks but low in conceptual tasks. Besides, the participants' grades in their exams correlate much more strongly with the estimated procedural knowledge scores than the estimated conceptual knowledge scores. On the other hand, the confirmatory factor analysis of the SEM confirmed the reliability, validity, and fitness of the measurement model, whereas the path analysis of the SEM supports the genetic view causal relationship that procedural knowledge of functions is necessary but not sufficient condition for conceptual knowledge. These results provide a theoretical foundation for improving mathematics education by working on the content of the assessments, the teachers' teaching approaches, and the students' learning strategies.

Keywords: Conceptual Knowledge, Procedural Knowledge, Functions, Genetic View, Assessments

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After independence, Morocco has undergone a series of educational reforms to meet the development challenges (Saoudi et al., 2020). From 2015 to 2030, a new reform strategy was implemented that promises an emphasis on the quality of education, namely the 2015-2030 Strategic Vision (CSEFRS, 2014; Saoudi et al., 2020). Mathematics education in Morocco has also embarked on a series of changes in both programs and pedagogical approaches. Indeed, the program of mathematics in Morocco has gone from a program that gives a lot of importance to the theoretical foundations of mathematics (logic, algebraic structures ...) to a program that considers mathematics as a tool and not an end, used for solving problems in various fields (Mawfik et al., 2003). At the pedagogical level, a shift from an objective-based to a competency-based approach to teaching mathematics was decided (Hamouchi et al., 2012). An objective-based approach is a traditional approach that considers the teacher responsible for formulating the learning objective, breaking it down into sub-objectives, and then seeking to use a stimulus to positively change the student's observable behavior, that is indicative of the learning objective being achieved (Skinner, 1968), without giving importance to what is going on in his head. Therefore, this approach is very efficient for the development of procedures and automatisms. While the current competency-based approach, adopted in 2008, aims to develop competencies to improve the quality of the teaching/learning process and the student's achievement (Hirtt, 2009).

The competency-based approach, based on the constructivist theory, enables students to master a competence, at their own pace, using their prerequisites and through an authentic and self-steered learning experience (Velde, 1999). To monitor the progress made in student learning throughout the process of implementing the reform, a national program for the learning evaluation (PNEA) of Moroccan students has been set up to assess students' language, mathematical and scientific competencies at the end of the last years of primary and secondary school and the first year of high school, according to the prescribed national education program (CSEFRS, 2016). Indeed, the PNEA surveys identified a growing gap between reforms aspired outcomes and students' achievement (CSEFRS, 2016). The PNEA (CSEFRS, 2016) survey reported that "students in the first year of high school are deficient in basic knowledge/ skills prescribed by the curriculum" This was moreover confirmed by international assessments such as TIMSS and PISA. Thus, Morocco remains among the last countries in terms of student achievement in Mathematics in TIMSS studies, and as well, the results of the PISA survey for the year 2018 indicate that Morocco's average score in mathematics is 368 points, over 100 points below the international average of 489, which confirmed that most Moroccan students are below average in basic academic competencies (Bourqia et al., 2018; OECD, 2018). There are many factors affecting student achievement, including primarily the interaction between teachers, students, and content (Bryk et al., 2010). In this study, we have tried to explain the key reasons behind this generalized weakness of the academic level of Moroccan students through an analysis of the relationship between students and knowledge, which thus offers a preliminary idea about other pedagogical aspects such as teachers' practices, assessments content, and students' learning strategies.

Competence is defined as a set of knowledge, skills, and behaviors (El Faddouli et al., 2011) needed to effectively perform tasks and solve problems at school or in various fields of real life (El Asame & Wakrim, 2018). We notice then that knowledge is the main component of competence, but the question that arises is: what mathematical knowledge do students need to develop competencies? To answer this question, Sáenz (2009) studied the role played by the type of school mathematical knowledge in the activation of competencies identified by PISA with particular attention to the importance of contextual knowledge. The results indicate that "designing teaching for the development of students' competencies is not radically different from working on concepts and procedures through the curriculum" (Sáenz, 2009), which means that the acquisition of conceptual and procedural knowledge (CK and PK) is of paramount importance to competencies development. Sáenz (2009) added that contextual knowledge, which consists of presenting school situation problems in a story context related to the real world, should be included in the teaching methodology of the contents. Besides, Rittle-Johnson & Koedinger (2005) claimed that organized knowledge requires integrating conceptual, procedural, and contextual knowledge within a domain, and Bransford et al.(1999) stated that organized knowledge enables students to solve original problems than having only memorized isolated. In addition, Lauritzen (2012) confirmed that students' ability to apply mathematical functions within economics depends on the two types of knowledge, conceptual and procedural knowledge. Therefore, students' competencies depend on their conceptual and procedural knowledge, and mathematical incompetence entails deficiencies in students' mathematical knowledge (PK and CK). In our study, we focused on Moroccan students' conceptual and procedural knowledge, which we believe would provide information explaining the low performance of students in national and international assessments.

According to Hiebert and Lefevre (1986), mathematical knowledge consists of both procedural and conceptual knowledge. Sfard (1991) used different terminology to name the two categories: operational and structural understanding, while Skemp (1978) distinguished between instrumental and relational knowledge. Conceptual knowledge can be defined as knowledge of the concepts and their interrelations in a domain, whereas procedural knowledge was defined as knowledge of the syntax and algorithms for completing mathematical tasks (Rittle-Johnson et al., 2001). Conceptual knowledge develops the ability of students to understand problems, choose suitable strategies, or generate a new one for solving original problems, and provides students with control mechanisms for examining the executed strategies and detecting errors (Byrnes & Wasik, 1991; Heibert & Lefevre, 1986). Procedural knowledge develops the ability of students to execute step-by-step actions and algorithms needed for solving problems (Heibert & Lefevre, 1986). Procedures require few cognitive resources and limited conscious attention and may not be learned meaningfully, contrary to conceptual knowledge (Lauritzen, 2012; Sáenz, 2009). They can easily be automated by solving similar problems. Linking the two knowledge types is beneficial since they serve different cognitive functions, and their absence can lead to mathematical incompetence (Heibert & Wearne, 1986). In fact, both procedural and conceptual knowledge is important, but it is incomplete that one can have a deep understanding of concepts without being able to perform procedures, or one can be able to execute calculations without understanding their meaning (Nesher, 1986; Rittle-Johnson & Koedinger, 2005). On the other hand, the connection of both procedural and conceptual knowledge contributes to procedural flexibility and efficiency (Star & Rittle-Johnson, 2008; Schneider et al., 2011) and increases the ability to apply mathematics in practice or in another academic field (Sáenz, 2009; Lauritzen, 2012).

The procedural-conceptual links can be developed through the teaching practices and learning strategies (Haapasalo & Kadijevich, 2000). Several research has confirmed that teachers' beliefs and knowledge influence classroom teaching and learning practices, therefore, a teacher who has a good understanding of mathematical knowledge can transform the content into knowledge that enables students to improve their competencies (Shulman, 1986; Muhtadi et al., 2017) . In the constructivist view, the teacher is responsible for facilitating cognitive restructuring and conceptual reorganizations by focusing on relational issues and giving more attention to reflections on procedures and solutions (Cobb, 1988), rather than focusing on rote learning of algorithms and mastery of skills. A similar focus is requested in assessments by proposing diversified tasks that evaluate conceptual and procedural knowledge instead of limiting the exams to similar problems that often emphasize procedural skills, especially since several studies have demonstrated that assessment practices have a strong impact on students learning strategies “by directing their attention to particular aspects of course content and by specifying ways of processing information” (Doyle, 1983). Indeed, learning strategies have been defined as cognitive processes that students undertake when studying for a course, and which may be superficial or deep strategies (Arend, 2007). Thus, we can say that students' approaches to learning reflect the nature and quality of teachers' practices and assessments.

Realizing that linking procedural and conceptual knowledge has many benefits for teaching and learning mathematics, it is obvious to ask how these two types of knowledge are related. Theory suggests four conflicting viewpoints on the causal relations between these two kinds of knowledge, each view is supported by some empirical studies (Haapasalo & Kadijevich, 2000). Thus, the relation might be unidirectional from conceptual to procedural knowledge or from procedural to conceptual knowledge, or bidirectional, or entirely the two kinds of knowledge are not related. Each viewpoint is described in detail in the following section.

The first and second objectives of the present study are related, which consist in developing an instrument to measure the procedural and conceptual knowledge of a mathematical concept and discovering the causal relationship between these two types of knowledge, supported by our study, in the Moroccan context. We chose to work with mathematical function concepts as they are one of the most important mathematical objects from kindergarten to graduate school (Breidenbach et al., 1992) and that has both conceptual and procedural features (Lauritzen, 2012). Moreover, the courses concerning numerical functions represent almost half of the Moroccan program in high school (MNEPS, 2007). To measure the conceptual and procedural knowledge of functions and to study the relationship between them, we need to develop reliable and valid measures since both types of knowledge cannot be measured directly (Haapasalo & Kadijevich, 2000). To meet these objectives, we have drawn particularly to the work of Lauritzen entitled "measuring conceptual and procedural knowledge of mathematical functions". Lauritzen's study (2012) investigates which of the four views can be supported using the structural equation modeling technique. The first aim of Lauritzen's study was to develop an appropriate set of tasks for measuring the conceptual and procedural knowledge of functions and the ability to apply functions, and the second one was to analyze their relationships through a large sample of first-year students in business school. The study developed one statistical model that consists of, on the one hand, the measurement part where conceptual knowledge, procedural knowledge, and ability to apply functions were modeled as latent factors, and on the other hand, the structural part where relations between different latent factors were investigated through linear regression equations (Lauritzen, 2012). Part of Lauritzen's study results showed that a large group of participants had good procedural knowledge but modest conceptual knowledge. Besides, its collected data supported the causal relation that procedural knowledge is a necessary but not sufficient condition for conceptual knowledge. In our study, we have first adapted the tasks developed by Lauritzen for measuring the two latent variables "procedural knowledge of functions" and "conceptual knowledge of functions" to the Moroccan curriculum. Second, we have used structural equation modeling (SEM) to investigate the causal relationship between these two kinds of knowledge for a large group of Moroccan high school students. It is noteworthy that in our SEM model we did not measure the latent variable "ability to apply functions" because, in Lauritzen's model, the interest of this variable was to measure the ability of business school students to apply functions in solving economic problems. However, our study focused on high school students who have not already experienced applying functions in other fields, especially in the economy.

Another goal of this study is to investigate whether students' achievement in mathematics analysis mainly reflects their performance on procedural knowledge, conceptual knowledge, or both. Student achievement is defined as the knowledge or skills learned in the school for a limited time evaluated by test scores assigned by teachers (Rice & Carter, 2016). Ourahay's study (2021) on summative evaluations in the Moroccan context shows that the assessments target low academic achievement and characterize, according to TIMSS, performances below the international average level. Students' achievements are reduced to the first cognitive levels of mathematical thinking, namely "knowing" and "applying" or "reproducing" (Heibert & Wearne, 1986; Bourqia et al., 2018; Ourahay, 2021). Through this research, we tried to verify to what extent these results correspond to the data of this study and their relation to the procedural and conceptual knowledge of students.

The details of the theoretical framework, as well as the key steps of this study, are further described in the next sections.

Relations between Conceptual and Procedural Knowledge

As already mentioned, conceptual knowledge denotes abstract knowledge of the concepts of a domain and their interrelations. Lauritzen (2012) differentiated two categories of relations between mathematical knowledge. At the primary level in which “the conceptual knowledge consists in recognizing the relationship between two pieces of information at the same abstraction level", and at the reflective level, "where relationships are constructed at a higher level of abstraction, less tied to context". In the case of functions, students at the primary level of conceptual knowledge should be able to shift back and forth between the different representations of functions (graphs, algebraic expressions, tables, and words) by understanding the isomorphism between them and that they represent the same mathematical object. While reflective knowledge is necessary for detecting whether mathematical properties are met or not in mathematical practices (e.g., getting a negative function value for a non-negative function) or in another context (e.g., economic applications)(Hiebert, 1986; Heibert & Wearne, 1986; Lauritzen, 2012). On the other hand, procedural knowledge has been separated into two categories: knowledge of forms and syntax which means that students can use symbols correctly, for example, they can readily recognize that this writing $f2(=)x$ is false and does not express a function, and knowledge of rules and algorithms for solving problems (e.g., calculation of function values). Procedures can easily be automatized by force of solving repeated problem types, and therefore they required few cognitive resources and limited conscious attention, and they can hardly be verbalized and justified by students (Heibert & Lefevre, 1986). Theory indicated that there are four possibilities of causal relations between conceptual and procedural knowledge: genetic view, dynamic interaction view, simultaneous activation view, and inactivation view. The details of each viewpoint are provided in the following sections.

Supporters of the inactivation view declared that conceptual and procedural knowledge are not related or have a very weak link than expected, especially as some students may have high conceptual knowledge and lack procedural skills or the reverse (Zucker, 1984, as cited in Lauritzen, 2012). Resnick & Omanson's study (1987) revealed that despite students having a good conceptual understanding of subtraction, they found great difficulty in performing subtraction calculations. Zucker (1984) worked on a large sample of 270 students to study the correlation between understanding decimal notations and performing procedures. As a result, the correlation relationship was statistically non-significant which means that the two kinds of knowledge should be treated independently.

The causal relations between procedural and conceptual knowledge may be bidirectional, which is referred to as the simultaneous activation view. In other words, procedural knowledge is a necessary and sufficient condition for conceptual knowledge which means that enhancing students’ procedural knowledge leads to better conceptual understanding and vice versa (Hiebert, 1986; Byrnes & Wasik, 1991). Lauritzen (2012) added that, in this viewpoint, "other explanatory variables for conceptual knowledge should be considered redundant". For the simultaneous activation view, computational errors in adding fractions are caused by a lack of conceptual knowledge of rational numbers (e.g., mathematical symbols are meaningless), and that developed conceptual knowledge allows detecting the presence of computational errors and procedures will be performed correctly. Therefore, the development of procedural and conceptual knowledge takes place simultaneously (Hiebert, 1986; Byrnes & Wasik, 1991).

The dynamic interaction view, also called concepts-first theory, suggests unidirectional relations from conceptual to procedural knowledge (Gelman & Williams, 1998; Halford, 2014). In other words, conceptual knowledge is a necessary but not sufficient condition for procedural knowledge. This view represents a synthesis of Inhleder & Piaget's theory (1958) that confirmed that acquiring new procedures begins with in-depth conceptual understanding. But once acquired, mastering these procedures can be done in two ways: first, discrimination and generalization processes as feedbacks of the environment specifying the contexts of when and where applying a given procedure (Byrnes & Wasik, 1991); second, the repeated practice of problems solving and the "proceduralization" process leads to automatized and efficient procedures. In addition, reflections on the results of procedures contribute to the enrichment of conceptual knowledge. Thus, the two kinds of knowledge affect each other diachronically and not synchronically (Inhelder & Piaget, 1958; Braine, 1988).

In contrast, the genetic view, also called the procedures-first theory, suggests unidirectional causal relations from procedural to conceptual knowledge (Siegler & Stern, 1998). In other words, procedural knowledge is a necessary but not sufficient condition for conceptual knowledge, and therefore students who develop rich procedural knowledge, do not necessarily develop conceptual knowledge. In this view, the concept is a process that is encapsulated as an object (Dubinsky, 1991; Sfard, 1991; Lauritzen, 2012) stated that the transition from operational to a structural understanding of a concept is done through three stages: interiorization, condensation, and reification. Interiorization is the first stage for learning a new mathematical notion. Students get acquainted with the processes (processes are operations performed on lower-level mathematical objects) that turn into a new concept at a later stage, for example, manipulating algebraic expressions that give rise to functions object (Sfard, 1991). The phase of condensation is a period when students become able to think about a process as a whole and encapsulate it as an object. This stage resembles transforming "a recurrent part of a computer program into an autonomous procedure". At the end of this stage, students should be able to generalize, compare, and combine processes (Sfard, 1991). Reification is an instantaneous qualitative change where a new concept will be encapsulated into a familiar object detached from the process that produced it. Therefore, a higher-level concept can now be developed such that the ex-reified construct is an input of its interiorization stage (Sfard, 1991). For example, the function is a building block for differentiation (Lauritzen, 2012). Sfard's (1991) theory is supported by the historical view confirming that the development of mathematical concepts should be in the same order as the development of concepts in history (e.g., the notion of numbers emerges through the process of counting). On the other hand, Lauritzen's (2012) study results support, by using structural equation modeling, the genetic view that procedural knowledge precedes conceptual knowledge of functions.

Functions' Conceptual and Procedural Knowledge

The function concept describes the relationship between input and output variables and can be represented in four different forms: words, tables, graphs, or algebraic expressions (Janvier, 1978). Understanding the different representation forms of functions, especially graphs and algebraic expressions, are among the main objectives of the mathematics curriculum for high school education in Morocco (Abouhanifa & Benkenz, 2018). The Moroccan educational guidelines state that care should be taken to present this concept in relation to other fields by solving engineering, physics, economics, and public life problems (MNEPS, 2007). Based on the constructivist theory, the guidelines insist that teachers are responsible for preparing the environment that helps students to construct the meaning of new function concepts in an explorative way (Boutin, 2000). For the content, students in the first year of Moroccan high schools should learn the concepts related to a domain of functions, even and odd functions, variations, minimum and maximum values, the graphical representation of some common functions $\left(x \mapsto ax^2 + bx + c \text{ and } x \mapsto \frac{ax+b}{cx+d}\right)$ and solving equations and inequalities graphically (MNEPS, 2007). The second year is a fundamental class where students have an opportunity to learn about composite functions, limits, derivation, infinite branches, concavity, and drawing the curve of any function. Students in their third and final year of high school learn about continuity, reciprocal functions, logarithmic functions, and exponential functions (MNEPS, 2007).

Functions are considered one of the best examples of a mathematical object for evaluating the relationship between procedural and conceptual knowledge (Lauritzen, 2012). The proposed tasks for measuring these two types of knowledge were limited to graph and algebraic expression forms. Students' procedural knowledge of functions' algebraic expression is reflected in their ability to execute a step-by-step procedure without necessarily having an in-depth understanding of functions (Lauritzen, 2012). For example, calculate functional values algebraically, and calculate the limits and derivative function. To measure the conceptual knowledge of functions' algebraic expressions, the designed tasks should present the function as a unit without giving its explicit algebraic expression, and therefore, students do not need to use algorithmic strategies for solving them (Sfard, 1991). For example, deduce the nature of the product of two functions (e.g., $f(x) \times g(x)$) presented by names (e.g., f and g) and defined only by some properties given as a text (e.g., f is a second-degree polynomial function and g is a first-degree polynomial function) (Lauritzen, 2012).

In the same vein, measuring the procedural knowledge of functions' graphical representation consists of tasks where students are asked to draw a graph or read values from the graph (Lauritzen, 2012). For measuring the conceptual knowledge of graphs, the proposed tasks should evaluate the ability of students to perform operations on a graph as a unit without having an idea about its corresponding algebraic expression (Lauritzen, 2012). For example, sketch the graph of the function $-f$ from the graph of f. Another aspect of functions' conceptual knowledge is reflected in students' ability to shift between the different representation forms, realizing the isomorphism between them and that they represent the same mathematical object (Duval, 1993; Lauritzen, 2012). For example, solve an algebraic equation using the graph of the corresponding function. Duval (1993) introduced the concept of registers of semiotic representations as "productions constituted by the use of signs belonging to a system of representation that has its constraints of meaning and operation". He claimed that for fully understanding the concept of functions, it is necessary to know all their different registers of semiotics representations (graphic, algebraic, and table) and to be able to convert the representations produced in a system into representations of another system (Duval, 1993).

On the other hand, the student's prior knowledge and experiences have been taken into consideration when creating the measurement tasks. Procedural knowledge is measured by routine tasks, while conceptual knowledge is often measured by unfamiliar problems where students must use their knowledge of functions to construct new solution strategies (Schneider & Stern, 2010).

The purpose of the current study is to investigate the conceptual and procedural knowledge of functions for a large group of Moroccan students using structural equation modeling (SEM). Relying on existing theory and empirical studies, the SEM is specified by combining (a) a measurement model, which measures latent variables (constructs) from observed variables (items), and (b) a structural model, which examines the causal relationships among latent variables. In our study, the measurement part of the model was used to measure two latent variables, procedural and conceptual knowledge of functions, through observed variables by developing a set of tasks adapted to the Moroccan curriculum. For the structural part, the hypothesized genetic view relation between procedural and conceptual knowledge was assessed. Otherwise, a correlation study was established between students' grades in mathematical analysis and their scores on procedural and conceptual knowledge. Thus, this study seeks to answer the three research questions. Firstly, how procedural, and conceptual knowledge of functions can be measured in the Moroccan context? Secondly, how procedural, and conceptual knowledge of functions do relate to each other? Lastly, do students' grades mainly reflect their performance on procedural knowledge or conceptual knowledge?

METHODS

Participants

The proposed test is addressed to students of the scientific branches of high school who have already learned all the essential concepts related to numerical functions and who must be generally average or well-performing so that they can have minimum knowledge of this concept. Indeed, participants were 337 Moroccan high school students ranging from 16 to 17 years and drawn from seven experimental science classes. The sample comprised 196 females and 141 males. All participants received the same content on the theme of numerical functions according to the Moroccan curriculum during two years of high school. Participants' grades, ranged from 0 to 20, from the Mathematical Analysis Exams were reported. The mean value was 15.08 with a standard deviation of 2.7, and the minimum and maximum values are 8 and 20. Thus, the participants' background in mathematics analysis could be described as good with some marked variations between them.

Conceptual Model

The principal research questions of this study are formulated as hypotheses that are included in one statistical model. Based on the Lauritzen model and existing theories, our model was set up and implemented in the IBM SPSS AMOS Graphic Version 24 to verify if the collected data supports the model. Figure 1 presents the research model that explains the relationship between measured and latent variables, and the relationship between two latent variables. The latent variables are conceptual knowledge and procedural knowledge that cannot be measured directly (Haapasalo & Kadijevich, 2000). Procedural knowledge of functions was measured through two observed variables (items): the algebraic procedures variable and the graphic procedures variable. Whereas students' conceptual knowledge of functions is reflected in their ability to solve problems about the different forms of a function (graph or algebraic expression) represented as units without using procedural steps, as well as their ability to understand the isomorphism between the different representations forms and easily switch between them.

Conceptual knowledge of functions was measured through four observed variables: algebraic interpretation, graphic interpretation, the relation between graphic and algebraic forms, and the ability to apply functions. As already discussed, Lauritzen has considered "Apply functions" as a latent variable that measured the ability to apply functions for solving economic problems and as a building block for differentiation. In this study, the ability to apply functions was considered as a part of conceptual knowledge of functions for two principal reasons: first, many researchers confirm that solving problems needs a depth understanding of mathematics concepts, for example, Cobb (1988) claimed that students who have constructed powerful conceptual structures are more able to solve problems in a wide variety of situations, in the same vein, Lauritzen (2012) claimed that procedural knowledge alone is insufficient for the student to be able to apply functions; second, the target sample in this study are high school students with absolutely no knowledge of economics or any other specific fields, especially that the teaching of mathematics in Morocco emphasizes the abstract side of concepts without making a connection with the real-world applications. Thus, the developed task for measuring the observed variable "Apply functions" involves a basic problem situation modeled by a linear function. More details about the tasks describing the above-mentioned observed variables will be given in the next section. On the other hand, the structural part of the model reflects the assumed relation between the two latent variables connected through linear regression equations where procedural knowledge serves as the independent variable and conceptual knowledge as the dependent variable (the genetic view causal direction).

Data Collection

The data in this study is quantitative, comprising the students' responses on a paper-and-pencil test containing 22 tasks, of which twelve tasks are from the Lauritzen test, and ten new tasks were developed according to the content of the Moroccan curriculum (see Appendix). The ten tasks fully respect the theoretical framework validated by the Lauritzen model with a change of examples. The validation process involves two mathematics teachers plus the research team. One of these two teachers participated in the correction of the students' tests. The results of the validation show that the test is adapted to the content of the mathematics curriculum in Morocco and can be used to measure the participants' knowledge of numerical functions. The second step was to verify that the questions were clear and concise, the time devoted was sufficient, and that students' solution strategies were coherent with the theoretical framework. Thus, a pilot test was applied to five students. Therefore, the content adequacy assessment was validated, and two hours was sufficient for students to answer all the test questions. On the other hand, the confirmatory factor analysis of the structural equation model was applied to assess the reliability and validity of the measurement instrument. The participants' grades in the Analysis exam were reported by their mathematics teachers. Participants were divided into five groups and passed the test for two hours. Before beginning, students were encouraged to answer the questions, even if they were unsure of their answers. It should be noted that the tasks for the different items were randomly ordered. The objective is to find out if participants answer the questions of the different types of knowledge according to the proposed order, or if they would choose the questions they are used to answering. The new proposed tasks of the test will be justified and described in the following paragraphs.

Questions Measuring Procedural Knowledge

The questions that measure procedural knowledge of functions are related to the graphic procedures (GrphPr) or the algebraic procedures (AlgPr). All these questions can be solved by following well-known and familiar algorithms, they test the students' ability to execute a plan (the third stage of Polya's model about problem-solving processes) (Polya, 1945) without a need of spending time understanding the problem statement and conceiving the solution plan (the first and second stage of Polya's model) or in choosing the appropriate solution strategy since the students are used to solve similar problems and are more likely to be attached to one particular strategy even if they know alternate approaches (Star & Rittle-Johnson, 2008); which means that any need for conceptual knowledge was avoided.

To measure the algebraic procedures item, tasks from the Lauritzen test (tasks 1 and 2 in Appendix) about calculating images and pre-images of a function and solving equations and inequations algebraically were proposed (Lauritzen, 2012). In addition, we have developed routine tasks, with different degrees of difficulty, on the calculation of limits, the calculation of derivative functions, and tasks for determining if a function is even or odd (tasks 3, 5, and 14 in Appendix). On the other hand, and for measuring the graphic procedures item, questions about sketching the graph of usual functions, identifying images, and pre-images of a function from its graph were proposed (tasks 4 and 18 in Appendix).

Questions Measuring Conceptual Knowledge

The conceptual knowledge of functions was measured via four items: graphic interpretation item (GrphInt), algebraic interpretation item (AlgInt), relations between graphic and algebraic representations item (RelationAlgGrph), and apply functions item (ApplyF) (Lauritzen, 2012). The designed tasks respect two main conditions: first, they can be solved without a need for procedural solution strategies; and second, that they are easy but unfamiliar in the way that they require more conscious thinking for understanding the problem statement and conceiving the solution plan (the first and second stage of Polya's model). In the graphic and algebraic interpretation items, the aspect of reification (Sfard, 1991) is present, and functions are treated as units. In the graphic interpretation item, the function is represented by its graph that contains enough information for answering the questions and lacks information for procedural solutions. For example, students were asked various questions to sketch the graph of $f(-x), -f(x), f(x)+g(x)$, the derivative of $f(x)$, based on the given graphs of $f(x)$ and $g(x)$ (Lauritzen, 2012)(see tasks 12, 13, 19 and 20 in Appendix). In the algebraic interpretation item, the function is represented by a name and defined by some properties given as a text with no information about its algebraic expression and graph (Lauritzen, 2012) (see tasks 10 and 11 in Appendix). Therefore, we developed a task where a function f is defined as an absolute value of h and h is given as an odd function defined on $\mathbb{R}$, students were asked to determine the domain and the parity of f, and the solutions to the inequality $f(x) \geq 0$. In addition, we have proposed two tasks that aim to evaluate, respectively, the ability of students to identify among a set of graphical representations the curves of functions as well as their ability to select among a set of equalities those which represent algebraic expressions of functions (see tasks 7 and 8 in Appendix).

The item "relation between graphic and algebraic representations" aimed at testing whether students can understand the relation between the algebraic and graphic representation and then solve problems by shifting fluidly between the two representations forms. Three tasks were taken from Lauritzen's test (see tasks 15, 16, and 17 in Appendix). In three others, the example functions proposed by Lauritzen have been modified (see tasks 6, 9, and 21 in Appendix). For example, a question was asked about solving the equation $f(x) = x + 2$, the given graph of f should be used as an intermediate tool for finding the intersection points of the graph, and the line of equation $y = x + 2$ and then deducing the algebraic solutions of equation $f(x) = x + 2$ (see task 9 in Appendix). The most challenged stage of Polya's model is the second one where students should interpret the equation $f(x) = x + 2$ as the intersection of two geometric objects (the graph and the line) and then conceive the plan of the solution strategy (find two points for drawing the line, determine the intersection, and deduce the algebraic solutions which are the abscissae of the graphically drawn intersection points).

To find out how students would react to a problem situation that they are not used to practicing in class, we designed a task, of the "apply function" item, where students should model a simple problem of melting ice by a linear function (see task 22 in Appendix). Thus, students should mobilize their conceptual knowledge about linear functions to succeed in the first and second Polya's model stages (Polya, 1945), understanding the problem statement and conceiving the solution plan. Solving this problem requires procedural knowledge to calculate the value of the slope and the y-intercept of the linear function, but it is a less challenging step because most of the students who succeed in the first and second stages do not find difficulty in the algebraic and numerical calculation, so we considered this item as a measured variable of the conceptual knowledge of functions.

Data Analysis

The tasks were scored on different ranges scales. The item's score is calculated by adding the scores of all the tasks that belong to it. The sum of scores of the items of each knowledge (procedural and conceptual) ranged from 0 to 22. Table 1 contains the scale of each item. The Cohen's kappa value for measuring the inter-rater reliability of each item was between 0.83 and 0.92, and the disagreements were solved by the coders. It is noteworthy that many students did not answer all the questions on the test, but no one left their paper blank, and they at least tried to answer the familiar tasks.

Table 1. Range of scores for each item

Latent Variables	Procedural Knowledge		Conceptual Knowledge			
Items	AlgPr	GrphPr	AlgInt	GrphInt	RelationAlgGrph	ApplyF
Scale	0-17	0-5	0-5	0-7	0-8	0-2

The model is over-identified (Byrne, 1998) with 8 degrees of freedom. The items were normally distributed. The structural equation modeling was used to test the hypothesis causal relationship between PKF and CKF. The method used for parameter estimation in structural equation modeling is the maximum likelihood estimator. Confirmatory factor analysis was utilized to verify the measurement quality by assessing the reliability and validity of the measurement model. We used the following measures for goodness-of-fit: CMIN/DF, CFI, TLI, RMSEA, to verify if the hypothesized model fits the data well. The Chi-square p-value was ignored because it is very sensitive to the sample size (more than 200) (Vandenberg, 2006). On the other hand, a correlation test (the Pearson correlation coefficient) between the students' grades in mathematics analysis assessments and their scores on PKF and CKF was applied to answer the third research question.

RESULTS AND DISCUSSION

In this section, we will provide the answers to the three research questions. Figure 1 shows the complete model with the standardized estimated parameters and errors term. Overall model analysis shows that all factor loadings are within the range from 0.62 to 0.91 and the relation between procedural and conceptual knowledge is significant. The measurement model part and the structural model part will be investigated separately in the next sections to answer the first and second research questions.

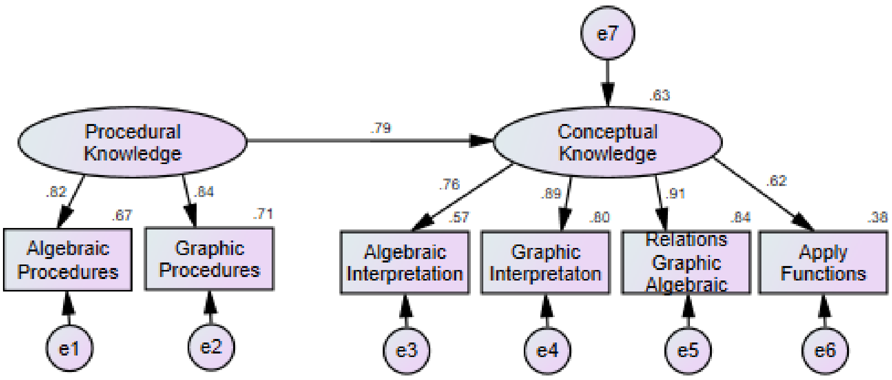

Figure 1. The structural equation *models*

Measurement Model Assessment

As shown in Table 2, the standardized factor loadings for every item exceed 0.7 except for the "Apply functions" item, which might be explained by the fact that students' conceptual knowledge is a necessary condition for applying functions (factor loading higher than 0.5), but not sufficient since students also need to apply procedures to solve problems involving functions. Lauritzen (2012) claimed that *"the ability to apply functions depends significantly on conceptual knowledge of functions" and that "procedural knowledge clearly affects the ability to apply functions when intermediated by conceptual knowledge."* On the other hand, the "algebraic procedures" item has more impact on procedural knowledge than the "graphic procedures" item. As well, the "graphic interpretations" item and the "relations between graphic and algebraic representations” item have similar greater impacts on conceptual knowledge than the “algebraic interpretation” item and much more than the “Apply functions” item.

Table 2. Factor loading of each Item

Latent Variable	Procedural Knowledge		Conceptual Knowledge			
Items	AlgPr	GrphPr	AlgInt	GrphInt	RelationsAlgGrph	ApplyF
Factor Loadings	0.816	0.841	0.756	0.892	0.914	0.619

To evaluate the quality of the measurement model, we measured reliability and validity. Reliability refers to the internal consistency of the measurement instrument, while construct validity is used to determine whether the instrument measures what is intended to measure. To demonstrate construct validity, we measured the convergent and discriminant validity. For each latent variable, the Composite Reliability (CR) score should be greater than 0.7 to measure reliability, while the Average Variance Extracted (AVE) value should be greater than 0.5 to confirm convergent validity (Hair et al., 2010). The AVE square root value of each latent variable should be larger than the correlation between them to confirm discriminant validity. As presented in Table 3, the composite reliability (CR) value of procedural knowledge (CR= 0.814) and conceptual knowledge (CR=0.877) exceeded 0.7. The items that measure each latent variable correlate strongly with each other since the convergent validity of each latent variable was confirmed with AVE values equal to 0.687 for procedural and 0.646 for conceptual knowledge. The square roots of the AVE (0.804 and 0.829) were higher than its correlation that equal to 0.793; thus, the observed variables (items) of procedural knowledge correlate more highly with each other than with the observed variables of conceptual knowledge and vice versa and therefore the discriminant validity was confirmed. For the model fit assessment, the following fit indices should satisfy these conditions to confirm that the model fits the data well: CMIN/DF smaller than 3.0 (Hair et al., 2010) and RMSEA smaller than 0.08 (Browne & Cudeck, 1989) while CFI and TLI should be higher than 0.9 (Bentler & Bonett, 1980). Therefore, the confirmatory factor analysis model showed good fit indexes since CMIN/DF=2.364, RMSEA=0.064, CFI=0.991, and TLI=0.983.

To answer the first research question, the quality of our measurement model of conceptual and procedural knowledge of functions in the Moroccan context is good, valid, and reliable. Therefore, the designed tasks purport to measure what they are supposed to measure and align well with the theoretical framework. Indeed, as already mentioned, tasks measuring procedural knowledge of functions should measure the ability of students to apply algebraic procedures such as calculating images and pre-images, calculating the derivative and limits, and thus, their ability to apply graphical procedures such as constructing the curve of a usual function. On the other hand, tasks measuring conceptual knowledge of functions measure the ability of students to consider a function as a unit, such as defining a function f as the absolute value of a function h without knowing any information about their algebraic expressions and then proposing questions about f. We can also measure conceptual knowledge of a function by evaluating the ability of students to easily switch from one representation to another (algebraic, graphical) and understand the isomorphism between the two representations. It remains to mention that the variable "apply functions" has a loading factor barely exceeding 0.5, which means that solving real problems needs certainly a depth understanding of mathematical concepts and their interrelationships, but not in itself sufficient since students need also procedural knowledge and contextual knowledge to develop problem-solving competencies (Saénz, 2009; Lauritzen, 2012).

Table 3. Validity and reliability analysis

	CR	AVE	Conceptual Knowledge	Procedural Knowledge
Conceptual Knowledge	0.877	0.646	0.804	
Procedural Knowledge	0.814	0.687	0.793	0.829

Structural Model Assessment

The results of the structural regression analysis model suggest that procedural knowledge of functions has a direct and positive effect on conceptual knowledge of functions (β = 0.79, t*value* = 13.34, ρ < 0.001). The findings yielded clear evidence for the genetic view that procedural knowledge of functions (PKF) is a necessary but not sufficient condition for its conceptual knowledge (CKF), at least in the case of this group of Moroccan students; Thus, supporting our second hypothesis. To have more transparent visual information on the relationship PKF-CKF, we created a scatterplot of participants' scores on procedural and conceptual knowledge of functions (Figure 2). For each student, the PKF score was calculated by summing the scores of the procedural knowledge items (algebraic procedures and graphical procedures) multiplied by the corresponding factor loading, and the same for the CKF score (Lauritzen, 2012). As we see in Figure 2, the scatterplot indicates a positive relationship between PKF and CKF. As the procedural knowledge of functions increases, the conceptual knowledge of functions also tends to increase, which confirms that procedural knowledge is a significant explanatory variable for conceptual knowledge in the case of Functions. The reverse is not true, as no one of the participants scored high on conceptual knowledge and lower on procedural knowledge, thus the dynamic activation view is absent in this study. In the same vein, the inactivation view was denied as the two kinds of knowledge were correlated. The students' scores at the diagonal may support the simultaneous activation view that conceptual and procedural knowledge develop iteratively. The results obtained on the relationship PKF-CKF are like those of Lauritzen's study (2012).

The genetic and simultaneous activation views on the relationship between procedural and conceptual knowledge give an idea about the pedagogical approach that should generally be adopted to teach mathematical functions, which is the development approach (Haapasalo & Kadijevich, 2000; Eronen & Haapasalo, 2010). In this approach, the teacher can opt for procedures at first, but without neglecting the conceptual understanding. However, traditional teachers exaggeratedly focus on teaching procedures, and even when students make a comprehension error that is evidence of a lack of conceptual knowledge, they consider them as a memorization problem and try to prevent the error by recalling procedures (Ma, 2020). Furthermore, the causal relationships between procedural and conceptual knowledge may vary according to the subjects taught; that is why it is necessary to teach these two types of knowledge with balance and understand that mathematical knowledge is developed iteratively between the two types of knowledge (simultaneous activation view) and that both are critical conditions for competencies development (Sáenz, 2009).

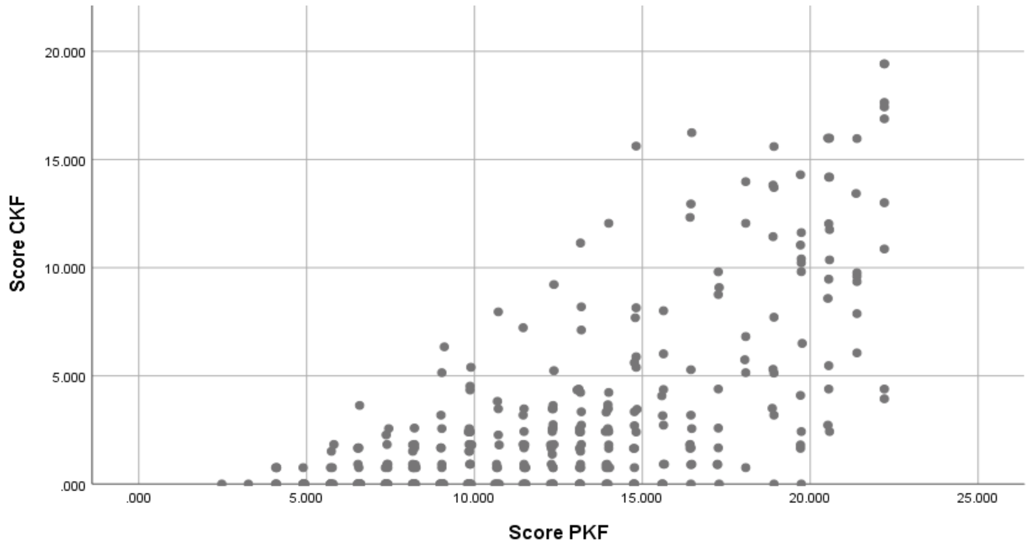

Figure 2. The scatterplot between PKF and CKF scores

Relationships between Mathematics Grades, Procedural, and Conceptual Knowledge Scores

The last purpose of the current study was to investigate whether students' scores on the mathematics analysis class exams primarily reflect their levels of procedural knowledge or conceptual knowledge (the third research question). For this purpose, the Pearson correlation test between students' grades and their scores of PKF and CKF was conducted. The results from Table 4 indicate that students' grades correlate significantly and positively with the PKF and CKF scores but with remarkably different coefficients of collinearity (0.79 for Grades-PKF and 0.52 for Grades-CKF), meaning that the students grades correlate much more strongly with the procedural knowledge scores than the conceptual knowledge scores.

As shown by the analysis of the scatterplots in Figure 3, as the students' grades increase, the procedural knowledge of functions scores increases as well, which means that students who scored high on their mathematical analysis exams scored high on the tasks measuring the procedural knowledge of functions. However, the increase in students' grades does not necessarily imply an increase in their scores on the tasks measuring the conceptual knowledge of functions, in other words, most observations don't "line up" and remain under the horizontal line of the equation CKF-score=5, which explains the weak positive correlation between students' grades and CKF scores. Indeed, in general, the scores of conceptual knowledges of functions only really started to rise for almost half of the students who achieved grades higher than 16.

Table 4. The correlation test results between grades and PKF and CKF scores

		Score PKF	Score CKF	Grades
Grades	Pearson Correlation	.790**	.529**	1
	Sig. (2-tailed)	.000	.000	

**. Correlation is significant at the 0.01 level (2-tailed).

We concluded that students' grades on their mathematical analysis exams do not reflect their conceptual knowledge of functions but especially their procedural knowledge of functions. Otherwise, the mathematical analysis exams in the Moroccan school emphasize the evaluation of procedures and give much less importance to the evaluation of conceptual knowledge of functions. Therefore, the students who get high grades in the class assessments do not necessarily have a deep conceptual understanding of functions. This is also supported by Ourahay's (2021) study, which reported that "the TIMSS survey grants 30% to the reasoning activity while the summative evaluation, conveyed by Moroccan mathematics teaching, only grants it 15%" and less than 10% on the baccalaureate exam. Therefore, the baccalaureate exams emphasize the technical and computational aspects of mathematics and orient students towards the use of automatisms, and adopt a guided resolution (CSEFRS, 2021; Ourahay, 2021).

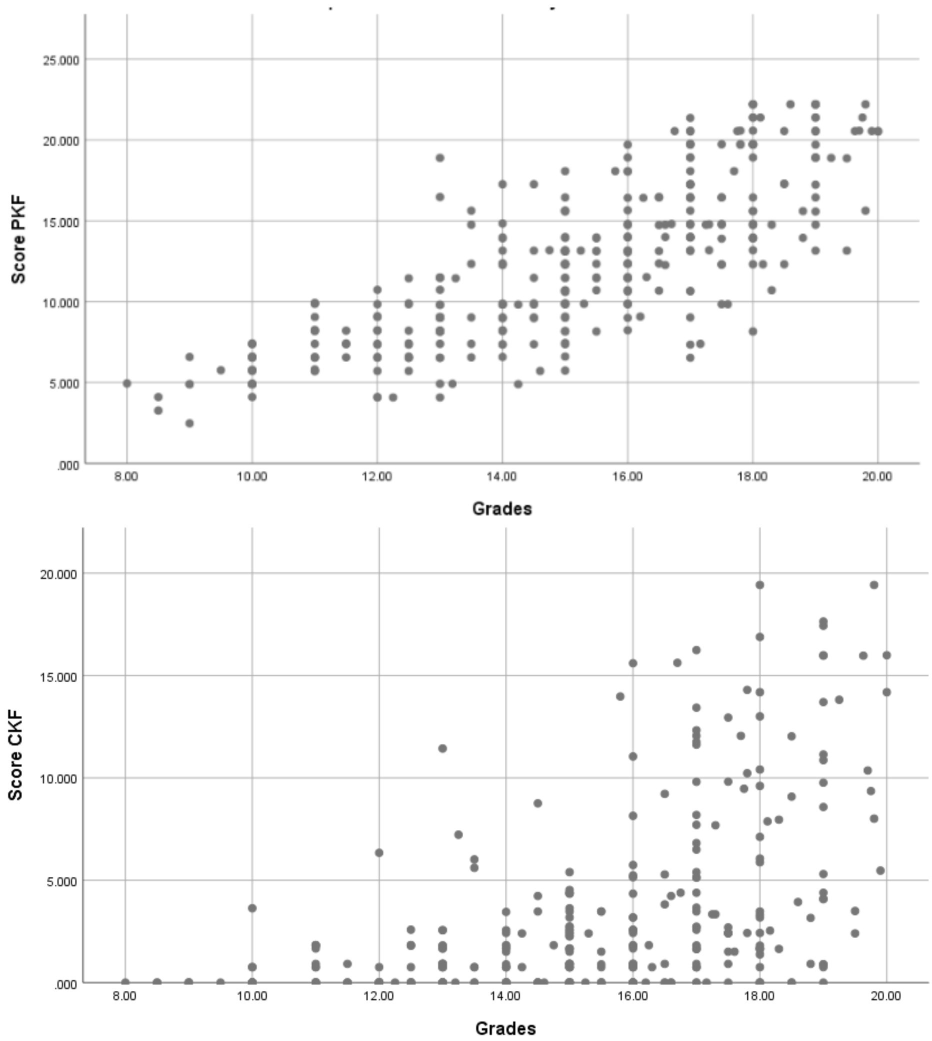

Figure 3. Scatterplot between students' grades and their PKF scores and CKF scores

The last reform of the mathematics curriculum in Morocco has introduced new approaches for teaching and learning mathematics within the framework of the competence-based approach based primarily on the constructivist theory. From the constructivist's view (Piaget, 1977), the student should construct his own knowledge by mobilizing his prior knowledge to solve new problematic situations, which allows him to make connections and to put meaning into new mathematical concepts and, therefore, enrich his cognitive structures (Boutin, 2000). This mathematical knowledge accompanied by a mastery of procedures is necessary for developing competencies (Sáenz, 2009) that are the current focus of the curricula of many countries, including Morocco. While in the traditional teaching approaches theories, transmission, and behaviorism, the teacher holds the information, and it is through his actions and words that the students learn. They emphasize the rote learning of procedures and the development of automatisms (Boutin, 2000). Pedagogical guidelines for teaching mathematics at Moroccan high school (MNEPS, 2007) have underlined the need to teach conceptual knowledge rather than superficial skills; they stated that: "the mathematical education of students should not be limited to the formal possession of definitions, theorems, results, and techniques, but rather to make these achievements alive and meaningful by employing and synthesizing them to meet challenges and solve problems." Indeed, the reality of most Moroccan students today, as proved by our study, did not meet these needs. The findings indicate that many students can perform procedures correctly but lack conceptual understanding. In other words, most students seem to be procedurally bounded learners since they focus on the mastery of procedures (how to do) without developing their conceptual knowledge (Why to do), as opposed to procedurally oriented learners that advance from procedural to conceptual knowledge if they receive the suitable teaching approaches (Ma, 2020). Several studies proved that the lack of conceptual understanding of mathematical objects, which means the lack of cognitive networks, will make solving unfamiliar problems, including “connection” and “reflection” tasks a very complex activity for students (Sáenz, 2009) since it requires high order cognitive process. Therefore, this explains very well the low performance of students in international assessments such as TIMSS and PISA, which place a lot of emphasis on these higher-order cognitive processes and on problem-solving in different contexts. On the other hand, we think that the assessments, the teaching approaches, and students' perceptions are the main factors that impact the students' learning approaches to be towards procedural knowledge or both procedural and conceptual knowledge.

At the certificate evaluations, Moroccan students are exposed to similar tasks that focus on performing procedures and lack conceptual knowledge (Ourahay, 2021). Therefore, even students who are procedurally bounded learners get high grades and sometimes higher than the conceptually oriented learners' grades. The more we decrease the "reasoning" tasks in assessments, the more we minimize the requirements of the success threshold (Ourahay, 2021). This explains why participants' grades in their exams correlated strongly with the estimated score of procedural knowledge of functions (PKF) and much less with the estimated score of conceptual knowledge of functions (CKF). The fact that the assessments focus on similar problems based on the execution of procedures has negatively impacted the teachers' approaches and the students' interests. As we have already mentioned, the literature confirmed that assessment practices have a strong impact on students learning strategies (Doyle, 1983; Arend, 2007; Ourahay, 2021). Thus, the determination of students to succeed and achieve good grades will push them to rely on rote learning by memorizing procedural steps and developing automatisms, and most of them will not be interested in understanding "the why" or solving problems in unknown contexts. For the same reason, teachers may often adopt teaching by objectives (behaviorism) or transmissive teaching as their teaching strategy. Ourahay (2021) stated that “these standardized assessments established over the years lead teachers and students to rely on the old assessment tests as the main teaching and learning resources, they lead them to undervalue mathematical reasoning and its skills, and consequently, the development of scientific thinking” (Ourahay, 2021). Therefore, if there is a general focus on teaching for understanding based on the competence approach, an adaptation of the assessments is an obligation. Thus, the assessments' tasks should measure both procedural and conceptual knowledge on the one hand and measure the ability to apply the required concepts for solving new problems (contextual knowledge) on the other hand. As Sáenz stated, “assessment of the mathematical competencies of participants (in PISA terms) includes assessment of the extent to which they have school mathematical knowledge (contextual, conceptual, and procedural) which they can productively apply to problem situations of any kind (personal, working, public, scientific, etc.)” (Sáenz, 2009).

The second factor that affects students’ mathematical knowledge is teachers’ teaching strategies, as claimed by several studies. Leikin & Levav-Waynberg (2007) revealed that teacher knowledge explains the gap between theory-based recommendations and school practices. We believe through the obtained results that teachers attach greater importance to the mastery of procedures than developing conceptual knowledge. Indeed, many teachers do not spend enough time building the meaning of new concepts and discussing relations, but they expose their students to numerous similar situations where procedures are applied and then ask them to follow the same procedural steps until they master them. As already mentioned, the error is not considered a learning opportunity but a lack of reinforcement by the teacher or exercises and memorization by students. It is avoided by re-explaining the procedural steps thoroughly rather than explaining the meanings of the procedures and correcting misconceptions (Boutin, 2000). This problem is claimed by several studies in different countries around the world. Thus, Sáenz claimed that “giving excessive importance to the algorithm over the underlying concept is particularly serious among those who are to be future teachers”, Tambwe (2019) found that “the majority of teachers (78%) were not able to prepare a competency-based lesson plan and even deliver lessons using competency-based approaches”, and Sáenz (2009) reported that there are “serious difficulties for future teachers when it comes to leading a process of teaching/learning with their students in line with the PISA proposals”. Traditional assessments can be one of the reasons of procedurally oriented teachers as already explained. Otherwise, many teachers claim that students have low performance and modest prior knowledge that are certainly not sufficient to develop conceptual knowledge (CSEFRS, 2021). Whereas Ma’s (2020) research confirmed that teachers’ knowledge level impacts their teaching approaches and that teachers with low conceptual knowledge were procedurally oriented in their teaching by emphasizing rote learning of procedures.

The results of USAID (2014) surveys in Morocco claimed that the candidates for the teaching profession have low academic skills. A further possibility is that even teachers who have a deeper understanding of mathematical concepts are not aware of the necessity of teaching conceptual knowledge and especially how to teach conceptual knowledge in combination with procedural knowledge. For these reasons, we believe that the main effort should be devoted to initial and in-service teachers’ training. However, the effort made to train teachers doesn’t meet the exigencies for quality education (CSEFRS, 2014). Tamani et al. (2021) stated that the results of their study “reveal the dissatisfaction of these trainee teachers with the qualifying training they have carried out “The USAID study in 2014 showed that most teachers (87%) of teachers who are in training and (70%) of teachers practicing in educational institutions are not satisfied with their training that is very theoretical and does not meet the requirements of their profession. Based on this research, here are some practical recommendations for improving the quality of teachers' training. We think that more attention should be given to initial and in-service training for teachers to learn more about the teaching approaches for conceptual and procedural knowledge. Teachers should be provided with functional knowledge and didactic tools needed to help students to develop competencies for solving problems in different contexts. We believe that teaching mathematics according to the competence approach requires rich and deep procedural and, above all, conceptual mathematical knowledge, two main knowledge to develop students' cognitive processes. The importance of contextual knowledge in linking academic mathematics (concepts and procedures) and the activation of students' skills (knowledge in action) to solve real-life problems; requires its integration into the methodology of teaching mathematical content at school. Besides, teachers should be trained in how to help students to construct their knowledge based on their own prior knowledge. The teacher should link a new concept to different situations and contexts and present it in different forms, which enables students to be aware of the multiple meanings of the concept, and therefore, have a deep understanding of this concept and its applications (Rico, 1997), in particular, in teaching the concept of mathematical functions.

Another issue that needs to be tackled is related to students' learning strategies affected by their beliefs and emotions about learning mathematics. Traditional assessments and procedurally bounded teaching approaches are two factors that lead students to believe that mathematics learning is achieved when the mastery of procedures is established (Ma, 2020). This makes them procedurally bounded students that have surface comprehension of mathematical concepts, and therefore they lose their self-confidence and their interest to even attempt to solve problems in unknown contexts (Ourahay, 2021). On the other hand, students' extrinsic motivation regarding exam grades leads them to put more effort into practicing the procedures and much less into understanding their meaning. A study by Benmansour (1999) showed that Moroccan students are "predominantly oriented towards obtaining good grades and gaining more social status" and that correlational analyses proved that students with "a stronger orientation towards grades reported higher perceptions of test anxiety, and greater use of passive strategies." She defined "passive strategies" as strategies that involve passive cognitive engagement like memorization of rules. Therefore, we believe that improving the quality of teachers and their pedagogical practices, as well as orienting assessments towards the different levels of cognitive processes and especially "reasoning" activities and towards the assessment of competencies instead of rote learning will certainly impact positively students' beliefs about mathematics and their learning strategies.

CONCLUSION

To measure the procedural and conceptual knowledge of functions and to clarify the relationship between them in the context of mathematics education in Morocco, a structural equation modeling (SEM) analysis was used. The results indicate that the quality of the measurement model, where conceptual knowledge and procedural knowledge of functions were modeled as latent factors, is good valid and reliable. In the structural part where relations between procedural and conceptual knowledge were investigated, the findings yielded clear evidence for the genetic view that procedural knowledge of functions is a necessary but not sufficient condition for conceptual knowledge of functions. Thus, this study's findings support the genetic view theory. On the other hand, the correlation tests between students' grades at their mathematical analysis assessments and their procedural and conceptual knowledge of functions scores indicate that students' grades correlate much more strongly with the procedural knowledge scores than the conceptual knowledge scores. Therefore, students' grades mainly reflect their performance on procedural knowledge, and students who get high grades do not necessarily have a rich conceptual knowledge.

These results have several applications and allow us to draw several conclusions about the Moroccan educational system. Firstly, many Moroccan students can perform procedures correctly but lack conceptual understanding. Secondly, at the evaluations, Moroccan students are exposed to similar tasks that focus on performing procedures and lack conceptual knowledge, and therefore, even students who are procedurally bounded learners get high grades. Thirdly, teachers' practices attach greater importance to the mastery of procedures than developing conceptual knowledge. Fourth, students' extrinsic motivation to obtain good grades leads them to put more effort into practicing the procedures and much less into understanding their meaning. These findings have been confirmed by another research but remain very limited. Therefore, future research projects should be devoted to teachers' knowledge and practices and students' learning strategies evaluation. On the other hand, studies on pedagogical interventions and their effects on the development of students' conceptual and procedural knowledge will be of great use.

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APPENDIX

1. Given the function : $\boldsymbol{f(x) = 2x^2 - 8x + 6}$
Calculate $\boldsymbol{f(1)}\ et\ \boldsymbol{f(2)}$

Solve $f(x) = 0$?
Solve $f(x) \leq 0$?

2. Given the function $f(x) = \sqrt{x^2 - 1}$

Mark the real numbers that belong to the domain of the function f:

0.765 1 π -6

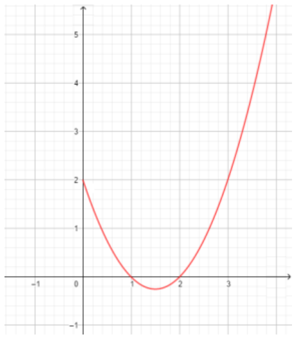
Mark the real numbers that belong to the images range of the function f :

0 $-\sqrt{3}$ $-\pi$ 2.6

3. Evaluate the following limits:

$\lim\limits_{x\to+\infty} \frac{6x^2-1}{3x+4}$ $\lim\limits_{x\to-\infty} \sqrt{x^2+1} + x$

4. The graph of f is shown below.



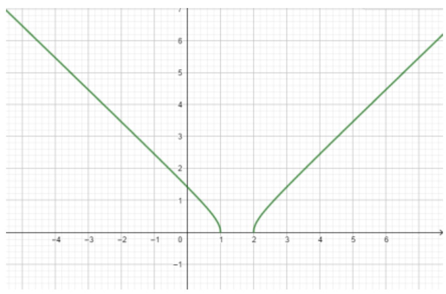
- Mark the real numbers that belong to the domain of the function f:

  0.765 $\sqrt{2}$ -6 π -1.345 1
- Mark the real numbers that belong to the images range of the function f :

  0 $\frac{1}{2}$ -1 $-\pi$ 2.6

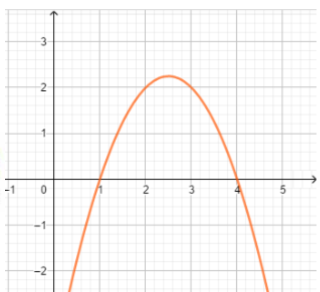
5. Determine whether the following functions are even or odd or neither even nor odd:
- $f(x) = -3x^2 + 1$
- $g(x) = \cos(x) + \frac{1}{x}$

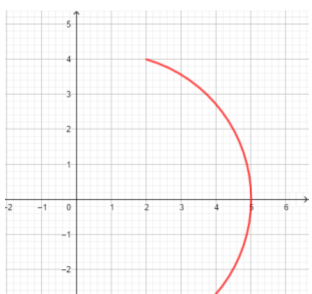
6. The graph of f is shown below

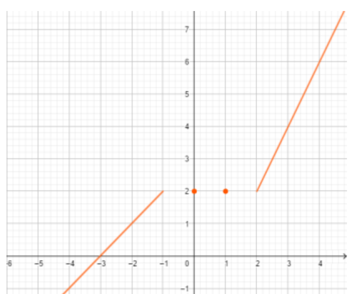


- State the domain of the function f
- State the domain on which f is differentiable
- Deduce the table of variations of the function f
- Is the function even or odd or neither even nor odd?

7. Determine which graphs show relations that are functions. Justify your answers







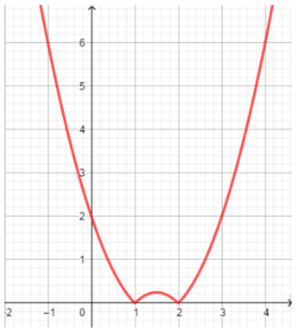
8. Determine which expressions show relations that are functions. Justify your answers

$\boldsymbol{x^2 + y^2 = 4}$ $\quad$ $\boldsymbol{y = \frac{x+5}{x-4}}$ $\quad$ $\boldsymbol{f(x) = 3}$ $\quad$ $\boldsymbol{f(x) = 2x + 3}$ $\quad$ $\boldsymbol{x^2 + 2x - y = 3}$

9. Let f be the function defined over $\mathbb{R}$ by the following graph

Solve the following equation and inequation graphically:

- $f(x) = x + 2$
- $f(x) > 0$

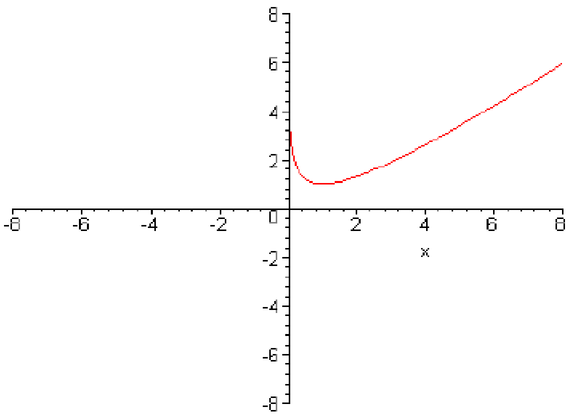


10. $\boldsymbol{f}$ is an odd function and the function h is a definite by: $h(x) = |f(x)|$.

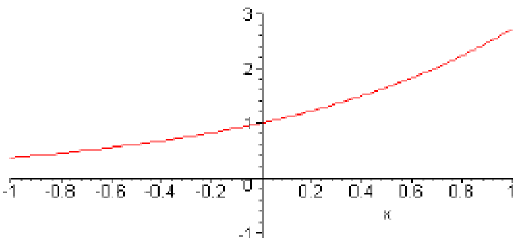
- Knowing that the domain of f is $\mathbb{R}$, deduce the domain of h
- The function h is even or odd?
- Deduce the solutions of the inequation $h(x) \geq 0$

11. Suppose $f(x)$ is a function of third degree and that g(x) is a function of second degree and that f(x) can be divided by g(x). What kind of function is $h(x) = \frac{f(x)}{g(x)}$?

12. The graph of $f(x)$ is shown below. Sketch the graph of $-f(x)$. You don't need to put more numbers on the axis. A rough sketch is enough



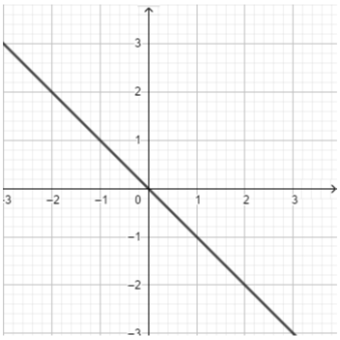
13. The graph of $f(x)$ is shown below. Sketch the graph of $f(-x)$. You don't need to put more numbers on the axis. A rough sketch is enough



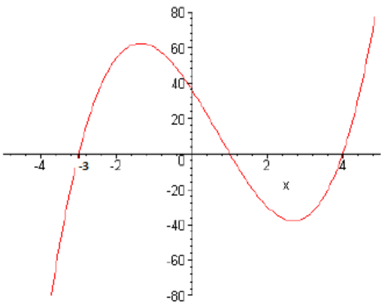
14. Calculate the derivative:

$$f(x) = x\sqrt{x} \qquad\qquad h(x) = \left(\frac{x}{2x-1}\right)^2$$

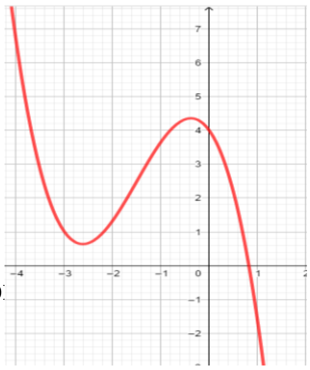
15. Let the function $f(x)$ be given by $f(x) = x^2 + 2x$. Suppose $g(x)$ is given by the graph below. What is the expression for $h(x) = f(x) \times g(x)$?



16. The graph of a function is shown below. Which of the following expression can the function be divided by: $x+1,\ x+3,\ x-3, x-4,\ x.$ Justify your answer



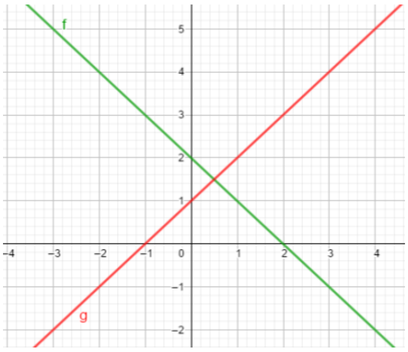
17. A function of third degree has the form $f(x) = ax^3 + bx^2 + cx + d$. The graph of $f(x)$ is sketched below. Find d.



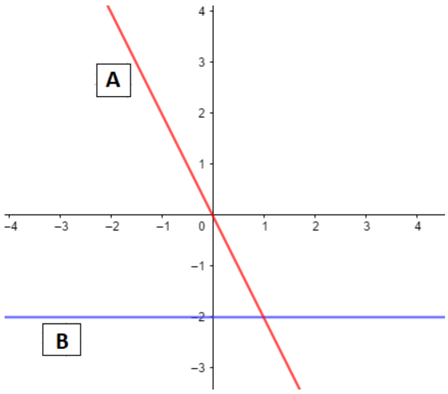
18. Sketch the graph of the following functio

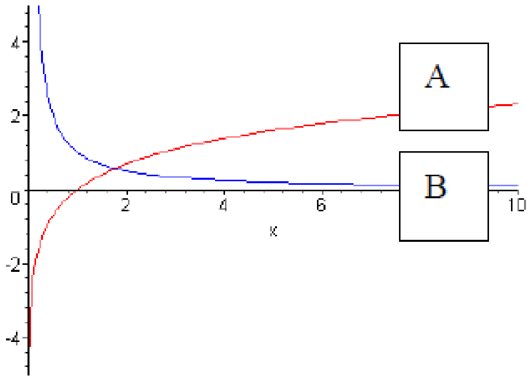
$f(x) = x^2 - 3x + 2$

19. The graphs of two functions are shown below. Sketch the graph of the sum of the two functions. You don't need to put more numbers on the axis. A rough sketch is enough.

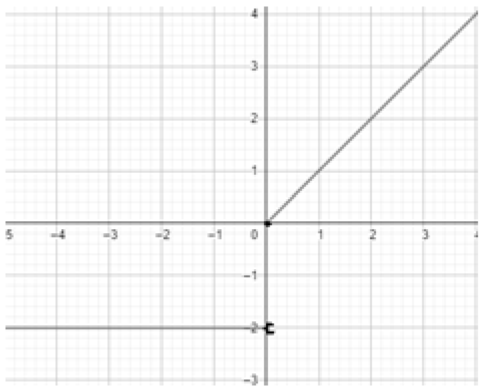


20. The graph of a function $f(x)$ and its derivative is shown in the same coordinate system. Decide whether A or B is the derivative.





21. The graph of $f(x)$ is shown below. Write down the expression for $f(x)$



22. During the coldest winter months, the city of Ifrane is covered with a 2 m thick layer of ice. When spring arrives, the warm air gradually melts the ice, and the thickness of the ice layer decreases constantly. After three weeks, this thickness is only 1.25 m.
Let S(t) be the thickness (in m) of the ice sheet as a function of time (expressed in weeks). Determine the expression of the function which describes the melting of this ice sheet.